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RELATIVISTIC SINGLE PARTICLE MOTION AND SPIN-ORBIT COUPLING
IN NUCLEI AND HYPERNUCLEI

R. Brockmann ^{*)}

Department of Physics, State University of New York
Stony Brook, N.Y. 11794

and

W. Weise

CERN - Geneva

A B S T R A C T

Within the framework of a relativistic mean field approach, a comparison is made between the single particle potentials and spin-orbit interactions for nucleons and Λ particles. The phenomenological observation that scalar and vector exchange interactions of the Λ turn out to be roughly a factor of three weaker than the corresponding nuclear forces is related back to properties of the ΛN interaction using boson exchange models. Fock terms related to K and K^* exchange for hypernuclei are also calculated. Their combined effect is found to be small.

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⁺⁾ Permanent address : Institute of Theoretical Physics, University of Regensburg, D-8400 Regensburg; work supported in part by German Bundesministerium für Forschung und Technologie under contract MEP 3200 X.

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1. Introduction

A new set of hypernuclear spectroscopic information has recently become available in terms of high precision (K^-, π^-) data obtained by the Heidelberg-Saclay-Strasbourg collaboration [1,2]. These experiments on p-shell and sd-shell hypernuclei have been improved in accuracy over previous measurements [3] to such a degree that it has become possible to investigate the detailed structure of the average field experienced by a Λ hyperon moving in a surrounding Fermi sea of nucleons, in very much the same way as this has been done previously for nucleons in a nucleus.

Various attempts have been made to interpret the structure of hypernuclear excited states as seen in (K, π) reactions. These investigations are partly based on the strangeness analogue concept [4-8], and partly utilize standard shell model techniques [9-11]. In a recent analysis, Bouyssy [11] has deduced a phenomenological shell model potential for the Λ particle of the form

$$V_{\Lambda}(r) = -v_{\Lambda} \frac{\rho(r)}{\rho_0} + v_{\Lambda}^{LS} \vec{\ell} \cdot \vec{\sigma} \left(\frac{\hbar}{m_{\pi} c} \right)^2 \frac{1}{\rho_0 r} \frac{d\rho(r)}{dr}, \quad (1)$$

where $\rho(r)$ is the nuclear density distribution, parametrized as

$$\rho(r) = \rho_0 \{1 + \exp[(r-R)/a]\}^{-1} \quad (2)$$

Bouyssy found

$$\begin{aligned} v_{\Lambda} &= (32 \pm 2) \text{ MeV}, \\ v_{\Lambda}^{LS} &= (2 \pm 1) \text{ MeV}, \end{aligned} \quad (2a)$$

the interesting observation being the almost vanishing spin-orbit coupling, quite unlike the large spin-orbit coupling for nucleons. The resulting shell model assignments for hypernuclear excited states have received further support by the angular distribution study of ref.[12]. In addition, Bouyssy's analysis

suggests a very small spin-spin part of the ΛN residual interaction. Attempts have been made in various directions to relate these phenomenological parameters to more fundamental properties of the elementary ΛN - two body interaction, e.g. within the framework of a Hartree-Fock approach [13].

Clearly, one of the basic issues is to understand the smallness of the ΛN spin-orbit interaction. In an earlier paper [14], we have suggested that the small size of the spin-orbit coupling might be related to the particular structure of the $2\pi^-$ and $3\pi^-$ exchange ΛN interaction. These conclusions were based on a relativistic Hartree picture. More recently, Noble [15] has pointed out the importance of the tensor couplings in vector meson exchange for the determination of the spin-orbit splitting. On the other hand, Fock terms should also contribute, and, among the variety of boson exchange mechanisms [16], K^* exchange has been proposed as a source of spin-orbit coupling as well [17]. An alternative position has been taken by Pirner [18], who finds a small ΛN spin-orbit interaction to be consistent with qualitative arguments derived from quark dynamics within QCD.

Any discussion of spin-orbit coupling which aims to go beyond pure phenomenology has to incorporate relativistic degrees of freedom in the single particle motion of bound particles [19, 20]. A transparent way to generate the nuclear spin-orbit interaction has been described in ref. [21], based on a relativistic Hartree-Fock approach starting from Boson-exchange two body interactions [22]. It has been shown there that an adaptation of Walecka's relativistic scalar-vector mean field theory [23] to finite nuclei, solving a self-consistent Dirac-Hartree equation, already gives satisfactory nucleon single particle energies and, in particular, the correct amount of spin-orbit splitting. Further investigations have reached similar conclusions [23,24], and extensions of the relativistic

Hartree approach to scattering energies [25,26] appear to be equally promising.

The degree of success to which a simple relativistic mean field theory works in describing the single particle properties of bound nucleons is certainly surprising, even more so because the masses and coupling constants of scalar and vector mesons required for this achievement are very close to those appearing in one-boson exchange models of the free NN interaction [21,22]. Unless there are massive cancellations between two-body correlations introduced by ladder summations of Brueckner type and Fock type self-energy loops, both of which are known to be individually large, these features would be very difficult to understand.

The plan for this paper is as follows: We shall first simply assume the validity of the concept of a local mean field, to be used in a Dirac single particle equation, to correlate as much as possible the shell model spectroscopic information for both nuclei and hypernuclei. In the second chapter, we shall develop the Dirac-Hartree phenomenology with scalar and vector mean fields and discuss the parameters of these mean fields for both nucleons and Λ particles. We shall then discuss the role of Fock terms. Further chapters are reserved for the question how these mean fields might be related back to basic features of the baryon-baryon interaction, where we shall stay as close as possible to recently developed one- and two-boson exchange interaction mechanisms.

2. Phenomenological scalar and vector mean fields for nucleons and Λ particles

It is useful first to develop a relativistic shell model phenomenology, based on a single particle Dirac equation for baryons moving in the nucleus. As in refs. [21-23], we shall assume that the average local single particle potential

consists of an attractive part, $U_s(r)$, which can be thought of as generated from scalar boson exchange, and repulsive part, $U_v(r)$, resulting from the time component of a vector boson exchange interaction⁺). The single particle Dirac spinors are denoted by

$$\psi_{nljm}(\vec{r}) = \begin{pmatrix} i G_{nlj}(r)/r \\ F_{nlj}(r) \vec{\sigma} \cdot \vec{r}/r \end{pmatrix} \varphi_{ljm}(\hat{r}), \quad (3)$$

where $\varphi_{ljm} = [Y_l \times \chi_{1/2}]_{jm}$, $\chi_{1/2}$ being Pauli spinors. The coupled Dirac equations for large and small components become

$$\begin{aligned} \frac{dF_\alpha}{dr} &= [M - E_\alpha + U_+(r)] G_\alpha(r) + \frac{\kappa}{r} F_\alpha(r), \\ \frac{dG_\alpha}{dr} &= [M + E_\alpha + U_-(r)] F_\alpha(r) - \frac{\kappa}{r} G_\alpha(r), \end{aligned} \quad (4)$$

where $\kappa = \pm(j+1/2)$ for $j = l \mp 1/2$, and

$$U_\pm(r) = U_s(r) \pm U_v(r). \quad (5)$$

Note that both U_s and U_v are individually large for nucleons, of the order of several hundred MeV in magnitude [22]. By eliminating the small components F_α and introducing

$$g_\alpha(r) = [M + E_\alpha + U_-(r)]^{-1/2} G_\alpha(r), \quad (6)$$

one arrives at the equivalent Schrödinger equation [22]

$$\frac{d^2 g_\alpha(r)}{dr^2} - \left[\frac{l(l+1)}{r^2} + 2M(V(r) + V^{LS}(r) - E_\alpha) \right] g_\alpha(r) = 0, \quad (7)$$

where $\varepsilon_\alpha = E_\alpha - M$. Here the Schrödinger single particle potential $V(r)$ is related to the Dirac potentials U_+ and U_- by

$$V(r) = U_+(r) [1 + U_-(r)/2M], \quad (8)$$

⁺) We assume at the moment U_s and U_v to be state independent. Refinements will be discussed later.

where we have dropped corrections of order ε_α/M as well as terms of order M^{-2} (see ref. [22] for the complete expression). The spin-orbit potential (again omitting small corrections of order ε_α/M) becomes

$$V^{LS}(r) = - \frac{\frac{d}{dr} U_-(r)}{2M [2M + U_-(r)]} \frac{\vec{\ell} \cdot \vec{\sigma}}{r}. \quad (9)$$

It is clear from eqs. (8) and (9) that the empirical single particle potential and the measured spin-orbit coupling are completely determined by the scalar and vector Dirac single particle potentials U_s and U_v . Let us first examine this for nucleons, where we compare with the phenomenological shell model potential of the form

$$V_N(r) = -v_N \frac{\rho(r)}{\rho_0} + v_N^{LS} \vec{\ell} \cdot \vec{\sigma} \left(\frac{\hbar}{m_\pi c} \right)^2 \frac{1}{\rho_0 r} \frac{d\rho}{dr}, \quad (10)$$

where $v_N \cong 50 - 60$ MeV, $v_N^{LS} \cong 10$ MeV. If we choose the parametrization

$U_{s,v}(r) = u_{s,v} \frac{\rho(r)}{\rho_0}$ and $U_\pm(r) = u_\pm \rho(r)/\rho_0$, then by comparison with eqs. (8-9) at $r = 0$ we find $u_- \cong -900$ MeV, $u_+ \cong -100$ MeV, hence

$$u_s \cong -500 \text{ MeV}, \quad u_v \cong 400 \text{ MeV}. \quad (11)$$

This should be taken as a rough estimate, but we mention that the U_s and U_v obtained from a more elaborate self-consistent Hartree calculation [22] are also of that magnitude. The typical feature here is that the relatively small single particle potential arises from the cancellation of a large attractive scalar potential and an almost equally large repulsive (time component of a) vector potential, while both these interactions add up to generate a sizeable spin-orbit force [21]. Note that with U_s alone (i.e. $U_v \equiv 0$), it would be impossible to reproduce both the empirical central and spin-orbit potentials at the same time.

Before we proceed to discuss how the quantities of eq. (11) can be related to properties of the nucleon-nucleon two-body interaction, we would like to extract, on the same phenomenological level, a similar set of parameters for a Λ particle in a hypernucleus, given the information provided by eqs. (1,2). We assume now that the Λ obeys the single particle Dirac equation, eq. (4), with the replacements $M \rightarrow M_\Lambda$, $U_{s,v} \rightarrow U_{s,v}^\Lambda$ and $U_\pm \rightarrow U_\pm^\Lambda$, respectively, and wish to determine the strength of the scalar and vector potentials $U_{s,v}^\Lambda(r) = u_{s,v}^\Lambda \frac{\rho(r)}{\rho_0}$. Using again eqs. (8,9) at $r \rightarrow 0$, we find that the observed ratios

$$v_N \approx 2 v_\Lambda \quad (12)$$

for the nucleon and Λ central potentials and

$$v_N^{LS} \approx (5 \div 10) v_\Lambda^{LS} \quad (13)$$

are compatible with a simple scaling for the corresponding Dirac single particle potentials,

$$U_{s,v}(\text{nucleon}) = \beta U_{s,v}(\Lambda), \quad (14)$$

where

$$\beta = 2.5 \div 3.$$

It remains now to be understood why those parts of the ΛN interaction which generate the scalar and vector mean field for a Λ particle are only about one third of the corresponding NN interactions.

3. Relativistic Hartree-Fock approach

Within a relativistic Hartree-Fock (HF) description [22], the relation between nucleon or Λ single particle energies and effective two-body forces between nucleons and Λ 's is determined by the Dirac-Hartree-Fock equation ⁺⁾

$$(-i\gamma_0 \vec{\gamma} \cdot \vec{\nabla} + \gamma_0 M - E_\alpha) \psi_\alpha(\vec{r}) + [U_\alpha \psi_\alpha](\vec{r}) = 0, \quad (15)$$

where

$$[U_\alpha \psi_\alpha](\vec{r}) = \sum_\beta \int d^3r' \bar{\psi}_\beta(\vec{r}') [W_{\alpha\alpha}(\vec{r}-\vec{r}') \psi_\beta(\vec{r}') \psi_\alpha(\vec{r}) - W_{\alpha\beta}(\vec{r}-\vec{r}') \psi_\beta(\vec{r}') \psi_\alpha(\vec{r})]. \quad (16)$$

In the construction of the two-body interaction $W_{\alpha\beta}$, we are guided by boson exchange mechanisms which, if written in momentum space,

$$W_{\alpha\beta}(\vec{q}) = \int d^3r e^{-i\vec{q} \cdot \vec{r}} W_{\alpha\beta}(\vec{r}), \quad (17)$$

are of the general form ⁺

$$W_{\alpha\beta}(\vec{q}) = \sum_i \int d\mu^2 \frac{\Gamma_i(\mu^2)}{(E_\alpha - E_\beta)^2 - \vec{q}^2 - \mu^2}, \quad (18)$$

where Γ_i can be any one of the following invariants

$$\Gamma_S(\mu^2) = \rho_S(\mu^2), \quad (19a)$$

$$\begin{aligned} \Gamma_V(\mu^2) = & \left[\gamma_\mu(1) \rho_V(\mu^2) \gamma^\mu(2) + q_\nu \sigma^{\mu\nu}(1) \frac{\rho_T(\mu^2)}{4M^2} q^\lambda \sigma_{\mu\lambda}(2) \right] \\ & + \frac{\rho_{VT}(\mu^2)}{2M} \left[\gamma_\mu(1) q_\nu \sigma^{\mu\nu}(2) + \sigma_{\mu\nu}(1) q^\nu \gamma^\mu(2) \right], \end{aligned} \quad (19b)$$

$$\Gamma_{PS}(\mu^2) = \gamma_5(1) \rho_{PS}(\mu^2) \gamma_5(2), \quad (19c)$$

$$\Gamma_{PV}(\mu^2) = q_\mu \gamma^\mu(1) \gamma_5(1) \frac{\rho_{PV}(\mu^2)}{4M^2} q_\nu \gamma^\nu(2) \gamma_5(2). \quad (19d)$$

Here the ρ_i are spectral functions representing mass distributions for various possible t-channel exchanges. For isovector bosons, an additional factor $\vec{\tau}_1 \cdot \vec{\tau}_2$

⁺) In all equations (15) to (18), α and β denote the quantum numbers of bound particles.

has to be supplied.

The self-consistent Hartree potential contained in eq. (16) is defined as

$$U_{\alpha}^H(r) = \sum_{\beta} \int d^3r' \bar{\psi}_{\beta}(\vec{r}') W_{\alpha\alpha}(\vec{r}-\vec{r}') \psi_{\beta}(\vec{r}'). \quad (20)$$

We note that for spin- and isospin saturated systems, U_{α}^H receives contributions only from scalar and vector exchange, eqs. (19a, b), of the effective two-body interaction, while all pieces of eq. (19) contribute to the remaining (non-local) Fock potential, i.e. the second part of eq. (16). In the simplest one-boson exchange models, the spectral functions ρ_i of eqs. (19) are of the form

$$\rho_i(\mu^2) = g_i^2 \delta(\mu^2 - m_i^2), \quad (21)$$

where g_i is a coupling constant and m_i are boson masses. If form factors of monopole type, $(\Lambda^2 - m_i^2)/(\Lambda^2 - t)$, are multiplied to the one-boson exchange potentials, this amounts to using spectral functions of the form

$$\rho_i(\mu^2) = g_i^2 [\delta(\mu^2 - m_i^2) - \delta(\mu^2 - \Lambda^2)]. \quad (22)$$

Improved interactions which treat the two-pion exchange degrees of freedom explicitly, like the Paris [27] or Stony-Brook [28] NN interactions, identify the spectral functions for isoscalar scalar exchange, ρ_S , and for isovector vector exchange, $\rho_{V,T}^P$, with the corresponding squared helicity amplitudes for $\pi\pi \rightarrow NN$, up to kinematic factors. The actual interaction W to be used in the HF calculation should not be the free two-body interaction, but rather a G matrix. Nevertheless, the spectral representation eq. (18) is a useful one, since ladder summations of the two-body potentials will again lead to this form, although

with modified mass spectra $\rho_i(\mu^2)$ to account for additional correlations.

Now, for nuclei, a simple self-consistent Hartree model using scalar and vector exchange interactions with

$$\rho_s(\mu^2) = g_s^2 \delta(\mu^2 - m_s^2), \quad (23)$$

$$\rho_v(\mu^2) = g_v^2 \delta(\mu^2 - m_v^2),$$

and all other $\rho_i = 0$, with $g_s^2/4\pi = 7.5$, $g_v^2/4\pi = 10.8$ and $m_s = 4m_\pi = 560$ MeV, $m_v = m_\omega = 780$ MeV, gives already surprisingly good results for single particle energies, including spin-orbit splitting, for a p-shell nucleus like $^{16}_0$ [21, 22]. Furthermore, the same model applied to $^{16}_{\Lambda}0$, with a simple scaling of coupling constants

$$g_{s,v}^2(\Lambda N) = \frac{1}{3} g_{s,v}^2(NN) \quad (24)$$

gave the reduction of both spin-orbit splitting and Λ single particle potential required by the phenomenology developed in section 2.

We wish to round up this picture by showing, in figs. 1-2, results of the same self-consistent Hartree calculation for ^{40}Ca and $^{40}_{\Lambda}\text{Ca}$. The single particle energies are plotted as a function of the scalar mass m_s , keeping the ratio g_s/m_s as well as all other parameters fixed. Note again the strongly reduced ΛN spin-orbit force in the d shell. It is also interesting to see that the ordering of levels in the sd shell depends on the range of the scalar two-body potential, a feature which can be traced back to the different node structure of the $2s_{1/2}$ wave function.

4. The Role of Fock terms

The question is of course whether inclusion of Fock terms in addition to the local scalar and vector mean field would destroy both the simplicity and the success of the Hartree-Dirac model. To study this, we investigate the Fock part of eq. (16) with a variety of Boson exchange interactions, for both nuclei and hypernuclei.

For nuclei, we include the following interactions in addition to the isoscalar scalar and vector exchange mentioned before:

- (i) One-pion exchange with pseudovector coupling according to eq. (19d), with

$$\rho_{PV,\pi}(\mu^2) = g_\pi^2 [\delta(\mu^2 - m_\pi^2) - \delta(\mu^2 - \Lambda_\pi^2)], \quad (25)$$

where $g_\pi^2/4\pi = 14.4$ and cutoff $\Lambda_\pi = 0.85$ GeV.

- (ii) ρ meson exchange according to eq. (19b), with

$$\rho_{V,\rho}(\mu^2) = g_\rho^2 [\delta(\mu^2 - m_\rho^2) - \delta(\mu^2 - \Lambda_\rho^2)], \quad (26)$$

$g_\rho^2/4\pi = 0.5$, $m_\rho = 770$ MeV, $\Lambda_\rho = 1.5$ GeV, and

$$\sqrt{\rho_{\pi,\rho} / \rho_{V,\rho}} = 6.6.$$

The parameters for π and ρ exchange are chosen consistent with ref. [29]. For a Λ -particle, only strangeness exchange interactions, as illustrated in Fig. 3, can contribute to Fock terms. We include K and K^* exchange, using pseudovector coupling for K exchange with

$$\rho_{PV,K}(\mu^2) = g_K^2 [\delta(\mu^2 - m_K^2) - \delta(\mu^2 - \Lambda_K^2)]. \quad (27)$$

The $K\Lambda$ coupling constant is chosen to be $g_K^2/4\pi = 16.6$, while $m_K = 495$ MeV,

and we have set $\Lambda_K = 1.1$ GeV. For K^* exchange,

$$\rho_{V,K^*}(\mu^2) = g_{K^*}^2 [\delta(\mu^2 - m_{K^*}^2) - \delta(\mu^2 - \Lambda_{K^*}^2)], \quad (28)$$

where $m_{K^*} = 892$ MeV. The K^*NA coupling strength has been adjusted at

$$g_{K^*}^2 / 4\pi = 1.06$$

with a ratio of tensor to vector coupling

$$\sqrt{\rho_{T,K^*} / \rho_{V,K^*}} = 4.5$$

and using $\Lambda_{K^*} = 1.5$ GeV. These parameters of K and K^* exchange have been chosen from the analysis of Nagels, Rijken and de Swart [29], except for the cutoffs, which are supposed to replace their use of a hard core at short distance. The parameters of these OBE interactions will be renormalized to some extent by additional correlations, but one can expect these interactions to give at least an impression of the magnitude of Fock terms and of their influence on the spin-orbit interaction. In fact, the model introduced here to calculate the nucleon Fock terms with isoscalar scalar and vector plus ρ and π exchange is similar to the one which gives reasonable effective masses in nuclear matter [31, 32] and the K , K^* exchange picture is then simply the strangeness exchange transcription of this model.

The actual calculation of the Fock terms proceeds according to the formulae given in Appendix A. For nuclei, the Fock potential calculated with isoscalar scalar and vector exchange behaves as expected, namely it amounts to roughly 1/4 of the Hartree potential, with opposite sign. The influence of π and ρ exchange is drastic. Given the Hartree wave functions for ^{16}O as described before, a first order calculation of π and ρ exchange Fock terms using these wave functions produces repulsive shifts of more than 30 MeV in the 1s state and about 20 MeV in the p-shell. The larger part of this repulsion

comes from π exchange. We note however that the tensor coupling of the ρ has an additional effect on the Hartree potential as well. This has not been calculated in the present context, but can be estimated to provide additional attraction of about 20 MeV in the Hartree potential. These numbers illustrate already that the step beyond the simplest mean field model must be accompanied by detailed considerations of cancellations among the various contributions, and it seems so far that correlations must be introduced to obtain stability, which is of course not surprising. Fully self-consistent Hartree-Fock calculations along these lines are in progress and will be reported elsewhere [33].

The situation is quite different for the hypernuclear case, where the only Fock term contributions come from K and K* exchange. According to results of the self-consistent HF calculations shown in table 1, the combined effect of K and K* exchange is obviously not large. In particular, the small spin-orbit splitting is not very much affected. This calculation has been performed with pseudovector K-baryon coupling. Pseudoscalar coupling enhances the K exchange contribution somewhat (it lowers the $s_{1/2}$ energy by additional 5 MeV, the $p_{3/2}$ shell by about 2 MeV and the $p_{1/2}$ shell by about 1.5 MeV). However, the overall conclusions concerning the magnitude of the spin-orbit coupling remain basically the same. We conclude therefore that the single particle motion of a Λ in a hypernucleus is considerably more simple than that of a nucleon in a nucleus. The Λ moves in a single particle potential which, to a good approximation, can be obtained already with a Hartree calculation. Fock terms (from the combination of K and K* exchange) are small.

5. The 2π and 3π exchange ΛN interaction

When discussing the possible origin of the simple scaling law, eq. (24), one is naturally led first to the spin-isospin-zero component of the two-pion exchange interaction, which is supposed to provide the driving attraction in the Hartree potential for either nucleons or Λ particles. In one-boson exchange potentials, the intermediate range attraction is usually provided by the ϵ "meson" (e.g. of mass 760 MeV, with a coupling constant $g_{\epsilon}^2/4\pi = 23.6$ according to ref. [39]). The ϵ is frequently replaced by a σ of smaller mass and smaller coupling constant, which plays a similar role if the ratio g/m is the same. The ϵ , if identified with the unitary singlet scalar meson, would have a universal coupling to all baryons, hence the scalar part of the ΛN Hartree potential $U_s(\Lambda)$ would be the same as $U_s(N)$. Consequently, it would be difficult to understand the reduction of the spin-orbit interaction for the Λ . On the other hand, the isoscalar and scalar exchange really arises from exchange of a correlated 2π system, as shown in Fig.4a. In the notation of eqs. (18, 19a), the spectral function for 2π exchange in the NN interaction is given by [34]:

$$\rho_s(\mu^2) = \frac{\pi}{2M^2} \sqrt{\frac{\mu^2 - 4m_\pi^2}{\mu^2}} |f_+^{J=0}(\mu^2)|^2, \quad (29)$$

where f_+^0 is the $J^\pi = 0^+$ helicity amplitude for $\bar{N}N \rightarrow \pi\pi$ [35]⁺⁾ . For the ΛN interaction, one of the f_+^0 is to be replaced by the corresponding helicity amplitude for $\bar{\Lambda}\Lambda \rightarrow \pi\pi$, and $2M^2$ is replaced by $M^2 + M_\Lambda^2$ in the kinematic factor.

Calculations of $f_+^0(\bar{\Lambda}\Lambda \rightarrow \pi\pi)$ have been carried out in refs. [36,37] with the result that $f_+^0(\bar{\Lambda}\Lambda \rightarrow \pi\pi)$ is greatly reduced in comparison with $f_+^0(\bar{N}N \rightarrow \pi\pi)$.

⁺⁾ We do not subtract here the iterated one-pion exchange, since it will appear explicitly in the Hartree potential U_s .

We would like to understand this feature in a simple model which identifies f_+^0 with the diagrams shown in Fig.4b. It is known that a satisfactory description of $f_+^0(N\bar{N} \rightarrow \pi\pi)$ can be obtained by considering intermediate nucleon and $\Delta(1232)$ states [38]. The analogous states which contribute to $f_+^0(\Lambda\bar{\Lambda} \rightarrow \pi\pi)$ are the $\Sigma(1190)(1/2^+, T=1)$ and the $\Sigma^*(1385)(3/2^+, T=1)$. Considering that the mass differences between N, Δ on one hand and $\Lambda, \Sigma, \Sigma^*$ on the other hand are of a similar order of magnitude, the primary difference between the $N\bar{N}$ and $\Lambda\bar{\Lambda}$ helicity amplitudes should come from the ratio of coupling constants, $g_{\Lambda\Sigma\pi}/g_{NN\pi}$ and $g_{\Lambda\Sigma^*\pi}/g_{N\Delta\pi}$. In evaluating these ratios, we adopt the standard SU(3) conventions of refs. [16, 17]. SU(3) gives

$$g_{\Lambda\Sigma\pi} = \frac{2\alpha_{PS}}{\sqrt{3}} g_{NN\pi},$$

where α_{PS} is the D/D+F ratio for pseudoscalar coupling. According to ref. [30], $\alpha_{PS} = 0.46$, although larger values ($\alpha_{PS} \sim 0.6$) also appear in the literature [39] ⁺. The coupling constants for transitions to spin 3/2 resonances are defined through their width,

$$\frac{g^{*2}}{4\pi} = \left[\frac{M^{*2} \Gamma(s)}{q^3} \right]_{s=M^{*2}}. \quad (30)$$

These constants are related to those which appear in the non-relativistic reduction of the spin 1/2 - spin 3/2 transition Hamiltonian,

$$\delta\mathcal{H} = i \frac{f^*}{m_\pi} \vec{S} \cdot \vec{q} [\pi(q) + \pi^*(-q)] + h.c. \quad (31)$$

by

$$f^* = \sqrt{\frac{3}{2}} g^* \frac{m_\pi}{M^*}, \quad (32)$$

⁺) We note that the $g_{\Lambda\Sigma\pi}$ obtained in ref. [30] is considerably smaller than the value $g_{\Lambda\Sigma\pi}^2/4\pi = 11$ quoted in ref. [39].

where M^* is the resonance mass. Nagels et al. [39] give values $g^{*2}(\pi\Lambda\Sigma^*)/4\pi = 6.7 - 7.7$, so that

$$f^{*2}(\pi\Lambda\Sigma^*)/4\pi = 0.10 - 0.12.$$

Given the Chew-Low value $f^{*2}(\pi N\Delta)/4\pi = 0.32$, we have $f^*(\pi\Lambda\Sigma^*)/f^*(\pi N\Delta) \simeq 0.56 - 0.6$. Therefore, within the model of Fig. 4, we arrive at the conclusion that there is, to a good approximation, a common reduction factor relating the helicity amplitudes $f_+^0(N\bar{N} \rightarrow \pi\pi)$ and $f_+^0(\Lambda\bar{\Lambda} \rightarrow \pi\pi)$, if the relatively weak $\Lambda\Sigma\pi$ coupling of ref. [30] is adopted: we find

$$f_+^0(N\bar{N} \rightarrow \pi\pi) = \beta_s f_+^0(\Lambda\bar{\Lambda} \rightarrow \pi\pi),$$

where β_s ranges between 2 and 3, depending on the SU(3) parameter α_{PS} .

The situation is more difficult for 3π exchange, as shown in Fig. 5. If the 3π exchange in the isoscalar $J^\pi = 1^-$ channel is identified altogether with a "bare" ω meson, the empirical ratio between ω -exchange NN- and ΛN -potential would be

$$V_\omega(NN) \simeq 1.2 V_\omega(\Lambda N)$$

according to ref. [30]. A ratio between the Hartree potentials $U_V(N)$ and $U_V(\Lambda)$ close to unity, however, would be hard to accommodate with the requirement of a small Λ spin-orbit coupling. On the other hand, a large part of isoscalar vector exchange is believed to be combined $\pi\rho$ exchange [38]. For the moment, let us assume the extreme situation that all of the 3π exchange comes from $\pi\rho$ exchange, as shown in Fig. 6. We have to compare the amplitudes $f_V(N\bar{N} \rightarrow \pi\rho)$ and $f_V(\Lambda\bar{\Lambda} \rightarrow \pi\rho)$ where the $\pi\rho$ system is coupled to the quantum numbers of the ω meson. Assuming that tensor coupling dominates the ρ -baryon interaction, suppose first that only spin 1/2 intermediate states (N or Σ) are considered. Then

$$f_V(N\bar{N} \rightarrow \pi\rho) \simeq \frac{g_{NN\pi}}{g_{\Lambda\Sigma\pi}} \frac{g_{NN\rho}^\tau}{g_{\Lambda\Sigma\rho}^\tau} f_V(\Lambda\bar{\Lambda} \rightarrow \pi\rho) \quad (35)$$

SU(3) gives

$$g_{\Lambda\Sigma\rho}^T = \frac{2\alpha_M}{\sqrt{3}} g_{NN\rho}^T, \quad (36)$$

where $\alpha_M = 0.5 - 0.6$ [40]. The empirical ratio determined by Nagels et al. [30] is $g_{NN\rho}^T = 1.3 g_{\Lambda\Sigma\rho}^T$. On the other hand, the same authors suggest $\alpha_M = 0.33$, which is probably too small. Spin 3/2 intermediate states (Δ or $\Sigma^*(1385)$) have to be considered as well. The $\rho\Lambda\Sigma^*$ coupling constant is not known, but the quark model would predict the tensor coupling ratio $g_{\rho\Lambda\Sigma^*}/g_{\rho N\Delta}$ to be equal to $g_{\rho\Lambda\Sigma}^T/g_{\rho NN}^T$. It is therefore suggestive to assume an approximate relation

$$f_V(N\bar{N} \rightarrow \pi\rho) = \beta_V f_V(\Lambda\bar{\Lambda} \rightarrow \pi\rho), \quad (37)$$

where $\beta_V = 3$ for $\alpha_{PS} = \alpha_M = 0.5$. However, a wide range of uncertainty must be associated with β_V . For $\alpha_M = 0.33$, β_V can be as large as 4.5, whereas $\alpha_{PS} = \alpha_M = 0.6$ would lead to $\beta_V = 2$.

The basic question is whether the ω meson couples to baryons more as an "elementary" particle, or whether the interaction is processed primarily by certain interacting subgroups of its 3π constituents, as shown in Fig. 6b. No conclusive answer to this question can be given, but the fact that SU(3) limits the coupling constant $g_{\omega NN}/4\pi$ for the "bare" ω to about 5, whereas a much larger coupling strength is required in boson-exchange models of the NN force, already indicates that much of the 3π exchange will probably proceed through mechanisms like those indicated in Fig. 6 [38]. Investigations along these lines by the Paris group [27] are presently directed towards such problems. In this context, we wish to refer to the analysis of Grein and Kroll [41] who have examined the dispersion relations for the spin-dependent forward NN amplitudes and find evidence for large non-resonant 3π exchange, suggesting an appreciable $\pi\rho$ exchange contribution.

To summarize this section, we have pointed out that a weak isoscalar scalar exchange ΛN interaction (two or three times weaker than the corresponding part of the NN interaction) finds support from commonly accepted 2π exchange mechanisms, if a weak $\pi\Lambda\Sigma$ coupling (consistent with $SU(3)$) according to Nagels et al. [30] is assumed, and if couplings to intermediate $\Sigma^*(1385)$ states are considered with $\pi\Lambda\Sigma^*$ coupling constants determined from the Σ^* decay width [39]. Furthermore, a reduction of the isoscalar vector exchange ΛN interaction in comparison to the NN interaction appears if the larger part of 3π exchange can be associated with $\pi\rho$ rather than bare ω exchange, although the reduction factor for vector exchange is quantitatively uncertain. Identification of isoscalar scalar and vector exchange with the exchange of the ϵ unitary singlet and the "bare" ω meson (with strong coupling to baryons) would lead to a ΛN isoscalar force comparable in strength with the isoscalar NN force. However, in such a picture, the observed reduction of the Λ single particle potential and spin-orbit interaction would be hard to accommodate.

We emphasize again that iterated one-pion exchange has purposely been included in our comparison of 2π exchange helicity amplitudes. By doing so, we have incorporated important pieces of the Hartree potential, like the second order tensor force, which would otherwise appear in a ladder iteration of the one-pion exchange interaction. It is obvious that the interactions to be used in the Hartree-Fock-Dirac equation, eq. (15-16), should be G matrices rather than potentials, and the use of the full helicity amplitudes (without subtraction of iterated Born terms) takes into account a large part of the second order G matrix. Given ratios of 2-3 between the driving 2π - and 3π -exchange NN and ΛN forces and assuming that higher order ladder summations will be of a similar nature for both kinds of interactions, these ratios will persist as well, at least qualitatively, for the corresponding Hartree potentials of eq. (20).

Alternative suggestions for reducing the Λ spin-orbit interaction have been made in refs. [15, 18]. Noble [15] has found that the tensor coupling from vector exchange, which gives an additional contribution to $U_{\pm}$ of eq. (15), acts such as to reduce $U_{\pm}$ by a large amount. This model rests on the assumption that a "bare" ω alone provides the vector exchange ΛN interaction.^{+) Pirner [18] has analyzed the problem within a certain selection of QCD mechanisms and finds that the spin-orbit force is reduced because the strange quark constituent of the Λ does not participate in the relevant interactions. It is at present impossible to relate these different pictures to one another. The examples indicate however that the smallness of the ΛN spin-orbit interaction, as seen in hypernuclear spectroscopy, is an important issue in the process of understanding the short range properties of baryon-baryon forces.}

6. Summary and Conclusions

The present analysis has confirmed the previous phenomenological observation [14] that the single particle levels of a Λ in a hypernucleus can be understood in a relativistic Hartree (mean field) theory with vector- and scalar boson exchange, assuming that the coupling of the corresponding ΛN interaction is weaker by about a factor of 3 as compared to the NN interaction from isoscalar scalar and vector exchange. In particular, the smallness of the Λ spin-orbit potential follows under these conditions.

Further developments using a self-consistent Dirac-Hartree-Fock scheme came to the conclusion that K and K^* exchange, which generate Fock terms, almost cancel and do not give significant contributions to the Λ single particle energies. Thus the phenomenological aspects of the single particle motion of a Λ in a hypernucleus can be summarized by saying that the essential features are well represented already in the Hartree approximation.

^{+) The proper reduction of the Λ -nucleus central potential can, however, not easily be achieved in this model.}

These observations have to be contrasted with the more complex situation in nuclei. The Dirac-Hartree model with (isoscalar) scalar and vector exchange works surprisingly well, but Fock terms from π and ρ exchange are large and repulsive, and it appears that additional correlations are necessary to stabilize the relativistic Hartree-Fock calculations. Incorporation of such correlations into the Dirac-HF approach is therefore an important program for further developments.

In an attempt to explain the reduced ΛN scalar and vector exchange interaction, we have tried to establish relations between the 2π and 3π exchange ΛN interaction and the corresponding pieces of the NN interaction, and found support for a reduced ΛN interaction, although quantitative predictions, especially for the 3π exchange degrees of freedom, are not possible, given the inherent uncertainties. A deeper understanding can probably only come from the (as yet non-existing) unification of quark pictures of the baryon-baryon force at small distance with the highly successful boson exchange models at intermediate and large distance.

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Appendix A: Formulae used in Dirac-Hartree-Fock calculations

Following the methods described in ref. [22] we obtain the Dirac-Hartree-Fock equations, including isoscalar scalar (s) and vector (v) exchange, together with π - and ρ -exchange:

$$\begin{aligned} \frac{dG_\alpha(r)}{dr} = & \left[M + E_\alpha + Y_\alpha^s(r) - Y_\alpha^v(r) \right] F_\alpha(r) - \frac{\kappa_\alpha}{r} G_\alpha(r) \\ & + W_\alpha^s(F, r) - W_\alpha^v(F, r) - W_\alpha^\rho(F, r) - V_\alpha^v(G, r) - V_\alpha^\rho(G, r) \\ & + U_\alpha^\rho(F, r) + U_\alpha^\pi(F, r) - X_\alpha^\rho(F, r) ; \end{aligned} \quad (A.1)$$

$$\begin{aligned} \frac{dF_\alpha(r)}{dr} = & \left[M - E_\alpha + Y_\alpha^s(r) + Y_\alpha^v(r) \right] G_\alpha(r) + \frac{\kappa_\alpha}{r} F_\alpha(r) \\ & + W_\alpha^s(G, r) + W_\alpha^v(G, r) + W_\alpha^\rho(G, r) + V_\alpha^v(F, r) + V_\alpha^\rho(F, r) \\ & + U_\alpha^\rho(G, r) + U_\alpha^\pi(G, r) + X_\alpha^\rho(G, r) . \end{aligned} \quad (A.2)$$

Here the quantities Y , W and V are all given explicitly in the Appendix of ref. [22]. The additional terms U and X are related to derivative couplings which appear in the pseudovector π NN Lagrangian, eq. (19d), and the tensor coupling of the ρ NN Lagrangian, eq. (19b):

$$\begin{aligned} U_\alpha^{\pi, \rho}(\{F\}, r) = & \pm 3 C_{\pi, \rho} \sum_{\beta} (2j_{\beta} + 1) \sum_{\substack{\ell \\ l_\alpha + l_\beta + \ell \\ \text{odd}}} \frac{\begin{pmatrix} j_\alpha & j_\beta & \ell \\ 0 & 0 & 0 \end{pmatrix}^2}{2\ell + 1} \times \\ & \times \sum_{\substack{j = \ell \pm 1 \\ j' = \ell \pm 1}} \left[(\kappa_\alpha + \kappa_\beta) \begin{Bmatrix} -F_\beta(r) \\ G_\beta(r) \end{Bmatrix} + a(j) \begin{Bmatrix} F_\beta(r) \\ G_\beta(r) \end{Bmatrix} \right] I_{\alpha\beta}^{\pi, \rho} \end{aligned} \quad (A.3)$$

where

$$I_{\alpha\beta}^{\pi,\rho} = \int_0^\infty dr' [(\kappa_\alpha + \kappa_\beta + a(j)) G_\alpha(r') G_\beta(r') \pm \pm (-\kappa_\alpha - \kappa_\beta + a(j)) F_\alpha(r') F_\beta(r')] j_{j_\alpha}(im_{\pi,\rho} r_z) h_{j_\beta}(im_{\pi,\rho} r_z), \quad (A.4)$$

where

$$a(j) = \begin{cases} \ell + 1 & \dots j = \ell + 1 \\ \ell & \dots j = \ell - 1 \end{cases},$$

and the plus and minus signs refer to π and ρ exchange, respectively.

Furthermore,

$$C_\pi = \frac{f_\pi^2}{4\pi} m_\pi \quad \text{with} \quad \frac{f_\pi^2}{4\pi} = 0.08,$$

while $C_\rho = \frac{f_\rho^2}{4\pi} \frac{m_\rho^3}{4M^2}$, where $\frac{f_\rho^2}{4\pi}$ is the ρ NN tensor coupling constant, with a tensor to vector ratio of 6.6.

Furthermore,

$$X_\alpha^\rho(\{F\}, r) = 3 C_\rho \sum_{\beta} (2j_\beta + 1) \sum_{\substack{\ell \\ \ell_\alpha + \ell_\beta + \ell \\ \text{even}}} (2\ell + 1) \left[2 \begin{pmatrix} \lambda_\alpha & \lambda_\beta & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 - \begin{pmatrix} j_\alpha & j_\beta & \ell \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \right] \times \quad (A.5)$$

$$\times \begin{Bmatrix} F_\beta(r) \\ G_\beta(r) \end{Bmatrix} \left[\int_0^\infty dr' \begin{Bmatrix} F_\alpha(r') F_\beta(r') \\ G_\alpha(r') G_\beta(r') \end{Bmatrix} j_\ell(im_\rho r_z) h_\ell^*(im_\rho r_z) + \right. \\ \left. + \begin{pmatrix} j_\alpha & j_\beta & \ell \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \int_0^\infty dr' \begin{Bmatrix} G_\alpha(r') G_\beta(r') \\ F_\alpha(r') F_\beta(r') \end{Bmatrix} j_\ell(im_\rho r_z) h_\ell^*(im_\rho r_z) \right]$$

where $\lambda_{\alpha,\beta} = \ell_{\alpha,\beta} \pm 1$ for $j_{\alpha,\beta} = \ell_{\alpha,\beta} \pm \frac{1}{2}$ if $X_\alpha^\rho(F, r)$, and $\lambda_{\alpha,\beta} = \ell_{\alpha,\beta}$ if $X_\alpha^\rho(G, r)$ is considered.

For a Λ in a hypernucleus, Fock terms only arise from K and K^* exchange, and the replacements $\pi \rightarrow K$, $\rho \rightarrow K^*$ are to be made, with corresponding change of masses and coupling constants, as described in the text.

	ϵ_{Λ} [MeV]		
	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$
Hartree	- 19.56	- 4.55	- 3.31
HF (S, V, K)	- 16.02	- 2.29	- 1.40
HF (S, V, K, K*)	- 19.02	- 3.57	- 2.52

Table 1: Single particle energies of a Λ in $^{16}_0$ resulting from a self-consistent relativistic Hartree-Fock (HF) calculation using scalar (s) and vector (v) exchange together with K and K* exchange as input. The parameters are:

$m_S = 550$ MeV, $m_V = 784$ MeV, $m_K = 495$ MeV, $m_{K^*} = 892$ MeV;
 $g_S(\Lambda)g_S(N)/4\pi = 2.5$, $g_V(\Lambda)g_V(N)/4\pi = 3.4$, $g_K^2/4\pi = 16.6$, $g_{K^*}^2/4\pi = 1.06$
 with a ratio between tensor and vector couplings of 4.5. Cutoff factors have been chosen as described in the text ($\Lambda_K = 1.1$ GeV, $\Lambda_{K^*} = 1.5$ GeV). First line: Hartree result; second and third line: full HF calculation showing selectively the effect of K and K* exchange. If $\Lambda_K = 0.85$ GeV is chosen, the results change by less than 10 %.

References

- [1] W. Brückner et al., preprint CERN (1979)
B. Povh, Proc. 8th Int. Conf. on High Energy Physics and Nuclear Structure
Vancouver (1979), Nucl. Phys. A 335 (1980) 233.
- [2] W. Brückner et al., Phys. Lett. 79 B (1978) 157.
- [3] W. Brückner et al., Phys. Lett. 62 B (1976) 481;
B. Povh, Ann. Rev. Nucl. Part. Sci. 28 (1978) 1.
- [4] A.K. Kerman and H.J. Lipkin, Ann. of Phys. 66 (1971) 738.
- [5] A. Gal, Adv. Nucl. Phys. 8 (1975) 1.
- [6] J.P. Schiffer and H.J. Lipkin, Phys. Rev. Lett. 35 (1975) 708.
- [7] R.H. Dalitz and A. Gal, Phys. Rev. Lett. 36 (1976) 362.
- [8] R.H. Dalitz and A. Gal, Phys. Lett. 64 B (1976) 154.
- [9] J. Hüfner, S.Y. Lee and H.A. Weidenmüller, Nucl. Phys. A 234 (1974) 429.
- [10] A. Bouyssy and J. Hüfner, Phys. Lett. 64 B (1976) 276.
- [11] A. Bouyssy, Nucl. Phys. A 290 (1977) 324; Phys. Lett. 84 B (1979) 41;
Phys. Lett. 91 B (1980) 15.
- [12] H.C. Chiang and J. Hüfner, Phys. Lett. 84 B (1979) 393.
- [13] M. Rayet, Ann. of Phys. 102 (1976) 226.
- [14] R. Brockmann and W. Weise, Phys. Lett. 69 B (1977) 167.
- [15] J.V. Noble, Phys. Lett. 89 B (1980) 325.
- [16] M.M. Nagels, T.A. Rijken and J.J. de Swart, Ann. of Phys. 79 (1973) 338.
- [17] A. Gal, J.M. Soper and R.H. Dalitz, Ann. of Phys. 63 (1971) 98.
- [18] H.J. Pirner, Phys. Lett. 85 B (1979) 190.
- [19] L.D. Miller, Ann. of Phys. 91 (1975) 40.
- [20] V.A. Krutov and L.N. Savushkin, J. Phys. A 6 (1973) **93**.
- [21] R. Brockmann and W. Weise, Phys. Rev. C 16 (1977) 1282.
- [22] R. Brockmann, Phys. Rev. C 18 (1978) 1510.
- [23] J.D. Walecka, Ann. of Phys. 83 (1974) 491.
- [24] B.D. Serot and J.D. Walecka, Phys. Lett. 86 B (1979) 146;
F.E. Serre and J.D. Walecka, Phys. Lett. 79 B (1978) 10.
- [25] M. Jaminon, C. Mahaux and P. Rochus, Phys. Rev. Lett. 43 (1979) 1097.
- [26] L.G. Arnold, B.C. Clark and R.L. Mercer, Phys. Rev. C 19 (1979) 917.
- [27] R. Vinh Mau et al., in: Mesons in Nuclei, M. Rho and D.H. Wilkinson,
eds., North-Holland (1979).
- [28] A.D. Jackson, D.O. Riska and B. Verwest, Nucl. Phys. A 249 (1975) 397.
- [29] K. Holinde, NORDITA preprint (1979), to be published in Phys. Reports.
- [30] M.M. Nagels, T.A. Rijken and J.H. de Swart, Phys. Rev. D 15 (1977) 2547;
Phys. Rev. D 20 (1979) 1633.
- [31] Y. Futami, H. Toki and W. Weise, Phys. Lett. 77 B (1978) 37.
- [32] G.E. Brown and M. Rho, Phys. Lett. 82 B (1979) 177.

- [33] R. Brockmann, to be published.
- [34] See e.g. G.E. Brown and A.D. Jackson, The Nucleon-Nucleon Interaction, North-Holland (1976).
- [35] G. Höhler and E. Pietarinen, Nucl. Phys. B 95 (1975) 210.
- [36] D.O. Riska, Nucl. Phys. B 56 (1973) 445.
- [37] M. Rimpault and R. Vinh Mau, Nuovo Cim. 35 (1965) 85.
- [38] J.W. Durso, M. Saarela, G.E. Brown and A.D. Jackson, Nucl. Phys. A 278 (1977) 445.
- [39] M.M. Nagels et al., Nucl. Phys. B 147 (1979) 189.
- [40] G. Gustafson, Phys. Lett. 65 B (1976) 363.
- [41] W. Grein and P. Kroll, Nucl. Phys. A 338 (1980) 332.

Figure Captions

Figure 1: Single particle energies for $^{40}_{\Lambda}\text{Ca}$ resulting from a self-consistent Dirac-Hartree calculation using a scalar and vector exchange interaction with the following parameters:
 $g_V^2/4\pi = 10.8$, $m_V = 780$ MeV; the dependence of ϵ on the scalar mass is shown explicitly. Here the ratio g_S/m_S has been fixed, with $g_S^2/4\pi = 7.5$ at $m_S = 560$ MeV.

Figure 2: Self-consistent Hartree-Dirac results of the single particle energies of a Λ hyperon in $^{40}_{\Lambda}\text{Ca}$. The same scalar and vector exchange model as in Fig. 1 has been used, except that the coupling constants $g_S^2/4\pi$ and $g_V^2/4\pi$ have been reduced by a factor of 3 compared to those in Fig. 1.

Figure 3: (a) K and K^* exchange ΛN interaction
(b) Fock term due to K and K^* exchange contributing to the single particle potential of a Λ in a hypernucleus.

Figure 4: (a) Two-pion exchange NN and ΛN interaction.
(b) Major contributions to the $N\bar{N} \rightarrow \pi\pi$ isoscalar s-wave helicity amplitude, and analogous construction of the $\Lambda\bar{\Lambda} \rightarrow \pi\pi$ s-wave helicity amplitude.

Figure 5: Three-pion exchange NN and ΛN interaction

Figure 6: (a) 3π exchange ΛN interaction viewed as combined $\pi\rho$ exchange
(b) ω -baryon coupling via intermediate 3π subsystem.

Figure 1

Figure 2

Figure 3

(a)

(b)

Figure 4

Figure 5

Figure 6