80-10-252
高山研圖書室

Ref.TH.2924-CERN

SPACE-TIME DESCRIPTION OF COLOUR SCREENING VIA JET CALCULUS TECHNIQUES

G. Marchesini

Istituto di Fisica dell'Università, Parma
and INFN, Sezione di Milano

L. Trentadue ^{*)}, ⁺⁾

Institute of Theoretical Physics
Stanford University, Stanford, CA 94305

and

G. Veneziano

CERN -- Geneva

A B S T R A C T

The branching process characterizing the perturbative development of a chromodynamic jet up to the formation of colour singlet clusters is studied in real space-time with jet calculus techniques. Whereas the space-time separation of generic partons increases indefinitely with energy (or momentum transfer), such a separation remains finite within each colour singlet system in its own rest frame. These results add further appeal and over-all consistency to the preconfinement picture.

^{*)} On leave of absence from Istituto di Fisica dell'Università, Parma and INFN, Sezione di Milano.

⁺⁾ Supported by a CNR-NATO Fellowship.

1. INTRODUCTION

Perturbative quantum chromodynamics (QCD) provides, in the leading logarithmic approximation (LLA), a simple, semiclassical description of the production of secondaries in the cascade process typical of hard jets. This description is provided (in axial gauges) by dressed tree amplitudes, where partons are degrading their mass (through repeated branching) from a value of order Q (the large parameter characterizing the hardness of the process) down to a scale Q_0 such that

$$\alpha_s(Q_0^2)/\pi \lesssim 1 \quad (1.1)$$

with $\alpha_s(q^2)$ the QCD running coupling constant. The resulting classical tree equations give the so-called jet calculus rules [1].

The subsequent development of a jet, from partons of (off-shell) mass Q_0 to actual hadrons, is a difficult, long-distance problem. This difficulty is, however, alleviated by the preconfinement property of QCD jets [2], according to which the final state is a collection of colour singlet (c.s.) systems of quarks, antiquarks, and gluons, whose mass remains of order Q_0 as Q increases indefinitely [2,3]. This result suggests that the non-perturbative confinement mechanism may act only locally in momentum space, so that the calculated parton distributions would give hadron distributions when convoluted with the experimentally known properties of resonances (masses and decays).

This important preconfinement feature stems essentially from two points. Since in LLA only tree amplitudes are involved, the colour quantum numbers are naturally ordered: the emitted quark, antiquark, and gluons which form a colour singlet are topologically nearby in the tree (fig. 1). Secondly, since not all possible states are allowed in the c.s., the resulting distribution is infrared singular (as $Q_0 \rightarrow 0$) and this provides the aforementioned damping [2-4] in the c.s. mass.

Another important feature of preconfinement is that colour connected (c.c.) distributions, i.e. distributions of the quarks or the gluons which screen the colour of the initial quark (cf. fig. 1), are peaked [3,4] at small x (the scaling variable) with vanishing average value: $\langle x \rangle^{c.c.} \approx Q_0^2/Q^2$.

It seems very natural to ask whether the preconfinement picture, described above in momentum space language, has a corresponding description in position space, i.e. in actual space-time. Indeed, recently, the time evolution of the jet has been studied [5,6] for instance in e^+e^- collisions. For dimensional and Lorentz boost reasons, one expects that a generic parton is emitted, in the centre of mass, after a time Q/Q_0^2 , i.e. growing with Q . However, the preconfinement picture in space-time would require that partons belonging to the same c.s. have a finite space-time separation in the c.s. rest frame. In fact one expects that, if unbalanced colour charges live for a long time, this would activate non-perturbative confining forces which would reshuffle completely the non-perturbative calculations. By using the aforementioned features of c.c. distributions, Caneschi and Schwimmer [5] argued that unscreened charges live an infinitesimal time.

In this paper we study more quantitatively the preconfinement picture from the space-time point of view through the techniques of jet calculus [1] and of c.c. distributions [3]. By using the branching decay mechanism of LLA we are able to compute the average time of emission of a generic parton. In the e^+e^- centre of mass this time turns out to be indeed of order xQ/Q_0^2 . The equations for the c.c. distributions allow us also to compute the lifetime of unscreened colour charges (at rest) and to prove that it is finite (of order $1/Q_0$) as Q grows.

Besides its theoretical interest, the knowledge of the space-time development of jets might turn out to have also some practical implication.

It has been argued [7], for instance, that the quark struck in a deep inelastic lepton-nucleus collision can travel and reinteract inside the nucleus as a bare quark (i.e. before turning into hadrons) to produce a $\mu^+\mu^-$ pair. The approach of ref. [7] can be criticized on the basis that, while travelling inside the nucleus, the quark produces a shower of secondary partons even within its perturbative evolution. Yet these secondaries are potential projectiles for hard reinteractions and could lead to specific A (atomic number) dependences for the final state [8]. Our techniques are able, in principle, to determine the space-time and momentum configurations of these secondaries.

The outline of this paper is as follows. In Section 2, after defining its physical meaning, we evaluate the average emission time for an off-shell parton in a quark or gluon jet. We do this by using the jet calculus rules and by regularizing the soft infrared divergences (for simplicity) in a light-like axial gauge. We show, in the Appendix, that the leading behaviour of the average time is gauge-independent (at least within a class of axial gauges). We discuss, in particular, the average time of emission in the regions $x \ll 1$ and $1 - x \ll 1$ and, in more detail, for the case of the non-singlet quark distribution.

In section 3 we turn to the question of time-of-screening for colour charges. We discuss first, in the language of $e^+e^- \rightarrow \text{hadrons}$, the time it takes for the colour of the original $q\bar{q}$ pair to be screened by the conversion of soft gluons into other pairs and, subsequently, we look at the relative times of emission of partons in the same (preconfined) generic colour singlet. We find both times to be finite [of $O(1/Q_0)$] as $Q^2 \rightarrow \infty$, in the relevant Lorentz frame.

2. AVERAGE EMISSION TIME FOR GENERIC PARTONS

In this section we develop the formalism needed to evaluate the average time for the emission of a parton in e^+e^- collisions at centre-of-mass energy $2E = \sqrt{Q^2}$. We start from the tree graphs of the LLA (see fig. 2), whose absolute square is summed by the Altarelli-Parisi (AP) equation [9], and which describes the x -spectrum of the parton b in the quark jet. The emission time is obtained, for each kinematical configuration, by summing (in a certain frame) the lifetimes for each elementary decay along the tree. The average time is then the average over all possible configurations with their AP weight.

Of course, the time it takes for producing on-shell partons out of a given jet is a quantity which depends on the infrared structure of QCD [i.e. on $\alpha_s(k^2)$ for small k^2], and, hopefully, QCD is such that partons *never* manage to go on-shell. In order to ask a meaningful question of perturbative QCD, we have to consider a quantity which refers to the time evolution of the jet in the perturbative stage, i.e. *before* partons get too close to their mass shell.

As usual, one can worry about the physical significance of any quantity related to off-shell "final" partons. Our point of view is to regard an off-shell parton of mass M and c.m. energy xE as a final state capable of depositing that amount of energy into a calorimeter (a cone) of opening angle $\delta \approx M/Q$.

The time $t(Q,M)$ we would like to compute is that spent in the jet evolution *before* the parton enters the calorimeter or, in other words, the time spent while bremsstrahlung takes place *outside* the calorimeter. Afterwards, the subsequent, non-perturbative evolution takes place *inside* the calorimeter and we shall not be concerned with it. The idea is illustrated in fig. 3.

Recalling that in the graphs of figs. 1 and 3 intermediate parton lines are dressed, one can interpret fig. 2 as a succession of actual decays of dressed partons. In this way one arrives [4,10] at the AP equation in the "real" form for the single-parton distribution D_a^b :

$$D_a^b(Q^2, M^2, x) = \int_{ab} \Delta_a(Q^2, M^2) \delta(x-1) + \int_{M^2}^{Q^2} \frac{dk^2}{k^2} \int_x^{1-\epsilon} \frac{dz}{z} \frac{\alpha_s(k^2(1-z))}{2\pi} \Delta_a(Q^2, k^2) P_a^{a'a''}(z) D_{a'}^b(k^2, M^2, x/z) \quad (2.1)$$

In eq. (2.1) $P_a^{bc}(z)$ is the real [1] AP decay probability and the inhomogeneous term represents the "completely virtual" contribution with Δ_a satisfying the equation

$$\frac{\partial}{\partial \ln k^2} \ln \Delta_a(k^2, M^2) = -\frac{1}{2} \sum_c \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_s(k^2(1-z))}{2\pi} P_a^{cc'}(z) \quad (2.2)$$

Finally, the parameter ϵ in (2.1) and (2.2), which prevents divergences of the integrals near $z = 0, 1$ is given in a general gauge by [4]

$$\epsilon = \max \left(Q_0^2/k^2, k^2/Q^2\xi^2 \right) \quad (2.3)$$

where ξ is the axial gauge parameter [4] (see also the Appendix) and Q_0 is a mass parameter [such that $\alpha_s(Q_0^2) \lesssim 1$], giving a precise perturbative definition of what we call a "dressed" parton. In other words, as it should be clear from Eq. (2.1),

the "completely virtual" contribution Δ_a includes an arbitrary amount of radiation with $p_T < Q_0$. If we had set $Q_0 = 0$ the argument of the running coupling constant could have gone down to values as small as $k^4/Q^2\xi^2$, i.e. arbitrarily close to zero. If instead $Q_0 \neq 0$, real and virtual contributions together cancel each other in the non-perturbative region and α_s always appears with an argument larger than Q_0^2 .

This is the mechanism used in ref. [4] in order to define and compute several quasi-exclusive quantities by perturbative methods. In our specific case, we shall need a further assumption on the proper lifetime of the so-defined dressed parton, i.e. on the average time which has elapsed between two "actual" decays (with $p_T > Q_0$). The assumption is that such a lifetime is just given by

$$\tau_i \simeq 1/\sqrt{k_i^2} \quad (2.4)$$

where k_i^2 is the virtual mass of the i^{th} parton ($k_i^2 > M^2 > Q_0^2$). In other words, provided that Q_0^2 satisfies (1.1), we assume that our "dressed parton" satisfies the ordinary indetermination principle^{*)} and that (2.4) is not affected by the inclusion of low p_T ($< Q_0$) bremsstrahlung in our definition of a parton.

Let us now proceed to the evaluation of the average time. Equation (2.1) can be trivially solved in the form of an infinite series (no hard emission, one emission, etc.) as shown graphically in fig. 2. The time weight for the r^{th} term of fig. 2 is

$$t_r = \sum_{i=1}^r \gamma_i \tau_i = \sum_{i=1}^r E_i/k_i^2 \quad (2.5)$$

with τ_i given in (2.4) and γ_i the Lorentz factor $E_i/\sqrt{k_i^2}$ for the generic frame in which t is computed. Owing to the ladder structure, it is easy to see that the double sum obtained introducing (2.5) into the expansion of (2.1) reconstructs two AP distributions as indicated in fig. 4, where the cross stands for the insertion of the lifetime factor Ex_c/k^2 , x_c being the fraction of energy^{**)} for the

*) Alternatively, eq. (2.4) can be obtained from the energy denominators $\Delta E_i \simeq 1/\tau_i \simeq E_i - |k_i|$ of non-covariant perturbation theory.

**) In the following x_c is identified with the scaling variable of the evolution equation which is, more precisely, the ratio of a ξ -dependent combination of energy and longitudinal momentum [4]. This makes no practical difference for x finite, but some care is needed for $x \rightarrow 0$.

decaying parton c. The average time is obtained by normalizing the result to the $D(x)$ function itself and one gets immediately

$$\langle t_q^b(Q^2, M^2, x) \rangle = T_q^b(Q^2, M^2, x) / D_q^b(Q^2, M^2, x)$$

$$T_q^b(Q^2, M^2, x) = \sum_{c,d} \int_{M^2}^{Q^2} \frac{dk^2}{k^2} \int_0^1 \frac{dx_c}{x_c} D_q^c(Q^2, k^2, x_c) \left(\frac{x_c E}{k^2} \right) \cdot (2.6)$$

$$\int_{x/x_c}^{1-\epsilon} dz \frac{1}{z} \frac{\alpha_s(k^2(1-z))}{2\pi} P_c^{dd'}(z) D_d^b(k^2, M^2, x/x_c z)$$

For actual estimates, it is useful to consider the moments of T_q^b , for which we immediately get

$$T_q^b(Q^2, M^2, n) = \int_{M^2}^{Q^2} \frac{dk^2}{k^2} \frac{E}{k^2} D_q^c(Q^2, k^2, n+1) R_c^d(k^2, n) D_d^b(k^2, M^2, n) \quad (2.7)$$

where

$$R_c^d(k^2, n) = \int_0^{1-\epsilon} dz z^n \frac{\alpha_s(k^2(1-z))}{2\pi} \quad (2.8)$$

with ϵ defined in eq. (2.3). Using eq. (2.1) (in its differential form) and eq. (2.2) one can rewrite eq. (2.7) as

$$T_q^b(Q^2, M^2, n) = \int_{M^2}^{Q^2} \frac{dk^2}{k^2} \frac{E}{k^2} D_q^c(Q^2, k^2, n+1) \Delta_c(k^2, M^2) \frac{\partial}{\partial \ln k^2} \left[\bar{\Delta}_c^{-1}(k^2, n) D_c^b(k^2, M^2, n) \right] \quad (2.9)$$

Before proceeding to make estimates, we want to remark that, although there is a gauge (ξ) dependence on T through ϵ [see eq. (2.3)], such dependence does not affect the leading behaviour of T , for which one can take the light cone ($\xi \rightarrow \infty$) value of ϵ :

$$\epsilon|_{\text{light cone}} = Q_0^2/k^2 \quad (2.10)$$

This is shown in detail in the Appendix.

We start by illustrating the estimate of $\langle t \rangle$ in the case of the "non-singlet" (valence) quark in a quark jet. In this case parton indices in (2.7) can be suppressed. Furthermore, noticing that, because of the time weight factor E/k^2 , most of the contribution to T comes from $k^2 \sim M^2$, one can make the approximation

$$D(k^2, M^2, n) \underset{k^2 \sim M^2}{\simeq} D(k^2, M^2, n+1) \quad (2.11)$$

and combining it with the other D factor in (2.7) one arrives at

$$T^{NS}(Q^2, M^2, n) \gtrsim D^{NS}(Q^2, M^2, n+1) \cdot \int_{M^2}^{Q^2} \frac{dk^1}{k^2} \frac{E}{k^2} R_q^2(k^2, n) \quad (2.12)$$

If we consider, for instance, the case $M^2 \sim Q_0^2$, the integral in (2.12) can be easily seen to behave as $[1/(n+1) + 1/(n+3)]$ giving

$$\langle t^{NS}(Q^2, M^2 \sim Q_0^2, x) \rangle \simeq \frac{E/Q_0^2 c_F}{2\pi b} I(Q_0^2) \left[\int_x^1 dz (1+x^2/z^2) D^{NS}(z) \right] \left(D^{NS}(x) \right)^{-1} \quad (2.13)$$

where $b = (33 - 2N_F)/12\pi$, $c_F = 4/3$ and

$$I(Q_0^2) = \int_{t_0 \equiv \ln Q_0^2/\Lambda^2}^{\infty} dt/t e^{-(t-t_0)} \underset{t_0 \gg 1}{\sim} 1/t_0 \quad (2.14)$$

Taking, for instance, $D^{NS}(z)$ to be of the form

$$D^{NS}(z) \sim \text{const. } z^{-\alpha} (1-z)^{\beta} ; 0 < \alpha < 1, \beta > 0 \quad (2.15)$$

with α, β at most slowly varying functions of Q^2 , one gets

$$\langle t^{NS}(Q^2, M^2 \sim Q_0^2, x) \rangle \underset{x \rightarrow 0}{\sim} \frac{E}{Q_0^2} \frac{c_F}{2\pi b} \frac{x^{\alpha}}{1-\alpha} I(Q_0^2) \quad (2.16)$$

and

$$\langle t^{NS}(Q^2, M^2 \sim Q_0^2, x) \rangle \underset{x \rightarrow 1}{\sim} \frac{E}{Q_0^2} \frac{c_F}{2\pi b} \frac{2(1-x)}{1+\beta} I(Q_0^2) \quad (2.17)$$

The behaviour of $\langle t \rangle$ for general values of x can be obtained by trivial numerical integrations. The above result shows that the average time increases with x for small x (partly because of the Lorentz boost effect), reaches some maximum and then drops again for $x \rightarrow 1$. The reason for this last behaviour is that, in this limit, there is no phase space available for hard bremsstrahlung, so that no cascade takes place.

For the case of a generic final parton (sea or gluons) the equations are less suitable for analytic estimates. Nevertheless, the behaviour of $\langle t \rangle$ can be evaluated again for $x \rightarrow 0, 1$. The case $x \rightarrow 1$ can be studied by looking at large moments ($n \gg 1$). Noticing that R_n is negligible for $k^2 < nQ_0^2$, one finds again

$$T(n) \underset{n \gg 1}{\sim} \frac{E}{Q_0^2} \frac{D(n)}{n} \quad (2.18)$$

so that a behaviour $\langle t \rangle \sim E/Q_0^2(1-x)$ also follows for a generic parton.

For $x \rightarrow 0$ we distinguish the case of $b = \text{gluon}$ and $b = \text{quark (antiquark)}$.

In the first case the last AP D-function in eq. (2.6) can be approximated by $\delta(1 - x/x_c z)$ since $k^2 \sim M^2$. Using then the $1/z$ behaviour of $P_c^{gc}(z)$ for $z \rightarrow 0$ one finds

$$\langle t_q^g(Q^2, M^2, x) \rangle \underset{x \rightarrow 0}{\sim} 1/D_q^g(x) \cdot \frac{1}{x} D_2^g(n=1) E/M^2 \quad (2.19)$$

From the known behaviour of $D_q^g(x)$ one gets, up to powers of $\log x$:

$$\langle t_q^g(x) \rangle \underset{x \rightarrow 0}{\sim} \gamma E/M^2 \exp(-\delta \sqrt{\log 1/x}) \quad (2.20)$$

with γ and δ calculable slowly varying functions of Q^2 and M^2 . Hence, as for the non-singlet, $\langle t(x) \rangle \rightarrow 0$ for $x \rightarrow 0$. For the case of $b = \text{quark}$ the singular behaviour of $P_c^{dd'}$ can only be used if $D_d^{b=q}$ in eq. (2.6) is expanded to 1st order in α_s so that $d = \text{gluon}$, giving an extra $\alpha_s(M^2)$ factor. Up to this extra factor, the behaviour of $\langle t_q^{q, \bar{q}} \rangle$ for $x \rightarrow 0$ is similar to that of eq. (2.20), i.e. it is quickly vanishing.

This conclusion fits nicely with the idea that wee partons are emitted earlier than the fast ones (so-called inside-outside cascade [11]). We will come back to this point in section 3, while discussing the colour structure of jets.

3. LIFETIME FOR UNSCREENED COLOUR CHARGES

In this section we use the formalism previously elaborated, to discuss features of preconfinement from the space-time point of view. In particular, we compute the lifetime of unscreened charges.

In the light-like gauge the $\bar{q}$ emitted in the annihilation of the photon (fig. 2) is not radiating and its colour charge will be screened by a particular quark (and in general a certain number of gluons) emitted in the upper blob of fig. 2. Let us then first ask what is the time it takes for these screening partons to be emitted in the upper blob. As explained in the Introduction, the pre-confinement picture requires this time to be finite, in the $\bar{q}$ rest frame. In this case, the average time calculation goes as in the previous section (fig. 4) except that now we have to impose the selection that the colour index of the parton b under consideration is connected to the colour index of the initial q produced in the annihilation of the photon only through the possible emission of gluons and not of q or $\bar{q}$.

Distributions with this selection rule, denoted by c.c. distributions (fig. 5), have been introduced in ref. [3] and have the general form [3,4] ($b = q, g$):

$$\Gamma_q^b(Q^2, Q_0^2, x) \simeq \Delta_q(Q^2, Q_0^2) \delta(x-1) - \frac{1}{x} Q^2 \frac{\partial}{\partial Q^2} \Delta_q(xQ^2, Q_0^2) \quad (3.1)$$

roughly independent of the parton index b . For the moments this gives

$$\Gamma^{c.c.}(Q^2, Q_0^2, n) \simeq (Q_0^2/Q^2)^n \left[1 + n \int_{Q_0^2}^{Q^2} d\ell^2 / \ell^2 \left(\ell^2/Q_0^2 \right)^n \Delta_q(\ell^2, Q_0^2) \right] \quad (3.2)$$

The average time for this case can then be computed as in eqs. (2.6) to (2.9), except that we must replace D_q^b distributions by Γ_q^b distributions. Hence

$$T^{c.c.}(Q, Q_0, n) = \sum_{c=q,g} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \Gamma_c^c(Q^2, k^2, n+1) \frac{E}{k^2} \Delta_c(k^2, Q_0^2) \frac{\partial}{\partial \ln k^2} [\Delta_c^{-1}(k^2) \Gamma(k^2, n)] \quad (3.3)$$

For instance, for the zero moment (average over x), we have

$$T^{c.c.}(Q, Q_0) \Big|_{n=0} \simeq \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \Gamma(Q^2, k^2, 1) \frac{E}{k^2} \frac{\partial}{\partial \ln k^2} \ln \Delta_q^{-1}(k^2, Q_0^2) = \langle t(Q, Q_0) \rangle^{c.c.} \quad (3.4)$$

where the average time $\langle t \rangle^{c.c.}$ is now obtained by normalizing with Γ and we used the sum rule $\Gamma(Q^2, Q_0^2, n=0) \equiv 1$. Noticing that $(\partial/\partial \ln k^2) \ln \Delta_q^{-1}$ in eq. (3.1) has a $\ln \ln k^2$ behaviour, we obtain the estimate:

$$\langle t^{c.c.}(Q, Q_0) \rangle \sim E/Q_0^2 \langle x \rangle_{c.c.} \approx E/Q^2 \quad (3.5)$$

since the average x taken by the c.c. parton $\langle x \rangle_{c.c.} = \Gamma(Q, Q_0, n=1)$ is vanishing as Q_0^2/Q^2 [cf. eq. (2)].

In contrast to the case studied in the previous section for a generic parton, the time of emission of a c.c. parton is depressed by the factor $\langle x \rangle_{c.c.}$. In the frame where the non-radiating $\bar{q}$ in fig. 2 is at rest, $E \approx Q^2/2Q_0$, the partons which screen its colour charges are then emitted after a finite time of the order of $1/Q_0$. The same result is obtained by replacing the proper time factor $E x_c/k^2$ by the more precise E_c/k^2 (see footnote on page 5) and noticing that $\langle E_c \rangle \approx Q_0$.

The dynamical mechanism which makes this time finite, the vanishing of $\langle x \rangle_{c.c.}$, is of course the same which forces c.s. to have finite masses. In fact, $\langle x \rangle_{c.c.} \approx Q_0^2/Q^2$ entails that the emitted c.c. partons have momenta very close to the momentum of the $\bar{q}$ to form a finite mass for their c.s. system.

Let us now consider the unscreened lifetime for partons belonging to a general c.s. emitted in the upper blob of fig. 2. Owing to the tree structure of the process, partons in the same c.s. originate from the branching of a single parton a in the way shown in fig. 6, where the intermediate parton a of mass $\sqrt{k^2}$ is emitted according to the AP distribution, and the emitted $q\bar{q}$ are c.c.. The time for the emission of the parton a has already been estimated in the previous section and grows linearly with Q .

Redefining the time origin so that parton a decays at time zero, we now estimate the time delay necessary for the $q\bar{q}$ pair in fig. 5 to be emitted by the subsequent evolution. This time is evaluated in the centre of mass of parton a , i.e. practically in the cluster's rest frame. From the result of eq. (3.5), averaging over the x -variables of both q and $\bar{q}$, we find

$$T^{q, \bar{q}}(k^2, Q_0^2) \sim \int_{\epsilon(k)}^{1-\epsilon(k)} dz \frac{\alpha_s(z(1-z)k^2)}{2\pi} P_a^{c.c'}(z) \frac{E}{Q_0^2} \langle x \rangle^{c.c.} \quad (3.6)$$

where we have used again the sum rule $\Gamma(n = 0) \equiv 1$. From the form of $\Gamma(n = 1)$ we obtain for the general c.s. the same result as in eq. (3.5), apart from a $\ln \ln k^2$ factor:

$$T^{q\bar{q}}(k^2, Q_0^2) \sim E/k^2$$

In the rest frame of the parton a , $E = k$ and $T \sim 1/k$. By convoluting this result with the preconfinement damping in k^2 we arrive at $t^{q\bar{q}} \sim 1/Q_0$. This result leads to the same conclusion as for the previous case of the first c.s.: unscreened charges live, in their rest frame, a finite time independent of Q .

The conclusion which is drawn from the results of sections 2 and 3 can be expressed as follows. The perturbative stage of the jet development occurs in an increasingly (with Q) large region of space and time. However, the final state can be seen as a collection of c.s. clusters which, besides having limited mass (i.e. limited relative momenta), also occupy in their own rest frame a finite region of space during a lifetime of order $1/Q_0$.

As time goes on beyond $1/Q_0$ it is impossible to argue about the fate of these clusters by perturbative techniques. If the coupling α_s does not increase with distance the clusters will inevitably disintegrate with each parton flying out independently. If, however, one goes rather abruptly from a weak to a strong coupling régime at some critical scale Q_0 , as indicated by recent lattice calculations [12], it is very appealing to think that our finite-mass, finite-size clusters will become the hadronic states we observe at macroscopic distances from the interaction region.

Acknowledgements

We are grateful to A. Bassetto, M. Ciafaloni, L. Caneschi and, in particular, to K. Konishi for useful discussions. One of us (L.T.) is grateful to the Theory Division at CERN for the hospitality extended to him while part of this work was done and to the Fondazione Angelo Della Riccia for financial support.

APPENDIX

We want to show here that our average time evaluations are independent of the gauge frame, within the collinear gauges. In the frame where the $q\bar{q}$ emitted at the photon vertex are along the z axis, let us take a general collinear gauge vector

$$\eta = (a+b, \ 0, \ a-b) \ , \quad \xi^2 = b/a \quad (A.1)$$

While throughout the paper we considered the light-like gauge corresponding to the limit $\xi \rightarrow \infty$, our results are, however, independent of ξ .

Consider the average time for emitting a generic parton given, for instance, in eq. (2.9). In this general gauge T_a^b depends on ξ only through $\Delta_c(k^2, Q_0, \xi)$, which is given in eq. (2.2). The form of the cut-off $\varepsilon(k^2, \xi)$ [eq. (2.3)] reduces to the light-like gauge cut-off $\varepsilon(k^2) \equiv Q_0^2/k^2$ for the range of $k^2 < \xi Q Q_0$. As discussed in the text the integration region of k^2 in eq. (2.9) is dominated by $k^2 \sim M^2$. Consequently, $\Delta_a(k^2, Q_0^2, \xi)$ reduces for any non-vanishing ξ to the light-like gauge case if $M^2 < \xi Q Q_0$. On the other hand, the first stage of the evolution from Q^2 to $Q Q_0$ takes a finite time of the order of $1/Q_0$ which cannot be determined in a gauge-independent way.

REFERENCES

- [1] K. Konishi, A. Ukawa and G. Veneziano, Nucl. Phys. B157, 45 (1979); Phys. Lett. 78B, 243 (1978) and 80B, 259 (1979).
- For a review of more recent results, see K. Konishi, preprint CERN TH. 2853 (1980).
- [2] D. Amati and G. Veneziano, Phys. Lett. 83B, 87 (1979).
- [3] A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Lett. 83B, 207 (1979); Nucl. Phys. B163, 477 (1980).
- [4] D. Amati, A. Bassetto, M. Ciafaloni, G. Marchesini and G. Veneziano, preprint CERN TH. 2831 (1980), to appear in Nuclear Physics B.
- [5] L. Caneschi and A. Schwimmer, Phys. Lett. 86B, 179 (1979).
- [6] C.B. Chiu and J. Swzed, Max-Planck preprint MPI-PAE-PTh. 15/79 (1979).
H. Minakata, Phys. Rev. Lett. 45, 325 (1980).
- [7] C. Michael and D.M. Webber, J. Phys. G 6, 243 (1980).
- [8] K. Konishi, preprint CERN TH. 2897 (1980).
- [9] G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977).
G. Parisi, Proc. 11th Rencontre de Moriond, Flaine, 1976 (ed. J. Tran Thanh Van) (LPTPE, Orsay, 1976), vol. 2, p. 83.
L.N. Lipatov, Yad. Fiz. 20, 181 (1974).
- [10] Yu.L. Dokshitzer, D.I. D'Yakonov and S.I. Troyan, Phys. Rep. 58, 271 (1980) and references therein.
- [11] J.D. Bjorken, Proc. 1973 Summer Institute of Particle Physics (SLAC-167, 1973); see, also, A. Casher, J. Kogut and L. Susskind, Phys. Rev. Lett. 31, 792 (1973); K. Konishi, ref. [1].
- [12] M. Creutz, Phys. Rev. D 21, 2308 (1980).

Figure captions

- Fig. 1 : Example of colour singlets (c.s.) in a quark jet and of colour connected (c.c.) partons.
- Fig. 2 : Graphs for the inclusive distribution of a parton of type b in e^+e^- collisions. In the light-like gauge the lower $\bar{q}$ is not radiating.
- Fig. 3 : Illustration of our definition of $t_q^b(Q,M,x)$.
- Fig. 4 : Amplitude for the average time evaluation. The cross stands for the time insertion Ex_c/k^2 .
- Fig. 5 : Typical contribution to the c.c. distribution Γ_q^q . Gluons are represented as colour-anticolour line propagators. While on the upper side any parton (q , $\bar{q}$, or g) can be emitted, on the lower side only gluons are emitted, so that the colour of the initial and the final quark is connected through emission of only gluons.
- Fig. 6 : Correlation function of $q\bar{q}$ belonging to the same colour singlet. Since the colour indices of $q\bar{q}$ are connected through the emission of gluons only, we have $acc' = qqg, \bar{q}g\bar{q}$, or ggg .

Fig. 1

Fig. 2

Fig. 3

$$\sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \text{---} = \text{---} \quad \begin{array}{c} x_c, k^2 \\ x, M^2 \end{array} =$$

$$= \text{---} \quad \begin{array}{c} q \quad c \quad d \quad b \end{array}$$

Fig. 4

$$\text{---} \quad \text{---} = \text{---}$$

Fig. 5

$$\text{---} \quad \begin{array}{c} q \\ \bar{q} \end{array} \quad a, k^2 \quad \begin{array}{c} c \\ c' \end{array} \quad \begin{array}{c} q \\ \bar{q} \end{array} \quad \text{c. s.}$$

Fig. 6