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ON A SENSITIVE ELLIPSOMETER TO DETECT THE VACUUM POLARIZATION
INDUCED BY A MAGNETIC FIELD

E. Iacopini, B. Smith, G. Stefanini and E. Zavattini

CERN, Geneva, Switzerland

ABSTRACT

An ellipsometer able to measure ellipticities down to 10^{-11} is being developed. We present here results of measurements made in order to test the main parts of the apparatus.

The feasibility and the limits of such an ellipsometer are discussed, in particular in view of measuring the polarization of the vacuum induced by a magnetic field (photon-photon interaction).

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1. INTRODUCTION

Within the framework of the classical theory of electromagnetism, no effects are expected on a light beam propagating in vacuum in a region where a magnetic field $\vec{B}_0$ is present. However, according to quantum electrodynamics (QED) a photon-photon interaction appears and it can be shown¹⁾ that the vacuum will act as a birefringent medium. When a linearly polarized light beam travels in a region where a magnetic field $\vec{B}_0$ is present ($B_0 \ll 0.4 \times 10^{10}$ tesla), with the propagation vector $\vec{k}$ orthogonal to $\vec{B}_0$, the indices of refraction are given by

$$n_L = 1 + 7AB_0^2 \quad (1)$$

$$n_T = 1 + 4AB_0^2, \quad (2)$$

from which

$$\Delta n = n_L - n_T = 3AB_0^2, \quad (3)$$

where*)

$$A = \frac{1}{90\pi} \alpha^2 \frac{\hbar^3}{m_e c^2} \simeq \frac{4}{3} \times 10^{-32} \text{ cm}^3/\text{erg}.$$

The indices n_L and n_T correspond to light linearly polarized parallel and orthogonal to $\vec{B}_0$, respectively. The expected deviation from Maxwell's theory is, however, very small. For field intensities produced in laboratory**), e.g. $B_0 = 10$ tesla, the deviation of the indices of refraction (1) and (2) from the unity is only of the order of 10^{-21} .

Vacuum polarization effects have been well established by the successful interpretation of various accurate results obtained in the field of experimental QED³⁾. However, performing an experiment in which the results depend *only* on the photon-photon interaction term is highly interesting.

*) Note that the contribution to the effect from the muon loop is $(m_\mu/m_e)^4 = 1.8 \times 10^9$ times smaller.

**) Much stronger fields are supposed to exist near the surface of neutron stars, making the photon-photon interaction a non-negligible process for astrophysics²⁾.

A recent suggestion to obtain Δn by classical optical methods⁴⁾ is to measure directly the ellipticity induced on the light beam by the magnetic field. We discuss here some measurements performed to test the feasibility of the experimental set-up outlined in ref. 4.

2. THE METHOD

The experimental apparatus is shown in fig. 1: a continuous-wave laser beam, directed along the z-axis, is linearly polarized by the prism (P) with the electrical vector along the x-axis. The light beam then passes through a Faraday cell (FC), which rotates the electrical vector around the propagation axis z by an angle

$$\Phi(t) = \Phi_0 \sin 2\pi f_F t . \quad (4)$$

The Faraday cell is followed by a quarter-wave plate (QW) mounted with its axis in the x direction. These two elements, FC and QW, introduce a modulated ellipticity $\Phi(t)$ on the initially linearly polarized light. The light beam is afterwards retained by multiple reflections in an optical cavity (OC), defined by a system of spherical mirrors M1 and M2, placed in high vacuum and traversed by a magnetic field $\vec{B}(t)$ turned on and off at a frequency f_M :

$$\vec{B}(t) = [B(t) \cos \gamma, B(t) \sin \gamma, 0] , \quad \gamma \approx 45^\circ . \quad (5)$$

During its multiple passages through the cavity, the light will acquire an additional ellipticity

$$\psi(t) = \pi \Delta n \frac{m\ell}{\lambda} = 3AB_0^2(t) \pi \frac{m\ell}{\lambda} , \quad (6)$$

where λ is the light wavelength, m is the number of reflections, and ℓ is the length of the magnetic field region. The ellipticity $\psi(t)$ will behave as a periodic function of time, with ψ_0 its maximum value.

After the m^{th} reflection, the light beam, of intensity I_0 , is extracted by means of a small mirror (M) and directed onto a Glan-Thompson prism analyser, oriented for maximum extinction of the transmitted ray. The intensities of the transmitted ray I_T and the reflected ray I_R are

$$I_T = I_0 \{ \sigma^2 + 2\Phi(t) [\beta + \psi(t)] + \Phi^2(t) + \psi^2(t) \} \quad (8)$$

$$I_R = I_0 - I_T \approx I_0 ,$$

where I_0 is the beam intensity prior to A, σ^2 is the extinction factor of the pair of prisms (A and P) and β is the misalignment angle between the axes of P and QW. The light signal I_T is converted into an electrical current $i(t)$ in a photomultiplier (PM).

The Fourier analysis of $i(t)$ shows, in the first approximation, the following components:

- i) a signal at the frequency f_M proportional to ψ_0^2 ;
- ii) two signals at the frequencies $f_F \pm f_M$, proportional to the modulation ellipticity Φ_0 , and to the ellipticity ψ_0 from which we wish to derive Δn ;
- iii) a signal at the frequency f_F depending on the misalignment angle β and proportional to Φ_0 ;
- iv) a signal at the frequency $2f_F$, proportional to Φ_0^2 ;
- v) a d.c. component proportional to σ^2 ;
- vi) the spectrum of the noise.

If we assume that the limiting noise is due to photon statistics, it can be shown that the signal-to-noise ratio (SNR) [components (ii) to (vi)] is given by the expression

$$\text{SNR} = \psi_0 \sqrt{\frac{W_0 \epsilon T \lambda}{hc}} \sqrt{\frac{2\Phi_0^2}{\sigma^2 + \Phi_0^2}} , \quad (9)$$

where W_0 is the light power prior to A, ϵ is the quantum efficiency of the PM, h is Planck's constant, c is the light velocity, and T is the measuring time.

Equation (9) shows that the modulation amplitude Φ_0 has to be related to the extinction factor σ^2 in order to optimize the SNR. For practical reasons a good choice appears to be

$$\Phi_0^2 \approx \sigma^2, \quad (10)$$

so that I_T is of the order of $I_0\sigma^2$, as follows from Eq. (8).

Equation (9) takes into account only the shot-noise. In the more general case, assuming Eq. (10), the measuring time T needed to achieve a given SNR can be written as

$$T = \frac{\xi^2 \text{ SNR}^2 hc}{\epsilon W_0 \psi_0^2 \lambda},$$

where ξ is the ratio between the noise in the signal $s(t)$ and the shot-noise due to photoelectron statistics. By inserting the wavelength of the most intense line of the argon laser spectrum ($\lambda_1 = 4880 \text{ \AA}$) in formula (6), and expressing the magnetic field in units of tesla, the light power W_0 in watts and the total optical path L in kilometres, one obtains for $\text{SNR} = 1$ and $\epsilon = 0.25$:

$$T = 70 \frac{\xi^2}{W_0 L^2} \left(\frac{10}{B_0} \right)^4 \text{ hours}. \quad (11)$$

Considering the high magnetic field intensity that we need, a realistic possibility for the frequency f_M seems to be

$$f_M \approx 5 \times 10^{-3} \text{ Hz}. \quad (12)$$

2.1 Optical cavity

From formula (11), one sees that with $B_0 = 10$ tesla, $W_0 = 1$ W, to detect the signal with a SNR of unity in one day, an optical path L of 1 km is needed. For this reason we intend to perform the experiment by placing an optical cavity in which the light beam is reflected for several hundred times in a magnet a few metres long. Of course this solution can be accepted only if the essential characteristics of the light beam are preserved to a tolerable degree: in particular, we must be sure that the polarization state of the light beam is not perturbed by the mirrors $M1$ and $M2$ of the cavity.

If $\text{ATT}(m)$ is the attenuation of the light due to the reflections, one has

$$\text{ATT}(m) = I(m)/I = e^{-m/m_0}, \quad (13)$$

where $I(m)$ is the light intensity after the m^{th} reflection and m_0 is the quality factor, related to the mirror reflectivity R by $m_0 = 1/(1-R)$.

Taking into account in formula (11) the reflection losses, one arrives at a measuring time T of

$$T = 70 \frac{\xi^2}{W_M \ell^2} \left(\frac{10}{B_0} \right)^4 \frac{e^{m/m_0}}{m^2} \text{ hours} , \quad (14)$$

where W_M is the light power (in watts) prior to M_1 . The above expression has a minimum for $m = 2m_0$; as shown in fig. 2 this minimum is quite flat, leaving the possibility to optimize for other parameters.

To make L a few kilometres, when ℓ is a few metres, $2m_0$ needs to be about 1000, i.e. the mirror reflectivity must be around 99.8%, a value that can be reached only with interferential mirrors.

It can be shown that in an optical cavity consisting of two spherical mirrors of the same focal length, the reflection points will be situated on an ellipse, a factor which limits the number of separated reflections considerably. Better use of the mirror surfaces is made if a cylindrical astigmatism is introduced into the system, for instance by mechanically deforming one of the mirrors, as suggested by Herriot et al.⁵⁾. This deformation displaces the reflection points from the original ellipse (or circle) into a more complicated pattern (a Lissajous figure).

A high number of reflections will inevitably depolarize the beam, mainly for the following three reasons:

- the divergence of the laser beam;
- the residual birefringence of the dielectric layers;
- the scattering processes on the surfaces.

The behaviour of the cavity has been experimentally investigated by mounting a system consisting of two concave spherical interferential mirrors^{*)} of 11 cm

*) Supplied by M.T.O., Metallisations et Traitements Optiques, 11, rue Ampère, B.P.6, 91302 Massy Cedex (France).

diameter and 10 m focal length, at a distance of about 2 m from each other. One of the mirrors had a 4 mm hole drilled in its centre, for injection of the light beam. The simplest way to extract the beam from the cavity was to use a small (4 mm diameter) mirror (M), held in position by a mechanical system giving the necessary degrees of freedom. The cylindrical astigmatism was created by deforming one of the mirrors (see fig. 3). In this manner we have easily obtained up to 1000 separated reflections on a surface of $6 \times 6 \text{ cm}^2$. In fig. 4 a photograph of the pattern of reflections on the entrance mirrors M1 is shown; the entrance angle of the light beam and the deformation amplitude of the mirror have been chosen so that it was possible to extract the beam easily after 500-700 reflections (see fig. 5).

The quality factor m_0 of the cavity has been measured in the following way. The direct laser light beam was attenuated by calibrated optical filters until it was equal in intensity to the light emerging from the cavity after m reflections. The filters employed give a value of the attenuation in the cavity. The measured attenuation $\text{ATT}(m)$ was then fitted with the exponential of formula (13), which gave a value of $m_0 = 250 \pm 10$. This value agrees quite well with the specification for the reflectivity given by the manufacturer ($R \geq 99.5\%$), and is valid for both lines $\lambda_1 = 5145 \text{ \AA}$ and $\lambda_2 = 4880 \text{ \AA}$ of the argon laser.

The depolarization of the beam has been measured by observing its influence on the extinction factor of a system of two crossed polarizers with the cavity mounted between the two prisms. A quarter-wave plate was used to compensate the residual ellipticity. The ray transmitted by the analyser was detected by a photomultiplier (EMI 9813).

To eliminate spot-size oscillations in the cavity, we have used a system of lenses to match the laser beam to the natural modes of the cavity. In this way the spot size remained constant during the reflections, with a diameter of about 2 mm. To reduce the effect of the thermal gradients in the air, the cavity was placed inside an aluminium tube. The whole optical system was mounted on a 3.8 m

long granite optical bench. Figure 6 shows the results of the measurements of the extinction factor σ^2 as a function of the number of reflections. Without any reflections we were able to obtain a σ^2 smaller than 10^{-7} *). After $2m_0$ reflections (which reduce the light intensity by e^2), the extinction factor had worsened to 2×10^{-6} .

The relevance of these results lies in the fact that they determine the following two experimental parameters for the set-up outlined in fig. 1:

- a) The modulation amplitude Φ_0 , which will take the value of about 10^{-3} because of formula (10). This corresponds to a Faraday cell angle of a maximum of 10^{-3} rad, a value that is readily obtained in the kHz frequency region.
- b) The intensity of the light impinging on the photodetector, which will be of the order of $1 \mu\text{W}$ if the beam intensity prior to A is 1 W . Such an intensity is within the linear range of sensitivity of a suitable PM tube.

2.2 Laser noise

The dominant noise source in the experimental apparatus is the laser itself. We have studied the light intensity fluctuations associated with the blue line ($4880 \text{ \AA}$) of a 2 W argon laser**). Since we intend to use ⁶⁾ $f_F \approx 1.5 \text{ kHz}$, we have investigated the frequency region below 2.5 kHz .

Figure 7 shows the laser noise amplitude at 1.5 kHz as a function of the light power impinging on the detector. The laser was working at full power and the required intensity reduction was obtained with neutral density filters. The shot-noise limit is indicated by the dashed line; it is reached at a few tens of nanowatts on the detector. Above this limit the noise increases proportionally to the light intensity. We have tested lasers also from other manufacturers (with

*) The prism polarizers were furnished by the Karl Lambrecht Corp., 4204 North Lincoln Avenue, Chicago, Illinois 60618, USA.

***) CR2, made by Coherent Radiation, 3210 Porter Drive, Palo Alto, CA 94304, USA.

power up to 20 W all lines) and we have found practically the same excess plasma noise.

Figure 8 shows the comparison between the Fourier spectrum of laser light and the Fourier spectrum of light from a stochastic source (in this case an incandescent lamp) between 0 and 2.5 kHz. In both cases the PM cathode current was fixed at 100 nA, corresponding to the expected working intensity of about 1 μ W at 4880 Å. The total laser noise exceeds, at this intensity level, the theoretical shot-noise limit by 15 dB, and this means that the ξ^2 of formula (11) takes the value of 70. In an effort to decrease this excess noise, two different methods have been employed, one working on the principle of light intensity stabilization (IS), and the other by common mode rejection (CMR). In the first method, one monitors the light intensity in a side branch derived by a 4% beam splitter. The photocurrent is amplified and applied as a correction voltage to a Pockels cell, mounted between a quarter-wave plate and a polarizer prism. With an a.c. coupled feedback loop, a noise reduction of about 20 dB in the frequency region of 10 Hz to 40 kHz has readily been obtained, with a light intensity loss of about 50%.

In the common mode rejection method (CMR) one uses the noise information contained in the reflected ray I_R , detected by a photocell. A variable optical filter equalizes the photocell d.c. current to the PM output current. Since I_R contains an extremely small proportion of the useful signal, the subtraction of one current from the other will get rid of the excess laser noise, without influencing the useful signal in any significant way. This method also reduces the light intensity fluctuations introduced by mechanical vibrations of the optical elements, in particular of the mirrors. We have achieved a noise reduction of about 20 dB, without any significant loss in light power. Figure 9 compares the spectrum resulting from the CMR circuit to the spectrum from an incandescent lamp. These two spectra were made with the PM operating at the same average output currents ($\sim 10 \mu$ A). The noise amplitude associated with the light

from the incandescent lamp has been checked to be consistent with the expected amplitude from an ideal shot-noise source.

From these measurements we conclude that the fluctuations in the signal $s(t)$ (see fig. 1) can be reduced (method IS) and compensated (method CMR) in such a way as to reach the limit set by the photon statistics. This means that in formula (11) we can put $\xi^2 = 1$, assuming that no further noise is added by the electronics for the data acquisition.

2.3 Signal detection and data analysis

The signal to be analysed is the current $s(t)$, coming out of the CMR circuit via an a.c. coupling.

In table 1 are given the Fourier amplitudes of the components of interest, normalized to the shot-noise for an integration time of 1 s. These values are obtained assuming $W_0 = 1$ W, $\sigma^2 = 10^{-6}$, $\beta = 10^{-7}$ rad and an ellipticity $\psi_0 = 10^{-11}$, which corresponds to the realistic choice of $B_0 = 7$ tesla, $\ell = 5$ m, and $m = 660$ reflections.

Phase sensitive detection is used to single out the significant spectral components, at frequencies $f_F \pm f_M$. Commercially available lock-in amplifiers have shown themselves to be unsatisfactory, because of overload from the f_F component and output drift. For this reason we have developed the set-up shown in fig. 10, to which we will refer later as a digital phase sensitive detector (DPSD).

This method of signal processing has been tested experimentally (see fig. 10). A photomultiplier PMM was illuminated with both an incandescent lamp (L), giving a constant illumination, and a light emitting diode (LED), producing modulated signals with the Fourier components in the same ratios as in table 1. The constant illumination was chosen in such a way as to give a current i_k of about 100 nA on the photomultiplier cathode, corresponding to 1 μ W of light at 4880 Å. For practical reasons f_S was selected to be 625 Hz, while we have used for f_M the value (12), i.e. $f_M = 0.005$ Hz. The digital filter time constant τ was set to 120 s. The data stored on magnetic tape were analysed by a Fourier transformation.

Figure 11 shows the Fourier spectrum of the data taken in a 40 h test run, in the frequency region near f_M . The amplitude of the peak ($SNR \approx 4$) corresponds to the expected value, given the LED modulation amplitude and the integration time chosen.

3. CONCLUSIONS

From the experimental results presented above, it can be concluded that it is possible to measure the ellipticity ψ_0 induced on a light beam by a magnetic field, provided that the field intensity is around 8 tesla and the light path in the magnet is several metres (which correspond to ellipticities just below 10^{-11})*).

We are therefore studying the feasibility of a 6 m long superconducting magnet with $B_0 = 8$ tesla, modulated at $f_M = 0.005$ Hz.

At present we are setting up the apparatus outlined in fig. 1 with a superconducting dipole of 4 tesla, 5 m long, now available at CERN, which can also be turned on and off at a frequency of about 5 mHz. This set-up will allow us to study the influences of a time variable strong magnetic field on the optical parts and on the electronics, and to understand the limits of the method.

Table 2 shows the expected ellipticity and measuring time T (in hours) for different possible settings of the most relevant parameters, assuming $\xi^2 = 1$, the photodetector efficiency $\varepsilon = 0.4$ (photodiode), and a mirror reflectivity $R = 99.7\%$, for a SNR of unity.

The value W_0 shown in table 2 implies the use of a laser with power W_L of about 15 W (single line). On the other hand, as shown by formula (11), the measuring time T goes inversely proportional to W_0 ; therefore it is possible to obtain a significant reduction of T by using a laser with a single line power W_L appreciably larger than 15 W. However, we have not considered such a (rather difficult) possibility because of fears that such a high light power might generate instabilities in the various optical elements.

*) The Cotton-Mouton effect in the residual gas present into the optical cavity is negligible, provided the pressure is below 10^{-9} Torr.

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Table 1

Relevant Fourier components
of the signal $s(t)$

Frequency	Amplitude (before the DPSD)
$f_F - f_M$	7×10^{-3}
$f_F + f_M$	7×10^{-3}
f_F	50
$2 f_F$	100
noise	1

Table 2

Expected measuring times

W_0 (W)	B_0 (tesla)	ℓ (m)	$m = 2n_0$	ψ_0 (rad)	T (h)
1	4	5	660	0.13×10^{-11}	140
1	8	6	660	0.67×10^{-11}	6

Figure captions

- Fig. 1 : Experimental apparatus (schematic).
- Fig. 2 : Measuring time T (normalized to its minimum value) versus the ratio m/m_0 (curve y).
- Fig. 3 : Mechanical system used to deform the mirror M_1 .
- Fig. 4 : Reflection pattern on the mirror M_1 . The entrance hole is visible at the centre.
- Fig. 5 : Over-all view of the optical cavity, showing the matching telescope, the prism polarizer P , and the pick-up mirror M . The beam, extracted after about 500 reflections, is visible on the white screen near M_2 .
- Fig. 6 : Extinction factor σ^2 as a function of the number of reflections in the cavity.
- Fig. 7 : Spectral noise y of the laser at 1.5 kHz [$y = 20 \log_{10} (\langle i_{\text{noise}} \rangle / i_{\text{dc}})$] as a function of the light power impinging on the light detector. The two arrows define the expected working region ($\approx 1 \mu\text{W}$).
- Fig. 8 : Noise spectra from two different light sources (for the same photocathode current, 100 nA):
a) Laser (4880 Å).
b) Incandescent lamp.
- Fig. 9 : Comparison between the noise from: (a) an incandescent lamp, and (b) the laser after the CMR circuit. The photocathode current was 100 nA and the laser wavelength was 4880 Å.
- Fig. 10 : Schematic layout of the digital phase sensitive detector (DPSD) as used in the test.
- Fig. 11 : Fourier amplitudes around f_M , as measured in the 40 h test run.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

DPSD

Fig. 10

Fig. 11