Ref.TH.2980-CERN

A SIMULTANEOUS FIT OF $b\bar{b}$, $c\bar{c}$, $s\bar{s}$ (bcs PAIRS)
AND $c\bar{s}$ SPECTRA

A. Martin

CERN--Geneva

ABSTRACT

A previous fit of the J/ψ and γ levels and leptonic widths by a power-like phenomenological potential is modified to include systems containing strange quarks. In spite of the smallness of the strange quark mass, the fit is astonishingly good.

1. INTRODUCTION

The present work is motivated by two facts. (i) Potential models of heavy quark systems, $b\bar{b}$ and $c\bar{c}$, work very well, in fact too well if one realizes that relativistic corrections are not really negligible¹⁾. In particular, a simple minded phenomenological potential

$$V = A + B r^\alpha \quad (1)$$

with $\alpha \approx 0.1$ reproduces all known levels of J/ψ and γ systems, as well as relative leptonic widths $\Gamma_e(ns)/\Gamma_e(ls)$ ²⁾. It is tempting to be more daring and to extend these considerations to lighter quark systems. Work in this direction, leading to bounds and estimates on quark mass differences, has already been done by Bertlmann and Ono³⁾. (ii) There is new experimental material on the $s\bar{s}$ system or new interpretations of existing data. A state interpreted as a radial excitation of the ϕ , called ϕ' has been discovered by the Orsay group at D.C.I. with a mass around 1.65 GeV⁴⁾. New evidence on the old E meson⁵⁾ has been obtained both in hadron collisions⁶⁾ and in radiative decay of the J/ψ ⁷⁾, with a mass 1.44 GeV. Very recently, this new data has been examined, and on the basis of the production and decays of this object it has been proposed that it is a 1^{++} $\ell = 1$ $s\bar{s}$ state⁸⁾. This quantum number attribution is not the one preferred by the discoverers of the $E(1.425)$ ⁵⁾ in $p\bar{p}$ annihilation, for whom the best choice was a 0^{++} state and the 1^{++} an unlikely second choice. However, in this paper we stick to the 1^{++} interpretation. It may be that the state observed in $p\bar{p}$ annihilation is another state. Without any calculation, it is very tempting to support the 1^{++} interpretation for the $E(1440)$ just by looking at the spacings of the sequences of states

$\phi(1.02)$	$E(1.44)$	$\phi'(1.65)$
and	$J/\psi(3.095)$	$\chi_{1^{++}}(3.510)$
		$\psi'(3.685)$

2. BASIC FORMULAE FOR THE FIT

For completeness, we recall that, in the Schrödinger picture, the masses of the states of a two body system of masses m_1, m_2 , and reduced mass $m/2$ bound by potential of type (1) are:

$$M(n, \ell, m_1, m_2) = m_1 + m_2 + A + \left(B^2 m^{-\alpha} \right)^{\frac{1}{\alpha+2}} C(n, \ell, \alpha) \quad (2)$$

with
$$m = \frac{2 m_1 m_2}{m_1 + m_2} \quad (3)$$

If one tries to reproduce the spectrum of a single system with such a potential, one sees that, with such a potential, the only quantities which can be determined from the fit are

$$m_1 + m_2 + A \quad \text{and} \quad B^2 m^{-\alpha} \quad (4)$$

In the special case of a $q\bar{q}$ system, only

$$2m + A \quad \text{and} \quad B^2 m^{-\alpha} \quad (4')$$

can be determined.

In principle, if we have two equal mass systems $q\bar{q}$ and $q'\bar{q}'$ with masses m and m' respectively, a fit by potential (1) will lead, if successful, to a complete determination of all parameters A, B, α, m_1 , and m_2 . This is what was done in Ref. 2) for $c\bar{c}$ and $b\bar{b}$, leading to the choices

$$\alpha = 0.104, \quad m_c = 1.45 \text{ GeV}, \quad m_b = 4.86 \text{ GeV}. \quad (5)$$

One of the successes of this fit was that it reproduced the mass of the recently observed⁹⁾ γ''' within 10 MeV. However, here we also want to describe the levels of the $s\bar{s}$ system with our model, and Bertlmann and Ono, from general considerations, give as a limit

$$1.23 < m_c - m_s < 1.46 \text{ GeV}, \quad (6)$$

so that the choice (5) would, unavoidably, lead to an extremely small mass for the strange quark, and make the non-relativistic treatment of $s\bar{s}$ untenable. Fortunately, the fit of $c\bar{c}$ and $b\bar{b}$ spectra is not terribly sensitive to a shift of the masses. The fit of $c\bar{c}$ can be maintained as well as before by keeping $2m + A$ and $B^2 m^{-\alpha}$ fixed. As one increases m_c , the difference $m_b - m_c$, adjusted to fit the ground state of $b\bar{b}$, varies very slowly, and the

agreement with the masses of higher states slowly deteriorates. We have adjusted m_c to get a 20 MeV deviation for γ''' , which we find tolerable.

As in Ref. 2), we have been forced by the accuracy of the fit to do something about hyperfine effects. We stick to the simple, but questionable assumption that the hyperfine splitting is controlled by a Fermi-type term. We believe that this is better than applying the standard rules to (1), which is only a phenomenological potential whose higher derivatives may be very different from those of the real potential. So we take

$$M_{\text{triplet}} - M_{\text{singlet}} = C \frac{|\psi(0)|^2}{m_1 m_2}, \quad (7)$$

and since anyway this is very crude, we use the WKB approximation for $\psi(0)$ (this is not the case, in the present paper, for the leptonic widths!) and get

$$M_{\text{triplet}} - M_{\text{singlet}} = 0.112 \text{ GeV} \frac{m_c^2}{m_1 m_2} \left(\frac{m}{m_c} \right)^{\frac{3}{2+\alpha}} \left(\frac{4n-1}{3} \right)^{2\frac{\alpha-1}{\alpha+2}}. \quad (8)$$

Anticipating the next section, let us indicate that the best fit of the 1S, 2S, 3S states of the $c\bar{c}$ system lead to

$$\alpha = 0.100 \quad (9)$$

The small change with respect to Ref. 2) is due to an improved accuracy of the calculations. Let us also indicate that we use (8) to find the centre of gravity of the $(s\bar{s})$ ground state from the ϕ mass, since the η' has an impure quark content.

3. RESULTS

We take

$$V = -8.064 + 6.8698 r^{0.100} \quad (\text{in GeV and GeV}^{-1})$$

and

$$m_c = 1.8, \quad m_b = 5.174, \quad m_d = 0.518 \quad (10)$$

The masses and relative leptonic widths are

		theory	experiment ⁹⁾	theory	experiment
for $c\bar{c}$	J/ ψ	3.095	3.095	1	1
	ψ'	3.687	3.684	0.40	0.46 ± 0.06
	ψ'''	4.032	4.030	0.25	0.16 ± 0.02
	Average P state	3.502	3.520		
	ψ'' (D state)	3.787 (no spin)	3.772		
	ψ^{iv}	4.280	4.414		
for $b\bar{b}$	Υ	9.46	9.46	1	1
	Υ'	10.025	10.02	0.51	0.45 ± 0.05
	Υ''	10.36	10.35	0.35	0.32 ± 0.05
	Υ'''	10.60	10.58	0.27	0.24 ± 0.05
	Υ^{iv}	10.76		0.21	
	1 P state	9.861			
	2 P state	10.242			
for $s\bar{s}$	ϕ	1.02	1.02		
	E(1 ⁺⁺)	1.42 (mean P state)	1.44		
	ϕ'	1.634	1.65		

For the $\phi - \phi'$ we cannot give the relative leptonic widths. All we can do is calculate the absolute leptonic width of the ϕ from that of the J/ ψ using the Van Royen-Weisskopf formula. This method should eliminate some common corrections, but cannot be expected to lead to perfect agreement. We find:

$$\begin{aligned}
 \Gamma_e(\phi) &= \frac{1}{4} \times (4.8 \pm 0.6 \text{ keV}) \left(\frac{M_{J\psi}}{M_\phi} \right)^2 \left(\frac{m_A}{m_C} \right)^{\frac{3}{2+\alpha}} \\
 &= 1.86 \pm 0.23 \text{ keV} .
 \end{aligned}
 \tag{11}$$

while experiment¹⁰⁾ gives

$$\Gamma_e(\phi) = 1.43 \pm 0.12 \text{ KeV}. \quad (12)$$

Considering that the leptonic width is much more sensitive to details of the wave function and relativistic effects than the energy levels, we find this kind of agreement acceptable.

for $c\bar{s}$

Here we have completely parameter-free predictions since the potential and the quark masses have already been fixed. The prediction for the centre of gravity of the ground state is

$$\left(\frac{1}{4} M_F + \frac{3}{4} M_{F^*} \right) = 2.0793 \text{ GeV} \quad (13)$$

which looks excellent. The separate predictions for F and F^* , based on blind application of Eq. (8) give

$$\begin{aligned} M_F &= 1.99 \text{ GeV} \\ M_{F^*} &= 2.11 \text{ GeV} \end{aligned} \quad (14)$$

The mass of the F is somewhat below the present best experimental value $2.01 \pm 0.01 \text{ GeV}$ ^{11),12)}. On the F^* the only existing result is the old Desy value¹³⁾, $2.14 \pm 0.06 \text{ GeV}$.

At the same time, we get a certain number of predictions on heavy (b,c,s) - light quark system, based on the belief that the binding of the light quark is, in first approximation, independent of the "heavy" quark, modulo recoil and hyperfine corrections. Using blindly the Quigg-Rosner formula¹⁴⁾,

$$M(x\bar{u}, 0^{-+}) - M_D = (m_x - m_c) \left(1 + \frac{0.033}{m_x} \right) \quad (15)$$

we get

$$M_B = 5.26 \text{ GeV}, \quad (16)$$

compatible with the extreme experimental limits

$$\frac{10.35}{2} < M_B < \frac{10.58}{2} \text{ GeV} \quad (17)$$

and

$$M_D - M_K = 1.36 \text{ GeV}, \quad (18)$$

in miraculous agreement with experiment (1.37). From formula (7), or its unknown substitute for light quarks, we get, again using the independence of the light quark wave function on the heavy quark

$$\frac{M_{D^*} - M_D}{M_{K^*} - M_K} = \frac{m_s}{m_c} = 0.29 \quad (19)$$

while experiment gives 0.36. This latter result coincides with that of A.D. Sakharov¹⁵⁾.

4. CONCLUDING REMARKS

The amount of experimental material "explained" by our naive phenomenological potential is amazing. Flavour independence is better than expected. Relativistic effects, already known to be non-negligible in the $c\bar{c}$ case, do not spoil the agreement of the $s\bar{s}$ spectrum. Even the leptonic width of the ϕ comes out reasonable. It is difficult to believe that this success is purely accidental. There must be an underlying truth behind the apparent success of the Schrödinger approach. Efforts by Cohen and Lipkin¹⁶⁾ go in this direction, but there is a long way to go before we get the full light.

ACKNOWLEDGEMENTS

The author acknowledges discussions with A. De Rujula and J.M. Richard at CERN, and stimulating contacts with P. Fishbane, M. Gell-Mann, P. Kaus and S. Meshkov at the Aspen Centre for Physics whose hospitality is gratefully acknowledged. Finally, the author thanks H. Grosse for help in writing a computer programme.

REFERENCES

1. H.J. Schnitzer, Phys. Rev. D18 (1978) 3482;
D. Beavis, S.Y. Chu, B.R. Desai and P. Kaus, Phys. Rev. D20 (1979) 763.
2. A. Martin, Phys. Lett. 93B (1980) 338;
Power potentials have previously been considered by M. Machacek and Y. Tomozawa,
Ann. Phys. 110 (1978) 407;
C. Quigg and J. Rosner, Comments Nucl. Part. Phys. 8 (1978) 11.
3. R. Bertlmann and S. Ono, University of Vienna preprint UW Th Ph-80-27.
4. J.C. Bizot, invited talk at the 20th International Conference on High Energy Physics,
Madison (1980), to be published in the Proceedings ; B. Durand, editor (American
Institute of Physics).
5. P. Baillon et al., Nuovo Cimento 50A (1967) 393.
6. C. Dionisi et al., Nucl. Phys. B169 (1980) 1;
C. Bromberg et al., paper submitted to the 20th International Conference on High
Energy Physics, Madison, Wisconsin.
7. G. Feldman, XV Recontres de Moriond, Les Arcs (1980);
D. Ashman, XV Recontres de Moriond, Les Arcs (1980).
8. C.E. Carlson, J.J. Coyne, P.M. Fishbane, F. Gross and S. Meshkov, National Bureau
of Standards preprint (1980).
9. K. Berkelmann, Rapporteur's talk at the 20th International Conference on High Energy
Physics, Madison (1980), to be published in the proceedings, B. Durand, editor
(American Institute of Physics).
10. J. Perez, Y. Jorba and F.M. Renard, Phys. Rep. 31C (1977) 1.
11. D. Aston et al., preprint CERN/EP/80-189 (1980).
12. G. Trilling, Rapporteur's talk at the 20th International Conference on High Energy
Physics, Madison (1980), to be published in the Proceedings, B. Durand editor
(American Institute of Physics).
13. U. Timm, Proceedings of the 1977 European Conference on Particle Physics, Budapest,
p. 727, L. Jenik and I. Montvay, editors, Central Institute for Physics,
Budapest.
14. C. Quigg and J. Rosner, Phys. Rep. 56 (1979) 168.
15. A.D. Sakharov, JETP Lett. 21 (1975) 9;
See also, A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12 (1975) 147.
16. I. Cohen and H.J. Lipkin, Phys. Lett. 93B (1980) 56.