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THE CERN MUON (g-2) EXPERIMENTS

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ABSTRACT

In this review we trace the evolution of the three muon (g-2) experiments which have played a considerable role in the history of CERN. In doing so, our discussion of the measurements is by no means comprehensive since full reports have already been published elsewhere. Instead we have endeavoured to place each in its physics context and to point out the technical developments that were necessary to achieve the succeeding steps in precision.

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1. INTRODUCTION

One of the major contributions of CERN, and one quite unique to this laboratory, has been the accurate measurement of the g -factor of the muon. The programme was started by a small group under Leon Lederman some 21 years ago, and subsequently there evolved a sequence of three experiments of increased precision. Although there were many changes of personnel during this period, the "($g-2$) group" has been almost a permanent feature at CERN, enjoying a buoyant life of uninterrupted activity until the last -- the publication of the final results in 1979. Thus for almost all of its existence the laboratory of CERN has been associated with this endeavour and it is most appropriate that it should be discussed in this 25th Anniversary publication.

In this report we shall not attempt to give an account of each experiment in minute detail; this is already extremely well done in the associated publications to which we will refer below. In addition to these original papers there also exist a number of articles giving more general discussion [1]. For the present article, however, we shall give the reader a short guided tour through this part of the history of CERN pointing out the landmarks of achievement along the way and also indicating the technical obstacles that had to be overcome at each stage in the development of these measurements.

The g -factor of the muon is a dimensionless number which relates its magnetic dipole moment to its intrinsic angular momentum. Thus the magnetic moment is written $\mu = g(e/2mc)(\hbar/2)$ and if the muon obeys the Dirac equation [2], then $g = 2$ exactly. In a more general sense we can say that the g -factor represents a fundamental property related to electromagnetism; if the particle participates in any other interactions which endow it with an internal structure then the value of its g -factor will reflect this departure from the point-like nature implied by the Dirac equation. The proton with a g -factor of 5.586 is an example of such a case. However, even in the absence of an intrinsic structure the quantum nature of the electromagnetic interaction itself also modifies the g -factor. This modification, as we shall see, is quite small and it has become conventional to define a magnetic moment anomaly " a " such that $g = 2(1 + a)$ which has given rise to the title ($g-2$) experiments.

The motivation for these experiments is twofold. The first aim is to check that the quantum theory of electromagnetism correctly predicts the value of the magnetic moment anomaly without need of modification. The second aim is to look for the effects of interactions outside electromagnetism. In particular, this second aim is associated with the mystery of the muon mass. In all respects the muon behaves just like the electron and yet its mass is 207 times larger. The question naturally arises as to what type of process generates that large mass. We should mention here, as will be amplified below, that some advantage accrues from this large mass in the use of the muon as a probe of effects at small distances.

2. THEORETICAL BACKGROUND

Having determined the magnetic moment anomaly as the terrain relevant to these experimental explorations, we do well to preface our journey with an examination of the map which has been constructed from the theory of quantum-electrodynamics (QED). To a large extent the development of the theoretical predictions has gone on in parallel with the evolution of the experimental measurements and in this area also physicists at CERN have made no small contribution.

Starting out from the prodigious achievement of the Dirac theory in predicting the value of $g = 2$, QED concerns itself directly with the anomaly. In the framework of the theory this latter arises from quantum fluctuations in the field associated with the emission and absorption of virtual photons and the polarization of the vacuum by these photons into virtual particle-antiparticle pairs. This self interaction between the particle and its field is always present even for free particles, and in QED, as in classical theory, it leads to infinite self energies. However, for QED the infinities are considered a part of the observed mass and charge of the particle so that the calculation of the effect of the fluctuations on other observable quantities such as the magnetic moment produces finite results.

The first-order contribution to the magnetic moment anomaly is due to the particle interacting with the external electromagnetic field while part of its original energy is carried by a previously emitted virtual photon which is

subsequently reabsorbed. Higher-order contributions involve more than one virtual photon and also vacuum polarization into virtual lepton pairs. All the virtual photons are emitted and absorbed within the region of interaction with the external field and so a factor e^2 is associated with each. As this is a small quantity it enables the magnetic moment anomaly to be built up as a power series in the dimensionless fine structure constant $\alpha = e^2/(\hbar c)$.

$$a_{\mu}^{\text{th}} = A(\alpha/\pi) + B(\alpha/\pi)^2 + C(\alpha/\pi)^3 \dots \quad (2.1)$$

The measurement of the (g-2) factor to higher and higher precision is then a means of checking the successive terms of this expansion.

The coefficient of the first term was originally calculated by Schwinger [3] and shown to be $A = 0.5$ from which it can be seen that the order of magnitude of the anomaly is 10^{-3} . The process giving rise to this contribution is illustrated diagrammatically in fig. 1a.

The second term comes from processes with two virtual photons. This is consequently the lowest level at which vacuum polarization occurs and through that the first term at which the magnetic moment anomaly of the muon is distinguished from that of the electron. The muon with its relatively large rest mass energy quite readily causes vacuum polarization into virtual electron-positron pairs while the effect that polarization into virtual muon pairs has on the (g-2) factor of the electron is negligible. The seven processes with two virtual photons which are independent of lepton (ℓ) mass are illustrated in fig. 1b, while the single process which distinguishes between the electron and muon anomalies is shown in fig. 1c. This result of the large mass difference between muons and electrons influences the values of all the higher-order coefficients in the expansion (2.1) such that while the theoretical prediction for the muon anomaly is

$$a_{\mu}^{\text{QED}} = (1165852 \pm 1.9) \times 10^{-9} \quad , \quad (2.2)$$

that for the electron is

$$a_{\mu}^{\text{QED}} = (1159652.4 \pm 0.4) \times 10^{-9} \quad . \quad (2.3)$$

In calculating these quantities the reciprocal of the fine structure given by Olsen and Williams [4]

$$\alpha^{-1} = 137.035987 \pm 29$$

has been used.

The production of these two numbers has involved many physicists in different centres throughout the world; it is a tremendous achievement of human ingenuity and a great triumph for the theory of QED. A full appreciation of that triumph can only be obtained by careful study of the original papers which are referred to and discussed in the reviews of Brodsky and Drell [5], Lautrup et al. [6], Calmet et al. [7], and Kinoshita [8].

So far only the change in the gyromagnetic ratio due to the interaction of the muon with its own electromagnetic field has been discussed, but any other field coupled to the particle should produce a similar effect. This we have indicated as the second motivation for performing the muon experiments. The large mass difference that we have referred to above is not explained by QED; according to that theory both particles obey exactly the same set of rules. The origin of this difference must be searched for elsewhere. In lowest order a coupling to some other field could contribute to the anomaly through a process of the type illustrated in fig. 1a with the internal virtual photon replaced by a virtual heavy boson. This contribution has been considered in some detail by Lautrup et al. [6]. If such an interaction had a coupling strength f and was mediated by a boson of mass M , then the effect on the anomaly would be

$$\Delta a_{\mu} = \left(\frac{f}{2\pi} \right)^2 \left(\frac{m_{\mu}}{M} \right)^2 L, \quad (2.4)$$

where the form of L depends upon whether the boson is scalar, pseudoscalar, vector or axial vector. Here the dependence upon the square of the lepton mass means, once again, that the muon is more sensitive to such an effect than is the electron.

However, before any discrepancy between QED and measurement can be laid at the door of such a hypothetical field, the effects of other known interactions must be taken into account. Although strongly interacting particles do not couple

directly to the muon, those that are charged do couple to the photon. Thus in addition to vacuum polarization into virtual lepton pairs, processes with virtual pion pairs for example must also occur. There is no theoretical model with which to calculate such effects, but fortunately they can be related to other experimental measurements. The coupling between the virtual photon and real hadron states is measured in e^+e^- colliding beam experiments and from the cross-section $\sigma_h(t)$ for the annihilation into hadrons the contribution of strong interactions to the muon anomaly can be calculated by dispersion theory (Bouchiat and Michel [9]):

$$\Delta a_\mu(\text{hadronic}) = \frac{m_\mu^2}{4\pi^3} \int_{4m_\pi^2}^{\infty} \sigma_h(t) k(t) dt, \quad (2.5)$$

where m_μ and m_π are the muon and pion masses, respectively, t is the square of the total centre-of-mass energy, and

$$k(t) = \int_0^1 \frac{x^2(1-x) dx}{m_\mu^2 x + t(1-x)} \rightarrow \frac{1}{3t} \text{ as } t \rightarrow \infty. \quad (2.6)$$

The process of e^+e^- annihilation into hadrons has been extensively studied and the cross-section is rather well defined from near threshold up to about 3 GeV centre-of-mass energy. Therefore the integration in eq. (2.5) can be carried out with reasonable accuracy. The most recent results are $\Delta a_\mu(\text{hadronic}) = (66.7 \pm 8.1) \times 10^{-9}$ by Barger et al. [10] and $(66.7 \pm 9.4) \times 10^{-9}$ by Calmet et al. [11]. As will be seen below the size of this contribution is about eight times the present experimental accuracy and its presence and order of magnitude have been confirmed.

The contribution of the four-fermion weak interaction is negligibly small ($\sim 10^{-12}$), but in the renormalizable gauge theories there are specific effects on the anomaly arising from the emission and reabsorption of virtual intermediate bosons. However, in general the size of the contribution is critically dependent upon the parameters of the theory and only in the simplest standard version, due to Weinberg [12] and Salam [13], are the parameters sufficiently well established to give predictions [14] for the muon anomaly of $(2.1 \pm 0.2) \times 10^{-9}$.

The addition of the contributions of the strong and weak interactions to the value of the muon anomaly calculated in QED yields for the most recent theoretical prediction

$$a_{\mu}^{\text{th}} = (1165921 \pm 8.3) \times 10^{-9} . \quad (2.7)$$

We will now turn to the experimental determination of this quantity and in order to emphasize the parallel development of theory and experiment we will place each measurement in its historical context indicating the contemporary state of the theoretical calculations.

3. BASIC PRINCIPLES OF THE (g-2) PRECESSION EXPERIMENTS

We start by considering a simple configuration in which the muon moves with a slow velocity on a circular orbit in a plane perpendicular to a uniform static magnetic field B. The momentum vector rotates at the cyclotron angular frequency $\vec{\omega}_c$ given by

$$\vec{\omega}_c = \frac{e\vec{B}}{mc} , \quad (3.1)$$

while the Larmor spin precession frequency is the same as for the particle at rest

$$\vec{\omega}_L = \frac{2\mu\vec{B}}{h} = g \left(\frac{e\vec{B}}{2mc} \right) = (1 + a_{\mu}) \left(\frac{e\vec{B}}{mc} \right) . \quad (3.2)$$

For $g = 2$ these two frequencies are equal and the muon would maintain its initial polarization direction with respect to its momentum vector. However, for $g > 2$ as predicted, the spin turns faster than the momentum vector; the relative frequency $\vec{\omega}_a$ being given by

$$\vec{\omega}_a = \vec{\omega}_L - \vec{\omega}_c = a_{\mu} \left(\frac{e}{mc} \right) \vec{B} . \quad (3.3)$$

This is the basic equation for the (g-2) precession experiments which have their origin in the electron experiment of Crane and co-workers [15]. The equation (3.3) underlies the basis of the experiments which is to contain the particle orbits in a known magnetic field and measure the angle between the spin and direction of motion as a function of time. The value of (e/mc) is deduced from the spin precession frequency of muons at rest. This latter result is obtained in a different

experiment, but the two values of B are both measured in terms of the proton magnetic resonance frequency and consequently the measurements of ω_a and ω_L can be used to deduce "a".

The fact that the anomaly is approximately 10^{-3} means that the muon must make about a thousand turns in the field for the relative spin direction to rotate through one cycle. Thus an accurate measurement of the anomaly will require the muons to be stored for many thousands of turns. This gives rise to a particular problem with muons since their lifetime at rest is only 2.2 μsec . To a certain extent this difficulty can be obviated by making use of relativistic time dilation and in the three CERN experiments there is a clear trend towards the use of muons of higher and higher energies. This also means, however, that we should re-examine the basic equations with which we started this discussion since they were derived specifically for low velocities.

The cyclotron frequency must now be written as

$$\vec{\omega}_c = \frac{e\vec{B}}{\gamma mc}, \quad (3.4)$$

where $\gamma = (1 - \beta^2)^{-1/2}$ and $\vec{\beta} = \vec{v}/c$.

The circular motion of the particle leads to a relativistic effect first pointed out by Thomas [16] in which the particle rest frame, when viewed from the laboratory, appears to rotate with a precession frequency $\vec{\omega}_T$ given by

$$\vec{\omega}_T = \left(1 - \frac{1}{\gamma}\right) \frac{e\vec{B}}{mc}. \quad (3.5)$$

This is in the opposite sense to the spin precession in the muon rest frame [eq. (3.2)] and so the net angular rotation frequency of the spin in the laboratory is

$$\begin{aligned} \vec{\omega}_S &\equiv \vec{\omega}_L - \vec{\omega}_T = (1 + a_\mu) \left(\frac{e\vec{B}}{mc} \right) - \left(1 - \frac{1}{\gamma} \right) \left(\frac{e\vec{B}}{mc} \right) \\ &= \left(a_\mu + \frac{1}{\gamma} \right) \left(\frac{e\vec{B}}{mc} \right). \end{aligned} \quad (3.6)$$

Examination of eqs. (3.4) and (3.6) reveals that the (g-2) precession frequency is unaffected by time dilation since the relativistic treatment leads to the same conclusion as our previous discussion, namely

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = a_\mu \left(\frac{e}{mc} \right) \vec{B}. \quad (3.7)$$

After setting the scene from which the first of the muon (g-2) experiments emerged in the next section, we shall go on to discuss each measurement in turn, dealing with the particular field configurations used to confine the muon orbits in each case. It should be clear at this point, however, that confinement is not possible with a uniform magnetic field alone and the fact that the practical realization of the experiments involves moving away from the ideal situation implies the possibility that corrections have to be applied to the above simple formulae. We shall also have to examine how the problems of injecting the muons into the storage region, ensuring the initial polarization of the sample, and measuring the spin rotation frequency were solved in each case.

4. ANTECEDENTS OF THE (g-2) MEASUREMENTS

Such has been the progress in understanding of the muon over the past twenty years that it is hard to give an accurate picture of the state of knowledge in 1958 when the feasibility of a muon (g-2) experiment was first studied at several laboratories (Panofsky [17]).

Some ten years earlier Kusch and Foley [18] had deduced the value of the electron g-factor from atomic beam magnetic resonance experiments carried out on several different alkali atoms. Their measurements had established the anomaly for the electron with a precision of about 5%, and in the same issue of the journal in which they published their final results Schwinger [3] had given the result of his calculation within the framework of QED.

Subsequently, Gardner and Purcell [19] made accurate measurements of the resonance frequency of protons and the cyclotron frequency of free electrons, both in the same magnetic field. When this was combined with the results of Koenig et al. [20], who compared the electron and proton magnetic moments, the g-value of the electron was obtained. In terms of the anomaly the accuracy was at the level of one percent and the result was in complete agreement with the theoretical work of Karplus and Kroll [21] who had calculated the coefficient of the next higher-order term, $(\alpha/\pi)^2$ in 1950.

Later measurements by Beringer and Heald [22] essentially confirmed the result of Koenig et al. with slightly improved precision, but the work of Franken and Liebes [23] gave a result in disagreement with that of Gardner and Purcell. This somewhat disrupted the sweet accord between theory and experiment for the electron anomaly, but the momentary disquiet was settled by the detection of a numerical error in the theoretical calculation of Karplus and Kroll. The recalculation by Sommerfield [24], Peterman [25], and Suura and Wichmann [26] restored the situation.

In the late fifties the (g-2) precession technique was being established for the direct measurement of the anomaly on free electrons with the pioneering work of Crane and his colleagues at the University of Michigan. This early work of Nelson et al. [27] and Schupp et al. [15] established the tradition of electron (g-2) measurements at Ann Arbor which has continued to the present day. The most recent result (Wesley and Rich [28]) defined the electron anomaly to three parts per million (ppm) and a proposal to increase the accuracy still further has been outlined by Rich [29].

In the ten years prior to 1958, the story of the electron g-factor was one of considerable activity on both the experimental and theoretical fronts, but the story of the muon could hardly have been in greater contrast. In 1948 Steinberger [30] had shown that the electrons from muon decay had a continuous energy spectrum indicating that the muon decayed into at least three particles. This work was conclusively confirmed in more detail by the study of Leighton et al. [31] of the decay of muons from cosmic rays stopped in a carbon plate in a cloud chamber. There had been a great deal of work pointing to the three-body decay, with Hincks and Pontecorvo [32] and Sard and Atthaus [33] showing that the neutral particles were not photons. Many of these early investigations are summarized in the review by Powell [34].

Throughout the period an accumulation of experimental evidence pointed to the muon behaving just like a heavy electron with intrinsic spin one half. Berestetskii et al. [35] had emphasized that QED theory would then imply an

anomalous magnetic moment a_μ for the muon of the same order as that for the electron, since the largest contribution, calculated by Schwinger [3], was the same for both. However, since the invariant momentum transfer was typically $q^2 \approx m_\mu^2$, an experiment on the muon would test the theory to much shorter distances. Further, it was pointed out by Schwinger [36] that the muon should have an extra interaction that would distinguish it from the electron and account for its larger mass. This new field would also influence the anomalous moment, changing its value from that predicted by QED. From this we see how in those early days the twin motivations for a muon (g-2) experiment were clear.

The means whereby such an experiment could be carried out, and in particular how the spin rotation could be detected, were much less clear. However, the way out of this difficulty was soon indicated by the revolutionary discovery of parity violation in beta decay. That parity was not conserved in the weak interaction had been predicted by Lee and Yang [37] and soon verified experimentally by Wu et al. [38]. Parity non-conservation was subsequently established in the pion-muon-electron decay sequence by the experiments of Garwin et al. [39] and Friedman and Telegdi [40]. The former experiment also established that to a precision of about 5% the muon g-factor was equal to 2 and the accuracy was improved by about an order of magnitude in a similar measurement made by Cassels et al. [41]. This aspect of Nature brought two gifts to the aid of the muon (g-2) measurement: the muons are born 100% polarized in the pion rest frame, and the asymmetry of the angular distribution of the electrons emitted in their subsequent decay enabled the polarization of the muon sample to be traced as a function of time. The scene was set and at CERN work started on the exploration of the feasibility of a (g-2) precession experiment with muons.

5. MEASUREMENTS ON POSITIVE MUONS AT THE CERN SYNCHRO-CYCLOTRON [42]

Following the early electron (g-2) precession experiments, the breakthrough experiment which made the direct attack on the magnetic moment anomaly for muons was that performed at the CERN Synchro-cyclotron by Charpak et al. [42]. The

design of the experiment fully exploited the initial muon polarization and final decay electron asymmetry in the framework of the idea that it should be possible to store muons in a conventional bending magnet which provided an approximately uniform vertical field.

A longitudinally polarized muon beam formed by forward decay of pions in flight inside the cyclotron was aimed at an absorber placed inside one end of the 6 m bending magnet. The muons lost energy in the absorber and consequently followed small orbits which could be contained within the magnetic field region. To prevent them re-entering the absorber after one turn, a small transverse gradient of the magnetic field was introduced, causing the orbits to drift along the length of the magnet. Vertical focusing was added by means of a parabolic term in the field. Thus by carefully shaping the magnetic field, muons were contained in the vertical direction while executing quasi circular orbits which walked along the horizontal axis of the magnet. The field in the median plane had the form

$$B_z(y) = B_0(1 + ay + by^2 + cy^3) , \quad (5.1)$$

where y is the transverse horizontal axis (the x axis is along the magnet and z is the vertical direction). The value of B_0 was about 1.6 T. The term ay produced the walking of an orbit of radius r in steps of $s \approx a\pi r^2$ per turn. The next term by^2 added vertical focusing such that the wavelength of the vertical oscillations was $2\pi/b^{1/2}$ while the final term cy^3 also contributed to the step size. This latter can be thought of as a correction for chromatic aberration since its inclusion allowed the steps to be made the same for a range of orbit radii. The full expression for the step size of an orbit with its centre on the line $y = 0, z = 0$ is $s = \pi r^2 [a + (3/4)cr^2]$, and so an appropriate choice of c enables the dependence on r or particle momentum to be effectively reduced.

The experimental configuration is shown in fig. 2 where the orbits in the three distinct regions of injection, storage and ejection can be seen. The magnet pole was 6 m long, 52 cm wide and the vertical gap was 14 cm. Muons entered through a magnetically shielded iron channel and hit the beryllium absorber which reduced their momentum from about 150 MeV/c to about 90 MeV/c. Here the step size

s was 1.2 cm per turn. There followed a slow transition to the long storage region where $s = 0.40$ cm in the field gradient $a = (1/B)(dB/dy) = 3.9 \times 10^{-4} \text{ cm}^{-1}$.

As the particles entered the storage region the reduction of the step size in the x direction led to a spread of the orbit centres in the y direction, as can be seen from a consideration of Liouville's theorem. However, the laws of electromagnetism also ensure that in weak gradients the flux through a wandering orbit is an invariant of the motion. This is a particular example of a more general principle of adiabatic invariance. Making use of this principle the magnet was shimmed so that the vertical field along the centre line was constant and this reduced the residual sideways excursions of the orbits to about ± 1 cm. In this process a flux coil of radius 19 cm corresponding to the optimum muon orbit size was passed along the magnet, and the measured changes in flux were calibrated against the effect of a 2 cm sideways displacement.

At the end of the storage region, a smooth transition was made to the ejection gradient, which was such as to give a step size of 11 cm per turn. The large gradient was necessary to ensure the successful ejection of the particles from the magnetic field region, thereby enabling their spin direction to be detected. Doubts were expressed at the time as to the feasibility of so ejecting the muons, and this can be appreciated by recalling the invariant flux rule cited above. This would imply that once the particles were trapped inside the magnet they would never emerge. However, the invariance principle only strictly applies to the case in which the field varies little over the dimension of the almost closed orbit and this is not the situation for the strongly distorted orbits of the ejection region.

After ejection the muons were stopped in a polarization analyser consisting of a non-depolarizing, non-conductive (methylene iodide) target, situated in a field free region. The target is shown at T in fig. 2 and in more detail in fig. 3.

The time t a muon spent in the field was determined by recording the coincidences in counters 123 at the input, and counters 4566'7 at the output; the interval being measured with respect to a 10 MHz crystal oscillator. Over-all,

storage times of 2 to 8 μ s were achieved in which the muons made up to 1000 turns or so, and the decay electron counting rate had an average value of 0.25 per second.

In principle, the spin direction could have been obtained from the ratio of counts seen in the forward and backward telescopes of the polarization analyser, but it would be difficult to ensure that they had equal efficiencies and solid angles. Therefore the more reliable method of using only one set of counters was chosen. To do this the spin direction of the muon was moved after the particle had been stopped. The flipping of the spin was done with a pulsed vertical magnetic field created by a current in an aluminium tape coil wound directly on the target of the analyser. In successive runs of about 1000 stopped muons, the spin was alternately flipped through $+90^\circ$ and -90° , and the counts in both the forward (77') and backward (66') telescopes were recorded. The spin movement was accomplished within 1 μ s after the arrival of the muon in the target, and then the decay electrons occurring within the succeeding 5 μ s were counted. From the decay electron counts in one of the telescopes $n^+(t)$ and $n^-(t)$, the asymmetry as a function of storage time t was calculated using the expression

$$A(t) = \frac{n^+(t) - n^-(t)}{n^+(t) + n^-(t)} . \quad (5.2)$$

The experimental data are shown in fig. 4 and as can be seen this asymmetry oscillated with the frequency ω_a . It could thus be fitted to a function of the form

$$A(t) = A_0 \sin (\omega_a t + \phi) . \quad (5.3)$$

The anomaly itself being given by [cf. eq. (3.7)]

$$\omega_a = a_\mu \left(\frac{e}{mc} \right) \bar{B} . \quad (5.4)$$

For a thorough discussion of the precautions which had to be taken in the determination of the mean field $\bar{B}$ seen by the muons, and the manner in which systematic errors in the initial phase ϕ were avoided, the reader is referred to the full report of the experiment given by Charpak et al. [42].

The over-all accuracy in the measurement of the mean magnetic field was about 5 parts in 10^4 , but the major part of the error in ω_a was due to statistics, with

uncertainties arising from the initial muon polarization, from scattering in the muon absorber, and from the final muon beam direction all perhaps making contributions.

The final experimental result was

$$a_{\mu}^{(1)} = (1162 \pm 5) \times 10^{-6} , \quad (5.5)$$

which agreed very well with the theoretical value current at that time

$$a_{\mu} = 1165 \times 10^{-6} . \quad (5.6)$$

This agreement enabled certain limits to be placed on the validity of QED as applied to muons. If the muon were not completely point-like in its behaviour, but had a form factor $F(q^2) = \Lambda_{\mu}^2 / (q^2 + \Lambda_{\mu}^2)$, it can be shown that

$$\frac{\Delta a_{\mu}}{a_{\mu}} = - \frac{4}{3} \left(\frac{m_{\mu}}{\Lambda_{\mu}} \right)^2 , \quad (5.7)$$

and the results of this first muon (g-2) experiment indicated that at the 95% confidence level $\Lambda_{\mu} > 1.3$ GeV.

Similarly a modification to the photon propagator at high q^2 or short distances is usually parametrized as

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} \left[1 \mp \frac{q^2}{(\Lambda_{\gamma}^{\pm})^2} \right] , \quad (5.8)$$

where the $\pm$ signs refer to the modifications with positive and negative metric; the adoption of the latter has been suggested by Lee and Wick [43] as a means of circumventing the violation of unitarity. The effect on the anomaly is given by

$$\frac{\Delta a_{\mu}}{a_{\mu}} = \pm \frac{2}{3} \left(\frac{m_{\mu}}{\Lambda_{\gamma}^{\pm}} \right)^2 , \quad (5.9)$$

and this allowed the 95% confidence limit on the photon propagator cut-off to be set at $\Lambda_{\gamma} > 1.0$ GeV.

We should note that the appearance of the square of the muon mass in both these expressions [eqs. (5.7) and (5.9)] confirms the previous statement that the muon is much more sensitive than the electron to departures from pure QED.

The result of this experiment was the first real evidence that the muon behaved so precisely as a structureless point-like QED particle; a heavy twin for the electron. Thus the two aims of looking for a limitation in QED and a clue to the μ -e mass difference remained as a motivation for an experiment of even greater precision.

In addition to measuring the muon magnetic moment anomaly, a subsidiary experiment was carried out to determine the muon electric dipole moment (EDM) and the result was reported in 1961 by Charpak et al. [44], but we defer discussion of this until section 8 below.

6. THE FIRST MUON STORAGE RING [45]

The role of the (g-2) experiment as the best test of QED at short distances had been established; to exploit this and to continue the search for a new interaction characteristic of the muon, it was desirable to design a new experiment which would press the accuracy of the measurement to finer limits. In order to do so, it was of paramount importance to increase the muon intensity, and in the second CERN (g-2) experiment this was done by producing pions in a target immediately adjacent to the storage ring. Thus the muons were created by pion decay inside the storage volume itself.

With the CERN PS available it was attractive to use high-energy muons, thereby taking advantage of relativistically dilated lifetimes. As the (g-2) precession frequency is unaffected by time dilation, a larger number of cycles could be observed. A combination of this factor and the increased intensity meant that some 30-40 periods of the (g-2) modulation could be seen.

The muons were stored in a ring magnet and their spin direction was determined from decays in flight. To do this, energy sensitive detectors were placed inside the ring. The decay electrons, being generally of lower energy than the stored muons, emerged inwards, and using the detectors to select only the most energetic of them was equivalent to biasing the acceptance in favour of those decays which were more or less forward in the muon rest frame. Thus the count rate exhibited a maximum each time the muon spin precessed through a position

parallel to the muon momentum. In fact, the observed decay electron count rate was modulated according to an expression of the form

$$N(t) = N_0 \exp(-t/\tau) [1 - A \cos(\omega_a t + \delta)], \quad (6.1)$$

where $\tau = \gamma\tau_0$ is the dilated muon lifetime, A is the over-all asymmetry which includes the polarization of the muon sample and δ is a phase factor. Fitting the decay spectrum to such a function by allowing the parameters N_0 , τ , A , ω_a , and δ to be free yielded the experimental result for the (g-2) frequency.

This first muon storage ring is shown schematically in fig. 5. Throughout a useful aperture of 4 cm vertical by 8 cm horizontal, the 5 m diameter magnet provided a field of 1.7 T which was shaped with a radial gradient to ensure the weak focusing of the particle orbits. This points to a particular difficulty with the design of these experiments and that is the conflict between the need to shape the field in order to contain the particles and the requirement that the field should be known with the utmost precision. In part this difficulty was overcome by making use of the fact that at injection the muons were localized in azimuth, since the protons which struck the target were in bunches 10 ns wide. This implied that at early times the decay electron count rate was modulated at the mean rotation frequency, allowing the mean radius of the stored muons to be calculated. The measured mean orbit position could then be used in conjunction with the magnetic field map to obtain a precise value of the average field.

The muons were derived from the forward decay of pions produced when the target was struck by 10.5 GeV/c protons from the CERN PS. The proton beam consisted of either two or three radio frequency bunches each 10 ns wide and spaced at 105 ns. As the rotation time around the storage ring was 52.5 ns, the resulting muon bunches were exactly in phase as they passed around the ring. The storage ring was designed for muons of momentum 1.27 GeV/c, but the mechanism of their production meant that pions of much higher momentum could contribute to the sample. Thus the over-all polarization of the muon population was diluted well below the value that could, in principle, be achieved by appropriately matched pion and muon momenta. This feature meant that the asymmetry of the (g-2) modulation was

relatively small ($\sim 12\%$). This can be clearly seen in the detected electron count rate which is shown in fig. 6 along with the high initial background which was a further disadvantage of the method of injection.

The energy threshold employed in the selection of the decay electron events was chosen so as to optimize the (g-2) signal. Changing the energy threshold affects the extent to which electrons produced at small angles in the muon rest frame are selected, and so alters both the magnitude of the count rate and its asymmetry. An analysis of the fitting of the data to eq. (6.1) shows that the expected error in ω_a is given by

$$\Delta\omega_a = 2^{1/2} (\tau N_e^{1/2})^{-1}, \quad (6.2)$$

where N_e is the total number of decay electrons recorded. Thus the appropriate energy threshold is that for which the product $N_e A^2$ is maximum, and this can be found by means of a fairly straightforward analytical calculation to be equal to 0.65 times the maximum decay electron energy (see, for example, the article by Combley and Picasso listed under ref. 1). For the 5 m storage ring this was equivalent to 750 MeV. In passing it should also be noted that the fractional error in ω_a can be reduced by choosing the highest possible magnetic field.

The storage ring field was mapped in the median plane with a proton resonance probe, at 288 azimuthal settings for each of 10 values of the radius. To transform this map into the mean field value seen by the total muon sample required a careful analysis of the muon radial distribution. As has been mentioned above, this was done by studying the fast rotation pattern of the decay electron count rate as the initially bunched muons sped around the ring. Because of the differing momenta or orbit radii of the muons, the bunches slowly spread until, after about 5 μ s, the particles were uniformly spread around the ring. From the resultant modulation of the decay electron count rate and its evolution with time, it was possible to reconstruct the mean radius of the muons and also a good approximation to the distribution of the population with respect to equilibrium orbit radius. This analysis yielded a value of the mean radius of 2494.3 ± 2.7 mm, but as can be seen from the data (fig. 6), the time during which this value was measured

represents a very small part of the total range of storage times used to observe the (g-2) precession signal. Furthermore, this early time period was not really usable for the (g-2) analysis owing to various backgrounds which swamped the spin precession modulation. Thus some care had to be taken to ensure firstly that the measured mean radius truly reflected the muon distribution and was not corrupted by the background, and secondly that the muon distribution was the same at later storage times. The first was checked by measuring at reduced intensity, thereby minimizing the systematic errors associated with background, while for the second a measurement of the change in mean radius between 3 μ s and 50 μ s showed that it was less than ± 1.1 mm. This gave confidence that the measured distribution at early time was closely similar to that of the muon population throughout the period of (g-2) data taking. A conservative over-all error in the mean radius of ± 3 mm was assigned implying an error of ± 160 ppm in the value of a_μ .

To calculate a_μ from the measured angular frequency ω_a and the mean magnetic field in terms of the proton resonance frequency $\bar{\omega}_p$, required the use of the ratio λ of the muon to proton magnetic moments. This latter was obtained from the experiment of Hutchinson et al. [46] on the muon and proton spin precession in the same field. Writing the mean Larmor frequency for protons as

$$\bar{\omega}_p = \frac{\bar{\omega}_\mu}{\lambda} = \frac{(1 + a_\mu)}{\lambda} \left(\frac{e}{mc} \right) \bar{B} , \quad (6.3)$$

and recalling eq. (3.7), allows the anomaly to be extracted from the ratio $R = \omega_a / \bar{\omega}_p$ through

$$a = R(\lambda - R)^{-1} . \quad (6.4)$$

The over-all average value obtained for muons of both polarities was

$$a_\mu^{(2)} = (116616 \pm 31) \times 10^{-8} , \quad (6.5)$$

the total error being made up of two major contributions. The uncertainty in the mean radius has already been discussed and this contributed $\pm 19 \times 10^{-8}$ to the error in a_μ , while the statistical error arising from the fit to the decay electron spectrum was equivalent to $\pm 23 \times 10^{-8}$ in a_μ . These two errors were assumed uncorrelated and added in quadrature.

At the time of the preliminary publication of this result by Bailey et al. [47], there was a slight disagreement with the current theoretical value amounting to 1.7 standard deviations, and this fact prompted a renewed attack on the calculation of the various QED contributions. From this the first evaluation of the light-by-light scattering process emerged; a contribution which had hitherto been estimated as small was revealed as significantly large in the pioneering work of Aldins et al. [48]. As a result of this work, the discrepancy was removed and the difference between experiment and theory for the muon (g-2) factor could be expressed as

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{th}} = 28 \pm 31 \times 10^{-8} . \quad (6.6)$$

Using the above formula for the shift in the anomaly due to a muon form factor [eq. (5.7)] or a photon propagator cut-off [eq. (5.9)], it can be seen that this result sets 95% confidence limits on $\Lambda_{\mu} > 7 \text{ GeV}$ and $\Lambda_{\gamma} > 5 \text{ GeV}$, respectively.

From the discussion of the fitting of the experimental data to the functional form [eq. (6.1)], it is clear that the observations can be used as a check of relativistic time dilation. The data accumulated in this second (g-2) experiment were used in this way to show that the prediction of the special theory of relativity was correct at the level of 1% accuracy. We will return to a discussion of the measurement of the muon lifetime in flight in section 9 below, but a full account of the results obtained with this storage ring including both (g-2) and lifetime measurements can be found in the report of Bailey et al. [45].

7. THE SECOND MUON STORAGE RING [49]

Thus at the beginning of the 1970's the world of muons, electrons, and photons seemed to lie securely within the realm of QED; for, in addition to the agreement cited above for the muon anomaly, the measurements of Wesley and Rich [28] on the electron magnetic moment anomaly had brought that quantity into agreement with theoretical prediction, while the calculations of Appelquist and Brodsky [50] had closed the gap that had previously appeared between theory and the Lamb shift measurements of Robiscoe and Shyn [51].

However, it was also clear at this time, from the calculations by Gourdin and de Rafael [52] and by Bramon, Etim and Greco [53] of the strong interaction contribution to the muon anomaly, that the muon measurement was just on the verge of seeing this effect. The tempting possibility of such a measurement reinforced the motivation for the continued quest for higher precision in muon ($g-2$) experiments. When this was added to the unresolved puzzle of the muon-electron mass difference and the still open question of the level at which the limitations of QED theory would be exposed, the case for pursuing this line of enquiry was irresistible. In fact, the means whereby an experiment of high precision could be made had already been outlined in the proposal to build a new muon storage ring at CERN which was presented in 1969 [54], and the progress made in resolving the various technical problems associated with the project had been reported in 1970 [55].

The third and most recent CERN muon ($g-2$) experiment very much evolved out of the experience gained with the second one. A source of major difficulty in that experiment had been the radial magnetic field gradient necessary to provide the vertical focusing; this had meant that the ($g-2$) frequency was a function of the mean radius of the muon orbit. This in turn had placed crucial dependence upon knowledge of the muon orbit distribution and, as we have discussed above, was responsible for a major part of the experimental error. In response to this difficulty the new storage ring was designed to provide a uniform magnetic field with vertical focusing of the muon orbits by electric quadrupoles.

A second difficulty with the previous ring had arisen from the method of injection. The large burst of particles coming from the target within the ring itself had upset the counting system and produced extensive background at early time. As we have mentioned above, the method also produced low initial longitudinal polarization due to the wide range of pion momenta which could contribute to the stored muon sample. In the new experiment a momentum selected pion beam was injected into the storage ring by means of a pulsed inflector; this gave the triple advantage of a much lower initial flash of unwanted particles, an

increase in muon intensity due to the proper matching to the acceptance of the storage ring, but most important, a high initial polarization of the muon sample ($\sim 95\%$).

Before describing the apparatus, we should discuss the effect that the chosen configuration of transverse fields ($\vec{\beta} \cdot \vec{B} = \vec{\beta} \cdot \vec{E} = 0$) has on the expression for the relative spin precession frequency [eq. (3.7)]. It is found that with the appropriate equations of motion (see, for example, Bargmann, Michel and Telegdi [56]) the cyclotron and spin angular rotation frequencies both contain an additional term proportional to the electric field:

$$\vec{\omega}'_c = \frac{e}{mc} \left[\frac{\vec{B}}{\gamma} - \left(\frac{\gamma}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \quad (7.1)$$

$$\vec{\omega}'_s = \frac{e}{mc} \left[\left(a_\mu + \frac{1}{\gamma} \right) \vec{B} + \left(\frac{1 - \gamma}{\gamma^2 - 1} - a_\mu \right) \vec{\beta} \times \vec{E} \right], \quad (7.2)$$

and these two expressions yield for the relative spin precession

$$\vec{\omega}'_a = \frac{e}{mc} \left[a_\mu \vec{B} + \left(\frac{1}{\gamma^2 - 1} - a_\mu \right) \vec{\beta} \times \vec{E} \right]. \quad (7.3)$$

It is clear from this latter equation that the effect of the confining electric field on the relative spin precession can be reduced to zero by a particular choice of the muon momentum equivalent to

$$\gamma = \left(1 + \frac{1}{a_\mu} \right)^{1/2} = 29.3. \quad (7.4)$$

The actual momentum is 3.098 GeV/c which was the value chosen for the new muon storage ring, thereby fully exploiting this fortunate cancellation and reducing the corrections necessary for the effect of the electric field to a few parts per million. Such corrections arose because not all the muons had exactly the so-called magic momentum.

The high relativistic γ factor dictated by these considerations brought the additional advantage of a proportionally increased muon lifetime which enabled the (g-2) precession signal to be followed for over a hundred periods of the modulation.

A plan view of the muon storage ring is shown in fig. 7; the uniform magnetic field of 1.47 T was provided by 40 bending magnets arranged so that their contiguous pole pieces formed a ring of 14 m diameter. The C-shaped magnets were open on the inside to allow the decay electrons to emerge and strike the energy sensitive detectors of which there were 22 deployed around the ring. This use of the complete circumference for detection purposes was an indication of the much cleaner injection process.

The pions were introduced into the ring through the channel of a pulsed coaxial line injector, which was fed with a 10 μ s current pulse of peak value 300 kA, sufficient to momentarily cancel the local magnetic field. The pion trajectories made almost a complete turn before leaving the storage volume. From a bunch of 10^6 pions about 200 stored muons were produced and from the decay of these about 35 electrons were detected above threshold.

The pole gap of the magnets was 14 cm high by 38 cm wide and each of the 40 magnets was individually supported on a ring-shaped concrete foundation. The magnetic field was derived from the current in four big concentric coils connected in series, the current being provided by a conventional rectifier unit with an active filter which ensured stability to five parts in 10^5 . To maximize the stability and reproducibility of the field in the storage ring, each of the 40 magnets was stabilized separately [57] by a control system which had an NMR probe and pick up coil as its sensors. The signals from these devices were used to automatically control the current through additional compensating coils which were wound around the yoke of individual magnets close to each pole tip. To bring the magnets into the same operating condition for each run, a special switching-on procedure was used which cycled the field up and down rapidly at first and then slowly settling to the working value. In this way the magnetic field was controlled to a precision of 1 ppm and the average field values deduced from maps taken at different times never varied by more than 2 ppm.

Two types of magnetic field map were taken. A limited mapping at some 400 points was made daily throughout the experimental runs. These measurements enabled

any drifts in the mean field to be followed in the interval between the full scale maps which were made before and after a sequence of runs. These complete maps consisted of field values measured at about a quarter of a million points throughout the storage volume and involved the removal of the vacuum chamber and quadrupole electrode system. This meant that the actual field seen by the circulating muons was slightly changed from the values obtained during the large scale mapping, and so these systematic effects due to the materials of the vacuum chamber and electrodes were carefully studied [49].

A section through the vacuum chamber and electrodes is also shown in fig. 7. The shape of the electrodes was devised to minimize the departures from the ideal quadrupole field. Each set of quadrupoles covered 36° in azimuth and there were eight such units. The two missing sectors were diametrically opposite each other. One region was close to the inflector to allow the pion orbits to enter the storage volume while the second region removed the first azimuthal harmonic in the focusing field, which would have given rise to very large closed orbit distortions. The voltages applied to the electrodes were in the form of flat-topped pulses about 1 ms long. Pulsed quadrupoles were used to overcome the ionization of the residual gas and consequent breakdown which had been found to occur in a d.c. system. This ionization was caused by trapped low-energy electrons spiralling up and down the magnetic field lines [58]. The vacuum which was usually less than 10^{-7} Torr, was maintained by ion getter pumps, thereby avoiding deterioration of the high voltage performance by contamination of the insulating surfaces with oil vapour.

As with the previous experiment, the muon orbit distribution was determined from the rotation structure of the decay electron spectra and the larger circumference of this second storage ring meant that the pattern persisted over a longer period. The mean radius was found to a precision of 0.5 mm while the r.m.s. spread in radii about the central value (7000 mm) was measured to 0.1 mm. The parameters of the muon distribution found in this way were used to obtain the average value of the magnetic field and the small correction due to the electric field.

In this discussion we have only been able to give the flavour of the various technical problems encountered; these and the experiment generally is much more fully dealt with in the final report on the muon storage ring [49].

The measured decay electron count rate is shown in fig. 8, which contains the total sample of 1.4×10^8 recorded counts. The (g-2) modulation is clearly visible out to a storage time of 0.5 ms; a comparison of this figure with the previous results shown in figs. 4 and 6 well illustrates the considerable progress made throughout this sequence of three experiments. As before, the experimental value for the frequency ω_a was obtained by fitting the data to a function of the basic form given in eq. (6.1). Nine separate runs were made over a period of two years and the values for the ratio R of the (g-2) frequency to the effective proton NMR frequency are shown in fig. 9 for these data sets, together with the average values for positive and negative muons which were

$$\begin{aligned} R(\mu^+) &= (3707173 \pm 36) \times 10^{-9} \\ R(\mu^-) &= (3707256 \pm 37) \times 10^{-9} . \end{aligned} \quad (7.5)$$

In both cases the statistical error of 10 ppm and the systematic error of 1.5 ppm have been added in quadrature. The over-all mean value is

$$R(\mu) = (3707213 \pm 27) \times 10^{-9} . \quad (7.6)$$

Using eq. (6.4) and the weighted average of the three most precise determinations of λ in refs. [59]-[61] which is

$$\lambda = 3.1833437(23) , \quad (7.7)$$

the following results for the muon anomalous moment were obtained,

$$\begin{aligned} a_{\mu^+} &= (1165911 \pm 11) \times 10^{-9} \\ a_{\mu^-} &= (1165937 \pm 12) \times 10^{-9} , \end{aligned} \quad (7.8)$$

and for the over-all average

$$a_{\mu}^{(3)} = (1165924 \pm 8.5) \times 10^{-9} . \quad (7.9)$$

This value is seen to be in excellent agreement with the total theoretical prediction previously stated in eq. (2.7).

From this result it can be concluded that the QED calculation of the muon anomaly is verified up to the sixth order. Referring to eq. (2.1) it can be deduced that the experimental uncertainty is equivalent to 1.2×10^{-5} in A, 3.5×10^{-3} in B, or 4.7% in C. The agreement with theory, however, also depends upon the inclusion of the hadronic contribution and so the latest result can be interpreted as a confirmation of the existence of hadronic vacuum polarization at the level of five standard deviations. Finally, the measurement has revealed no evidence for a special coupling to the muon and the proposed effect of weak interactions is still well below the experimental sensitivity.

Looking at the possible modifications to QED in a little more detail, then similar quantitative statements to those made for the previous two experiments can be made in this case. When considering a hypothetical muon form factor, then the latest experimental result sets the 95% confidence limit $\Lambda_\mu > 36$ GeV while the equivalent limit for modifications to the photon propagator is $\Lambda_\gamma > 20.7$ GeV.

We have mentioned in the case of the first and second muon (g-2) experiments that these also incorporated a measurement of the muon electric dipole moment and a check on relativistic time dilation, respectively. The muon storage ring used in the final (g-2) experiment was also used to refine the precision of both of these measurements and these we discuss in the following two brief sections.

8. MUON ELECTRIC DIPOLE MOMENT

In our formulation of the motion of the muon spin in an electric and magnetic field, we have neglected the possibility that this particle has an electric dipole moment (EDM). Treating an EDM analogously to a magnetic dipole moment, we can write

$$\mu = g \left(\frac{e}{2mc} \right) \frac{\hbar}{2} ; \quad d = f \left(\frac{e}{2mc} \right) \frac{\hbar}{2} . \quad (8.1)$$

Assuming as before that we have transverse fields ($\vec{\beta} \cdot \vec{B} = \vec{\beta} \cdot \vec{E} = 0$), then the classical relativistic equations for the spin motion in the laboratory fields $\vec{B}$ and $\vec{E}$ contain a term additional to those given in eq. (7.2) for $\vec{\omega}'_s$, explicitly

$$\vec{\omega}''_s = \vec{\omega}'_s + f \left(\frac{e}{2mc} \right) [\vec{E} + \vec{\beta} \times \vec{E}] . \quad (8.2)$$

This extra term is carried over into the equation for the angular frequency of the spin precession relative to the momentum vector. Since for the actual fields under consideration here the magnetic field term is dominant, the relative spin precession is now composed of two orthogonal axial vectors and this is illustrated in fig. 10. The total precession is the combination of that due to the magnetic and that due to the electric dipole moments, and its axis is consequently tilted with respect to the normal to the orbit plane as can be seen from this figure. The angle of tilt δ is given in the small angle approximation by $\delta = (f\beta)/(2a_\mu)$ and can be detected by looking for the time variation of the vertical component of the muon polarization with the same frequency as the (g-2) signal.

In the first muon (g-2) experiment such a measurement was made by using a horizontal pulsed magnetic field at the analysing target instead of the vertical field which was applied in the case of the magnetic moment measurement. Thus the vertical component of the spin was alternately flipped into the forward and backward directions. The analysis of the data was similar to that described above for the (g-2) experiment and the resulting limit on the muon EDM reported by Charpak et al. [44] was

$$d_\mu \lesssim (0.6 \pm 1.1) \times 10^{-17} \text{ e}\cdot\text{cm} . \quad (8.3)$$

The technique used in the recent storage ring experiment to search for a tilt in the plane of the spin precession was slightly different. In this case the electron detectors were used in conjunction with scintillation counters which labelled the decay electrons according to whether they struck the counters above or below the median plane. Any oscillating vertical component of the polarization gives a signal in quadrature with the (g-2) modulation, as can be seen from fig. 10. Thus the modulation recorded by each half of the detector would have a slightly different phase. The technique and possible sources of systematic errors are fully discussed by Bailey et al. [62]. Separate measurements on μ^+ and μ^- gave

$$\begin{aligned} d_{\mu^+} &= (8.6 \pm 4.5) \times 10^{-19} \text{ e}\cdot\text{cm} \\ d_{\mu^-} &= (0.8 \pm 4.3) \times 10^{-19} \text{ e}\cdot\text{cm} . \end{aligned} \quad (8.4)$$

Assuming opposite EDM's for particle and antiparticle, the combined result was

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e}\cdot\text{cm} . \quad (8.5)$$

Note that this would imply a 26 ppm change in the (g-2) frequency. So if QED is correct an EDM lower limit on d_μ can be set.

Measurements of particle electric dipole moments are of fundamental importance since the existence of such a static property would imply the lack of invariance for the electromagnetic interaction under both P and T [63]. However, this limit for the muons is some five orders of magnitude away from the very stringent limits which have been placed on the electric dipole moment of the neutron [64] and of the electron [65]. This large difference is mainly due to the fact that unlike the muons these two particles have been studied in neutral systems.

9. MUON LIFETIME IN FLIGHT

From the experimental data shown in figs. 6 and 8 it can be seen that the exponential fall off in count rate due to the muon lifetime is characteristic of the muon (g-2) data. The experiments thus provide a means of making accurate measurements of the muon lifetime in a circular orbit, thereby enabling a direct and stringent test of Einstein's theory of special relativity to be made. As a bonus such measurements also shed light on the so-called twin paradox, give an upper limit to the granularity of space time, and test the CPT invariance of weak interactions.

From our discussion of the analysis of the decay electron spectra observed in the storage ring experiments, it is clear that the muon lifetime in flight emerges as one of the fitted parameters and in the first storage ring experiment this fitted value was $(26.37 \pm 0.05) \mu\text{s}$, compared to the expected value of $26.69 \mu\text{s}$; this latter had been calculated from the lifetime at rest, which at that time was known to a precision of about 5 parts in 10^4 . By comparing values over several runs, Bailey et al. [45] were able to express the difference between experimental measurement and theoretical calculation as

$$\begin{aligned} \tau_{\text{exp}} - \tau_{\text{th}} &= -(0.30 \pm 0.03) \mu\text{s} \\ &= -(1.1 \pm 0.1)\% , \end{aligned} \tag{9.1}$$

where the error is purely statistical.

The fact that the measured lifetime was shorter than the expected value was ascribed to the slow loss of muons due to field imperfections, and in the most recent storage ring experiment the stability of the muon sample against such losses was ensured by using a scraping system that shifted the muon orbits at early times in order to "scrape off" those muons most likely to be lost.

The rotation frequency ω_r of the muons was obtained from the fast rotation analysis of the bunch structure at early storage times and from the equation

$$\bar{\gamma} = \lambda \bar{\omega}_p / (1 + a_\mu) \omega_r , \quad (9.2)$$

a value of the mean gamma factor $\bar{\gamma}$, for the muon sample of 29.327 ± 4 was obtained. The best value for the lifetime at rest is $2.19711 \pm 8 \mu\text{s}$ (Balandin et al. [66]), which then gives the expected lifetime in flight as $64.435 \pm 9 \mu\text{s}$, compared with the experimental result of $64.378 \pm 26 \mu\text{s}$. Thus the transformation of time was validated to an accuracy of $-(0.9 \pm 0.4) \times 10^{-3}$. For a full discussion of the technical details of the measurement and of the careful study which was carried out with respect to the possible systematic distortion of the decay spectrum, the reader is referred to the original publication by Bailey et al. [67].

The dilated lifetime was measured for muons of both signs, and since from CPT invariance these should be equal, the experimental results gave the best test of this theorem as applied to the weak interaction of muons. In this connection it should be noted that the Lorentz γ factor was the same for both μ^+ and μ^- to a much higher precision than the quoted lifetime errors. The limits found were

$$3.0 \times 10^{-3} > \frac{\tau_{\mu^+} - \tau_{\mu^-}}{\tau_{\mu}} > -1.4 \times 10^{-3} . \quad (9.3)$$

For comparison the measurement of the muon (g-2) factor may also be used to test the CPT prediction that $g_{\mu^+} = g_{\mu^-}$, in this case testing the theorem as it applies to the electromagnetic interaction. The 95% confidence limits in this instance are much more stringent,

$$7 \times 10^{-9} > \frac{g_{\mu^+} - g_{\mu^-}}{g_{\mu}} > -58 \times 10^{-9} . \quad (9.4)$$

10. CONCLUDING REMARKS

After some 21 years of $(g-2)$ measurements on the muon at CERN, a great deal of territory has been brought within the civilized domain of QED theory, and the precision of the most recent result defines the limits within which that domain is secure against any future theoretical excursions. As we have stressed above, any modification to the photon propagator or new coupling common to both muons and electrons would imply a perturbation of a_μ by a factor (m_μ/m_e) larger than for a_e . Thus in the absence of possible coupling particular to the electron, the present muon result ensures that a_e is a "pure QED quantity" down to the level of three parts in 10^{10} .

However, all the effort expended in this activity has brought us no nearer to understanding the mystery of the muon mass. No evidence of a special coupling to the muon has been found. On more general observational grounds it is known that the neutrinos distinguish between the charged leptons. The neutrinos clearly know the difference in the sense that the electron, the muon and the new lepton of mass $1.8 \text{ GeV}/c^2$, discovered by Perl et al. [68], each have their own associated neutral massless fermion; perhaps it is in this area that enquiry should be made for an answer to the charged lepton mass splittings.

For the present, however, the thread which has linked many experimenters together in the common cause of measuring the muon $(g-2)$ factor at CERN is now broken and those who have shared this experience have gone their separate ways. It remains to be seen whether or not future refinement of the theory of the weak, electromagnetic, and strong interactions will call for the discerning scrutiny of further measurements of even greater precision.

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Figure captions

- Fig. 1 : Processes contributing to the lepton anomalous magnetic moment in QED theory; a) process giving rise to the first term in eq. (2.1); b) the seven processes with two virtual photons which are independent of lepton (ℓ) mass; and c) the leading process which distinguishes between electron and muon anomalies. The diagrams (b) and (c) together make up the second term in eq. (2.1).
- Fig. 2 : The storage of muons in the 6 m bending magnet used in the first CERN (g-2) experiment. The field gradient makes the orbits walk to the right and at the end a large gradient is used to eject the particles so that they are stopped in the polarization analyser. Injected and ejected muons which stopped in the analyser were signalled by a coincidence between detectors 123 and 466'57, respectively. The decay electrons were separated into forward $[77'4(66')]$ and backward $[66'4(77')]$ events and collected in 0.1 μ s time bins as a function of storage time.
- Fig. 3 : An enlarged view of the polarization analyser. The spin direction of a muon which stopped in the liquid methylene iodide (E) was flipped through 90° by a current pulse in the aluminium strip coil (G). Backward or forward decay electrons were detected by scintillation counter telescopes 66' and 77', respectively. The static magnetic field was minimized by a double iron shield and a mumetal shield (A).
- Fig. 4 : The asymmetry A of the observed decay electron counts as a function of storage time t.

- Fig. 5 : Plan view of the 5 m diameter magnet used in the first muon storage ring at CERN. The momentum of the muons was 1.3 GeV/c and these particles were derived from a pulse of 10 GeV protons which produced pions at the target. A fraction of the latter subsequently decayed in flight inside the storage region. The proton beam and target are indicated in the figure as is the shielding needed to protect the decay electron counters.
- Fig. 6 : Decay electron count rate modulated by the (g-2) frequency as observed in the second CERN muon experiment.
- Fig. 7 : Plan view of the 14 m diameter magnet which formed the second muon storage ring at CERN. Cross-sections of the magnet and electric quadrupoles are also shown. The muon momentum was 3.098 GeV/c. The line of the 3.1 GeV/c pion beam is shown entering the ring through the pulsed inflector.
- Fig. 8 : The decay electron count rate as a function of time (in microseconds) after injection, from the most recent CERN muon (g-2) experiment. The time range for each line is shown to the right. The distribution contains a total record of 1.4×10^8 electron counts.
- Fig. 9 : Individual values of the ratio $R = \omega_a / \vec{\omega}_p$ for the nine experimental periods of the third experiment together with the weighted averages.
- Fig. 10 : Angular frequency vector diagram for the spin precession relative to the momentum when the particle has both electric and magnetic dipole moments. The plane of the precession is tilted through an angle $\delta = \omega_{\text{edm}} / \omega_a = f\beta / (2a)$.

a)

b)

c)

Fig. 1

Fig. 2

0 6 cm

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10