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SYMMETRY RESTORATION AND GAUGE FIELD DYNAMICS IN THE CP^{n-1} MODEL

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A B S T R A C T

The CP^{n-1} model has a local $U(1)$ phase invariance for which the gauge field is composite. We show that the generation of dynamics for this field (as occurs in the $1/N$ expansion) is a consequence of the restoration of the local $U(1)$ symmetry.

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1. INTRODUCTION

The CP^{n-1} model¹⁾ has the following Lagrangian

$$\mathcal{L} = \overline{D_\mu z_i} D^\mu z_i - \lambda \left(\bar{z}^i z_i - \frac{1}{f} \right) \quad (1.1)$$

where $D_\mu z_i = \partial_\mu z_i + i f A_\mu z_i$, and λ is a Lagrange multiplier field. The index i is an $SU(n)$ index so that (1.1) has a rigid $SU(n)$ symmetry in addition to the local $U(1)$ phase invariance

$$\begin{aligned} \delta z_i &= -i \theta(x) z_i \\ \delta A_\mu &= \frac{1}{f} \partial_\mu \theta(x) \end{aligned} \quad (1.2)$$

The equations of motion for A_μ and λ imply

$$\begin{aligned} A_\mu &= \frac{i}{2} \bar{z}^i \overleftrightarrow{\partial}_\mu z_i \\ \bar{z}^i z_i - \frac{1}{f} &= 0 \end{aligned} \quad (1.3)$$

It follows that in perturbation theory in the coupling constant f both the local $U(1)$ and the rigid $SU(n)$ symmetries are spontaneously broken by $\langle Z_i \rangle \neq 0$, while A_μ is also a composite field with no dynamics. But since Eqs. (1.3) are operator equations in the quantum theory we have in general, for example,

$$\langle \bar{z}^i z_i \rangle_c + \langle \bar{z}^i \rangle \langle z_i \rangle - \frac{1}{f} = 0 \quad (1.4)$$

from which we see that $\langle Z_i \rangle$ may vanish provided that $\langle \bar{z}^i z_i \rangle_c \neq 0$, i.e., provided there is dynamical mass generation for the Z_i particles. It is equally possible that the operator $\bar{z}^i \overleftrightarrow{\partial}_\mu z_i$ will acquire a one-particle vector pole and thus generate dynamics for A_μ . In fact, in the $1/N$ expansion in two space-time dimensions, to leading²⁾ and next-to-leading order³⁾, we do have both $\langle Z_i \rangle \neq 0$ and a dynamical A_μ . The purpose of this paper is to show that the appearance of a dynamical $U(1)$ gauge field is a consequence of $\langle Z_i \rangle \neq 0$ or, more precisely, of the restoration of the local $U(1)$ symmetry. We will give two independent arguments that lead to this connection. The first is perhaps not rigorous, but is instructive.

2. SCALAR VERSUS VECTOR POLE IN A_μ

Although the gauge field A_μ is composite, the $U(1)$ invariance of (1.1) must still be fixed as for any other gauge theory. Let us suppose that we fix the gauge by requiring $\partial_\mu A^\mu \sim 0$, that is, we choose a gauge weighted, in the usual way, around $\partial_\mu A^\mu = 0$. The effective Lagrangian is

$$\mathcal{L}_{\text{eff.}} = \overline{D_\mu \bar{z}_i} D^\mu z_i - \lambda \left(\bar{z}_i z_i - \frac{1}{f} \right) - \partial_\mu B A^\mu + \frac{\alpha}{2} B^2 - i \partial_\mu \bar{c} \partial^\mu c \quad (2.1)$$

where $c, \bar{c}$ are the Faddeev-Popov (F.P.) ghosts and α is an arbitrary constant. The effective action is invariant under the following BRS transformation⁴⁾

$$\begin{aligned} \delta z_i &= [i\Lambda Q_B, z_i] = -if\Lambda c z_i \\ \delta A_\mu &= [i\Lambda Q_B, A_\mu] = \Lambda \partial_\mu c \\ \delta \bar{c} &= [i\Lambda Q_B, \bar{c}] = i\Lambda B \\ \delta c &= \delta B = 0 \end{aligned} \quad (2.2)$$

where Λ is the anticommuting BRS constant and Q_B the BRS charge satisfying $Q_B^2 = 0$. From the second equation of (2.2) follows the relation between asymptotic fields:

$$[i\Lambda Q_B, A_\mu^{\text{as.}}(x)] = \Lambda \partial_\mu c^{\text{as.}}(x) \quad (2.3)$$

Now, as c is a free field satisfying $\square c = 0$, $c^{\text{as.}} \neq 0$. Therefore, the left-hand side of (2.3) cannot vanish, i.e., A_μ must have a massless asymptotic field, either vector or scalar.

This result is easily understood when the $U(1)$ symmetry is spontaneously broken. If $Z_n = \sigma + i\psi$ and we choose $\langle \sigma \rangle \neq 0$, then A_μ contains the linear term $\langle \sigma \rangle \partial_\mu \psi$ and so

$$A_\mu^{\text{as.}}(x) = \langle \sigma \rangle \partial_\mu \psi^{\text{as.}}(x) \quad (2.4)$$

But ψ is the massless Goldstone field for the broken $U(1)$ invariance and so (2.3) is fulfilled by virtue of a massless scalar pole in A_μ .

If $\langle \sigma \rangle = 0$ the massless scalar pole in A_μ disappears because the $U(1)$ symmetry is restored so that it is not clear in this case how (2.3) can be satisfied. Equation (2.3) can be satisfied, however, if the massless scalar pole in A_μ is replaced by a massless vector pole, which implies the generation of Maxwell dynamics for A_μ .

This argument seems to depend crucially on the choice of a gauge for which the F.P. ghosts propagate [i.e., such that $c^{\text{as.}}(x) \neq 0$]. Since there is initially no kinetic term for A_μ , it is possible to choose a gauge by fixing one of the Z_i . For example, if $\langle \sigma \rangle \neq 0$, the $U(1)$ gauge can be fixed by requiring $\psi = 0$ and in this case the F.P. ghosts do not propagate. However, such a gauge

choice is permissible only if the $U(1)$ symmetry is indeed spontaneously broken. If the symmetry is restored, then $\langle \sigma \rangle = 0$ and $\psi = 0$ fails to fix the gauge. In other words, $\langle Z_1 \rangle = 0$ is a field configuration for which a Gribov-type ambiguity occurs⁵⁾ in "unitary" gauges such as $\psi = 0$. For perturbation theory about $\langle \sigma \rangle \neq 0$ this ambiguity is innocuous, but if we have dynamical restoration of the $U(1)$ symmetry the ambiguity is important. The only gauges that avoid this ambiguity, and are therefore permissible in the case of dynamical restoration of symmetry, are those for which the F.P. ghosts are propagating. The simplest example is just the $\partial_\mu A^\mu \sim 0$ gauge. Henceforth we shall use this gauge exclusively.

3. SYMMETRY RESTORATION AND THE VECTOR POLE

Goldstone's theorem states that if, and only if, the current of some continuous symmetry contains a massless pole the symmetry is spontaneously broken, so that if j_μ does not contain a massless pole the symmetry is unbroken. Only if the symmetry is unbroken will the charge operator $Q = \int d^{d-1} x j_0(x)$ be well-defined. How does this apply to the $U(1)$ symmetry of the CP^{n-1} model? The Noether current

$$j_\mu = i \bar{z}^i \overleftrightarrow{D}_\mu z_i \quad (3.1)$$

satisfies, by the A_μ field equation

$$j_\mu - \partial_\mu B = 0 \quad (3.2)$$

so that by virtue of the conservation of j_μ , $\partial_\mu j^\mu = 0$, B satisfies the massless wave equation $\square B = 0$. Therefore, although the charge

$$Q^{\text{Noether}} = \int d^{d-1} x j_0(x) = i \int d^{d-1} x (\bar{z}^i \overleftrightarrow{D}_0 z_i) \quad (3.3)$$

can be shown formally to generate the $U(1)$ phase transformation by use of the canonical commutation relations (CCR), it is not a well-defined operator because of the massless B contribution $j_\mu = \partial_\mu B$. But we cannot immediately conclude that the $U(1)$ symmetry is broken because there is the possibility of constructing another conserved current by adding a superpotential, i.e.,

$$j'_\mu = j_\mu + \partial^\nu f_{\nu\mu} \quad (3.4)$$

where $f_{\mu\nu}$ is any local antisymmetric tensor field. Both Q and Q' generate the same transformation on any local field. The question is now whether we can find an $f_{\mu\nu}$ such that the massless pole in j_μ is absent in j'_μ . The only candidate in our case is $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. We could therefore define the $U(1)$ charge by

$$Q^{(1)} = \int d^4x \left[j_0(x) - \xi \partial^i F_{0i}(x) \right] \quad (3.5)$$

where ξ is an arbitrary number. If A_μ has no massless vector pole then the massless pole in j_μ cannot be cancelled for any choice of ξ . In particular, if $A_\mu^{\text{as}}(x) = \partial_\mu S(x)$ then $F_{\mu\nu}^{\text{as}}(x) = 0$. In this case the $U(1)$ symmetry is spontaneously broken.

If, on the other hand, A_μ has a massless vector pole, i.e.,

$$A_\mu(x) \xrightarrow{|x_0| \rightarrow \infty} A_\mu^{\text{as}}(x) \quad ; \quad F_{\mu\nu}^{\text{as}}(x) \neq 0 \quad (3.6)$$

then we can show that $\partial^\nu F_{\mu\nu}$ contains the massless field B

$$\partial^\nu F_{\mu\nu} \xrightarrow{|x_d| \rightarrow \infty} \omega \partial_\mu B \quad (3.7)$$

for some non-vanishing constant ω . The proof is as follows⁶⁾: from the CCR and BRS invariance we have

$$\begin{aligned} [A_\mu(x), B(y)] &= [A_\mu^{\text{as}}(x), B(y)] = -i \partial_\mu D(x-y) \\ [B(x), B(y)] &= 0 \end{aligned} \quad (3.8)$$

By Lorentz covariance we generally have⁶⁾, for any massless asymptotic field of A_μ^{as} ,

$$[A_\mu^{\text{as}}(x), A_\nu^{\text{as}}(y)] = -i \left\{ a g_{\mu\nu} D(x-y) + b \partial_\mu \partial_\nu D(x-y) + c \partial_\mu \partial_\nu E(x-y) \right\} \quad (3.9)$$

where a, b, c are constants and $E(x)$ is a dipole invariant function such that $\square E(x) = D(x)$. Equation (3.8) leads to

$$\begin{aligned} [\square A_\mu^{\text{as}}(x), A_\nu^{\text{as}}(y)] &= -i c \partial_\mu \partial_\nu D(x-y) \\ [\partial^\mu A_\mu^{\text{as}}(x), A_\nu^{\text{as}}(y)] &= -(a+c) \partial_\nu D(x-y) \end{aligned} \quad (3.10)$$

and hence,

$$[\partial^\nu F_{\mu\nu}^{\text{as}}(x), A_\mu^{\text{as}}(y)] = -a \partial_\mu \partial_\rho D(x-y) \quad (3.11)$$

Similarly,

$$[\partial^\nu F_{\mu\nu}^{\text{as}}(x), B(y)] = 0 \quad (3.12)$$

so that from (3.8), (3.11) and (3.12) we see that

$$\begin{aligned} [\partial^\nu F_{\mu\nu}^{as}(x) - a \partial_\mu B(x), A_\rho^{as}(y)] &= 0 \\ [\partial^\nu F_{\mu\nu}^{as}(x) - a \partial_\mu B(x), B(y)] &= 0 \end{aligned} \quad (3.13)$$

from which follows (3.7) with $\omega = a$. Therefore, provided $a \neq 0$, the choice of $\xi = a^{-1}$ in (3.5) will ensure a well-defined $U(1)$ charge, and the $U(1)$ symmetry will be unbroken. But the condition that $a \neq 0$ is just that A_μ has a massless vector pole, as can be seen from (3.9). Therefore we conclude that whenever the $U(1)$ symmetry is restored, the composite $U(1)$ gauge field will acquire the dynamics of the Maxwell field.

4. COMMENTS

Coleman's theorem⁷⁾ requires that in two dimensions the rigid $SU(n)$ invariance be restored. Within the $1/N$ expansion this is seen to happen as $\langle Z_1 \rangle = 0$. Concurrently, the local $U(1)$ is also restored and as we have seen, this implies a massless vector pole for A_μ . In general, however, there is no necessary connection between restoration of the local $U(1)$ and rigid $SU(n)$ symmetries. The following order parameters

$$\begin{aligned} \bar{z}_i & \\ \phi^a &= \bar{z}^i (\lambda^a)_{ij} z_j \\ \Phi &= \epsilon^{i_1 i_2 \dots i_n} z_{i_1}(x_1) z_{i_2}(x_2) \dots z_{i_n}(x_n) \end{aligned} \quad (4.1)$$

discriminate between the following three possibilities:

- i) $\langle Z_1 \rangle \neq 0 \rightarrow U(1)$ and $SU(n)$ are broken;
- ii) $\langle \phi^a \rangle \neq 0$ but $\langle Z_1 \rangle = \langle \Phi \rangle = 0 \rightarrow SU(n)$ broken but $U(1)$ unbroken;
- iii) $\langle \Phi \rangle = 0$ but $\langle Z_1 \rangle = \langle \phi^a \rangle = 0 \rightarrow U(1)$ broken but $SU(n)$ unbroken.

For example, if we add massless fermions to the CP^{n-1} model, the local $U(1)$ invariance is spontaneously broken as there is no longer a massless vector pole in A_μ , but the $SU(n)$ symmetry remains unbroken. In this case, A_μ may still have the dynamics of a massive field, although our arguments do not require this. All that we can say is that if the local $U(1)$ symmetry is restored there does exist a massless $U(1)$ propagating gauge field.

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