Ref.TH.3001-CERN

VIRTUAL PHOTON STRUCTURE

T. Uematsu *)
DESY - Hamburg

and

T.F. Walsh **)
CERN -- Geneva

A B S T R A C T

The structure functions of a real photon are calculable in QCD. The leading contribution is proportional to $\ln Q^2$, with a coefficient reflecting the gluon flux in a real photon. We investigate this leading term for non-zero target photon mass. In an appropriate limit the gluon content in a virtual photon is found to vanish. The gluon radiative corrections of QCD can thus be turned off by tuning the target photon mass.

*) Alexander von Humboldt Fellow. On leave from the Research Institute for Fundamental Physics, Kyoto University, Kyoto Japan.

**) Permanent address : DESY, Notkestr. 85, Hamburg, Germany.

Ref.TH.3001-CERN

2 December 1980

The real photon structure functions, measurable in $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ ¹⁾, are a particularly clean test of QCD. There is an anomalous pointlike term growing as $\ln Q^2$ and large as $x = Q^2/2k \cdot q \rightarrow 1$ ²⁾ (see Fig. 1). The x dependent coefficient of $\ln Q^2$ in $F_2^Y(x, Q^2)$ is calculable in QCD³⁾. It differs from the Born approximation ignoring gluon radiation²⁾. (Of the other linearly polarized γ structure functions, F_L^Y is hardly changed from the Born approximation, and F_X^Y is not changed at all as $Q^2 \rightarrow \infty$ ⁴⁾.)

Some of the theoretical difficulties in the QCD interpretation of deep inelastic lepton-nucleon scattering are either altogether absent or quite different in form for a photon target. Target mass effects⁵⁾, important for the nucleon, are trivially absent for target $\kappa^2 = -k^2 = 0$. Less trivially, higher twist effects are also probably important for a nucleon target⁶⁾. They must look different for a photon target. For the nucleon this effect is an expansion in powers as $(\mu_n^2/Q^2)^n$, for dimensional scales μ_n and powers n . μ_n will be typically of order of the inverse nuclear charge radius, $\mu \sim m_p$. However, the pointlike term in F_2^Y involves no charge radius. Photon radius effects appear due to recoil from gluon emissions. Since these involve high transverse momentum, it is likely that higher twist effects will be suppressed by powers of the strong coupling g_s . Also, the only dimensional parameter which can appear is QCD's Λ . If Λ is smaller than is usually believed⁶⁾, so are higher twist effects for the photon target^{*)}. (There is a hadronic piece in F_2^Y , but this is negligible at large Q^2 and x . Our argument is not affected by this term.)

For these reasons we believe that the QCD prediction for $F_2^Y(x, Q^2, \kappa^2 = -k^2 = 0)$ is of special interest. It is then tempting to ask what happens for non-zero photon target mass, $\kappa^2 = -k^2 > 0$. There are many structure functions for $q^2, k^2 \neq 0$ ⁷⁾. But we can define an off-shell extension of F_2^Y by

$$\begin{aligned} F_2^Y(x, Q^2, \kappa^2) &\equiv 8\pi\alpha x [W_{++}^{++} + W_{+-}^{+-}] \\ &= x \sum e_{q_i}^2 (q_i(x, Q^2, \kappa^2) + \bar{q}_i(x, Q^2, \kappa^2)) \end{aligned} \quad (1)$$

*) These features may make $F_2^Y(x, Q^2)$ useful in obtaining an unambiguous value for QCD's Λ from a deep inelastic process.

where the W 's are transverse photon helicity amplitudes ^{4),7)}. For $\kappa^2 = 0$, F_2^Y in (1) is just the conventional F_2^Y to leading order in $\ln Q^2$. [We can ignore ambiguities in the definition of $F_2^Y(x, Q^2, \kappa^2)$ in the leading log approximation. It may be useful for some purposes to define F_2^Y differently than by (1). Our results will still hold.]

In this paper we will ask what happens for F_2^Y in the limit where

$$\Lambda^2 \ll \kappa^2 \ll Q^2 \quad (2)$$

In this limit we can ignore target mass effects $\sim \kappa^2/Q^2$. We find that the leading logarithm contribution to F_2^Y is calculable in the limit (2). The result can be expressed in terms of moments of the quark and gluon distributions in a virtual photon,

$$q(m) \text{ [or } G(m)] = \int_0^1 dx x^{m-1} q(x) \text{ [or } G(x)]$$

We set the quark mass to zero. The non-singlet, singlet, and gluon moments are then

$$q_{NS}(m) = \frac{\alpha}{2\pi} 3e_{NS}^2 f(m) \frac{\left\{ \ln \frac{Q^2}{\Lambda^2} - \ln \frac{\kappa^2}{\Lambda^2} \left[\ln \kappa^2/\Lambda^2 / \ln Q^2/\Lambda^2 \right]^{d_{qq}(m)} \right\}}{1 + d_{qq}(m)} \quad (3)$$

and

$$\begin{aligned} q_S(m) = \frac{\alpha}{2\pi} 3e_S^2 f(m) & \left[\frac{d_{qq}(m) - d_-(m)}{d_+(m) - d_-(m)} \frac{1}{1 + d_+(m)} \left\{ \ln \frac{Q^2}{\Lambda^2} \right. \right. \\ & \left. \left. - \ln \frac{\kappa^2}{\Lambda^2} \left(\frac{\ln \kappa^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{d_+(m)} \right\} \right. \\ & \left. - \frac{d_{qq}(m) - d_+(m)}{d_+(m) - d_-(m)} \frac{1}{1 - d_+(m)} \left\{ \ln \frac{Q^2}{\Lambda^2} \right. \right. \\ & \left. \left. - \ln \frac{\kappa^2}{\Lambda^2} \left(\frac{\ln \kappa^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{d_-(m)} \right\} \right] \quad (4) \end{aligned}$$

$$\begin{aligned}
 G(m) = & \frac{\alpha}{2\pi} 3e_s^2 f(m) \frac{dG_q(m)}{d_+(m)-d_-(m)} \left[\frac{1}{1+d_+(m)} \left\{ \ln \frac{Q^2}{\Lambda^2} \right. \right. \\
 & - \ln \frac{\kappa^2}{\Lambda^2} \left(\frac{\ln \kappa^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{d_+(m)} \left. \right\} - \frac{1}{1+d_-(m)} \left\{ \ln \frac{Q^2}{\Lambda^2} \right. \\
 & \left. \left. - \ln \frac{\kappa^2}{\Lambda^2} \left(\frac{\ln \kappa^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{d_-(m)} \right\} \right]
 \end{aligned}
 \tag{5}$$

where, for N_F flavours,

$$e_s^2 = \langle e_q^2 \rangle = N_F^{-1} \sum e_q^2, \quad e_{NS}^2 = e_q^2 - \langle e_q^2 \rangle$$

and thus $q(m) = q_{NS}(m) + q_S(m)$. f is defined as

$$f(m) = \frac{m^2 + m + 2}{m(m+1)(m+2)}$$

the moments of the Born $\gamma(k) \rightarrow q\bar{q}$ distribution. The notation for the d_{ij} is that of Llewellyn Smith ³⁾. Formally, we recover the real photon case for $\kappa^2 = \Lambda^2$.

From Eqs. (3) - (5), we now see that in the limit

$$1 \ll \ln \frac{Q^2}{\kappa^2} \ll \ln \frac{\kappa^2}{\Lambda^2} \tag{6}$$

we have

$$\begin{aligned}
 q_{NS}(m) & \approx \frac{\alpha}{2\pi} 3e_{NS}^2 f(m) \ln \frac{Q^2}{\kappa^2} \\
 q_S(m) & \approx \frac{\alpha}{2\pi} 3e_S^2 f(m) \ln \frac{Q^2}{\kappa^2} \\
 G(m) & \approx 0
 \end{aligned}
 \tag{7}$$

Remarkably, these are just the contributions from the Born approximation ignoring gluon radiation to leading order. Thus in this kinematic region the QCD radiative corrections to the pointlike γ structure functions are turned off by tuning κ^2 away from zero. The approach to the limit (7) is given by Eqs. (3)-(5).

In the quark and gluon distributions (3)-(5) we have ignored non-leading log contributions as well as power corrections of the form κ^2/Q^2 , Λ^2/Q^2 (target mass effects and higher twists) ^{*)}. Nevertheless, it is interesting to see how $\kappa^2 \neq 0$ affects F_2^γ by inverting the moments (3)-(5). In Fig. 2 we show F_2^γ for $Q^2 = 30 \text{ GeV}^2$ and various choices of κ^2 , Λ^2 ^{**)}. For small Λ an effect is already present for $Q^2 \sim 10 \text{ GeV}^2$, $\kappa^2 \sim 1 \text{ GeV}^2$. (The curves $\kappa^2 = \Lambda^2$ reproduce the real photon result.) For $\kappa^2 \neq 0$ it is, strictly speaking, necessary to modify x to a variable covering the full range $0 \leq x \leq 1$. We interpret the variable in Fig. 2 this way, e.g., it is a ξ scaling variable. The effects of this are minor for $\kappa^2/Q^2 \ll 1$.

It is important to realize that (7) holds when the logarithmic interval $\ln Q^2/\Lambda^2 - \ln \kappa^2/\Lambda^2$ is large. Our condition is that this be small compared to $\ln \kappa^2/\Lambda^2$. An appropriate limit would be $\kappa^2, Q^2 \rightarrow \infty$ with $\ln Q^2/\kappa^2 \propto (\ln \kappa^2/\Lambda^2)^{1-r}$, $0 < r < 1$. Physically, our result says that in this limit we can neglect gluon radiation in the logarithmic interval $\ln Q^2/\Lambda^2 - \ln \kappa^2/\Lambda^2$. The logarithmic spacing of gluons must then grow approximately as $\ln \kappa^2/\Lambda^2$. This suggests another interpretation of (7). Suppose that we start at fixed large κ^2 and increase $Q^2 > \kappa^2$. At first the logarithmic interval $\ln Q^2/\Lambda^2 - \ln \kappa^2/\Lambda^2$ is too small for QCD radiative corrections to build up. As $Q^2 \rightarrow \infty$ for fixed large κ^2 a full ladder of gluon emissions develops. (In an axial gauge the leading log contribution comes from ladder diagrams ³⁾.) The characteristic unit in this development is $\ln \kappa^2/\Lambda^2$.

We now sketch the derivation of Eqs. (3)-(5). It uses the Altarelli-Parisi equations ^{10),11)}. We treat the non-singlet case for simplicity. Introduce the variables $t = \ln Q^2/\Lambda^2$, $t_\kappa = \ln \kappa^2/\Lambda^2$ and $t' = \ln Q^2/\kappa^2$. The evolution of the non-singlet distribution is given by ¹¹⁾ :

*) Quark mass effects are unimportant if final states without charm are selected. See Ref. 8) for a discussion of $m_q \neq 0$ (these authors also briefly discuss $\kappa^2 \neq 0$). κ^2/Q^2 corrections have been discussed in Ref. 9).

**) The inverse Mellin transform integral was done numerically.

$$\frac{\partial q_{NS}(m)}{\partial t'} = 3e_{NS}^2 \frac{\alpha}{2\pi} f(m) - \frac{1}{t' + t_k} d_{qq}(m) q_{NS}(m) \quad (8)$$

The solution to this inhomogeneous equation is

$$q_{NS}(m) = C(t') \left(\frac{t' + t_k}{t_k} \right)^{-d_{qq}(m)} \quad (9)$$

(To this must be added a solution of the homogeneous equation, but this contribution is negligible as $Q^2 \rightarrow \infty$.) The boundary condition on C is $C(t'=0) = 0$. The evolution of C is given by

$$\frac{\partial C(t')}{\partial t'} = 3e_{NS}^2 \frac{\alpha}{2\pi} f(m) \left(\frac{t' + t_k}{t_k} \right)^{d_{qq}(m)} \quad (10)$$

with solution

$$C(t') = 3e_{NS}^2 \frac{\alpha}{2\pi} f(m) \left[\frac{(t' + t_k)^{d_{qq}(m)+1} - t_k^{d_{qq}(m)+1}}{(1 + d_{qq}(m)) t_k^{d_{qq}(m)}} \right] \quad (11)$$

Rewritten, (9) and (11) form just the expression for $q_{NS}(m)$ given in Eq. (3). The derivation of $q_S(m)$ and $G(m)$ follows the same route.

We have also derived Eqs. (3)-(5) using operator product methods. It might be worth noting here that $\kappa^2 \neq 0$ effects can be incorporated into the operator product analysis. This then yields also the non-leading logarithm contributions for $\kappa^2 \neq 0$. [For $\kappa^2 = 0$ this was done in Ref. 12).] To the two-loop level the leading contribution dominates by a factor $1/\bar{g}^2(Q^2)$ over the non-leading part. However, the non-leading corrections depend on the precise way F_2^Y is defined. As we have already noted, we need not worry about this detail for the leading logarithm contribution.

ACKNOWLEDGEMENTS

One of us (T.U.) would like to thank the Humboldt foundation for financial support, and also the DESY Theory Division for its hospitality. We both extend our thanks to the CERN Theory Division, where this paper was written. We also thank G. Martinelli for advice on the Mellin transform inversion.

REFERENCES

- 1) T.F. Walsh - Phys.Letters 36B (1971) 121 ;
S.J. Brodsky, T. Kinoshita and H. Terazawa - Phys.Rev.Letters 27 (1971) 280.
- 2) T.F. Walsh and P.M. Zerwas - Phys.Letters 44B (1973) 195 ;
R.L. Kingsley - Nuclear Phys. B60 (1973) 43 ;
R.P. Worden - Phys.Letters 51B (1974) 57 ;
P.M. Zerwas - Phys.Rev. D10 (1974) 1485.
- 3) E. Witten - Nuclear Phys. B120 (1977) 189 ;
C.H. Llewellyn Smith - Phys.Letters 79B (1978) 83 ;
W.R. Frazer and J. Gunion - Phys.Rev. D20 (1979) 147 ;
R.J. De Witt, L.M. Jones, J.D. Sullivan, D.E. Willen and H.W. Wyld - Phys. Rev. D19 (1979) 2046.
- 4) C. Peterson, T.F. Walsh and P.M. Zerwas - Nuclear Phys. B174 (1980) 424 ;
W.R. Frazer and G. Rossi - Phys.Rev. D21 (1980) 2710 ;
F. Delduc et al. - Nuclear Phys. B174 (1980) 147, 157 ;
K. Sasaki - Phys.Rev. D22 (1980) 2143.
- 5) O. Nachtmann - Nuclear Phys. B63 (1973) 237 ;
H. Georgi and H.D. Politzer - Phys.Rev. D14 (1976) 1829 ;
S. Wandzura - Nuclear Phys. B122 (1977) 412.
- 6) L.F. Abbott and R.M. Barnett - SLAC-PUB-2227
- 7) R.W. Brown and I.J. Muzinich - Phys.Rev. D4 (1971) 1496 ;
C.E. Carlson and W.-K. Tung - Phys.Rev. D4 (1971) 2873.
- 8) G. Hill and G. Ross - Nuclear Phys. B148 (1979) 373.
- 9) W.R. Frazer and J. Gunion - Phys.Rev.Letters 45 (1980) 1138.
- 10) G. Altarelli and G. Parisi - Nuclear Phys. B126 (1977) 298.
- 11) R.J. De Witt et al - Ref. 3), and C. Peterson et al. - Ref. 4).
We use the notation of this latter reference.
- 12) W. Bardeen and A. Buras - Phys.Rev. D20 (1979) 166 ;
D.W. Duke and J.F. Owens - FSU Preprint HEP 802401 (January 1980).

FIGURE CAPTIONS

Figure 1 Deep inelastic scattering on a virtual photon in $e^+e^- \rightarrow e^+e^- + \text{hadrons}$.
We assume that $\kappa^2 = -k^2 \ll Q^2 = -q^2$.

Figure 2 Behaviour of the structure function $F_2^Y(x, Q^2, \kappa^2)$ at $Q^2 = 30 \text{ GeV}^2$.
 $\kappa^2 = \Lambda^2$ is the lowest order QCD result and "Box" labels the Born approximation neglecting gluon radiation. For $\kappa^2 \neq 0$, the variable x is assumed modified to cover the range $0 \leq x \leq 1$. (For example, x could be the ξ scaling variable.) The vertical axis is in units of

$$\frac{F_2^Y(x, Q^2, \kappa^2)}{\langle e^4 \rangle 3 N_f \frac{\alpha}{\pi} \ln Q^2 / \kappa^2}$$

FIG. 1

Fig. 2