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ON DIPOLE TRANSITIONS IN RELATIVISTIC QUARKONIUM MODELS

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A B S T R A C T

We discuss dipole transitions of Dirac and Klein-Gordon charmonium and bottonium models. The relativistic wave functions and therefore also the dipole rates turn out to be substantially different from non-relativistic models.

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In the charmonium system the electric dipole transition $\psi' \rightarrow \gamma P_j$ are experimentally well known to be $(16 \pm 5) \text{ keV}$ for each transition¹⁾. To our knowledge, up to today there is only one theoretical study of the charmonium system yielding rates between 15 and 25 keV²⁾. This work uses a Salpeter equation with a static potential which incorporates all kinematic relativistic effects. Most of the theoretical studies of the charmonium system take the non-relativistic approach using a Schrödinger equation. In any confining Schrödinger potential model, however, these transition rates are known³⁾ to be $\approx 90\%$ of the upper bounds given by the dipole sum rules⁴⁾:

$$\Gamma(\psi' \rightarrow \gamma P_j) \approx 0.9 \cdot (40, 56, 64) \text{ keV} \cdot 1.6 \text{ GeV}/m_c \text{ for } j=(2,1,0) \quad (1)$$

This is clearly much too large as compared to experiment and also very different from the numbers of the relativistic Salpeter model of Ref. 2). In this paper we want to discuss some relativistic effects which will be shown to make up a sizeable fraction of the difference between experiment and Eq. (1).

A number of corrections to the electric dipole (E1) transitions have been discussed in the literature⁵⁾⁻⁷⁾, but most of them are quite small:

- i) higher multipoles contribute $((k \cdot r)/2)^2 \approx 5\%$,
- ii) finite size effects (as a correction to the long photon wavelength approximation) have been shown to be smaller than 5%,
- iii) recoil corrections are similar to ii),
- iv) an anomalous quark magnetic moment may change the rates significantly if it is large and negative,
- v) relativistic corrections to the non-relativistic wave function are hitherto unknown; only corrections to energy levels (Breit's equation) have been discussed in some detail (and have proved to be very important).

In the remainder of this paper we will show that the type v) corrections are probably the most important ones for transitions in the charmonium system. This is not immediately evident, since a naïve estimate of the relativistic correction to the kinetic energy (or the particle momentum) indicates only small effects:

$$\langle p^4 \rangle / 4m_c^3 \approx (\langle p^2 \rangle / m_c) \cdot (v/2c)^2 \approx E_{\text{kin}} \cdot 10\%.$$

The easiest way to study relativistic corrections to a wave function is to study the wave functions of an equation which contains the desired relativistic terms. We have therefore solved the potential model

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} \gamma_0 + \lambda r \quad (2)$$

using the Klein-Gordon (KG)^{*)} and the Dirac equation. In their radial forms for the equal mass $c\bar{c}$ system they read

$$R'' = \left[\frac{\ell(\ell+1)}{r^2} + (m_c + \frac{\lambda}{2} r)^2 - (\frac{M}{2} + \frac{2}{3} \frac{\alpha_s}{r})^2 \right] R \quad (3)$$

for the KG equation and

$$\begin{aligned} g' &= -\frac{x}{r} g + \left[\frac{M}{2} + \frac{2}{3} \frac{\alpha_s}{r} + m_c + \frac{\lambda}{2} r \right] f \\ f' &= \frac{x}{r} f - \left[\frac{M}{2} + \frac{2}{3} \frac{\alpha_s}{r} - m_c - \frac{\lambda}{2} r \right] g \end{aligned} \quad (4)$$

for the Dirac equation. The radial wave functions are normalized to $\int_0^\infty dr R^2(r) = 1$ and $\int_0^\infty dr (f^2(r) + g^2(r)) = 1$, respectively. The orbital wave functions of the KG equation are familiar from the Schrödinger theory, those of the Dirac equation are the standard ones⁸⁾ with $\kappa = -\ell - 1$ for $j = \ell + 1/2$ and $\kappa = \ell$ for $j = \ell - 1/2$, where h is the total spin of the Dirac wave function. While the KG equation is for spinless particles from the beginning, we have to average over states with the same ℓ but different j in the Dirac equation because j is not a good quantum number for an equal mass system like charmonium. Both equations still use the potential in static approximation and neither of them contains realistic spin-spin and spin-orbit interactions, but both contain the relativistic terms in the kinetic energy, the quadratic terms of the potentials etc., and both are easy to programme and give results after short computation time. The corrections induced by spin interactions and retardation (of the Coulomb part of the potential) may be viewed as treatable via these terms in the Breit equation which are not contained here. Please note that these terms are of the same magnitude as those we aim for in this paper. Both equations have

^{*)} The Dirac matrices of (2) are meaningless in the KG equation. Here the notation shall indicate that the Coulomb part goes with the energy and the linear part with the mass.

been studied before in order to fit the charmonium system⁹⁾⁻¹⁰⁾, but we are not aware of any work which obtained the relativistic wave functions besides the energy eigenvalues.

The spectra of the KG and Dirac equations are not identical because of i) the interaction of the Dirac magnetic moment with the static Coulomb field⁸⁾ and ii) a new interaction with the scalar confinement field, which arises because the momentum operator and the distance dependent mass $m(r) = m_0 + \lambda \cdot r$ no longer commute. Examples of spectra of the Dirac and KG equation are shown in the Table. We do not expect the spin interactions i) and ii) to be a realistic description of a fermion-antifermion system like charmonium. It, therefore, seems more appealing to omit all spin interactions and regard the KG equation as being the more adequate description of charmonium (better: of the centres of gravity of charmonium spin multiplets). For the following discussion, however, the differences between the KG and the Dirac equation play no rôle, because the quantities δ , which we will define in Eq. (5) are identical in both cases. For comparison with the Schrödinger theory, we have included three Schrödinger spectra in the Table. One readily notes that the wave functions of the Schrödinger theory are too large, because the Schrödinger theory underestimates the momentum corresponding to a given kinetic energy.

In order to study the dipole transitions we consider the quantities

$$\begin{aligned} \delta_1 &= \frac{1}{2} (E(2S) - E(1P)) m_c \langle r \rangle_{2S,1P}^2 \\ \delta_2 &= \frac{1}{3} (E(1P) - E(1S)) m_c \langle r \rangle_{1P,1S}^2 \end{aligned} \quad (5)$$

For the harmonic oscillator, which saturates the dipole sum rules, $\delta_1 = \delta_2 = 1$, δ_1 can be considered as a correction factor to the dipole sum rule transition rate. In the non-relativistic Schrödinger potential model of our Table ($\alpha_s = 0.5$) $\delta_2 = 0.91$ and $\delta_1 = 0.86$, generally $0.9 < \delta_1 < 1$ in monomial confining potential models³⁾. The study of the KG and Dirac equations reveals the relativistic nature of the charmonium wave functions:

$$\begin{aligned} \delta_1 &\approx 0.6 \\ \delta_2 &\approx 0.7 \end{aligned} \quad \text{in } c\bar{c}. \quad (6)$$

δ_2 is much more stable against changes of the potential shapes and magnitudes than δ_1 which, for example, depends quite strongly on the amount of Coulomb potential in our ansatz (2). Using (1) and (6) we obtain

$$\Gamma(\psi' \rightarrow \gamma P_j) \approx (25, 34, 38) \text{ keV} \frac{1.6 \text{ GeV}}{m_c} \text{ for } j = (2, 1, 0) \quad (7)$$

which has to be compared to $(16 \pm 5) \text{ KeV}$ for each transition measured experimentally. One can read off the Table that the dipole matrix elements between relativistic wave functions are indeed much smaller than those between the corresponding non-relativistic wave functions. This is probably also reflected by the numbers of Ref. 2). A further comparison with Ref. 2), however, is very difficult, since these authors used a potential which does not confine and has a shape different from (2) in the relevant regime of distances. It is conceivable that other terms of order $(v/c)^2$ which are kept in Ref. 2) but not in the KG equation give corrections of the same order of magnitude as (6). Those are supposedly different for different P_j states and might help to make the rates of (7) equal. It therefore seems possible to avoid a large negative anomalous magnetic moment of the c quark⁷⁾ as well as the necessity of introducing a large P_j wave- $D\bar{D}$ or $\psi' - D\bar{D}$ mixing¹¹⁾.

For the transition $P_j \rightarrow \gamma J/\psi$ we use δ_2 of (6) and the dipole sum rule numbers $\Gamma(P_j \rightarrow \gamma J/\psi) = (490, 370, 180) \text{ keV} \cdot 1.6 \text{ GeV}/m_c$ for $j = (2, 1, 0)$ to obtain

$$\Gamma(P_j \rightarrow \gamma J/\psi) \approx (340, 260, 130) \text{ keV} \frac{1.6 \text{ GeV}}{m_c} \text{ for } j = (2, 1, 0) \quad (8)$$

There certainly are corrections to (8) as to (7), however, they might be smaller. An indication of this is the stability of δ_2 (but not δ_1) against changes of the potential. The reason for this is also obvious: $\psi' \rightarrow \gamma P_j$ involves one wave function with a radial node, and the transition is very sensitive to the exact location of this node, while $P_j \rightarrow \gamma J/\psi$ involves no node. This latter transition is only smaller than in non-relativistic models because the wave functions in general are smaller. We note an agreement between (8) and the corresponding rates given in Ref. 2), Eq. (15g).

The relativistic corrections to the wave functions will also affect the dipole transition rates in the $b\bar{b}$ system. Fitting the $b\bar{b}$ system with (2): $\alpha_s = 0.365$, $\lambda = 0.978 \text{ GeV/fm}$, we obtain

$$\begin{aligned}\delta_1(b\bar{b}) &\approx 0.65 \\ \delta_2(b\bar{b}) &\approx 0.80\end{aligned}\tag{9}$$

which leads to

$$\Gamma(\Upsilon' \rightarrow \gamma {}^3P_j(b\bar{b})) \approx \frac{2j+1}{3} \frac{k^3}{k_{c.o.g.}} 10^{-10} \text{ keV}^{-1} .\tag{10}$$

Assuming $M({}^3P_j) = (9.905, 9.875, 9.835)$ one finds $\Gamma(\Psi' \rightarrow \gamma {}^3P_j(b\bar{b})) \approx (1.9, 2.3, 1.6) \text{ keV}$ or branching fractions of 5.7% to 8.2%. This is more than scaling estimates using charmonium numbers as input¹²⁾, but less than the non-relativistic model. One has to keep in mind, however, that the greatest uncertainties come from our ignorance of the P state masses. Only when these are known can Eq. (10) be tested.

In conclusion we note that relativistic wave functions (from the Klein-Gordon or Dirac equations) lead to electric dipole transition rates which are well below the non-relativistic estimates, not only in charmonium, but also in bottonium. Moreover, the magnitude of this effect in charmonium indicates that most of the discrepancies between non-relativistic theory and experiment may be due to relativistic effects. Unfortunately, we cannot make a more quantitative statement here because our model does not correctly include the relativistic spin interactions, not to speak of retardation effects. It appears necessary to study a Bethe-Salpeter equation including at least the retardation effects of the one-gluon exchange.

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Table

Some spectra of the Dirac and KG equations, (3) and (4), and of the Schrödinger equation with the static potential $V(r) = -4/3 \alpha_s/r + \lambda r + V_0$. All masses are in Gev and all lengths in fm.

	Dirac	KG	Schroedinger		
m_c [GeV]	1.358	1.370	1.370	1.370	1.370
α_s	0.39	0.39	0.39	0.50	0.60
λ [GeV/fm]	1.067	1.057	0.895	0.789	0.684
V_0 [GeV]	0	0	-0.025	0.119	0.256
$M(1S)$	3.067	3.067	3.067	3.067	3.067
$M(1P)$	3.500	3.521	3.492	3.516	3.538
$M(2S)$	3.672	3.673	3.673	3.673	3.673
$M(1D)$	3.791	3.808	3.789	3.805	3.817
$M(3S)$	4.097	4.096	4.121	4.100	4.077
$M(2D)$	4.179	4.192	4.201	4.190	4.174
$M(4S)$	4.446	4.445	4.503	4.459	4.411
$M(3D)$	4.510	4.521	4.565	4.529	4.487
$\langle 1S r 1S \rangle$	0.334	0.334	0.395	0.379	0.365
$\langle 1S r 1P \rangle$	0.369	0.369	0.434	0.416	0.398
$\langle 1P r 1P \rangle$	0.558	0.558	0.648	0.654	0.663
$\langle 1P r 2S \rangle$	-0.452	-0.480	-0.534	-0.558	-0.583
$\langle 2S r 2S \rangle$	0.663	0.663	0.792	0.802	0.814
$\langle 1D r 1D \rangle$	0.725	0.725	0.850	0.870	0.895

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