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FROM MESONS TO BARYONS

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ABSTRACT

We discuss the possibility of extrapolating the phenomenology of the meson spectrum in potential models to the rich domain of the baryons. As an illustration, we have computed the mass of the Ω^- using a recent empirical quark-antiquark potential.

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In a recent preprint¹⁾, A. Martin proposed a very simple phenomenological potential which fits very well the spectrum of mesons with hidden or naked beauty, charm or strangeness. The quark-antiquark interaction of Ref. 1) consists of

$$V(r) = A + B r^\beta \quad (1)$$

with

$$A = -8.064 \quad B = 6.8698 \quad \beta = 0.100 \quad (2)$$

and is used with quark masses

$$m_c = 1.800 \quad m_b = 5.174 \quad m_s = 0.518 \quad (3)$$

all units being appropriate powers of GeV.

Clearly, the functional dependence in Eq. (1) has no direct and simple theoretical meaning. It may only be understood very qualitatively as a convenient interpolation between an elusive Coulombic short range²⁾ behaviour and a linear behaviour screened by pair creation effects³⁾. From the phenomenological viewpoint, however, the success of the fit in Ref. 1) is rather impressive and makes it tempting to extrapolate the investigation to states built with more than two quarks and in particular to the case of baryons.

The present phenomenology of the baryon spectrum in potential models, although rather elaborate⁴⁾, is completely disconnected from the meson sector. Exceptions are some attempts to derive the meson ($Q\bar{Q}$) and the baryon (QQQ) potential energy from a common framework, as was done for instance in an instanton⁵⁾, string⁶⁾ or bag model⁷⁾ picture.

If the $Q\bar{Q}$ force is purely phenomenological, there is no unambiguous way of extrapolating it to the QQQ case. The rule generally adopted consists of considering a pairwise quark-quark potential

$$V_{qq}(r) = \frac{1}{2} V_{q\bar{q}}(r) \quad (4)$$

This is subject to important criticisms. The factor $1/2$ arises from the analogy with the one gluon contribution. It is not a priori obvious that colour octet exchange should also saturate the confining part. Moreover, multibody forces may very well be present. In perturbation theory, a three-body interaction appears at the order α_s^2 as a consequence of the three gluon vertex. Within the string⁶⁾, the bag model⁷⁾ or in lattice gauge theories⁸⁾, there is also a three-body type of linear potential governing the long-range interaction in baryons.

There exist, on the other hand, more optimistic comments on the rule (4)

- i) The long range three-body interaction arising in the string or the bag model can be approximated by a sum of pairwise terms satisfying approximately the relation (4). More precisely, in these models one gets for large separations^{8),9)}

$$V_{q\bar{q}}(r) = \lambda r$$

$$V_{qaa} \simeq \sum_{i,j} V_{qa}^{\text{eff}}(r_{ij}) = \sum_{i,j} 0.54 \lambda r_{ij} \quad (5)$$

- ii) The intermediate range potential, if attributed to instanton contributions in the dilute gas approximation, seems to satisfy the relation (4) for the case of baryons⁵⁾. Multibody forces show up only for at least the four quark states¹⁰⁾.
- iii) As analysed by Feinberg and Sucher¹¹⁾, if one seriously considers a quark-quark potential $V_{QQ} = V^1 + \tilde{\lambda}_1 \tilde{\lambda}_2 V^8$ to estimate the forces between hadrons, the colour singlet exchange part V^1 should be much weaker than the colour octet component V^8 at large distances, unless one can never separate two hadrons. The octet part V^8 by itself already introduces disturbing enough Van der Waals type of forces¹²⁾.
- iv) Previous phenomenological analyses^{13),14),15)} show little or no departure from the rule (4).

We therefore now adopt this relation (4) as an educated guess in order to compute the masses of heavy baryons from empirical $Q\bar{Q}$ potentials. We are then faced with the non-relativistic three-body problem using an interaction of the type

$$V = \frac{1}{2} \sum_{i,j} (A + B r_{ij}^{\beta}) \quad (6)$$

or

$$V = \frac{1}{2} \sum_{i,j} (A + B \ln r_{ij}) \quad (7)$$

As already pointed out^{7),16)}, the hyperspherical formalism¹⁷⁾ is particularly efficient here. In this method, the standard Jacobi variables $\vec{\xi}_1 = \vec{x}_2 - \vec{x}_1$ and $\vec{\xi}_2 = (2\vec{x}_3 - \vec{x}_1 - \vec{x}_2)/\sqrt{3}$ are replaced by an over-all distance ξ and a set of five angles Ω given by

$$\xi^2 = \vec{\xi}_1^2 + \vec{\xi}_2^2$$

and

$$\Omega = \{ \omega(\hat{\xi}_1), \omega(\hat{\xi}_2), \text{tg}^{-1}(\xi_1/\xi_2) \} \quad (8)$$

Here, we consider for simplicity only the case of equal masses. The wave functions

and the potential are expanded on the hyperspherical harmonics $P_{[L]}(\Omega)$. $[L]$ denotes here the values of the "grand" orbital L and of the associated magnetic numbers, but, for simplicity, the brackets will now be omitted. With the simple power law (6), the matrix elements of the potential

$$V_{LL'}(\xi) = \int d\Omega \, P_L^*(\Omega) V(\xi, \Omega) P_{L'}(\Omega) \quad (9)$$

can be calculated analytically. This gives

$$V_{LL'} = A_{LL'} + B_{LL'} \xi^\beta \quad (10)$$

with $A_{LL'} = 3/2A\delta_{LL'}$, and, for instance,

$$B_{00} = \frac{12}{\pi} B \frac{\Gamma(\frac{3}{2} + \frac{\beta}{2}) \Gamma(\frac{3}{2})}{\Gamma(3 + \frac{\beta}{2})} \quad (11)$$

For a logarithmic potential (8), one has

$$V_{LL'} = A_{LL'} + B_{LL'} \ln \xi \quad (12)$$

with $B_{LL'} = 3/2B \delta_{LL'}$, and, for instance,

$$A_{00} = \frac{3}{2} A - \frac{3}{2} B \left(\ln 2 - \frac{1}{4} \right) \quad (13)$$

For almost any value of the exponent β , the potential has the nice property of being almost diagonal and scalar in the hyperspherical basis. For instance, in the case $\beta = 0.1$, the first off-diagonal corrections $V_{0L}(\xi)$ $L = 4, 6, 8$ to the hyperscalar approximation $V_{00}(\xi)$ are governed by

$$\begin{aligned} B_{04} &\simeq 0.007 B_{00} \\ B_{06} &\simeq -0.004 B_{00} \\ B_{08} &\simeq -0.002 B_{00} \end{aligned} \quad (14)$$

This means that each QQQ state is dominated by a single hyperspherical harmonic, i.e.,

$$\psi(x_1, x_2, x_3) = \frac{u_L(\xi)}{\xi^{5/2}} P_L(\Omega) \quad (15)$$

whose associated hyperradial wave function u_L satisfies the simple differential equation

$$u_L'' - \frac{(L+3/2)(L+5/2)}{\xi^2} u_L + m(E - V_{00}(\xi)) u_L = 0 \quad (16)$$

For instance, solving only the above equation for the ground state and the first radial excitation of the sss system with the parameters (2) and (3) gives masses $M = 1.621$ GeV and $M^*(sss) = 2.139$ GeV. Instead of (16), we now solve two coupled equations, introducing successively the coupling of the $L = 0$ component with the $L = 4, 6$ or 8 scalar and symmetrical harmonics, as given in Ref. 18). The computed shift ΔE_L and the percentage of admixture p_L into the wave function are displayed in Table 1. Their value clearly indicates that the hyperspherical expansion converges rapidly and that using a single harmonic is already a good approximation.

Several simple scaling laws à la Quigg and Rosner¹⁹⁾ can be written down for baryons in the case of power law or logarithmic interactions. The hyperscalar approximation of the potential also gives rise to interesting relations between meson and baryon masses. In the case of the logarithmic potential⁷⁾, for instance, one can easily show that

$$\frac{1}{3} M_{qqq}(L) - \frac{1}{2} M_{q\bar{q}}(L + \frac{3}{2}) = \frac{B}{8} - \frac{B}{4} \ln 6 \quad (17)$$

The mass of the ground state baryon is related to that of a fictitious meson with $\ell = 3/2$, whose mass in turn can be interpolated from the experimental values which have integer ℓ . Alternatively, if one uses the numerical result $M_{Q\bar{Q}}(3/2) \approx M_{Q\bar{Q}}(0) + 0.800B$, the above equation leads us to

$$\delta M = \frac{1}{3} M_{qqq}(0) - \frac{1}{2} M_{q\bar{q}}(0) \approx 0.077 B \quad (18)$$

to be compared with the result of Cohen and Lipkin¹⁵⁾ $\delta M = B/2 \ln 2/\sqrt{3} \approx 0.072B$, derived using the virial theorem and some guess at the radial dependence of the wave functions. Note that our figure 0.077 is necessarily too high for the ground state because of the hyperscalar approximation. The exact numerical result turns out to be $\delta M \approx 0.075B$ when non-hyperscalar corrections are computed, in the same way as done in Table 1 for the $r^{0.1}$ case.

The spectrum of the heavy baryons obtained from Eq. (16) is displayed in Table 2 for the sss and ccc cases. We first used the potential of A. Martin¹⁾, as given in Eqs. (1)-(3). The second spectrum comes from the spin-dependent part of the potential of Badhuri et al.¹³⁾. Finally, in the ccc case, the results of a bag model calculation⁷⁾ are shown for comparison. Inspection of Table 2

indicates that various kinds of potentials give very similar predictions for triple charmed baryons, as soon as the parameters are adjusted to give realistic masses in the mesonic sector. Indeed, all these potentials, in spite of their rather different functional dependence, have almost the same shape in the region $\xi = 0.2 - 1$ fm which is crucial for the lowest part of the spectrum. For the sss case, the smooth confinement in $r^{0.1}$ of the first potential leads to lower excitation levels than the linear potential. Here, indeed, we test the interaction at larger distances.

We now discuss the spin-dependent corrections, restricting ourselves to the case of the Ω^- . Following Ref. 1), the potential (1) is not taken seriously as a guide to derive the associated spin-dependent components (an alternative viewpoint is adopted in Ref. 20). Instead, the spin-spin splitting is assumed to be of the DGG type²¹⁾, i.e.,

$$\Delta M^m = - C \tilde{\lambda}_1 \tilde{\lambda}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{|\phi(0)|^2}{m_1 m_2} \quad (19)$$

where C is a new parameter, adjusted to reproduce

$$\Delta M(cc\bar{c}) = M(J/\psi) - M(\eta_c) = 0.112 \text{ GeV} \quad (20)$$

The generalization of Eq. (19) to a symmetric baryon is

$$\Delta M^B = - \frac{C}{m^2} \left(\sum_{i,j} \tilde{\lambda}_i \tilde{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \int d^3 \vec{\xi}_2 |\phi(\vec{\xi}_1=0, \vec{\xi}_2)|^2 \quad (21)$$

In the hyperscalar approximation (16), this means for the Ω^-

$$\Delta M(\Omega^-) = \frac{3}{8} \frac{m_c^2}{m_s^2} \frac{\int_0^\infty \mathcal{U}_0^L(sss, \xi) \xi^{-3} d\xi}{|\phi(cc\bar{c}, 0)|^2} \Delta M(cc\bar{c}) \quad (22)$$

Using the parameters (2) and (3), one gets $\Delta M(\Omega^-) \approx 0.045$ GeV. Such a small value is a twofold consequence of the assumed colour structure of the potential. There is an explicit $1/2$ reduction of $\tilde{\lambda}_i \tilde{\lambda}_j$ when going from mesons to baryons in the generalized Breit-Fermi formulae (19) and (21). The same factor $1/2$ in the spin-independent potential, as per Eq. (4), makes it almost 50% less probable to find two given quarks at the same place in the Ω^- than the quark and the antiquark in the ϕ meson.

Badhuri et al.¹³⁾ have parametrized the spin-spin potential by a smeared δ function, as done previously by others²²⁾. This allows one to include, non-perturbatively, the spin effects in the Schrödinger equation. With their parameters, we get $M(\Omega^-) = 1.686$ GeV in the hyperscalar approximation, to be compared

with the value $M(\text{sss}) = 1.624 \text{ GeV}$ in Table 2, with the central force only. Our result for $M(\Omega^-)$ is lower by 13 MeV than the figure displayed by Badhuri et al¹³⁾. Since we have checked the exactness of their computation, this indicates that the hyperscalar approximation (16) works better here than the effective two-body equation of Feshbach and Rubinow²³⁾ which they use. Both methods are indeed variational, so the smallest figure is the best one for the ground state.

Let us now briefly summarize:

- i) The simple rule $V_{\text{QQ}} = 1/2 V_{\text{Q}\bar{\text{Q}}}$ is far from being fully justified theoretically. It seems, however, compatible with the presently known experimental results on baryons.
- ii) The hyperspherical formalism is very well suited for the three quark problem.
- iii) In particular, the recent potential of Ref. 1) gives, for the Ω^- , a mass

$$M = 1.621 - 0.004 + 0.045 = 1.662 \text{ GeV}$$

when one puts together the hyperscalar approximation, the corrections to it and the spin-spin splitting respectively. This has to be compared with the experimental value $M = 1.672 \text{ GeV}$. Why such a simple non-relativistic potential works so well for mesons is already a puzzle¹⁾. Why its most naive extrapolation to the Ω^- is so successful is an additional challenging problem.

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TABLE 1

	L = 4	L = 6	L = 8
ΔE_L	-2.9	-0.6	-0.06
P_L	0.0017	0.0002	0.00001
ΔE_L^*	-8.6	-2.0	-0.2
P_L^*	0.014	0.002	0.0001

Corrections to the hyperscalar approximation for the Ω^- and its first radial excitation (*). The energy shifts ΔE_L (in MeV) and the percentage of admixture P_L are shown for the corrections corresponding to grand orbital $L = 4, 6$ or 8 .

TABLE 2

			sss		ccc		
J^P	L	n	Martin ¹⁾	Badhuri et al. ¹³⁾ (recalculated)	Martin ¹⁾	Badhuri et al. ¹³⁾ (recalculated)	Bag Model ⁷⁾
$\frac{3}{2}^+$	0	0	1.621	1.624	4.766	4.797	4.791
$\frac{3}{2}^+$	0	1	2.139	2.244	5.254	5.278	5.302
$\frac{1}{2}^- \frac{3}{2}^-$	1	0	1.937	2.019	5.112	5.134	5.137
$\frac{1}{2}^+ \frac{3}{2}^+ \frac{5}{2}^+$	2	0	2.274	2.358	5.358	5.402	5.420

Heavy baryon spectrum (in GeV) in the hyperscalar approximation and without spin corrections for various potentials. L is the grand orbital, n the number of modes.

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