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"COSMOLOGICAL IMPLICATIONS OF GRAND UNIFIED THEORIES"

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ABSTRACT

Grand Unified Theories (GUTs), in addition to their spectacular predictions testable in the realm of terrestrial laboratories (such as proton decay), seem to play an essential role in the evaluation of the Infant Universe. For example, Grand Unified Interactions may be responsible for the creation of the observed net baryon number or (partly?) for the apparent smoothness of the Universe. All these cosmological implications of GUTs are analyzed in some detail, after a lengthy resumé of the basic principles of GUTs and their important consequences for particle physics.

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COSMOLOGICAL IMPLICATIONS OF GRAND UNIFIED THEORIES

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INTRODUCTION

In our search for unity of all fundamental interactions observed in nature, Grand Unified Theories (GUTs)¹⁻⁴ seem to be a major discovery. GUTs suggest that strong and electroweak interactions originate from a single set of interactions characterized by a single, basic, gauge coupling constant, while the "matter fields" (quarks and leptons) are members of one multiplet.

The most dramatic and spectacular outcome of such grand ideas⁵ (other consequences being the explanation of charge quantization, clarification of many key points of fermion spectroscopy, etc.) is, by far, the prediction that *protons are not for ever*. Quarks and leptons, being in the same multiplet, cannot avoid transforming into each other and thus breaking both baryon and lepton number conservation, although in such a very feeble way that it takes a long, long time⁶ ($10^{31\pm 2}$ years) for protons to die. The observed longevity* of the proton [$\tau_p > O(10^{30}$ years)] indeed entails that the grand unification scale is cosmologically high^{3,4,6} (10^{15} GeV) so it should not come as a surprise that grand unified interactions may have played an important role in the "earliest" stages of the evolution of the Universe ($T > 10^{15}$ GeV or $t < 10^{-35}$ s). Indeed, the most striking consequence of GUTs with respect to cosmology *again* involves B-number violating interactions and provides a possible explanation⁸ of the observed net baryon number of the Universe. If true, GUTs solve⁸ one of the biggest problems of cosmology of all time: Why is there no antimatter around? and Why is there much more "light" than matter?

Amazingly enough, the interactions that are responsible for our creation will eventually dissolve us *back* to our parts, *back into the soup* [photons, neutrinos, electrons, i.e. the only stable (?) particles known to exist (?)].

Elementary particle interactions have played an essential role during the whole evolution of the Universe from its *very beginning* until now. Standard strong and electroweak interactions are responsible for the primordial nucleosynthesis⁹ (^4He , ^2D , ...) as well as for the creation of heavy elements (C, N, Fe, ...) in the interior of stars⁹, while grand unified interactions are responsible⁸ for the

*Learned, Reines and Soni⁷), and Steinberg, private communication (1980), quote limits on the nucleon lifetime of 2 and 3×10^{30} years, respectively, if the decay modes resemble those expected in the SU(5) model of Ref. 2.

creation of a net baryon number (quarkosynthesis) and possibly¹⁰⁾ (??) for the smoothness of the Universe.

These lectures, mainly devoted to the cosmological implications of GUTs, also include the essential ingredients of GUTs and some of their important applications to particle physics⁵⁾.

Section I contains some basic points concerning the structure of the standard strong and electroweak interactions prior to grand unification. A detailed exposé of GUTs is attempted in Section II, including their basic principles and their consequences for particle physics. The minimal, simplest GUT, $SU(5)$ ²⁾ is analysed in some detail and it will be used throughout these lectures as the GUT prototype. Finally, Section III contains the most important cosmological implications¹¹⁾ of GUTs, including baryon number generation in the early Universe (in rather lengthy detail); dissipative processes in the very early Universe; grand unified monopoles, etc.

The interface between GUTs and cosmology is given in Table 1.

I. A GLIMPSE OF STRONG AND ELECTROWEAK INTERACTIONS

During our long and still continuing struggle to break nature's complicated code concerning the fundamental structure of the world, we have realized that gauge interactions seem to play an essential role. All (?) fundamental (?) interactions observed in nature [strong, electroweak, gravitational, (?)] look like gauge interactions. Indeed, intense experimental and theoretical efforts during the last decade have shown that we get a phenomenologically satisfactory description¹²⁾ of the "low-energy" world [$E < 100$ GeV (?)] by assuming that $SU(3)_{\text{colour}}$ is the gauge group responsible for strong interactions, while $SU(2) \times U(1)$ is the gauge group capable of describing electroweak interactions. Fermions (quarks and leptons) are thrown into the simplest possible representations of these groups. Quarks come in three colours; both their left- and right-handed parts are triplets (3) under $SU(3)_C$, while left-handed quarks are doublets (2) and right-handed quarks are singlets (1) under $SU(2)$. The leptons do not participate in strong interactions, i.e. they are colour [$SU(3)_C$] singlets (1), but they do participate in electroweak interactions behaving just like the quarks do: left-handed leptons are $SU(2)$ doublets (2), right-handed leptons [except the neutrinos (?)] are $SU(2)$ singlets (1). So under the group $G_1 \equiv SU(3)_C \times SU(2) \times U(1)$, the fermions exhibit the following structure:

$$\text{generation (or family)} \equiv \left\{ \begin{array}{lll} \begin{pmatrix} u \\ d \end{pmatrix}_L^{\text{red}} & \begin{pmatrix} u \\ d \end{pmatrix}_L^{\text{yellow}} & \begin{pmatrix} u \\ d \end{pmatrix}_L^{\text{blue}} ; & u_R^r, u_R^y, u_R^b \\ & & & d_R^r, d_R^y, d_R^b \\ \\ & \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & & ; e_R \end{array} \right. \quad (1)$$

and there are at least two more generations like (1), including (c, s, ν_μ , μ) and (t, b, ν_τ , τ).

After the prescription of the fermion ("matter") content of the theory, we now proceed to describe the gauge symmetry breaking. Because of renormalizability, the only way available to break a gauge symmetry is to do it "spontaneously". Spontaneous breakdown may occur either dynamically or by the explicit use of Higgs particles¹³⁾. (A systematic study of the properties and production mechanisms of these particles is reported in Ref. 13.) In both cases, however, there are "constraints" to be satisfied. Experimental data¹⁴⁾ show that

TABLE 1 Grand Connection

$$\rho \equiv \left(\frac{m_W}{m_Z \cos \theta_W} \right)^2 \approx 1, \quad (2)$$

which immediately implies that, in the explicit Higgs picture, the SU(2) breaking *necessarily* occurs only through one [or more (?)] [SU(2)] doublet(s) of Higgs fields. The use of other Higgs representations of "strength" comparable to that of the Higgs doublet(s) is excluded. In the "observed" low-energy SU(3)_c × SU(2) × U(1) group, SU(2) seems to be the only broken gauge symmetry, while SU(3)_c and U(1)_{em} remain exact internal local symmetries, probably the only ones (?). We assume that

$$G_1 \equiv \text{SU}(3)_c \times \text{SU}(2) \times \text{U}(1) \xrightarrow{m_W \sim 100 \text{ GeV}} G_2 \equiv \text{SU}(3)_c \times \text{U}(1)_{\text{em}} \quad (3)$$

(m_W indicates also the mass of the gauge bosons mediating weak interactions) through a Higgs [SU(2)] doublet (or doublets) in the explicit Higgs picture.

Despite the fact that the above picture is capable of providing a correct phenomenological description¹²⁾ of all the observed "low-energy" physics, most of the fundamental questions still remain unanswered. Here are some of them¹⁵⁾:

- Why is charge quantized? (i.e. Why does $Q_{\text{proton}} + Q_{\text{electron}} = 0 (< 10^{-20})$?)
- What kind of unification is this if we have three independent coupling constants, g_3, g_2, g_1 [α_3 (or α_s), α_2, α_1 , where $\alpha_i \equiv g_i^2/4\pi$, $i = 1, 2, 3$] each corresponding to one of the "observed" gauge groups?
- What is the relationship between quarks and leptons (charges, masses, helicities, etc.)?
- Why are there three generations *identical* in representation content but so *vastly* different in mass?
- Why is the inter-generation mixing (i.e. Cabibbo angles) small?
- How many generations are there?
- Why do charged and neutral weak currents violate parity?
- etc.

All these unanswered questions clearly indicate that SU(3) × SU(2) × U(1) is not enough, and we must surely go beyond the "standard" model, as we do in the following sections.

II. GRAND UNIFIED THEORIES (GUTs)

Let me tell you about grand unification. Grand Unified Theories¹⁻⁴⁾ have a rich structure; thus it is hoped that we may be able to answer a lot of the above-mentioned questions. GUTs are different!

II.1 Grand Principles and Great Results

The basic hypothesis of the "grand synthesis" is that there exists a *simple*^{*} group $G \supset G_1$, which is characterized by a single coupling constant g , and that all interactions are generated by G . Quarks and leptons are, in general, members of the same multiplet(s) of the group G . Then, at some superhigh energy scale ($m_X \sim 10^{15}$), G suffers a breaking down to G_1 , and then around $m_W \sim 100$ GeV breaks down in the usual way [see (3)] to $G_2 \equiv SU(3)_c \times U(1)_{em}$:

$$\begin{aligned} G &\xrightarrow{m_X (\sim 10^{15} \text{ GeV})} G_1 [\equiv SU(3)_c \times SU(2) \times U(1)] \xrightarrow{m_W (\sim 10^2 \text{ GeV})} \\ &\longrightarrow G_2 [\equiv SU(3)_c \times U(1)_{em}] \end{aligned} \quad (4)$$

(m_X indicates also the mass of the gauge bosons mediating grand unified interactions). If such a scenario²⁻⁴⁾ is realized in nature, then a few things follow immediately.

1) Since G is a simple group, electric charge is quantized. Actually, the charge operator Q is a generator of the group G and is thus traceless; so when it acts on any representation of G containing both quarks and leptons, it is bound to give some relation between quark and lepton charges. So it is no longer a mystery why $Q_p + Q_{e^-} = 0$. GUTs even entail that the fine structure constant α be small¹⁶⁾ ($< 1/25$). Furthermore, simple relationships are expected to be found between the different coupling constants ($g_1, g_2, g_3, \dots$) since there is only *one free* gauge coupling constant g for all interactions. It should be stressed that the equality of all coupling constants is true of superhigh energies, where G is supposed to be an exact symmetry. As one comes down below m_X (the grand unification scale) to low energies Q , the couplings $g_1(Q), g_2(Q), g_3(Q)$ evolve differently, so that at "low energies" we observe coupling constants for different interactions which have vastly different magnitudes³⁾. For example, the $SU(2) \times U(1)$ mixing angle $\sin^2 \theta_W$ in simple GUTs is given by

$$\sin^2 \theta_W = \frac{\frac{3}{5} g_1^2(Q)}{g_2^2(Q) + \frac{3}{5} g_1^2(Q)}, \quad (5)$$

which is²⁾ $\frac{3}{8}$ in the symmetry limit at high energies where $g_2 = g_1$. At low energies, when $Q \approx m_W$, one finds³⁻⁵⁾ [see also Eq. (33)]

$$\sin^2 \theta_W (m_W) \approx 0.20 - 0.21, \quad (6)$$

to be compared with the experimental value¹⁴⁾

$$\sin^2 \theta_W (Q \ll m_W) \approx 0.230 \pm 0.015. \quad (7)$$

To make a direct comparison between (6) and (7) requires a complete computation of radiative corrections at low energies, which has not yet been done. However, the agreement between (6) and (7) for a number which *a priori* could have been anywhere between 0 and 1 is surely impressive.

^{*}The grand unified group is simple, either in the strict mathematical sense of having no invariant subgroups, or in the somewhat extended sense of being a direct product of isomorphic simple gauge groups together with a discrete group which connects these factors irreducibly. In any case, we have only *one free* gauge coupling constant.

2) Since quarks and leptons sit in the same representation(s) of the group G , it is natural to find correlations between their helicities and similar responses to weak interactions which are contained in the G interactions.

3) The fact that quarks and leptons share the same representation(s) of G has another important consequence; namely, there are relationships between quark and lepton masses. At superhigh energies where G is a good symmetry, quark and lepton masses are more or less equal (modulo *some possible* Clebsch-Gordan coefficients). At lower energies though, when G is badly broken, higher-order corrections due to the "standard" $SU(3) \times SU(2) \times U(1)$ interactions modify these "symmetry relations" in an appropriate way -- making quarks heavier than leptons, mainly because quarks have strong interactions and leptons do not. Quantitatively, one finds^{4,17)} that if $m_b \approx m_\tau$ at superhigh energies, as happens in many models, then [see also Eq. (36)]

$$\frac{m_b(Q=T)}{m_\tau} \approx \left[\frac{\alpha_s(Q=T)}{\alpha_s(Q=m_X)} \right]^{12/(33-2f)}, \quad (8)$$

where $\alpha_s(\equiv \alpha_3)$ is the strong coupling constant evaluated at the mass of the upilon (≈ 9.5 GeV), $m_X \approx 10^{15}$ GeV, and f is the number of flavours [in Section III, $\alpha_s(Q=m_X) \equiv \alpha_X$ for simplification]. Using experimental data to determine α_s , and taking $f=6$, we get^{4,17)} from Eq. (8):

$$\frac{m_b}{m_\tau} \approx 2.7 - 3, \quad (9)$$

which is the observed value ($m_\tau \approx 2m_b \approx 9.5$ GeV; $m_\tau \approx 1.8$ GeV).

4) A very important point to make here is that Eq. (8) is sensitive to the number of flavours (f). Two-loop corrections¹⁷⁾ to Eq. (8) determine uniquely^{4,17)} that $f > 6$ would imply an unaccepted m_b/m_τ value [an uncomfortable increase¹⁷⁾]. So GUTs not only explain why quarks are heavier than leptons, but also determine that *there are only three generations*^{4,17)}. This piece of news is most welcome to cosmologists, who have determined¹⁸⁾ independently that the observed "He abundance of the Universe requires that there be not more than three "light" neutrinos ($m_\nu \ll 1$ MeV), i.e. no more than three generations. [We remind the reader that in the simple GUTs considered here, the number of generations (1) = number of "light" neutrinos = $\frac{1}{2}$ (number of "ordinary" -- not superheavy -- quarks) = $\frac{1}{2}$ (number of "ordinary" flavours).]

This parallelism is meaningful only if GUTs imply $m_\nu \ll 1$ MeV for all their neutrinos. Recall that the present laboratory limits are

$$m_{\nu_e} \lesssim 35 \text{ eV}, \quad m_{\nu_\mu} \lesssim 600 \text{ MeV}, \quad m_{\nu_\tau} \lesssim 200 \text{ MeV}. \quad (10)$$

Most fashionable GUTs do indeed predict $m_\nu \ll 1$ MeV, with a common tendency¹⁹⁾ to find

$$\frac{m_\nu}{m_W} \approx \frac{m_W}{m_X} \rightarrow m_\nu \sim (10^{-5} \text{ to } 1) \text{ eV}, \quad (11)$$

the second hierarchy emerging naturally, given the (unnatural) first one (4). Neutrino masses in the lower end of the range (11) are best detected by astrophysical means, such as oscillations in cosmic-ray or solar neutrinos²⁰⁾ or time-delays in

pulses from supernova collapses. Neutrino masses ≥ 1 eV fall within the sensitivity of present laboratory experiments^{21)*}, as well as having possible astrophysical implications through their aggregation in galactic haloes and/or clusters. The astrophysical and cosmological constraints on such neutrinos are reviewed by others at this school.

5) Notice that the renormalization effects (6) and (9) are essentially independent of the choice of the grand unified group since they depend only on the "low-energy" interactions at energy scales below m_X . We assume the "desert hypothesis" that no new interactions arise between m_W and m_X . A detailed analysis of these effects is presented in Section II in the framework of the SU(5) model; but, of course, the analysis applies to any Grand Unified Theory under the above-mentioned assumptions.

6) Since GUTs put quarks and leptons into the same multiplet, and since gauge theories contain vector bosons linking all particles in a multiplet, there will in general be some interactions, changing quarks into leptons, which violate baryon number (B) and lepton number (L) conservation. This is by far the most dramatic and amazing consequence of GUTs, and most certainly B- and L-violation play a central role in the possible inter-connection between GUTs and cosmology. The B- and L-number violations are in accord with the gauge dogma: exact symmetries in nature are always "local" since, for instance, non-perturbative gravitational effects break any global (but not local) symmetry²²⁾.

In general, such B-violating forces will make the proton unstable, so we have to watch out that protons do not decay too quickly! The presently available experimental lower bound is⁷⁾

$$\tau_p \geq 0(10^{30} \text{ years}), \quad (12)$$

which is much stronger than the naive lower bound coming from the present age of the Universe ($\sim 10^{10}$ years) or from the fact that we personally are not "hot soup" ($\sim 10^{16}$). Since

$$\tau_p \propto \frac{m_X^4}{m_p^5}, \quad (13)$$

it was realized early on that the proton longevity, as expressed quantitatively through Eq. (12), betokens very massive ($m_X > 10^{14}$ years) leptoquark bosons X to violate baryon number conservation in a large class of models. Precise determinations^{3,4,6,23)} of m_X from other sources [see Eq. (37)] show that the actual value of m_X should lie in the range of^{6,23)} 10^{14} - 10^{15} GeV; thus, through Eq. (13), in a large class of grand unified models the proton decays with a lifetime^{6,23)} [see also Eq. (43)]

$$\tau_p \sim 10^{31 \pm 2} \text{ years}. \quad (14)$$

Such a value is close to the present experimental lower bound (12) and thus, if true, the disintegration of the proton is expected to be seen soon²⁴⁾.

Grand unification indeed occurs at superhigh (grand) energies. Since B-violating forces also play an essential role⁸⁾ in baryon number generation in the Universe,

*Lyubimov and co-workers²¹⁾ see a ν_e mass between 14 and 46 eV, while the ν_e/ν_μ induced event ratio in the 1979 CERN beam dump experiments was not unity.

we may conclude that interactions (or particles mediating these interactions) responsible for our existence are driving us to our extinction; or, more poetically, *if the proton was stable it would not exist.*

So far, we have seen that without using any particular grand unified model, we have been able to answer successfully many of the questions of Section I, relying just on the basic principles of grand unification. Of course, there are striking differences between individual grand unified models, but there are many good properties in common, such as the ones mentioned above, that are shared by almost all the models. Enough of generalities.

We now turn to studying, in detail, a specific GUT, the Georgi-Glashow SU(5) model²⁾, which is the first proposed model within our framework. It has all the merits necessary for being correct, and it certainly serves as an excellent prototype for a wide class of grand unified models. Let us see how things work in SU(5). (The reader who has had enough of our grand unification jazz may by-pass the next subsection and move to Section III.)

II.2 SU(5)

Let us see first how Georgi and Glashow got their SU(5) model²⁾. One looks for a "simple gauge group". The gauge group has to contain $SU(3)_C \times SU(2) \times U(1)$, i.e. it has to have at least rank 4. Let us concentrate for now on the groups of rank 4 that have just one coupling constant. There are nine of them: $[SU(2)]^4$, $[SO(5)]^2$, $[SU(3)]^2$, $[G_2]^2$, $SO(8)$, $SO(9)$, $S_p(8)$, F_4 , and $SU(5)$.

The first two are out because they do not contain an $SU(3)$ subgroup. The $SU(3)_C \times SU(2)$ representation content of a generation [Eq. (1)] is:

$$(3,2) + 2(\bar{3},1) + (1,2) + (1,1) . \quad (15)$$

Now notice that the representation (15) is complex and only $[SU(3)]^2$ and $SU(5)$ among all the nine groups admit complex representations. But the choice $[SU(3)]^2$ is impossible because leptons and quarks, having different colour properties, must be in different representations; the electromagnetic charge is a generator of the weak $SU(3)$ which does not allow fractional charges, and the sum of quark charge would have to be zero. Therefore the only possible rank 4 group is $SU(5)$.

Let us now see how a generation redistributes itself under $SU(5)$. We observe that

$$\begin{aligned} 5 &= (3,1) + (1,2) \\ 10 &= (\bar{3},1) + (1,1) + (3,2) , \end{aligned} \quad (16)$$

where 5 is the fundamental representation of $SU(5)$, 10 is the antisymmetric part of the product of two 5's ($5 \times 5 = 15 + 10$), and we have exposed their $SU(3)_C \times SU(2)$ representation content. But a $\bar{5} + 10$ (16) is identical to (15), i.e. has exactly the $SU(3)_{\text{colour}} \times SU(2)$ content that we are looking for! So we get [r, y, b are $SU(3)_C$ indices]:

$$\bar{5} : \begin{pmatrix} \bar{d}^r \\ \bar{d}^y \\ \bar{d}^b \\ e^- \\ \nu_e \end{pmatrix}_L ; \quad 10 : \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}^b & -\bar{u}^y & -u^r & -d^r \\ -\bar{u}^b & 0 & \bar{u}^r & -u^y & -d^y \\ \bar{u}^y & -\bar{u}^r & 0 & -\bar{u}^b & -d^b \\ u^r & u^y & u^b & 0 & -e^+ \\ d^r & d^y & d^b & e^+ & 0 \end{pmatrix}_L \quad (17)$$

Obviously for each generation we have a different $\bar{5} + 10$ representation. We make now the following comments.

1) There are no Adler-Bell-Jackiw anomalies. The anomalies of the irreducible representations (17) ($\bar{5} + 10$) cancel²⁵). Actually, SU(5) is the only group of *any rank* with a $(\bar{5} + 10) = 15$ anomaly-free representation with the correct $SU(3)_c \times SU(2) \times U(1)$ content! This is a very important statement because it means that if nature chooses the generation (1) as the basic fundamental unit, and if all new quarks and new leptons follow the same pattern (new generations of the same origin), then SU(5) is unique.

2) There is another gratifying observation: the electric charge operator Q has to be a generator of SU(5) (because of charge quantization) and thus Q is traceless ($\text{Tr } Q = 0$). This fact has the following consequences:

i) Take the 5, then $(\text{Tr } Q)_5 = 0$ implies

$$-3Q_d + Q_{e^-} = 0, \quad \text{i.e.} \quad \begin{aligned} Q_d &= \frac{1}{3} Q_{e^-} \\ Q_u &= \frac{2}{3} Q_{e^+} \end{aligned} \quad (18)$$

and in general, if the colour group was SU(n), then

$$Q_d = \frac{1}{n} Q_{e^-}, \quad Q_u = \frac{n-1}{n} Q_{e^+}. \quad (19)$$

Thus there is a remarkable relation (I have not used at all the Gell-Mann-Nishijima formula) between the charge of quarks and the number of colours.

ii) Since $(\text{Tr } Q)_{\text{quarks}} \neq 0$, and we need a traceless charge operator, it is a necessity to put quarks and leptons in the same irreducible representation. But then we definitely create new kinds of exotic interactions (leptoquark or diquark currents) which in general violate B-number and L-number conservation, and *they make the proton unstable!*

iii) The tracelessness of the charge operator is essential in the cancellation of anomalies, i.e. to keep the theory renormalizable. So in GUTs we find an interesting interrelation between charge quantization, anomaly-free theory (renormalizable), violation of baryon and lepton number (proton decay), and the fractional charge of the quarks.

3) We also note¹⁵) that since $(\nu_e e)_L$ and $\bar{d}_L(\nu d_R^*)$ are contained in the same multiplet [see Eq. (17)], only d_L is still available to have SU(2) weak interactions. Indeed the other multiplet (10_L) contains the $(u d)_L$ doublet, so it is no longer a mystery why $(u d)_L$ goes with $(\nu_e e)_L$ and not, say $(\nu_e e)_R$.

Let us now turn to the boson structure of the model. It is clear that at present energies the world does not seem to be described by the SU(5) gauge group. This means that SU(5) is badly broken at present, and will eventually become an exact symmetry at very, very high energies. We may envisage the breaking of SU(5) in two stages [see Eq. (4)]:

$$\begin{aligned} \text{SU}(5) &\rightarrow \text{SU}(3)_c \times \text{SU}(2) \times \text{U}(1) \rightarrow \text{SU}(3)_c \times \text{U}(1), \\ \Phi &\equiv \underline{24} \text{ (adjoint)} \quad \quad \quad H \equiv \underline{5} \text{ (vector)} \end{aligned} \quad (20)$$

where the first stage happens through a 24 of Higgs [transforming like the adjoint representation of SU(5)], and the second one through a 5 of Higgs [transforming like the fundamental representation of SU(5)]. At the first stage, 12 gauge bosons get masses, namely a colour triplet and weak isodoublet:

$$\begin{pmatrix} X \\ Y \end{pmatrix}_{r,b,y} \quad \text{with} \quad \begin{aligned} Q &= \frac{1}{3} \\ Q &= \frac{1}{3} \end{aligned} \quad (21)$$

together with their antiparticles, while the other 12, the familiar 8 gluons (g_i , $i = 1, \dots, 8$), 3 weak gauge bosons ($W^\pm, Z^0$), and the photon (γ), remain massless.

At the next stage the three weak gauge bosons get masses, so we are left (as we should be) with eight massless gluons plus a massless photon. As we saw, since the 12 "new" gauge bosons mediate bizarre interactions that may lead to proton decay (disintegration of the world!), we had better make them very heavy ($\sim 10^{15}$ GeV); which, in turn, means that the vacuum expectation value (v.e.v.) of the 24 has to be "astronomically" higher than the v.e.v. of the 5, which corresponds to the usual value of ~ 250 GeV. The $\underline{24}(\equiv \Phi)$ has a vacuum expectation value⁴⁾

$$\langle 0 | \Phi | 0 \rangle = O(10^{15} \text{ GeV}) \begin{pmatrix} 1 & & & & \\ & 1 & & & 0 \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ 0 & & & & -\frac{3}{2} \end{pmatrix} \quad (22)$$

and generates m_X and m_Y , while the $\underline{5}(\equiv H)$ has a vacuum expectation value⁴⁾

$$\langle 0 | H | 0 \rangle = O(10^2 \text{ GeV}) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (23)$$

[note that it is SU(4) symmetric] and generates m_f (fermion masses), m_W and m_Z . We also mention, in passing, that the colour triplet part of the $\underline{5}(\equiv H)$ of Higgs [see Eq. (16)], call it H_X , together with the surviving 12 Higgs bosons of the $\underline{24}(\equiv \Phi)$ of Higgs, become⁴⁾ superheavy with masses $O(m_X)$. Actually, since H_X is also capable of mediating proton decay, there is a lower bound on its mass²⁶⁾,

$$m_{H_X} \geq 10^{11} \text{ GeV}, \quad (24)$$

which is smaller than the corresponding one for m_X ($\geq 10^{14}$ GeV) since the Higgs-fermion-fermion coupling constant ($\sim m_f/m_W$) is smaller than the gauge coupling constant. The latest estimates⁶⁾ from low-energy phenomenology determine that:

$$m_X \sim 10^{14}-10^{15} \text{ GeV} ; \quad \frac{m_{HX}}{m_X} \approx 10^{0 \pm 1} . \quad (25)$$

How do the fermions get masses in SU(5)? By looking at the representations (17) it is obvious that the only way that fermions can get masses is through the $\bar{5} \times 10$ and 10×10 coupling. Since $\bar{5} \times 10 = 45 + 5$ and $10 \times 10 = \bar{45} + \bar{5} + 50$, the Higgs multiplet(s) should belong to the irreducible representations {5,45} of SU(5). Now how do we choose which is (are) the relevant multiplet(s)? First, by invoking the principle of economy and simplicity, we may choose the 5. Also notice that in any case we need a 5 of Higgs for the second stage of symmetry breaking; and if we have it, why do we not use it? [This corresponds to the successful use, see Eq. (2), of a doublet of Higgs in the SU(2) $\times$ U(1) model to break the symmetry down to the U(1), and at the same time to give masses to the three weak gauge bosons and to fermions.] Then putting everything together, we have the obvious choice of a 5 of Higgs to give masses to everybody (except, of course, the super-heavy gauge bosons, who get their masses through the 24 of Higgs).

In such a case it is easy to see the following things happening:

- 1) From the $(\bar{5})_f \times (5)_{\text{Higgs}} \times (10)_f$ coupling, only the q_c (generic name for any $Q = -1/3$ quark) and ℓ^- get masses, and actually they are equal²⁻⁴⁾ [see Eq. (23)]:

$$m_{q_c} = m_{\ell^-} . \quad (26)$$

Had we used the 45 of Higgs, then we would get⁵⁾

$$m_{q_c} = 1/3 m_{\ell^-} . \quad (27)$$

- 2) From the $(10)_f \times (5)_{\text{Higgs}} \times (10)_f$ coupling, only the q_A (generic name for any $Q = 2/3$ quark) get masses. This is rather unfortunate because we cannot get relations between q_A and q_c (or ℓ^-) masses. So Eq. (26) remains intact and the masses of $Q = 2/3$ quarks remain arbitrary parameters together with various Cabibbo angles.

It should be stressed that the above mass relations, as well as the equality of all coupling constants, are true at superhigh energies, where SU(5) is supposed to be an exact symmetry. Then one may ask, What is the connection between measurable (at present energies) quantities, such as coupling constants and quark masses, and the corresponding quantities at superhigh energies? One even may worry about the whole approach, since the strong (QCD) and weak/e.m. coupling constants are very different:

$$\alpha_s(Q^2 = 10 \text{ GeV}^2) \approx 0.2-0.3$$

$$\alpha = \frac{1}{137} . \quad (28)$$

Also at present energies the $Q = -1/3$ quarks (q_c) are much heavier than charged leptons (ℓ^-). Fortunately there is a simple answer to all these problems. The renormalization group assures us that coupling constants²⁷⁾ and quark masses²⁸⁾

vary with the q^2 at which they are probed, and asymptotic freedom suggests that the strong interaction coupling^{3,4)} decreases as q^2 increases; simultaneously, a U(1) coupling increases slightly with q^2 . The qualitative behaviour of the couplings is shown in Fig. 1. The same is true for the masses. So maybe the dissimilar

Fig. 1. Qualitative picture of the evolution of the SU(3), SU(2), and U(1) couplings in a grand unified theory such as SU(5).

values (28) can be brought together at some sufficiently high q^2 that would correspond to the m_X^2 of superheavy gauge bosons X that unify the strong, weak, and electromagnetic interactions. To get from the grand unified mass (GUM) m_X to present energies μ , it is necessary to know how the SU(3), SU(2), and U(1) coupling constants (g_3, g_2, g_1) vary. Assuming always $g_i \ll 1$, we get^{3,4)}

$$\frac{1}{4\pi \alpha_{\text{GUM}}} \equiv \frac{1}{g_{\text{GUM}}^2} = \frac{1}{g_i^2(\mu)} + 2\beta_i \ln \frac{\mu}{m_X}, \quad (i = 1, 2, 3), \quad (29)$$

where the β_i are defined by

$$\mu \frac{\partial}{\partial \mu} g_i(\mu) \simeq \beta_i g_i^3(\mu) + O(g_i^5), \quad (30)$$

and if we neglect Higgs contributions and finite mass effects, then

$$\beta_1 = \frac{f}{24\pi^2}, \quad \beta_2 = \frac{-1}{16\pi^2} \left[\frac{22}{3} - \frac{2}{3} f \right], \quad \beta_3 = \frac{-1}{16\pi^2} \left[11 - \frac{2}{3} f \right], \quad (31)$$

where f is the number of quark (lepton) flavours [$f = 6$ is favourable in this model^{4,17)}]. It is then easy to show that^{3,4)}

$$\ln \left(\frac{m_X}{\mu} \right) = \frac{\pi}{11\alpha} \left[1 - \frac{\alpha(\mu)}{\alpha_s(\mu)} \cdot \frac{8}{3} \right] \quad (32)$$

and

$$\sin^2 \theta_W(\mu) = \frac{1}{6} + \frac{5}{9} \frac{\alpha(\mu)}{\alpha_s(\mu)}. \quad (33)$$

(We use the notation $\alpha_i \equiv g_i^2/4\pi$, $i = 1, 2, 3$ and $\alpha_3 = \alpha_s$.) At high energies, where SU(5) is exact, it is simple group theory that tells us²⁾ [see Eq. (5)] that

$$\sin^2 \theta_W = \frac{3}{8}. \quad (34)$$

Not only can we estimate coupling constant renormalization^{3,4)} but also mass renormalization^{4,17)}, and using the anomalous dimension²⁸⁾ of the fermion mass operator

$$\gamma_m = \frac{-g_3^2}{2\pi^2} + \frac{3g_2^2}{64\pi^2} - \frac{3}{8\pi^2} g_1^2, \quad (35)$$

we find^{4,17)}

$$\ln \left[\frac{m_q(\mu)}{m_\ell(\mu)} \right] = \ln \left[\frac{m_q(m_X)}{m_\ell(m_X)} \right] + \frac{4}{11 - \frac{2}{3}f} \ln \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right] + \frac{3}{2f} \ln \left[\frac{\alpha_1(\mu)}{\alpha_{\text{GUM}}} \right]. \quad (36)$$

So we have to determine m_X , $\sin^2 \theta_W(\mu)$ and $m_q(\mu)$ by using Eqs. (32), (33), and (36). As an input we need $\alpha_s(\mu)$, $\alpha(\mu)$, m_μ , m_τ , and $m_{q_c}(m_X)/m_{\ell-}(m_X)$, which we take to be equal to one, as indicated by Eq. (26). We must be very careful about several details^{6,23)} of the calculation, such as threshold effects²³⁾ at $Q \approx m_X$ and $Q \approx m_W$, variation²³⁾ of α between $Q^2 \approx 0$ ($\alpha \approx 1/137$ from the Josephson effect) to $Q^2 \approx 10^4 \text{ GeV}^2$ ($\alpha \approx 1/128$), etc. For the most recent thorough analysis of all these effects and precise determination of m_X and all other desirable quantities, the reader should consult Ref. 6.

We would like to stress once more that the above detailed analysis of the renormalization effects in SU(5) applies to any grand unified theory which has no new interactions between m_W and m_X .

We find^{4,6,17)}

$$\left. \begin{array}{l} \alpha_s(10^4 \text{ GeV}^2) \approx 0.11-0.15 \\ \alpha(10^4 \text{ GeV}^2) \approx \frac{1}{128} \\ m_\mu = 0.105 \text{ GeV} \\ m_\tau \approx 1.8 \text{ GeV} \\ m_{q_c}(m_X) = m_{\ell-}(m_X) \\ f = 6. \end{array} \right\} \begin{array}{l} \alpha_X \approx \alpha_{\text{GUM}} \approx 0.024 \approx \frac{1}{42} \\ m_X = (6 \pm 3) \times 10^{14} \text{ GeV} \\ \sin^2 \theta_W \approx 0.20-0.21 \\ m_s = (0.38-0.50) \text{ GeV} \\ m_b \approx (4.8-5.4) \text{ GeV} \end{array} \quad (37)$$

The results (37) are certainly impressive, since the predicted values for $\sin^2 \theta_W$ and for the quark masses are very close to their experimental values* [see Eqs. (7)]

*The approximate successes of the b and s quark mass estimates constitute the only evidence in favour of the conventional "generation" assignments such as (17), the so-called "mass connection"^{5,29)}.

and (9)]. However, before opening the champagne, a few words of caution concerning the "light" quark masses (m_d, m_s). Even though the absolute value of m_d at low Q^2 cannot be calculated (strong interactions become uncontrollable) and the qualitative smallness of m_d is correct, we do have the renormalization group-invariant prediction, from Eq. (26),

$$\frac{m_d}{m_s} = \frac{m_\ell}{m_\mu} \approx \frac{1}{200}, \quad (38)$$

which conflicts with current algebra/chiral symmetry estimates that yield

$$\frac{m_d}{m_s} \approx O\left(\frac{1}{20}\right) \text{ not } \frac{1}{200}. \quad (39)$$

Their failure may either reflect³⁰⁾ some small "slop" in the quark mass matrix which is of the order of a few MeV and does not respect SU(5) symmetry, or it may reflect the existence of a more complicated³¹⁾ Higgs structure* in SU(5), i.e. using a combination of 5 and 45 [see Eqs. (26) and (27)], or it may even indicate that SU(5) should be abandoned in favour of some higher group.

Having determined [Eq. (37)] the value of m_X , we are now ready to calculate⁶⁾ the proton lifetime. In SU(5), the X and Y bosons acquire essentially identical masses⁴⁾ [$m_X - m_Y \approx O(m_W)$] which yield an effective four-fermion interaction of strength⁴⁾

$$\frac{G_{GU}}{\sqrt{2}} = \frac{g^2}{8m_X^2} \quad (40)$$

which takes the form^{4,6,29)}

$$\frac{1}{4} \mathcal{L}_{GU} = \frac{1}{\sqrt{2}} G_{GU} \left[\left(\epsilon_{ijk} \bar{u}_k^c \gamma_\mu u_{jL} \right) \left(2\bar{e}_L^+ \gamma^\mu d_{iL} + \bar{e}_R^+ \gamma^\mu d_{iR} \right) \right. \\ \left. + \left(\epsilon_{ijk} \bar{u}_k^c \gamma_\mu d_{jL} \right) \left(\bar{\nu}_e^c \gamma^\mu d_{iR} \right) + \text{H.C.} \right]. \quad (41)$$

Then the "classic"⁴⁾ method of estimating the proton decay rate is to argue that the $q + q \rightarrow \bar{\ell} + \bar{q}$ annihilation diagrams of Fig. 2 dominate, and evaluate these

Fig. 2. The graphs dominating (Ref. 4) proton decay in simple grand unified models

*We will see later that such a complication may also be needed to get an appreciable net baryon number density.

using non-relativistic SU(6) for the initial-state wave function and a calculable non-leptonic short-distance enhancement factor, and make the "parton model" assumption that the sum over final-state mesons approximates an inclusive annihilation cross-section. All these are analysed in great detail in Refs. 4 and 6. We then find that⁶⁾

$$\tau_{p, \text{bound } n} \sim (0.6 \text{ to } 25) \times 10^{30} \left(\frac{m_X}{5 \times 10^{14} \text{ GeV}} \right)^4 \text{ years} , \quad (42)$$

and thus we finally conclude⁶⁾ [with the use of formulae (37) and (42)] that

$$\tau_p = 8 \times 10^{30 \pm 2} \text{ years} \quad (43)$$

in the simple SU(5) model with three generations and no significant new physics before the grand unification point. Experimentalists²⁴⁾ will be cheered by the estimate (43) in the simple SU(5) model²⁻⁴⁾. However, we must bear in mind that the above estimate is subject to all kinds of uncertainties⁶⁾, which have (un)fortunately a tendency to increase τ_p . So if experimentalists²⁴⁾ do not find decaying protons with the "canonical" value of τ_p , they will at least have the first experimental evidence for something more exciting than the simple SU(5) model²⁻⁴⁾.

Attempts³²⁾ have also been made to estimate branching ratios into different final states. Several methods³²⁾ [SU(6), bag model, ...] have been followed, with predictions that are qualitatively similar but vary quantitatively. Nevertheless, the consensus is³²⁾ that the decays $p \rightarrow e^+\pi^0$ and $n \rightarrow e^+\pi^-$ have non-negligible branching ratios:

$$B(p \rightarrow e^+\pi^0) \approx \frac{1}{5} - \frac{1}{3} \quad \text{and} \quad B(n \rightarrow e^+\pi^-) \sim \frac{1}{3} - \frac{1}{2} , \quad (44)$$

something which is especially welcomed by experimentalists, because then the signal for proton decay will be rather clean.

Theorists should also feel happy about the big numbers of (44) for the following reasons. Despite the fact that B and L are violated in GUTs, a linear combination of them, namely B-L, seems to be *exactly* conserved in the simple SU(5) model, and violated *in higher orders* in other³³⁾ simple GUTs such as O(10) or E₆, etc. Basically what happens is³⁴⁾ that SU(3) × SU(2) × U(1) invariance imposes this B-L conservation to leading order, which then implies a strong selection rule³⁴⁾, $\Delta B = \Delta L$, which means that³⁴⁾

$$B \rightarrow \ell^+ X ; \quad B \not\rightarrow \ell^- X . \quad (45)$$

Thus if the scale at which $G \rightarrow \text{SU}(3)_C \times \text{SU}(2) \times \text{U}(1)$ is $O(m_X)$, then one expects³⁴⁾, for example,

$$\frac{p \rightarrow e^-\pi^+\pi^+}{p \rightarrow e^+\pi^0} \quad \text{or} \quad \frac{n \rightarrow e^-\pi^+}{n \rightarrow e^+\pi^-} \sim \left(\frac{m_W}{m_X} \right)^2 \sim 10^{-24} . \quad (46)$$

So if the whole picture developed up to now makes sense, it is better that the experimentalists²⁴⁾ should not (almost ever!) see p (or n) $\rightarrow e^- X$. The experimental observation of such a "peculiar" decay will signal³⁴⁾ the existence of a new intermediate scale (M) between m_W and m_X , such that $(m_W/M)^2 \gg 10^{-24}$. But then all hell will break loose, since a lot of the subsequent cosmological implications of GUTs will not hold water³⁵⁾. The existence or not of some linear combination of B and L (such as B-L) which survives as a global symmetry has important consequences³⁵⁾ in cosmology (baryon number generation, nucleosynthesis, etc., see Section III).

There will be much to be learned from the experimental study of the different nucleon decay modes. There is another interesting consequence of the exact B-L conservation in SU(5). The neutrinos are strictly massless. This happens because formula (17) implies that the neutrino has no right-handed components, i.e. no Dirac mass term ($\bar{\nu}_R \nu_L$) is allowed, and exact B-L conservation prohibits the existence of any Majorana mass term ($\nu_L \nu_L$) since it has $\Delta(B-L) = \Delta L = 2 \neq 0$. Of course one may invoke³⁶⁾ post-SU(5) interactions and/or other SU(5) fermion representations (1, 24, ...), so that the exact B-L conservation ceases to be true and thus the neutrinos may acquire masses³⁶⁾. But, it is safe to say that the experimental discovery of $m_\nu \neq 0$ will directly rule out the SU(5) model in its simplest, minimal and originally proposed form²⁾. In other GUTs³³⁾, such as O(10), E_6 , etc., since there is no exact B-L conservation the neutrinos naturally acquire¹⁹⁾ a tiny but non-zero mass as discussed before [see Eq. (11)]. The discovery of a neutrino mass in the range (10^{-5} to a few) eV [see Eq. (11)] will be the "green light" for some GUTs more complicated than the simplest SU(5) model.

Actually there are other characteristic differences* between SU(5) and O(10), E_6 . Here are some of them³³⁾: i) O(10) and E_6 are automatically anomaly-free, despite the fact that they have complex representations [this is a unique property of all $O(4n+2)$ $n \geq 2$, orthogonal groups, and E_6], while in SU(5) there is a kind of accidental cancellation between the anomalies of the $\bar{5}$ and the 10. ii) O(10) and E_6 are intrinsically L-R symmetric, which means that the observed parity violation in weak interactions is a low-energy phenomenon (i.e. spontaneous) that will disappear at superhigh energies [perhaps a necessary condition if we want to unify with gravity, which respects L-R symmetry (?)], while SU(5) is intrinsically L-R asymmetric i.e. parity violation survives also at superhigh energies. iii) Finally in O(10) and E_6 each generation belongs to an *irreducible* representation [16 of O(10) or 27 of E_6], while in SU(5) each generation belongs to a *reducible representation* [$\bar{5} + 10$, see Eq. (17)].

Having established the credentials of GUTs for our cosmological friends, and taking on trust their "standard" Big-Bang model**, we are fully equipped to discuss the amalgamation of grand unification with cosmology and its striking consequences.

III. COSMOLOGICAL ECHOES OF GRAND UNIFICATION***

III.1 The Grand Connection

Do cosmology and grand unified theories of elementary particle interactions have anything useful to say to each other? The numerous successes of GUTs make many theorists perceive that strong and electroweak interactions can be understood in principle. Nevertheless, GUTs invoke energy scales of $O(10^{15})$ GeV [see Eqs. (25) and (37)] which seem vertiginous to many physicists. It is not imaginable to reach these energies in laboratory experiments, and experimentalists are therefore forced to look for very indirect and feeble side-effects of grand unification, such as proton decay. Even these valiant efforts may be brought to nought by a (logarithmically) modest increase in the grand unification mass-scale⁶⁾ [see Eqs. (32) and (42)]. However, this mass-scale may be achieved directly in cosmological and astrophysical situations. For example, black hole explosions could in principle

*It should be stressed that the phenomenological successes of SU(5) [see Eq. (37)] are shared by O(10) and E_6 .

**The "standard" Big-Bang model has been covered extensively by other lecturers at this school. Familiarity with the B-B model at the level of Ref. 9 will be assumed in the subsequent discussions.

***Large parts of this section are also contained in Ref. 11.

achieve temperatures up to the Planck temperature of 10^{32} K, corresponding to energies of 10^{19} GeV, while temperatures corresponding to particle energies of 10^{15} GeV or more are generally thought to have occurred very early in the Big-Bang when the Universe was about 10^{-37} s old. It may indeed turn out that the Big-Bang has been the only place to do direct experiments on GUTs (albeit rather uncontrolled, at least by us). In this section we review progress on some interfaces between cosmology and grand unification, asking whether cosmology tells us important things about GUTs, and vice versa.

TABLE 2 Important Problems

Grand Unification	Cosmology
Fermion spectroscopy	Baryon to photon ratio, and lepton numbers
Grand unification mass $m_X = ?$ and coupling constant $\alpha_{GUT} = ?$	Homogeneity and isotropy
SU(5) or a more complicated GUT?	Galaxy formation
Mechanism of symmetry breaking	Simultaneity of expansion of causally separated parts of the Universe
Connection with gravitation?	How many black holes and/or monopoles in the Universe?

Shown in Table 2 are a few of the important problems in grand unification and cosmology: neither listing is exhaustive. The problems concerning grand unification were discussed in detail in Section II, and before turning to the cosmological problems let me stress once more that the biggest open problem in gauge theories is perhaps the mechanism of spontaneous symmetry breaking -- Is it realized through explicit Higgs fields? or through dynamical symmetry breaking? or a combination of the two? Eventually one must of course include gravitation in a truly unified theory of all the fundamental interactions, although present attempts at grand unification generally postpone this problem *sine die*.

One of the most striking aspects of the Universe is that we are here. The Universe is not a strictly neutral soup of photons, neutrinos, and antineutrinos, but also contains a small seasoning of protons, neutrons, and electrons, out of which the stars and planets and people are formed. Thus, one of the outstanding problems in cosmology has been to understand why there are just a few baryons in the Universe: $n_B/n_\gamma = 10^{-9 \pm 1}$; and why there seem to be no antibaryons³⁷⁾, at least on the scale of our local galactic cluster. Related unknowns are the densities (chemical potentials) of other quantities not yet known to be violated, such as electric charge and the various lepton numbers (L_e , L_μ , L_τ). Another fundamental problem is to understand why the Universe is apparently so homogeneous and isotropic, for example in the distribution of luminous matter within it. The reader is actually living (?) evidence that some inhomogeneities exist, of which the most notable are galaxies, whose formation is not yet fully understood. But the Universe looks quite regular on a large scale, and a particularly striking and baffling aspect of this homogeneity and isotropy of the Universe is the apparent necessity, within the standard Big-Bang model, for causally separated parts of the Universe to have chosen to behave in the same way at the same time. Presumably it was not coincidence, and the causality arguments must somehow be thwarted by some deviation from our "standard"

ideas on particle interactions or on the evolutionary history of the Universe. Finally, on a more mundane level, our unified theories of the fundamental interactions predict the existence of massive classical solutions³⁸⁾, grand unified monopoles, whose abundance is unknown³⁹⁾ but must be strongly suppressed in order to avoid a conflict with the standard Big-Bang model. This might prove to be a serious embarrassment for simple grand unified models, although we ourselves doubt it.

Here we will not consider, let alone answer, all the above problems, but will address ourselves to three topics where the cosmology-GUT interface seems particularly active. The first topic is baryon number generation in GUTs⁸⁾. These provide a qualitative framework for generating a net baryon excess through C-, CP-, and B-violating interactions dropping out of thermal equilibrium very early in the Big-Bang. When one tries to put this scenario on a quantitative basis, one discovers⁴⁰⁻⁴²⁾ that the grand unification mass should not be much lower than 10^{15} GeV, or α_{GUT} much larger than $1/40$, in agreement with GUTs findings [Eq. (37)], and that a theory⁴³⁾ larger than the minimal SU(5) GUT with just two Higgs representations (5 and 24) is probably required, in accordance with proposals³¹⁾ advanced for curing Eq. (39). The second topic is that of dissipative processes, very early in the evolution of the Universe, due to the relatively long mean free times of elementary particles possibly providing forms of viscosity and thermal conductivity. These suppress¹⁰⁾ some forms of inhomogeneity, but permit the growth of long wavelength compressional perturbations which have been suggested⁴⁴⁾ as the origin of galaxy formation. The third topic is the cosmological density of grand unified monopoles³⁹⁾. It has been suggested⁴⁵⁾ that this may be brought acceptably low by postulating a first-order phase transition with large super-cooling, or that one may have to postulate⁴⁶⁾ a GUT larger than SU(5), or that symmetry breaking may have to be dynamical⁴⁷⁾. In fact, none of these may be necessary⁴⁸⁾.

It will be seen from the above synopsis that on many issues cosmology and grand unification do indeed have things to say to each other, although the utility of some of the connections is not yet clear.

III.2 The Evolution of the Very Early Universe

The Universe now contains many photons, principally in the 3 K microwave background radiation, and relatively few baryons by comparison:

$$n_B/n_\gamma = 10^{-9 \pm 1} . \quad (47)$$

Strictly speaking, one should compare the baryon number density (n_B) of the Universe with its entropy density (s). Most of the entropy of the present Universe is contained in the microwave radiation background and in the associated neutrino and antineutrino backgrounds. With a microwave background temperature of 3 K, the photon entropy density is $s_\gamma \approx 250/\text{cm}^3$ (for us $k_{\text{Boltzmann}} = 1$ always). The entropy density of each species of neutrino and antineutrino is calculated to be⁹⁾ $(7/22)s_\gamma$. The quantity s_γ is very roughly equal to n_γ , but it is better to deal with entropy than photon number densities, because as long as the expansion of the Universe is adiabatic, the total entropy in any co-moving volume remains fixed, while processes in the early Universe, such as e^+e^- annihilation, can affect the number of photons. In the following we will use either n_γ or s_γ (or even s), but the reader should keep in mind the above remarks.

The baryon number density (n_B) is not so well known, but limits on the deceleration of the cosmic expansion restrict⁹⁾ n_B to be less than about $3 \times 10^{-6}/\text{cm}^3$, while the mass density of visible galaxies provides a lower bound on n_B (assuming that there is not much antimatter in the present Universe, as we argue below) of about

$3 \times 10^{-8}/\text{cm}^3$. The ratio n_B/s_γ (or n_B/n_γ) is thus in the range 10^{-10} to 10^{-8} as indicated in Eq. (47). This ratio remains constant throughout the history of the Universe, as long as the baryon number is conserved and the Universe expands adiabatically. Furthermore, there are no known concentrations of antimatter, and the absence of high-energy photons, which could be the products of $B\bar{B}$ annihilation, indicates that our local cluster of galaxies does not contain any significant fraction of antimatter³⁷⁾. The cosmic-ray antiprotons recently observed⁴⁹⁾ do not constitute evidence for cosmological antimatter, as their flux is consistent with that expected from secondary $\bar{p}$ production by primary nucleons. Before the advent of GUTs there was no satisfactory explanation of the ratio (47) or of the apparent matter-antimatter asymmetry. Some held that the Universe is in fact symmetric and contains equal amounts of matter and antimatter, but failed to explain satisfactorily how their concentrations could have got separated in the present Universe⁵⁰⁾. Others hypothesized that the Universe had in fact started out with only baryons, and the large number of photons in Eq. (47) was subsequently generated by dissipative processes. Yet others just assumed that the Universe had always been slightly asymmetric. It now seems possible to believe that the matter-antimatter asymmetry and the small number (47) can be naturally explained⁸⁾ in GUTs. Three crucial ingredients are necessary in order to realize this trick. One is clearly that one has a theory with baryon number B -violating interactions. Another is that some component of these interactions violates both C and CP , as making either of these transformations reverses the sign of the net baryon number which must therefore be zero if the Universe evolves from an initially symmetric state. Finally, the B -, C -, and CP -violating interactions must get out of thermal equilibrium, as in such a state the arrow of time is lost and CPT invariance guarantees that the net baryon number is zero^{51,40,42)}. We will now see how these three conditions are fulfilled with the expansion and cooling of the Universe introducing the necessary departure from thermal equilibrium while GUTs provide the necessary B -, C -, and CP -violation.

We will be interested in epochs of the Universe's evolution when it was very hot, with a temperature of (10^{32} to 10^{23}) K, equivalent to a particle energy $O(10^{19}$ to $10^{10})$ GeV. The upper limit is given by the Planck temperature T_P corresponding to a mass m_P related to Newton's gravitational constant by $m_P \equiv (G)^{-1/2} \approx 1.2 \times 10^{19}$ GeV. The quantum gravitational interactions between particles at or above T_P or m_P are $O(1)$, conventional classical approximations break down, and we cannot even speculate sensibly any more. Below 10^{19} GeV, quantum gravitational effects presumably decrease as a power of the temperature T , and we can use classical solutions of the Einstein equations, typically Robertson-Walker-Friedman (RWF) metrics, to describe the large-scale structure of gravitational effects. The expansion of the Universe in the presence of curvature k is then described⁹⁾ by

$$\frac{\ddot{R}}{R} = -4\pi G \left(\rho + \frac{p}{3} \right) \quad (48a)$$

$$\left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{R^2}, \quad (48b)$$

where R is the RWF scale parameter and ρ and p are the energy density and pressure of matter in the Universe. At very high temperatures, when all particles are ultrarelativistic, we have $p = \frac{1}{3}\rho$ and

$$\rho = g_T B T^4, \quad (49)$$

where g_T is the effective number of particle helicity states (1 for a scalar, 2 for a massless vector, 7/4 for a two-component spinor, etc.) and B is a constant taking the value

$$B = \frac{\pi^2}{30} \quad (50)$$

if the particles are in kinetic equilibrium. Solving the evolution equations (48) with the particle energy density (49) yields an age-temperature relation for the Universe:

$$\frac{t(T)}{t_P} = \left(\frac{3}{32\pi g_T B} \right)^{1/2} \left(\frac{T_P}{T} \right)^2, \quad (51)$$

where t_P is the Planck time $O(10^{-42})$ s related to the Planck energy of 10^{19} GeV by the usual uncertainty relation.

The relativistic particles in the hot but cooling and expanding Universe have interactions of which the most important are $(2 \leftrightarrow 2)$ scattering processes (Fig. 3a) and $(1 \leftrightarrow 2)$ decays and inverse decays (Fig. 3b). Decays and inverse decays of a

Fig. 3 Examples of important fundamental interactions: a) $(2 \leftrightarrow 2)$ scattering; b) $(1 \leftrightarrow 2)$ decays and inverse decays.

massive particle X will be characterized by a time-dilated version of the free decay time:

$$\tau \propto \frac{(T^2 + m_X^2)^{1/2}}{\alpha_X m_X^2} \quad (52)$$

which is very long in the limit that $T \gg$ the mass m_X . [For α_X , see discussion after Eq. (8).] In fact, many or all of the particles in GUTs acquire their masses from spontaneous symmetry breaking, so that they are functions of temperature and are zero above some critical temperature T_c which is generally $\gtrsim m_X$. This entails a further lengthening of τ compared with formula (52). At low temperatures, $T \ll m_X$, the density of massive particles is suppressed by a Boltzmann factor $\exp(-m_X/T)$, and there is a similar reduction in the probability for two light particles to come together to make a massive particle. This means that a true measure of the mean free time $\tau_{1 \leftrightarrow 2}$ between $1 \leftrightarrow 2$ interactions is related to τ in (52) by

$$\tau_{1 \leftrightarrow 2} \approx \tau \times \exp(-m_X/T). \quad (53)$$

Dimensional arguments would suggest that in the ultrarelativistic limit, $2 \leftrightarrow 2$ scattering in a GUT with a dimensionless coupling constant α_{GUM} [see Eqs. (29) and (40)] would behave as

$$\sigma_{2 \leftrightarrow 2} \approx \frac{\alpha_{\text{GUM}}^2}{T^2} \quad (54)$$

possibly modified by logarithmic factors, and for processes involving vector boson exchange the best guess [see Eq. (37)] is that $\alpha_{\text{GUM}} \sim 1/40$. In fact, if the scattering particles were free, one would have from the t-channel exchange of a massive boson X,

$$\sigma_{2 \leftrightarrow 2} \approx \frac{\alpha_{\text{GUM}}^2}{m_X^2} \quad (55)$$

at high energies; but, in the primordial plasma we are considering, there is a reduction⁵²⁾ of σ due to screening effects. These reduce the cross-section to at most $\alpha_{\text{GUM}}^2 \lambda_D^2$, where the Debye length⁵²⁾

$$\lambda_D \approx \left(\frac{T}{32\pi^2 \alpha_{\text{GUM}} n} \right)^{1/2}, \quad (56)$$

and n is some relevant number density:

$$n = g_T A T^3 \quad (57)$$

with $A \approx 0.12$ for a Boltzmann distribution. If (?) the relevant n corresponds to $g_T = 1$, then for $\alpha_{\text{GUM}} \approx 1/40$, $\lambda_D \approx 1/T$, and the dimensional cross-section formula (54) is applicable. If we consider $(2 \leftrightarrow 2)$ scattering (Fig. 3a) via a boson whose mass m_X is comparable to the temperature T , then the cross-section (52) is suppressed by propagator effects

$$\sigma_{2 \leftrightarrow 2} \approx \frac{\alpha_{\text{GUM}}^2 T^2}{(T^2 + m_X^2)^2}, \quad (58)$$

which means that the scattering is greatly suppressed at temperatures $T \ll m_X$. From the cross-section (54) one can deduce a mean free time $\tau_{2 \leftrightarrow 2}$ between collisions:

$$\tau_{2 \leftrightarrow 2} = \frac{1}{n \sigma(2 \leftrightarrow 2)}, \quad (59)$$

and we see from Eqs. (54) and (57) that

$$\tau_{2 \leftrightarrow 2} \propto \frac{1}{\alpha_{\text{GUM}}^2} \frac{1}{T} \quad (60a)$$

at very high temperatures. At very low temperatures, $T \ll m_X$, Eqs. (57) and (58) imply that for $(2 \leftrightarrow 2)$ scattering through massive particle exchange, Eq. (60a) is modified to become

$$\tau_{2 \leftrightarrow 2} \propto \frac{1}{\alpha_{\text{GUM}}^2} \frac{m_X^4}{T^5}. \quad (60b)$$

In the neighbourhood of $T \sim m_X$ neither formula (60a) nor (60b) is appropriate. If the X exchange takes place in a crossed channel, there will be a simple interpolating form for $\tau_{2 \leftrightarrow 2}$. However, if the X exchange takes place in the *direct* channel, there is a possibility of enhancing $\sigma_{2 \leftrightarrow 2}$ and reducing $\tau_{2 \leftrightarrow 2}$ correspondingly, because of scattering through X bosons close to their mass shell⁵²⁾. It is convenient to explicitly remove these pole contributions from $\sigma_{2 \leftrightarrow 2}$ and lump them together with the $(1 \leftrightarrow 2)$ decays and inverse decays to which they are directly related by unitarity, leaving behind a reduced $(2 \leftrightarrow 2)$ cross-section which leads to a smooth interpolation of $\tau_{2 \leftrightarrow 2}$ between formulas (60a) and (60b).

We can now approximately determine the epochs during which various interactions are in equilibrium, by looking at the ratio between the expansion time t [Eq. (51)] and the various different mean free times τ . If $t/\tau < 1$, the corresponding interaction is out of equilibrium; whereas if $t/\tau > 1$ then the interaction is in equilibrium. We have several different cases to consider: the results are illustrated in Fig. 4. Hereafter unless explicitly stated to the contrary, we will use "natural" units in which Planck units (m_P , T_P , t_P , ...) are unity.

Fig. 4 The ratios of the universal expansion time (51) to the mean free times for different interactions⁵³⁾ discussed in the text. Note the possibility that no known interactions were in equilibrium at $10^{19} \text{ GeV} > T > 10^{15} \text{ GeV}$, the probable period of equilibrium for baryon number violating interactions at $T \sim 10^{15} \text{ GeV}$, and their subsequent non-equilibrium period.

Quantum gravitational effects. If we assume that at $T < T_p = 1$, $\sigma_{\text{grav}} \leq G$ [and $\sigma_{\text{grav}} \approx G^2 T^2$ seems likely to be a realistic estimate for $(2 \leftrightarrow 2)$ scattering], then

$$\tau_{\text{grav}} \geq \frac{1}{GT^3} \quad (\text{for example } 1/G^2 T^5), \quad (61)$$

whereas $t \sim T^{-2}$ so that

$$\left. \frac{t}{\tau} \right|_{\text{grav}} \lesssim T, \quad (62)$$

and quantum gravitational interactions are expected to be relatively unimportant and out of equilibrium for $T < 10^{19}$ GeV.

Strong, weak, and electromagnetic interactions. Since the masses of particles mediating these interactions are negligible on the energy scales we are considering, the cross-section (54) should be applicable, and we expect

$$(t/\tau)_{s,w,em} \approx \frac{\alpha_{\text{GUT}}^2}{T}. \quad (63)$$

This ratio is $O(10^{-4})$ at $T \sim 10^{19}$ GeV, meaning that these interactions should be out of thermal equilibrium at that temperature, a state of affairs which should persist until $T \sim \alpha^2 \sim 10^{15}$ GeV, after which strong, weak, and electromagnetic interactions should be in equilibrium. This equilibrium state should be maintained until much lower temperatures, below those at which the weak interaction symmetry is spontaneously broken and the colour interactions are shielded by long-distance non-perturbative effects.

Grand unification interactions. By these we mean the interactions of (or involving exchanges of) the superheavy X bosons of a GUT. As far as the $(2 \leftrightarrow 2)$ interactions are concerned, we expect at $T \gg m_X$ a behaviour like that of the strong, weak, and electromagnetic interactions (63), while at temperatures $T \ll m_X$ we expect from Eq. (60b) that

$$\left(\frac{t}{\tau} \right)_{2 \leftrightarrow 2 \text{ GUT}} \approx \frac{\alpha_X^2 T^3}{m_X^4} \quad (64)$$

with a smooth interpolation between Eqs. (63) and (64) at $T \sim m_X$. This function is therefore $O(1)$ at $T \sim m_X \sim \alpha_X^2$. For $(1 \leftrightarrow 2)$ interactions, we expect from Eqs. (52) and (53) that

$$\left(\frac{t}{\tau} \right)_{1 \leftrightarrow 2 \text{ GUT}} \approx \frac{\alpha_X m_X^2}{(T^2 + m_X^2)^{1/2} T^2} e^{-m_X/T}. \quad (65)$$

This function is clearly $\ll 1$ at $T \gg m_X$ and at $T \ll m_X$, peaking for T near m_X at a value of order $(\alpha_X/m_X) \approx 10^2$. The ratios $(t/\tau)_{1 \leftrightarrow 2, 2 \leftrightarrow 2 \text{ GUT}}$ are plotted⁵³⁾ in Fig. 4, and we see that they suggest that for an epoch where T was of order m_X the interactions involving X bosons may have been in equilibrium.

One can make similar estimates for the interactions of the superheavy Higgs bosons H_X expected in GUTs. Shown in Fig. 4 are the curves corresponding to $(2 \leftrightarrow 2)$ interactions via Higgs exchange and $(1 \leftrightarrow 2)$ Higgs decays and inverse decays, assuming a Higgs mass $\sim 10^{14}$ GeV [see Eq. (25)] and a Higgs coupling $\alpha_H \approx 10^{-3}$. Although

these parameters are clearly subject to some uncertainty, we see a tendency to have a period in which H_X processes are in equilibrium.

Given the general qualitative features of the expansion time to mean free time ratios shown in Fig. 4, there are three qualitatively different scenarios⁵³⁾ for generating the baryon asymmetry observed in the Universe, illustrated in Fig. 5.

Fig. 5 The various scenarios⁵³⁾ (A), (B), and (C) described in the text for baryon number generation in the context of GUTs

(A) The Universe might have started at $T \sim 10^{19}$ GeV with a baryon-antibaryon asymmetry $\gg 10^{-9}$, or such a large asymmetry might have been generated by interactions at $T > m_X \sim 10^{15}$ GeV. This baryon asymmetry would then have been reduced during the period of grand unified equilibrium around $T \sim 10^{15}$ GeV (in accordance with general theorems^{42,51)} to the value $10^{-9 \pm 1}$ observed today.

(B) Again one considers the possibility that the baryon asymmetry may have been large at temperatures $> m_X$, but that grand unified interactions in equilibrium may then have pushed the number much below $10^{-9 \pm 1}$, only to have X and H_X interactions in the post-equilibrium period regenerate the required number $10^{-9 \pm 1}$.

(C) Perhaps the Universe really did start off at $T > 10^{15}$ GeV with a baryon asymmetry which was either zero or very small, $< 10^{-9 \pm 1}$. Then as X and H_X interactions dropped out of thermal equilibrium after 10^{15} GeV, the required baryon number of $10^{-9 \pm 1}$ per photon could have been generated.

Examining these different alternatives, we see that options (B) and (C) give us the virtue of naturalness: the final asymmetry is determined by GUT considerations independent of the initial conditions. In contrast, option (A) requires a very delicate fine-tuning⁴²⁾ of the interaction rates if the total wash-out of the baryon number is to be avoided. It does not require a C- and CP-violating component in the B-violating GUT interactions, but we have strong reasons to believe that such a component exists. In the rest of this discussion we will discuss the baryon asymmetry⁸⁾ generated by C- and CP-violating interactions of GUTs in options (B) and (C).

III.3 Mechanisms for Baryon Number Generation

The generation of the net baryon number (47) may have been quite complicated, with several different baryon-number violating processes contributing and/or competing. However, it seems likely that the dominant driving mechanism may have been the decays and inverse decays of some superheavy boson^{41,51)}, either a gauge vector X or a Higgs scalar H_X ⁴¹⁾. In most simple^{2,33)} GUTs the dominant decays of these particles are expected (Fig. 6) to be into pairs of quarks or into antilepton and antiquark:

$$X \text{ or } H_X \rightarrow (q + q) \text{ or } (\bar{q} + \bar{\ell}) . \quad (66)$$

Fig. 6 Dominant decay modes of superheavy vector bosons X and Higgs bosons H_X

The total decay rates of particles and antiparticles must be the same because of CPT:

$$\begin{aligned} \Gamma_{\text{tot}}(X) &= \Gamma(X \rightarrow qq) + \Gamma(X \rightarrow \bar{q}\bar{\ell}) + \dots \\ \Gamma_{\text{tot}}(\bar{X}) &= \Gamma(\bar{X} \rightarrow \bar{q}q) + \Gamma(\bar{X} \rightarrow q\ell) + \dots \end{aligned} \quad (67)$$

but the partial decay rates into conjugate channels may differ if C and CP are both violated:

$$B \equiv B(X \rightarrow \bar{q}\bar{\ell}) \equiv \frac{\Gamma(X \rightarrow \bar{q}\bar{\ell})}{\Gamma_{\text{tot}}(X)} \neq \bar{B} \equiv B(\bar{X} \rightarrow q\ell) \equiv \frac{\Gamma(\bar{X} \rightarrow q\ell)}{\Gamma_{\text{tot}}(\bar{X})} . \quad (68)$$

Then if one naively assumes that one starts with an identical population of X and $\bar{X}$ bosons (or of H_X and $H_{\bar{X}}$), their decays would generate a net baryon to photon number density ratio⁴¹⁾

$$\frac{n_B}{n_\gamma} \approx \left(\frac{g_X}{g_{\text{tot}}} \right) (\bar{B} - B) \approx (10^{-2}) \Delta B , \quad (69)$$

where g_X is the number of helicity states of X bosons and g_{tot} is the total number of helicity states. We will see later that this naive decay mechanism is somewhat

over-simplified*, but it suffices for discussions of some qualitative features of the baryon generation problem.

It is clear that no difference ΔB in the branching ratios of particles and anti-particles can arise in lowest order. Such a non-zero ΔB can first arise in fourth order, through final-state interactions (see Fig. 7) involving bosons which violate B conservation**. The resulting difference in branching ratios is of order⁴¹⁾

$$\Delta B \approx \text{Im Tr}(abcd) / \sum_a \text{Tr}(aa^*) , \quad (70)$$

where a, b, c, and d are the coupling matrices introduced in Fig. 7. In fact, in

Fig. 7 A sample fourth-order interference diagram which could [see Eq. (70)] contribute to ΔB and hence to the baryon asymmetry⁴¹⁾

simple models the first C- and CP-violating ΔB often occurs only in higher orders. In a large class of GUTs all the CP violation originates from the fermion-fermion-Higgs Yukawa couplings. As in the simplest case [Eq. (70)], all leading CP-violating rate differences such as ΔB are related to products of Yukawa coupling matrices traced along fermion lines. These traces may arise from thermal averaging over initial states as well as from sums over possible final states, and occur for scattering processes as well as for $(1 \leftrightarrow 2)$ decays and inverse decays. It is then important, in the context of any particular model, to determine the lowest-order trace of Yukawa couplings which violates C and CP. The lowest-order contributions to the baryon asymmetry can then be got from this trace by connecting up the Yukawa vertices by Higgs lines in all possible ways, and then cutting the resulting diagrams in all possible ways corresponding to processes with different initial and final states and different possible arrangements of the final-state interactions. As an example, we consider⁵³⁾ the "minimal" SU(5) model²⁾ with the usual $\bar{5} + 10$ fermions and one 24 and one $(\bar{5} + 5)$ pair of Higgs representations (see Section II.2). In this model the first CP-violating trace is eighth order in the Yukawa coupling (see Fig. 8a). Many different ways of connecting up the Higgs vertices are possible, of which one example is shown in Fig. 8b. This diagram and its fellows may then be cut in many different ways corresponding to different physical processes, as indicated in Figs. 8c,d. Each one of these cuts corresponds to a direct CP-violating interaction rate $[(1 \leftrightarrow 2) \text{ or } (2 \leftrightarrow 2), \text{ etc.}]$ which may contribute to the total baryon asymmetry generated. Suppose we consider the contribution of the cut in Fig. 8c which gives a CP-violating difference ΔB in decay branching

*An estimate of the dilution of the baryon number generated due to subsequent entropy generation in an homogeneous and isotropic Universe is given by Steigman, Olive and Schramm⁵⁴⁾.

**There is a simple theorem⁴¹⁾ which shows that $B - \bar{B}$ can receive *no contribution* from graphs which are of first order in baryon-violating interactions, even if the graphs involve an arbitrary number of baryon-conserving interactions. For details see Ref. 41.

Fig. 8 a) One of the lowest-order C-, CP-, and B-violating fermion traces in minimal SU(5). b) The Higgs-fermion-fermion vertices of the trace connected up in one of many possible ways. c) The previous diagram cut in such a way as to contribute a CP-violating ΔB in Higgs decays. d) The same diagram cut so as to make a CP-violating difference between Higgs-fermion scattering cross-sections (cf. Fig. 3a). The dashed lines are $\underline{5}$ Higgs bosons, the solid lines $\underline{10}$ fermions, the zig-zag lines $\underline{5}$ fermions, and the dotted lines are unitarity cuts.

ratios. Each Yukawa vertex is $O(m_f/m_W \times g)$, and the CP-violating effect is proportional to GIM-like products of differences in quark masses. If we assume that there are just three generations of $\underline{5} + \underline{10}$ fermions and that $m_t \gg m_b \gg m_c \gg$ other fermion masses, then we have a CP-violating

$$\Delta B \lesssim \frac{1}{10} \alpha_X \frac{m_b^4 m_t^3 m_c}{m_W^8} / (m_t^2/m_W^2) \alpha_X, \quad (71)$$

where the $1/10$ is an upper limit on the product of mixing angles and phase factors, and we have neglected various numerical factors from phase space, etc., as well as combinatorial factors from the large number of similar diagrams. Putting in the known quark masses we find

$$\Delta B \lesssim 10^{-14} \frac{m_t}{m_W} \lesssim 10^{-14} \quad (72)$$

which Eq. (69) demonstrates to be too small to generate the observed baryon asymmetry of order $10^{-9 \pm 1}$. A model more complicated than minimal SU(5) (more Higgs representations? superheavy fermions?) is necessary^{41,43} in order to generate the observed number, and can easily be constructed (see Section III.4). (Note that any process involving a gauge vector boson in this model, whether in an initial or final state or in a virtual interaction, has CP violation only in at least one order of α higher.)

Before discussing in more detail the mechanics of computing the baryon asymmetry in specific models, it is perhaps worth making a few general remarks about the nature of the CP violation necessary for generating a non-zero baryon number. It

is clear that one needs a "hard" form of CP violation which persists up to high energies of $\sim 10^{15}$ GeV, and therefore that models where CP violation is generated by spontaneous breakdown on a weak interaction scale of order 100 GeV are not suitable. Furthermore, models where CP is violated spontaneously at $O(10^{15})$ GeV are also not acceptable. At these very early epochs, causally connected domains in the Universe are very small, and in a scenario of spontaneous CP violation there would be no correlation between the signs and perhaps magnitudes of the baryon number generated in different domains. At subsequent epochs, previously separated domains become connected, and the net baryon asymmetry we see today would tend to be a statistically averaged -- and hence much diluted -- version of the asymmetry possible in any individual domain. It therefore seems that a scenario of spontaneous CP violation is difficult to maintain in the absence of acausal effects, and that some form of intrinsic CP violation such as that found in the minimal SU(5) model^{2,29)} is to be preferred. One might wonder whether the sign of the baryon asymmetry can be related to the sign of CP violation in the $K^0-\bar{K}^0$ system. Quite apart from the computational problems of getting numbers for either of these physical quantities from parameters of a fundamental Lagrangian, it seems unlikely in general that the same CP-violating parameter controls both phenomena. For example, in the standard SU(5) with N_g (e.g. 3) generations, the generalized Kobayashi-Maskawa matrix characterizing low-energy weak interactions has $(N_g - 1)^2$ (e.g. 4) parameters, of which $\frac{1}{2}(N_g - 1)(N_g - 2)$ (e.g. 1) are CP-violating phases "observable" via $K^0-\bar{K}^0$ phenomena, while at high energies there are $(N_g - 1)$ (e.g. 2) additional CP-violating phases^{29,53)}, one of which determines the dominating CP-violating rate (71). In fact, even in the minimal SU(5) model, CP violation and hence a net baryon asymmetry are possible with only four quarks, although the number generated increases if more quarks, or other structure, is introduced, and a realistic model probably needs at least six quarks*.

There are several obstacles remaining before a quantitative calculation of the baryon asymmetry can be made. The first is that because the minimal SU(5) model is too small, one must go^{41,43)} to a more complicated model with more free parameters, which hinders the attainment of numerical precision. Secondly, the naive decay scenario outlined above is incomplete. This is because, as can be seen from Fig. 4, $(2 \leftrightarrow 2)$ interactions are still significant and near equilibrium around the time of the heavy particle decays. Since the net baryon asymmetry must be zero in equilibrium, this suggests that these $(2 \leftrightarrow 2)$ interactions have a tendency to reduce the net asymmetry generated by $(1 \leftrightarrow 2)$ interactions. This washing-out has been computed numerically by various groups^{42,56)} integrating up the Boltzmann equations using both $(1 \leftrightarrow 2)$ and $(2 \leftrightarrow 2)$ interactions. They find that the amount of wash-out is very sensitive both to the masses of the superheavy bosons and to the grand unified coupling α_X . We see from Fig. 9 that the efficiency of B generation via vector bosons X with $\alpha_X \sim 1/40$ is 100% efficient only for $m_X \geq 10^{17}$ GeV, while for Higgs bosons H_X with $\alpha_{H_X} \sim 10^{-3}$ complete efficiency persists down to $m_{H_X} \approx 10^{15}$ GeV. If one requires an efficiency $\geq 0.1\%$ in order to get a large enough n_B/n_Y , then one needs

$$m_X \geq 10^{15} \text{ GeV} \quad \text{or} \quad m_{H_X} \geq 10^{13} \text{ GeV} . \quad (73)$$

For comparison, we recall that the latest best estimates⁶⁾ from low-energy phenomenology are [see Eqs. (37) and (25)]

$$m_X \approx 6 \times 10^{14} \text{ GeV} , \quad m_{H_X}/m_X \approx 10^{0 \pm 1} , \quad (74)$$

*Anthropocentric constraints on the baryon number asymmetry are discussed in Ref. 55.

Fig. 9 A diagram from Ref. 52 illustrating how the net baryon number obtained from a CP-violating difference in decay branching ratios may be less than expected due to the wash-out by $(2 \leftrightarrow 2)$ interactions.

which are just consistent with the constraints (73). The greater amount of washing-out for vector boson processes is another reason (in addition to their intrinsically higher order of CP violation) why one might expect the dominant contribution to the baryon asymmetry to come from processes involving superheavy Higgses. There is one complication with these wash-out calculations that should be mentioned. It is that in models with reducible fermion representations for each generation of fermions, such as SU(5) with its $\bar{5} + 10$ representations, the vector bosons which are the dominant $(2 \leftrightarrow 2)$ interactions cannot wash out all combinations of quark asymmetries^{35,56}) because of helicity conservation. This means that in such "reducible" models the constraints (73) may be relaxed somewhat, although the degree of relaxation has not yet been computed. Another important problem is the interplay of washing out and CP violation in a situation³⁵) with several different varieties of superheavy boson (more than one stage of GUT symmetry breaking, $m_X \neq m_{H_X}$, etc.). Under certain restrictions on α_X and m_X the heavier bosons may establish equilibrium for the lighter bosons⁴¹), or the asymmetries from different sources may just add^{35,56}). Other calculational complications arise from possible deviations from the simple isotropic and homogeneous, monotonically expanding and cooling Universe which we have been considering. For example, there may be significant inhomogeneities -- primordial black holes -- whose explosions at or after a time corresponding to a universal temperature of $\sim 10^{15}$ GeV might contribute significantly⁵⁷) to the net baryon asymmetry. (Conversely, a satisfactory computation of n_B/n_γ in a completely homogeneous and isotropic scenario might impose constraints on the spectrum of primordial black holes.) Related complications may arise from the observation⁵³) (see Fig. 4) that no known particle interactions may have been in equilibrium at temperatures between 10^{19} and 10^{15} GeV. This means that particles need not have been in a Boltzmann distribution in this epoch, and that dissipative phenomena may have been important, as will be discussed in the next section. Finally, as will be discussed in Section III.6, it has recently been suggested⁴⁵), in the context of grand unified monopole production early in the Universe, that the Universe may have had a complicated thermal history during the grand unified phase transition. It may have supercooled in the symmetric phase down to a temperature $\ll 10^{15}$ GeV, and then have been reheated by the latent heat released in a first-order phase transition. This would complicate considerably the previous naive discussion (see Fig. 4) of the expansion rate, interaction rates, and thermal equilibrium. Because of all the complications mentioned here -- and doubtless others -- it seems reasonable to conclude that the GUT mechanism for generating the baryon asymmetry [Eq. (47)] is still in a qualitative stage, although the process of quantification is making significant advances.

III.4 A Concrete Proposal

Next we present a deliberately oversimplified scenario for baryon number generation, where we ignore most of the complications mentioned above, just to show how a detailed calculation concerning n_B/n_γ may go and what kind of answers we get out of it.

In Fig. 10, all the different phases relevant to our scenario are presented in detail. Let us look more closely at this figure. For temperatures $T > T_X$ we

Fig. 10 A schematic view of the scenario described in the text.

assume that everything is at random [out of equilibrium (?)], but as the Universe expands and cools down to temperatures $T \sim T_X$, where the expansion rate [$\sim 1/t$, see Eq. (51)] becomes comparable to the decay rate Γ_X [$\Gamma_X \sim 1/\tau$, see Eq. (52)] of the heaviest superheavy gauge boson around, we assume that the X interactions establish thermal equilibrium for everybody. The X particles act as equilibrators. Of course, the reader may replace this equilibration procedure with his (her) favourite one. All previously existing (primeval or otherwise created) net baryon number is assumed to be washed out, since B-violating interactions involving X particles are in equilibrium. In order to have enough X particles around to create equilibrium, we need the expansion rate curve to cut the decay rate curve at some temperature, $T_X > m_X$, as happens in Fig. 10, otherwise the Boltzmann factor $\exp(-m_X/T)$ will make the number of X particles uncomfortably small. This condition is satisfied, as may easily be seen from Eq. (65) (with $T \gg m_X$) if

$$m_X < O(g_T^{1/2} \alpha_X m_P) , \quad (75)$$

which, for the simple SU(5) model, implies $m_X < O(10^{17}-10^{18} \text{ GeV})$, certainly consistent with Eq. (74). For example, if $m_X \approx 6 \times 10^{14} \text{ GeV}$, as given by Eq. (74), we get

$$T_X \sim O(10^{16} \text{ GeV}) . \quad (76)$$

As the Universe cools down, particles with masses (M) between m_X and m_A (see Fig. 10), will play their games and will eventually die out, with declining temperature T , because of the $e^{-M/T}$ factor in the range $m_A \ll T \ll M$, so we are no longer concerned with them. A denotes by definition the lightest superheavy particles around, possessing B -violating interactions and able to produce appreciable amounts of net baryon number. We call such particles *myrtons*.

For reasons already explained⁴¹⁾, superheavy gauge bosons seem rather impotent in creating a net baryon number, so most probably myrtons should be identified with superheavy Higgs bosons or superheavy fermions*. Let us follow the rate of the A particles, the myrtons. Because of the *assumed* equilibration created by the X particles, the myrtons and antimyrtons are considered to be in equal numbers, and for temperatures $T \gg m_A$, their number is proportional to the number of photons ($\sim T^3$):

$$n_A = n_{\bar{A}} \sim n_\gamma \sim T^3. \quad (77)$$

The strict equality between myrtons and antimyrtons, as well as the proportionality of myrtons to the number of photons around, remains true up to temperatures $T \sim T_A$, where the expansion rate of the Universe becomes comparable to the decay rate of myrtons, Γ_A . This happens because for $m_A < T < T_X$, myrtons behave like free massless particles participating in the expansion of the Universe, so that Eq. (77) is preserved with $T \propto 1/R$. A free expansion of massless particles may not change the particle distribution functions. For temperatures $T_A < T < m_A$, we demand that $\tau > t$ (see Fig. 10), i.e. myrtons cannot decay, so they sit around still satisfying Eq. (77). But now they are wildly out of equilibrium, since for $T < m_A$ equilibrium would demand the appearance of the Boltzmann factor $\exp(-m_A/T)$, which may not in fact be there since no annihilation of myrtons takes place. When the temperature reaches T_A , myrtons start decaying, but clearly the inverse decays cannot take place since they are blocked by the Boltzmann factor $\exp(-m_A/T)$, with $T < T_A \ll m_A$. So, we arrive at the phase where we have, at some temperature $T_A \ll m_A$, myrtons and antimyrtons satisfying Eq. (77) [i.e. a lot of them around ($\sim n_\gamma$) and in *equal* numbers] and virtually decaying freely.

From now on, everything is smooth sailing in the realm of particle physics.

Let us consider a specific example, the simple $SU(5)$ model²⁾, where we may identify myrtons (A) with the colour triplet part (3), called H_X of the (5) of Higgs [see Section II.3, discussion after Eq. (23)]. In such a case the highly desirable condition, $\Gamma_A < \text{expansion rate}$, or equivalently $\tau_A > t$ (for having enough myrtons around and for blocking their inverse decay), implies

$$m_{H_X} > O(10^{-1}-10^{-2})\alpha_{H_X} m_P, \quad (78)$$

where we have again used Eqs. (51) and (52) (with $T \ll m_{H_X}$) and replaced $\alpha_X \rightarrow \alpha_{H_X} \sim 10^{-3}$, as discussed before. Actually we can do a more precise job. Since H_X is assumed to be the lightest superheavy particle around, to lowest order it may decay only to light stuff -- quarks and leptons -- because of *colour conservation*; thus

$$\frac{1}{\tau_{H_X}} = \Gamma_{H_X} \approx \frac{m_{H_X}}{8\pi} [3m_t^2 + 2m_b^2] \sqrt{2} G_F, \quad (79)$$

*The importance of superheavy fermions in the net baryon number producing processes was first stressed by Georgi and Nanopoulos, Ref. 19, p. 70, footnote.

which for $m_t \approx 20$ GeV becomes $\Gamma_{H_X} \sim 10^{-3} m_H$ (with $G_F = 10^{-5} \text{ GeV}^{-2}$, the Fermi coupling constant). In this case, Eq. (79) used in conjunction with Eq. (51) gives the condition $m_{H_X} > O(10^{-5} m_P) \approx O(10^{14} \text{ GeV})$, which lies within the range of Eq. (74). Taking, for example, $m_{H_X} \sim m_X \sim 6 \times 10^{14} \text{ GeV}$, we get

$$T_{H_X} \sim O(10^{14} \text{ GeV}) . \quad (80)$$

To recapitulate, in the simple SU(5) model the necessary conditions for starting our scenario rolling are

$$O(10^{14} \text{ GeV}) < m_{H_X}, \quad m_X < O(10^{17} \text{ GeV}) , \quad (81)$$

certainly in agreement with the latest estimates from low-energy phenomenology as given by Eq. (74).

Assuming that the basic conditions for the creation of net baryon number are met, we now need to calculate ΔB [see Eq. (69)] in order to find out how many baryons we produce. Since we have to consider *at least* one-loop diagrams interfering with tree diagrams (as shown in Fig. 7),

$$\Delta B \leq O(\alpha^2 \epsilon) , \quad (82)$$

where ϵ is some CP-violating parameter assumed* to be in range $\epsilon \sim (10^{-3}-10^{-1})$. Then Eqs. (69) and (82) imply

$$\frac{n_B}{n_Y} \leq O(10^{-9 \pm 1}) \quad (83)$$

to be compared with Eq. (47). The reader may draw her (his) own conclusions.

In order to get a precise value of ΔB , we again need a specific model. The SU(5) model, suitably modified³¹⁾ to cure Eq. (39) through the use of a slightly more complicated Higgs system than the minimal one** (5, 24), gives^{40, 41)}

$$\Delta B \approx \frac{1}{32\pi} (\sqrt{2} G_F) m_b^2 \epsilon \approx 10^{-6} \epsilon , \quad (84)$$

in accordance with Eq. (82) and thus implying Eq. (83).

A few comments are in order.

- i) It is interesting to notice that, as mentioned many times in our discussion, an appreciable baryon number creation through our mechanism demands a more complicated SU(5) structure, which is also needed if we want to avoid wrong relations such as the one given by Eq. (39). Is this accidental?
- ii) Equation (83) shows that ΔB is proportional to the m^2 of the heaviest $Q = -1/3$ quark around.

*We remind the reader of the fact that the observed CP violation in the neutral K-system gives $\epsilon \approx 10^{-3}$.

**We assume in this example that there are no superheavy fermions around; or that if they are around, they do not play an important role in the net baryon number generation by being, for example, heavier than H_X .

Here we are dealing with superhigh energies, so Eq. (26) should be used^{4,17)} and not Eq. (9), and an $m_b \approx m_\tau \approx 2$ GeV fits just perfectly. A much heavier $Q = -\frac{1}{3}$ quark will make n_B/n_γ too big, while if only four quarks existed, since $(m_s/m_b)^2 \sim 10^{-3}-10^{-4}$, n_B/n_γ will be uncomfortably small. Actually there are reasons for believing⁵⁵⁾ that the baryon-to-photon ratio could not be much different from its observed value (otherwise matter and radiation would still be in equilibrium, etc.). Thus in view of the lightness of the s and d quarks, the existence of a third generation is needed⁵⁵⁾ not only for creating more CP-violating phases (than if there were only 4 quarks in the GUT) but also for providing a $Q = -\frac{1}{3}$ quark of appropriately large mass^{4,17)}. Maybe (wildly speculating) the observed n_B/n_γ ratio puts limits on the possible mass spectrum of the next $Q = -\frac{1}{3}$ quarks to be discovered (?), although it seems that possibly we are close to the end of the mass scale allowed⁵⁵⁾. These words sound familiar. Just remember the nucleosynthesis limits¹⁸⁾ on light neutrinos ($m_\nu \ll 1$ MeV) or the quark-lepton mass relations' (9) limits^{4,17)} on the number of flavours. Everything fits -- let there be *only* three generations!

Before getting to other cosmological implications of GUTs, a few words* about L-violation. As mentioned in Section II, the lepton number L is expected (together with the baryon number B) to be violated in GUTs. Then we might expect, in general, a net n_L comparable to n_B ⁵⁸⁾. However sometimes, as in the simple SU(5), B-L is conserved (globally) and so neither n_B nor n_L has good reason to be small unless the net n_B and n_L were both initially zero (a symmetric Universe?); but this is the exception⁵⁹⁾ and not the rule⁵⁸⁾. In any case it seems⁵⁸⁾ that $n_L \sim n_B$ is a natural output of GUTs, i.e. $n_\nu/n_\gamma \sim 10^{-8} \ll 1$; in other words there is no neutrino degeneracy.

Thus we see that GUTs help the standard Big-Bang model on its way to explaining the observed primordial ⁴He abundance in the Universe ($\sim 25\%$ by mass), by strengthening (or justifying) two big (Bang) assumptions:

- a) there are no more than three light ($m_\nu \ll 1$ MeV) neutrinos [because there are only six flavours^{4,17)}];
- b) there is no neutrino degeneracy $n_\nu/n_\gamma \sim 10^{-8} \ll 1$ [because $n_B \sim n_L$ ⁵⁸⁾].

III.5 Non-Equilibrium in the Very Early Universe?

It is interesting and perhaps significant that when the temperature of the Universe was between 10^{19} GeV and roughly 10^{15} GeV, no "known" particle interactions may have been in equilibrium^{53,60)}: during that epoch, one has approximately

$$\frac{\text{mean free time}}{\text{expansion time}} \approx \frac{T}{\alpha^2} > 1. \quad (85)$$

This means that particles need not have been Boltzmann distributed, and that dissipative phenomena¹⁰⁾ may have been significant. What might the primordial distribution of particles have been? In a Robertson-Walker-Friedman Universe, when the temperature was of order 10^{19} GeV, the horizon diameter $D_H = 4t$ would have been of the same order as the Planck distance $\approx 1/m_P$. In the absence of acausal effects -- which should, however, perhaps be expected in the context of quantum gravity -- it is difficult to imagine how coherence could have been established over distances $> D_H$ with the same ease as over distances $< D_H$. (It is difficult to imagine particles with a wavelength size larger than the horizon size.) In this case, one

*Other lecturers at this School will cover this subject in detail.

might expect particles with short wavelengths⁶⁰⁾, $\lambda < 1/m_p$, or high energies, $E \gtrsim O(mp)$, to have a spectrum distorted by comparison with a naive Boltzmann distribution. As an extreme case, one could even imagine a cut-off of the low-energy tail of the spectrum at $E < O(mp)$. If such an approximate cut-off occurred, one can estimate that $(1 \leftrightarrow 2)$ production processes would have been unlikely to fill in the spectrum at $T > \text{few} \times 10^{16}$ GeV, while $(2 \leftrightarrow 3)$ interactions are unlikely⁶⁰⁾ to have done the job before $T \approx 10^{16}$ GeV⁶⁰⁾. Therefore, if the masses of the X bosons were significantly higher than the fashionable 10^{15} GeV, it is an open question whether their distributions were ever fully thermalized. In the absence of thermalization, various theorems^{42,51)} about the absence of a net baryon number become inapplicable, and the computation of n_B/n_γ becomes more complicated. This complication is unlikely to have arisen if m_X was as low as the expected 10^{15} GeV, but could be significant if m_X were significantly larger.

Equation (85) implies that during this very early epoch, particles would have had mean free paths that were large compared with the horizon size, and therefore dissipative phenomena¹⁰⁾ may have occurred as illustrated in Fig. 11. Generically, the shear viscosity η of an imperfect fluid is⁶¹⁾ of order

$$\eta \approx T^4 \tau, \quad (86a)$$

and taking the earlier estimates of the mean free time τ , one gets¹⁰⁾

$$\eta \approx (10^2 \text{ to } 10^3) T^3 \quad (86b)$$

when the temperature exceeds 10^{15} GeV. Similarly, the thermal conductivity χ of an imperfect fluid is of order

$$\chi \sim T^3 \tau, \quad (87a)$$

which becomes¹⁰⁾

$$\chi \approx (10^3 \text{ to } 10^4) T^2 \quad (87b)$$

in this very early epoch. By contrast, the bulk viscosity is zero for a fluid of ultrarelativistic particles. The grand unified viscosity (86) and thermal conductivity (87) may have played a role in smoothing out primordial inhomogeneities or, perhaps correcting initial anisotropies as illustrated in Fig. 11. Because of causality, their effects should be limited to within the size of an horizon: this

Fig. 11 Illustration of particles with long mean free paths transferring energy and momentum, and hence possibly giving rise to dissipation through viscosity and/or thermal conductivity.

may be sufficient¹⁰⁾ to damp high-frequency modes of oscillation⁹⁾ (radiative, rotational, or compressional) which might otherwise have contained an energy density comparable to that in elementary particles. It should be noted that the numerical factors of order 1 in the formulae for η and χ used in Ref. 10 in fact depend on the precise angular form of the cross-section σ being used, and that some formulae given in the literature are not exactly correct. See, for example, Straumann⁶¹⁾. Also the exponent in formula (24) of Ref. 10 should be a factor of 2 larger*.

It has recently been emphasized by Sato⁶¹⁾ that the conventional viscosity approximation for discussing dissipative effects is only valid when $\tau < t$. In our case (86) this is only strictly valid when the temperature is in the lower part of the range $10^{19} \text{ GeV} > T > 10^{15} \text{ GeV}$. This means that the results of Ref. 10 can only be regarded as qualitative. However, we believe that the basic physical picture of Fig. 11 remains valid even if $\tau > t$: the point is that particles are then able to "leak" out of an inhomogeneity.

These dissipative effects also tend to homogenize the Universe locally, preparatory to the generation of baryon number: in particular they may tend⁶²⁾ to hinder the production of primordial black holes $\leq$ the horizon size.

However, viscous effects do not prevent the initiation and growth of very low frequency, large-scale, compressional inhomogeneities, which Press⁴⁴⁾ has speculated may have originated at the time of the grand unified phase transition and subsequently grown in intensity and evolved into galaxies. In view of the logarithmically long growth-time between 10^{15} GeV and the epoch at which galaxy formation is conventionally initiated, this may be a promising way⁴⁴⁾ of avoiding the normal problems in explaining galaxy formation.

It is worth mentioning [Weinberg⁹⁾] that in the framework of theories of elementary particle interactions such as GUTs, energy density fluctuations are mainly adiabatic [$(n_B/n_Y) \sim (n_B/s)$ is given in terms of purely microscopic quantities]. This fact has some interesting experimental consequences, for in adiabatic fluctuations there is a simple relation between temperature fluctuations and number (or energy) density fluctuations which, at the onset of recombination, becomes

$$\left. \frac{\delta T_Y}{T_Y} \right|_R = \frac{1}{3} \left. \frac{\delta n_B}{n_B} \right|_R = \frac{1}{3} \left. \frac{\delta \rho}{\rho} \right|_R \approx 10^{-3} - 10^{-4} . \quad (88)$$

Thus fractional fluctuations are to be expected in the observed microwave background temperature of order $10^{-3} - 10^{-4}$ over angles of $30''$ (corresponding to a typical galaxy of mass $10^{11} M_\odot$), and if Thomson scattering during or after recombination does not smooth out these temperature fluctuations, they should be observable. Since the measurement of such small fluctuations with a $30''$ angular scale is well within the reach of present techniques, it may be interesting and worth while to look for them.

III.6 Grand Unified Monopoles

These appear³⁸⁾ near the mass-scale at which a primitive $U(1)$ factor in the gauge group of the world gets absorbed into a simple group. For example, in the case of the minimal GUT $SU(5)$, $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ at around 10^{15} GeV , one expects

*We thank N. Straumann for correspondence and discussions on these points.

Fig. 12 Impressions of a) Higgs fields in causally separated domains pointing in different (internal group) directions causing b) a monopole to be formed where they meet. Note that the value of the Higgs field goes to zero in the centre.

to encounter topologically non-trivial and stable states (cf. Fig. 12a) called magnetic monopoles M with masses

$$m_M \approx \frac{m_X}{\alpha_X} \approx 10^{16} \text{ GeV} . \quad (89)$$

These monopoles may have been produced copiously in the very early Universe, and if so their topological stability may have caused them to survive to lower temperatures in unacceptably large numbers³⁹⁾. It is convenient to introduce a dimensionless density for these objects, which is

$$r \equiv n/T^3 \quad (90)$$

and is basically the monopole-to-photon ratio. From the success of the conventional calculations¹⁸⁾ of nucleosynthesis at temperatures $T \sim 1 \text{ MeV}$, one deduces that monopoles did not dominate the energy density at that epoch, so that

$$r_{1 \text{ MeV}} < 10^{-19} . \quad (91a)$$

The absence of a collapse in the present Hubble expansion due to a high density of relic monopoles means that

$$r_{3 \text{ K}} < 10^{-25} . \quad (91b)$$

It has been estimated that conventional $M\bar{M}$ capture and annihilation in an homogeneous Universe could not have reduced r below 10^{-10} . The Universe now contains inhomogeneities (e.g. you, me, and galaxies), but excess annihilation in these could not have explained the number (91a), and probably not the number (91b) either. We therefore conclude that *either* very few monopoles [$\leq$ Eqs. (91)] were produced *and/or* the conventional capture and annihilation rates are not applicable.

The conventional picture^{39,45)} of monopole production in the early Universe goes somewhat as follows. At very high temperatures the Higgs fields have zero vacuum expectation value and tend to run around in random directions as in Fig. 13a. At low temperatures the Higgs fields will congeal with a fixed, non-zero v.e.v., but in general pointing in different internal group directions in different domains of space, as illustrated in Fig. 12b. Where these domains come together there may be a mismatch with the Higgses pointing in different directions so that the lowest

Fig. 13 Illustration of the Higgs field value as a function of temperature. a) At high temperatures $T > T_c$, $|H|$ is expected to be zero. b) For intermediate temperatures, $T_c > T > T_g$, the minimum of the potential $V(|H|, T)$ is at $|H| \neq 0$, but the thermal fluctuations of $|H|$ are so large that it can pass over the central hump. c) At low temperatures $|H| \neq 0$ and the Higgs field no longer fluctuates over the hump.

energy configuration has a zero field value in the centre, as in Fig. 12a. Such a "hole" is a topologically stable monopole³⁸⁾ for which one must pay the energetic price (89).

A popular line of arguments⁴⁵⁾ is to suggest that the number of "holes" is of the same order⁶³⁾ as the number of causally separated domains at the temperature where the monopoles freeze out. To get an acceptably low number [Eqs. (91)], one of the arguments tries to reduce as far as possible the number of different domains by setting this freeze-out at the lowest possible temperature, postulating a first-order phase transition and strong supercooling⁴⁵⁾. When the phase transition takes place, the latent heat then reheats the Universe back to a temperature closer to 10^{15} GeV, after which it expands and cools in the normal way. Apart from the suppression of monopole production, an interesting feature of this scenario⁶⁴⁾ is that the Universe expands exponentially during the supercooled epoch, as the energy density and pressure are then dominated by the vacuum energy density. This has two attractive implications⁶⁴⁾, and one which is potentially catastrophic. One piece of good news is that the classic "simultaneity" problem is avoided, because before this period of exponential expansion all points in the Universe we now see -- and much more besides -- were sufficiently close to be in causal contact and hence all expanding in the same way at the same time. Another piece of good news is that during this exponential expansion any initial curvature [$k \neq 0$ in the RWF solution (48)] would have become flatter, rather like a balloon being blown up. This might explain why the present density of the Universe is so close to the critical density for eventual gravitational collapse. The bad news is that it is very difficult⁶⁴⁾

to see how the transition to a broken phase of the GUT could ever have occurred. The basic problem is that while bubbles of asymmetric Universe expand at the speed of light, the Universe as a whole is expanding too quickly for them to catch up. Unless this problem is solved, the scenario is doomed.

The necessity for a strongly first-order phase transition has in any case been criticized⁴⁸⁾. The concept of a Higgs field pointing in any given direction is gauge-dependent, and the concept of domains "pointing in a fixed direction" is not on as solid a basis as in a ferromagnet. A monopole is of course a gauge-independent object, and the proposal⁴⁸⁾ is to calculate their density assuming a second-order or weakly first-order transition and the validity of thermal equilibrium arguments. In a second-order phase transition the lowest-energy state starts having a non-zero value of the Higgs field at the critical temperature T_c (see Fig. 13a). However, for a range of temperatures below that, the thermal fluctuations are sufficient to knock the Higgs field out of its minimum and over the central "hump" in the Higgs potential, as indicated in Fig. 13b. Therefore, in this range of temperatures it does not even really make sense to talk of a conserved topological quantum number, or of monopoles. At a temperature $T_G < T_c$ the thermal fluctuations are no longer sufficient to jiggle the Higgs, and it congeals into the non-zero valley of the Higgs potential, as in Fig. 13c. This Ginzburg temperature T_G ^{39,65)} is basically determined by the energy density in a correlation volume of radius $O(1/m_{\text{Higgs}})$ being less than the free energy density of order $(\langle O|H|O \rangle)^4$:

$$T_G \approx (\text{geometrical factor}) \times \frac{1}{m_H^3} \times (\langle O|H|O \rangle)^4. \quad (92)$$

Down to this temperature the Higgs field is still moving around quickly enough,

$$\tau \sim \frac{1}{m_H} < t \sim T_G^{-2}, \quad (93)$$

so that thermal equilibrium might be relevant and one would expect

$$r \approx O[\exp(-m_H/T_G)] = O\left\{\exp\left[-C(m_H/m_X)\right]\right\}, \quad (94)$$

where⁴⁸⁾ in the minimal SU(5) model

$$C \approx O(30). \quad (95)$$

Bearing in mind the uncertainties in estimating C and the possibility that $m_H/m_X > 1$ (dynamical symmetry breaking?) this argument could well give r low enough to be compatible with the limits (91). Some people dislike this argument⁴⁷⁾ on the grounds that since it gives much less than one monopole per causally connected volume, it seems to require the existence of some acausal correlation in the Higgs field. However, as remarked above, gauge invariance may require some of the arguments about causally unrelated domains to be relaxed.

A final remark about the present density of monopoles concerns the classic calculations of capture and annihilation rates which assumed no correlations in the spatial distributions of monopoles and antimonopoles. It has been pointed out⁴⁶⁾ that in some GUTs some of the monopoles may be bound together by a string of Z^0 magnetic flux which enhances the annihilation rate. An example⁶⁶⁾ of a model where this may play a role in suppressing monopoles is in SO(10) broken down through the chain $SO(10) \rightarrow SO(6) \times SO(4) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$. Another mechanism⁶⁷⁾ for suppressing monopoles proposes to reduce the cosmological density of monopoles by having electromagnetic gauge invariance spontaneously broken at temperatures $T \geq 1$ TeV. Open problems in this scenario include its successful embedding in a

grand unified theory, and the computation of the net charge density of the Universe due to C-, CP-, and Q-violating effects out of thermal equilibrium. However, all other GUTs conserve Q and hence do not explain why the present charge density of the Universe is so small.

My present feeling is that while none of the above arguments provides a totally convincing mechanism for disposing of the grand unified monopole problem, I do not propose to lose any sleep worrying about it. But it would be worth improving the present experimental upper limits^{8,9} on the relic monopole density by a few orders of magnitude: after all, perhaps ...?

Summarizing Section III we see that GUTs shed light on many "dark corners" of the "standard" Big-Bang model.

- 1) They provide¹⁰ enough viscosity (?) (through grand unified interactions) to homogenize and isotropize a possibly chaotic Universe before the temperature gets down to $O(10^{15}$ GeV). At the same time, B- and L-violating interactions coming into equilibrium wash out any "premature" (produced primordially or otherwise) net baryon or lepton number.
- 2) They create⁸ (through appropriate B- and L-violating interactions) a net baryon and lepton number in the Universe in quantitative^{8,11,12} (?) agreement with observations.
- 3) They create (?) very low frequency, large-scale, compressional inhomogeneities at the time of the grand unified phase transition; which inhomogeneities, subsequently grown in intensity, may evolve into galaxies¹⁴.
- 4) They support^{4,17,18} the tacit assumptions which are involved in the successful nucleosynthesis calculations [at most three kinds of light ($m_\nu \ll 1$ MeV), non-degenerate ($n_\nu/n_\gamma \sim 10^{-8}$) neutrinos].

CONCLUSIONS

There is indeed a flourishing cosmic connection between grand unification and cosmology, which has several aspects that are more or less strong. The principal points concern the following:

Neutrino Masses and Numbers

There are phenomenological reasons in GUTs to expect only three or four "light" neutrinos, and theoretical reasons to expect that they might have masses up to a few electronvolts. The success of nucleosynthesis calculations seems to require three or four "light" neutrinos, and masses up to $O(100)$ eV are allowed by cosmology. Such massive "light" neutrinos may even help alleviate the missing-light problem of clusters of galaxies.

Baryon Number Generation

GUTs provide a framework and a qualitative mechanism for generating the baryon asymmetry observed in the Universe. For it to work, the masses of the superheavy bosons should not be too small, nor their coupling constants too large. Present GUTs are more or less consistent with these constraints, but it seems that a larger GUT than the minimal SU(5) may be necessary if one is to generate a large enough number.

Grand Unified Monopoles

First estimates of their production rates early in the Universe gave values much above the allowed limits. Ideas for getting rid of them have been proposed, one of which has other interesting cosmological implications (simultaneity, asymptotic flatness) but also has serious problems. It may well be that the grand unified monopole problem has a relatively banal solution. In the meantime: why not look for them?

FAREWELL TO THE UNIVERSE

Are grand unified theories correct? Is the Universe open? An affirmative answer to these questions (which is highly probable) will give a very macabre answer to the question, What is the future of the Universe? *DEATH!* If the Universe is open, it will continue to expand for ever; and as it will be expanding, all the matter it contains will gradually decay, perhaps within a time-scale of 10^{32} years. Apart from sporadically exploding black holes, the only contents of the Universe left will be the completely stable particles (photons, electrons-positrons, and neutrinos). We will be back in the soup! This is a rather melancholic vision of the future of our Universe, but it is very likely to be true. Perhaps even the Universe cannot escape the basic law of prosperity and decline. It may be called the Rise and Fall (decay!) of the baryon number. How strange it is that interactions that create us are driving us to our extinction.

How lucky we are that the present age of the Universe is around 10^{10} years and not 10^{30} years.

* * *

And I felt as if the whole Universe
had given me a nudge!

Lawrence Durrell

Clea ("The Alexandria Quartet")

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