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TEMPERATURE DEPENDENCE OF THE BAG CONSTANT AND
THE EFFECTIVE LAGRANGIAN FOR GAUGE FIELDS AT FINITE TEMPERATURES

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ABSTRACT

We calculate the free energy for a gauge field at finite temperature in the presence of a constant gauge magnetic field in the one-loop approximation, omitting unstable modes. The effect of finite temperature on the approximate "true" Savvidy ground state of QCD is derived. A phase transition to the perturbative ground state at $T_{cr} = 1.3B^{1/4} \sim 190$ MeV is found.

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In order to understand the phase transition from the hadronic phase to the quark plasma¹⁾, the behaviour of the true vacuum of quantum chromodynamics as a function of temperature must be understood. At zero temperature, the perturbative vacuum has an energy density B which is higher than that of the true vacuum - this difference vanishes at high temperatures and the confining state is destroyed²⁾. Consequently B must be viewed as a temperature-dependent quantity. Our aim is to determine this dependence in a non-perturbative approach.

In this note we will, therefore, derive the temperature dependence of the free energy for the gauge field in a particular model of the true vacuum state and also study its behaviour nearer to the phase transition point at which the perturbative state is restored as the lowest energy state. Beyond this critical temperature, individual hadrons cannot exist, as the confining phase has been dissolved²⁾. This concept of maximal hadronic temperature occurs already in the bootstrap model of hadronic matter³⁾ and it has already been suggested that it corresponds to a transition to the quark-gluon plasma⁴⁾. Recently this transition has been considered in more detail within the concept of perturbative QCD^{5,1)}.

Our point of departure will be the effective action of gauge theories in the one-loop approximation with an arbitrary gauge magnetic field⁶⁻¹²⁾. This problem, at zero temperature, has recently attracted much attention as the effective action exhibits a minimum at a non-zero value of the magnetic field strength. It was pointed out by Savvidy⁸⁾ and by Nielsen¹¹⁾ that a "ferromagnetic" state with constant field strength H could be considered as an approximation to the true ground state of quantized Yang-Mills theories. Subsequently, it was found¹³⁻¹⁵⁾ that the Savvidy state contains unstable modes which lead to a more complicated structure of the ground state¹⁶⁻¹⁹⁾. Nielsen and Ninomiya¹⁷⁾ found an example of such an approximate vacuum state with domain structure.

While it is not yet clear whether this approach will ultimately lead to the construction of a Lorentz-invariant vacuum state, we assume here that the Savvidy state⁸⁾ incorporates the dominant mechanism for the energy difference between the perturbative vacuum and the true ground state, that is, the lowering in energy of the gluonic zero-point fluctuations with long wavelengths²⁰⁾. In this case, the Savvidy state could be successfully applied to predict properties which are less sensitive to the precise symmetry structure and more dependent on the energy balance.

We now turn to the computation of the free energy of a non-Abelian gauge field in the one-loop approximation in the presence of a gauge magnetic field. In our approach, the c-number gauge potential A_μ^a and the corresponding gauge field $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$ (chosen to be constant for the ease of our calculation) play the rôle of an order parameter characterizing the ground state. As discussed above, the resulting effective action has (at zero temperature) a minimum at finite magnetic field strength. The effective action, or, for finite temperature, the effective free energy (thermodynamic potential Ω in the absence of quarks) provides us with a convenient means of finding the best state which otherwise cannot be done in any order of perturbation theory²¹⁾.

In order to obtain Ω in the one-loop approximation, one needs the propagator of gauge field modes in the background potential. For these we will use the proper-time method of Schwinger²²⁾. The technique of our calculations is closely related to the work of Dittrich²³⁾, who obtained the effective Lagrangian of QED at finite temperature. Indeed, the $T = 0$ formalism is easily extended employing Matsubara's temperature-dependent Green's functions²⁴⁾ $G(\vec{x}, \tau)$ where τ is the "imaginary-time" co-ordinate that runs from 0 to $\beta = T^{-1}$. A necessary condition for the application of this approach is that the system is stationary. This is fulfilled if the c-number gauge field satisfies the condition

$$\beta^\mu F_{\mu\nu}^a = 0 \quad (1)$$

where β^μ is the (time-like) temperature four-vector. From condition (1), one easily derives the relations $H^2 - E^2 > 0$ and $\vec{E} \cdot \vec{H} = 0$ which provide the reference-frame independent characterization of a pure magnetic field. In the particular Lorentz-frame where $\beta^\mu = (\beta, 0, 0, 0)$, there is only a gauge magnetic field $\vec{H}$ and $\vec{E} = 0$.

Instead of the vacuum persistence amplitude of Schwinger²²⁾, we can now consider the change in the grand partition function $Z = \exp(-\beta\Omega)$ under an infinitesimal variation in the order parameter $F_{\mu\nu}^a$ of the finite temperature state. With a slight generalization of the result of Bernard²⁵⁾ and in analogy to the zero temperature expression derived by Honerkamp²⁶⁾, we find that $\delta \ln Z(A)$ is related to the (temperature-dependent) Green's functions of the gluon and (Faddeev-Popov) ghost modes, $\mathcal{D}$ and $\mathcal{G}_{FP}$ respectively:

$$\delta \ln Z = \frac{1}{2} \beta \text{Tr}_{xQL} \delta (\ln \mathcal{D} - 2 \ln \mathcal{G}_{FP}) \quad (2a)$$

where the trace runs over the four space variables, x_μ , and internal (i.e. Lorentz tensor, L, and colour, Q) indices. We note that $\ln Z$, as formulated here, is a gauge-invariant quantity, although the individual contributions from gluons and ghosts depend on the chosen gauge. In the so-called background gauge^{7,26)} for small fluctuations $\hat{A}_\mu^a$ around the c-number potential A , the Green's functions are given by

$$(\mathcal{D}^{-1})_{\mu\nu}^{ab} = [\pi^2 - 2igF]_{\mu\nu}^{ab} \quad (3a)$$

$$(\mathcal{Q}_{FP}^{-1})^{ab} = (\pi^2)^{ab} \quad (3b)$$

Here $\pi_\mu^{ab} = i\delta^{ab}\partial_\mu + gA_\mu^{ab}$ is the gauge-covariant momentum operator, and the potential vector and field strength tensor have been converted into colour matrices by means of the structure constants f^{acb} of the gauge group: $A_\mu^{ab} = if^{acb}A_\mu^c$. There is one more complication originating in the gluon field modes with spin parallel to the background magnetic field H and occupying the lowest Landau orbit. For small momenta they lead to negative contributions in the trace of $\pi^2 - 2igF$ in Eq. (2). Following Nielsen and Olesen, we omit in our calculations these unstable modes which at $T = 0$ are responsible for the decay of the homogeneous magnetic field into a more complicated structure¹³⁾. In our calculations we now include a step function θ that ensures that these modes are not counted, and consider the associated quantity Z' . Only with an improved form of the non-perturbative vacuum¹⁸⁾ can this omission be properly rectified.

$$\delta \ln Z' = \frac{1}{2} \beta \text{Tr}_{xQL} \delta \{ \ln [\mathcal{D} \theta (\pi^2 - 2igF)] - 2 \ln \mathcal{Q}_{FP} \} \quad (2b)$$

Applying Schwinger's proper-time formalism²²⁾, Eq. (2b) may be integrated and the following integral representation for $\ln Z$ emerges:

$$\ln Z'[A, T] = \frac{1}{2} \beta \int_0^\infty \frac{ds}{s} \text{Tr}_{xQ} \left\{ \text{Tr}_L \exp[-is(\pi^2 - 2igF)] \theta(\pi^2 - 2igF) - 2e^{-is\pi^2} \right\}$$

Recalling the integral representation of the step function

$$\Theta(\vec{\pi}^2 - 2igF) = - \int_{-\infty}^{\infty} \frac{dt}{2\pi i} \frac{e^{-it(\vec{\pi}^2 - 2igF)}}{t + i\eta}$$

and that $\Pi^2 = -\Pi_0^2 + \vec{\Pi}^2$ we find that

$$\ln Z'[A, T] = \frac{1}{2} \beta \int_0^\infty \frac{ds}{s} \text{Tr}_Q \text{Tr}_x e^{is\Pi_0^2}.$$

$$\left\{ \int_{-\infty}^{+\infty} - \frac{dt}{i2\pi(t+i\eta)} e^{-i(s+t)\vec{\Pi}^2} \text{Tr}_L e^{-i(s+t)(-i2gF)} - 2e^{-is\vec{\Pi}^2} \right\} \quad (4)$$

For the time-like operator, $\Pi_0 = i\partial_0$ in Eq. (4), the usual result

$$\text{Tr}_{x_0} [e^{is\Pi_0^2}] = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{is\omega^2} = \frac{1}{2\pi} \sqrt{\frac{\pi i}{s}} \quad (5)$$

must now be replaced²³⁾ by a sum over the Matsubara frequencies

$$\text{Tr}_{x_0} [e^{is\Pi_0^2}] = \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{is\omega_n^2}, \quad \omega_n = \frac{i\pi}{\beta} 2n \quad (6)$$

The result is readily expressed in terms of the Jacobi theta function²⁷⁾:

$$\frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{is\omega_n^2} = \frac{1}{\beta} J_\theta(0, 4i\pi s/\beta^2) = \sqrt{\frac{1}{4\pi is}} J_\theta(0, \frac{\beta^2}{4i\pi s}) \quad (7)$$

This makes the dependence on temperature of Eq. (4) explicit. The Lorentz trace in Eq. (4) can be easily evaluated using the eigenvalues of the matrix F given by Schwinger²²⁾:

$$\text{Tr}_L e^{-i(s+t)(-2igF)} = 2(1 + \cos 2gH(s+t)) = 4(1 - \sin^2 gH(s+t)) \quad (8)$$

We now compute the spatial trace, following Schwinger²²⁾, by introducing s as a proper time co-ordinate and by deriving a differential equation for the operator $\Pi(s) = \exp(is\Pi^2)\Pi \exp(-is\Pi^2)$. In this way we arrive at the relation:

$$\text{Tr}_x [\Pi_\mu \Pi_\nu e^{-is\Pi^2}] = -ig F_\mu^\sigma [\exp(2gFs) - 1]_{\sigma\nu}^{-1} \text{Tr}_x [e^{-is\Pi^2}]. \quad (9)$$

At this point we have to restrict the Lorentz indices to be space-like, since in the case of a pure magnetic field, the inverse of $[\exp(2gFs)-1]$ exists only for the space-like part of the Lorentz matrix²⁸⁾. Eq. (9) permits us to derive Schwinger's differential equation for the vector part of the momentum operator entering into Eq. (4):

$$\frac{d}{ds} \ln \text{Tr}_x [e^{-is\vec{\Pi}^2}] = - \sum_{\mu, \sigma=1,3} g F_\mu^\sigma [\exp(2gFs) - 1]_{\sigma\mu}^{-1}. \quad (10)$$

The solution is:

$$\text{Tr}_x [e^{-is\vec{\Pi}^2}] = \left(\frac{1}{2\pi} \sqrt{\frac{\pi}{is}} \right)^3 \int d^3x \exp \left(-\frac{1}{2} \text{Tr}'_L \ln \frac{\sinh gFs}{gFs} \right)$$

where Tr'_L indicates that the trace runs only over space-like indices of the Lorentz matrix F . Thus as in Eq. (8), we find

$$\text{Tr}_x [e^{-is\vec{\Pi}^2}] = \left(\frac{1}{2\pi} \sqrt{\frac{\pi}{is}} \right)^3 \int d^3x \frac{gHs}{\sin gHs} \quad (11)$$

With Eqs. (6), (8) and (11), we find for Eq. (4)

$$\begin{aligned} \ln Z' = & \frac{1}{\beta} \int_0^\infty \frac{ds}{s} \text{Tr}_Q \beta^{-1} \sum_n e^{is\omega_n^2} \int d^3x \left\{ - \int_{-\infty}^\infty \frac{dt}{2\pi i(t+i\eta)} \left(\frac{1}{2\pi} \sqrt{\frac{\pi}{i(s+t)}} \right)^3 \right. \\ & \left. \cdot 4gH(s+t) \left(\frac{1}{\sin gH(s+t)} - \frac{e^{igH(s+t)}}{2i} + \frac{e^{-igH(s+t)}}{2i} \right) - \left(\frac{1}{2\pi} \sqrt{\frac{\pi}{is}} \right)^3 \frac{2gHs}{\sin gHs} \right\} \end{aligned} \quad (12)$$

There are three terms to be integrated over t . For the first and last terms, the integrals are closed in the lower half-plane, enclosing the pole at $t = -i\eta$ (the poles of the sine-function are in the upper half-plane). The integral of the middle term is closed in the upper half-plane - it does not contribute. We thus find

$$\ln Z'[H, T] = -\beta \text{Tr}_Q \frac{1}{(4\pi)^2} \int d^3x \int_0^\infty \frac{ds}{s^3} g H s \left(\frac{1}{\sin g H s} - i e^{-ig H s} \right) J_\theta \left(0, \frac{\beta^2}{4i\pi s} \right). \quad (13)$$

This expression is analogous to the effective action obtained by Batalin et al.⁷⁾ and Yildiz and Cox¹²⁾ at $T = 0$ for magnetic gauge fields after the magnetic modes described above in Eq. (2b) have been subtracted by the procedure of Nielsen and Olesen¹³⁾. Let us now recall the known zero temperature properties of Eq. (13); the Jacobi theta function can be written in the form

$$J_\theta \left(0, \frac{\beta^2}{4i\pi s} \right) = 1 + 2 \sum_{n=1}^{\infty} \exp \left(- \frac{\beta^2 n^2}{4i s} \right)$$

For $T = 0$ the integral over s in Eq. (12) diverges and must be regularized by an ultra-violet cut off, i.e., by inserting a finite lower limit δ for the s -integration¹²⁾. The result is

$$\ln Z'(H, T=0) = -\beta V \frac{1}{(4\pi)^2} \text{Tr}_Q \left[\frac{11}{6} (gH)^2 \ln H_0 \delta + \frac{1}{2\delta^2} + \frac{11}{6} (gH)^2 \ln \frac{H}{H_0} \right] \quad (14)$$

where V is the volume occupied by H , and H_0 is an arbitrary field strength scale. The quadratically divergent term can be absorbed into the zero-point energy, while the logarithmically divergent term is combined with the Lagrangian of the free gauge field, $+ H^2/2C_1$, to renormalize the coupling constant (see, e.g., Ref. 12)).

$$\frac{1}{g_R^2} = \frac{1}{g^2} + \frac{11C_1}{48\pi^2} \ln(H_0 \delta) \quad (15)$$

Here C_1 is the eigenvalue of the Casimir operator in the adjoint representation ($C_1 \delta^{ab} = f^{aik} f^{bik}$). The final result, including the free gauge field contribution, is^{8,12,13)}

$$F'(H, T=0) = F_{(0)}(H, T=0) - \beta^{-1} V^{-1} \ln Z'(H, T=0) = \frac{1}{(4\pi)^2} \text{Tr}_Q \left[\frac{8\pi^2}{C_1 g_R^2} (gH)^2 + \frac{11}{6} (gH)^2 \ln \frac{H}{H_0} \right]. \quad (16)$$

where the free energy per unit volume (= thermodynamic potential per unit volume) $F = -\beta^{-1} \ln Z/V$ has been introduced here.

For $SU(2)$ we have $C_1 = 2$ and the colour trace gives a factor of 2 independent of the orientation of H^{ab} in colour space. For higher groups $SU(N)$ the result becomes much more complicated, as discussed in detail by Ambjørn, Nielsen and Olesen¹⁶⁾. Only in special colour directions does the trace give just a factor that cancels C_1 . To keep the following discussion as simple as possible, we shall take $SU(3)$ as the gauge group and assume that the magnetic field points in the λ_8 direction. Then $C_1 = 3$ and Tr_Q also gives just a factor three.

As was first observed by Savvidy⁸⁾, the expression (16) has a minimum at a non-zero magnetic field strength $H_{\min}$, which, under our above assumptions for $SU(3)$, is given by:

$$F'_{\min}(T=0) = -\frac{11}{64\pi^2} (gH_0)^2 \exp\left(-1 - \frac{32\pi^2}{11g_R^2}\right) < 0. \quad (17)$$

This value corresponds in our approach to the true vacuum, thus we now set

$$F'_{\min} = -B. \quad (18)$$

This assumption allows us to eliminate H_0 and g_R in terms of the "bag constant" B . B was first introduced in the context of the MIT bag model²⁹⁾, and its value will be chosen here as that used there: $B = (145 \text{ MeV})^4$.

To compute the finite temperature part of Eq. (13), we rotate the integrand to the negative imaginary s -axis and obtain

$$F'(H, T) - F'(H, 0) = -\text{Tr}_Q \int_0^\infty \frac{ds}{s^3} gHs \left(\frac{1}{\sinh gHs} - e^{-gHs} \right) 2 \sum_{n=1}^\infty e^{-\frac{\beta^2 n^2}{4s}}. \quad (19)$$

We note that we could have used this choice of s in the first place, but in doing so, we would have obscured the proper time formalism²²⁾.

Eq. (19) is our original contribution to the subject of this paper, apart from several technical improvements and clarifications which are not immediately apparent. We will now turn to the discussion of the phenomenological consequences which emerge from Eqs. (16) and (19). With $\sqrt{3}gH/2$ being the eigenvalue of H_{ab} , when the magnetic field points in the λ_8 direction, we find

$$F'(H, T) = \frac{1}{(4\pi)^2} 3(gH)^2 \left[\frac{11}{12} \left(\ln \frac{11}{64\pi^2} - 1 + \ln \frac{(gH)^2}{B} \right) + \frac{8}{\tau} \sum_{n=1}^{\infty} \frac{1}{n} K_1(n\tau) - 2 \int_0^{\infty} \frac{dx}{\sinh x} \sum_{n=1}^{\infty} \exp\left(-\frac{1}{4} n^2 \tau^2 x\right) \right]. \quad (20)$$

where

$$\tau = (\sqrt{3} gH/2)^{1/2} / T. \quad (21)$$

We first note that in the high temperature limit the last term in Eq. (20) dominates and we find

$$F'(H, T \rightarrow \infty) - F(H, 0) = -\frac{8}{45\pi^2} (\pi T)^4 \quad (22)$$

the thermal black body radiation term, as expected for eight coloured gluons.

For large values of τ (small temperatures), the two temperature-dependent terms fall off exponentially as can be seen in Fig. 1, showing the functions

$$f_1(\tau) = \frac{45\tau^4}{8\pi^4} \int_0^{\infty} \frac{dx}{\sinh x} \sum_{n=1}^{\infty} \exp\left(-\frac{1}{4} n^2 \tau^2 x\right) \quad (23a)$$

$$f_2(\tau) = \tau^{3/2} \sum_{n=1}^{\infty} \frac{1}{n} K_1(n\tau). \quad (23b)$$

This low-temperature behaviour in the Savvidy state

$$F'_{min}(T \rightarrow 0) \sim B^{5/8} T^{3/2} \exp\left(-\left(\frac{48\pi^2}{11} B\right)^{1/4} / T\right) \quad (24)$$

indicates that there is a gap of magnitude $(48\pi^2/11 B)^{1/4} \sim 675 \text{ MeV}$ in the excitation spectrum of the gauge field in the absence of quarks.

In Fig. 2 we have plotted the free energy F' per volume in units of B for various values of the temperature $T/B^{1/4}$ as a function of $gH/B^{1/2}$, after subtracting the T -dependent quantity at $H = 0$. The Savvidy minimum (at $T = 0$) becomes shallower with the growing temperature. For $T > T_{\text{cr}} = 1.27 B^{1/4} (\sim 183 \text{ MeV})$, the minimum at non-zero magnetic field lies energetically higher than the minimum at $gH = 0$, and it vanishes completely for $T > 1.5 B^{1/4} \sim 215 \text{ MeV}$. The depth of the Savvidy minimum of Eq. (20), that is, the temperature-dependent bag constant $B(T)$, is shown in Fig. 3 as a function of the temperature. As is obvious, T_{cr} is the phase transition point at which the (non-perturbative) Savvidy minimum becomes less favourable than the normal perturbative ground state at $gH = 0$. We note that $B(T)$ can be well approximated by the formula

$$B(T) \sim -B \left[1 - \left(\frac{T}{T_0} \right)^4 \right], \quad T_0 = 1.12 B^{1/4} \quad (25)$$

for temperatures below $0.8 B^{1/4}$.

We would like to interrupt this discussion of our results now to draw the reader's attention to the fact that we have found a first-order phase transition in a theory with dynamical symmetry breaking. This is in conspicuous contrast to the opinion stated in the article of Dolan and Jackiw³⁰⁾ that, in general, dynamically-broken symmetries could not be thermally restored. The reason why we find a phase transition in QCD (in our approximation to the ground state) is obviously that the symmetry breaking is caused by the infra-red behaviour of the theory, and not by the high-energy behaviour as in some models³¹⁾. Also it is clear that the Schwinger model cannot serve as a counter-example³⁰⁾ because thermal excitations cannot restore the two missing spatial dimensions.

Another interesting feature of our results is that the barrier between the Savvidy minimum and the perturbative ground state vanishes rapidly for $T > T_{\text{cr}}$. We therefore expect that the phase transition proceeds rapidly once the critical temperature is reached. On the other hand, when one starts out in the perturbative vacuum, at $gH = 0$, there is only a very small barrier to the Savvidy minimum when the temperature falls below $0.7 B^{1/4} (100 \text{ MeV})$.

The physical picture emerging from our results is this: in the environment of a hot hadronic gas, the effective bag constant $B(T)$ approaches zero with

increasing temperature. This causes hadrons to expand against the weakening vacuum pressure, but this expansion is very slight at first and amounts only to an increase of 20% in the baryon radius at $T \sim B^{\frac{1}{4}}(150 \text{ MeV})$. When the temperature exceeds $T_{\text{cr}} = 1.27B^{\frac{1}{4}} \sim 183 \text{ MeV}$, the true vacuum between hadrons undergoes a phase transition to a non-confining, perturbative phase, and the hot hadronic phase dissolves into a quark-gluon plasma. When the quark gas is cooled down again below T_{cr} , hadrons are frozen out again. We note that a comprehensive treatment of this process requires the inclusion of the quark degrees of freedom in the computation of F' , and as hadrons have a small size as compared to the now infinite range of H , also an improvement of the Savvidy ground state.

In summary - we have shown in this paper, by an explicit calculation of the thermodynamic potential Ω in the non-perturbative sector of $SU(3)$ gauge theory, how the confining phase dissolves at high temperatures, and a phase transition to perturbative states occurs at $T_{\text{cr}} = 1.3B^{\frac{1}{4}}$. We have given explicitly the temperature dependence of the bag constant.

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Figure Captions

- Fig. 1 : Both contributions f_1 and f_2 (Eqs. (23a) and (23b)) to the free energy F' per unit volume as a function of $\tau = (\sqrt{3}/2 \text{ gH}/T)^{\frac{1}{2}}$.
- Fig. 2 : The free energy F' per unit volume in units of B with $T/B^{\frac{1}{4}}$ as parameter.
- Fig. 3 : Temperature dependence of the bag constant - the Savvidy minimum of F' . At $T = 1.5B^{\frac{1}{4}}$ the "true" vacuum state disappears; the dashed curve is the approximate formula for small T , Eq. (25).

Fig. 1

Fig. 2

Fig. 3

19.