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RADIATIVE MUON CAPTURE FOR $N = Z$ NUCLEI

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A B S T R A C T

The commonly used elementary Hamiltonian of radiative muon capture has been confirmed while the alternative Hwang-Primakoff approach is shown not to be gauge invariant.

In the inclusive process on $N = Z$ nuclei, the closure approximation is avoided by using a realistic nuclear excitation spectrum.

The study is exemplified by a detailed application to ^{40}Ca . Predictions are given for the high energy photon spectrum, circular polarization and asymmetry with respect to the muon polarization for various values of the pseudoscalar coupling constant g_p . A semi-quantitative agreement is found with the ^pdata on the spectrum; more precise experiments are necessary to determine g_p .

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1. INTRODUCTION

The interest in radiative muon capture (RMC)

$$\mu^- + p \rightarrow n + \nu + \gamma$$

stems from the fact that the high energy part of the photon spectrum is particularly sensitive to the value of the induced pseudoscalar coupling constant g_p . In principle, RMC is suitable for a study of its momentum dependence, since at very low photon energy one essentially recovers the momentum transfer of ordinary muon capture, namely $\bar{q}^2 \approx -\mu^2$, whereas at the high energy end of the photon spectrum where no energy is released to the neutrino $\bar{q}^2 \approx +\mu^2$.

One is forced to consider only the part of very energetic photons in the spectrum for both experimental (neutron contamination) and theoretical reasons connected with the use of a reliable nuclear Hamiltonian¹⁾. Therefore one determines the value of the pseudoscalar coupling constant at time-like momentum transfer, in contrast to the space-like one of ordinary muon capture.

The theory of RMC even in the simplest case of capture on a proton is not altogether settled. In fact, recently an alternative form²⁾ of the elementary amplitude has been proposed so that even the problem of the basic Hamiltonian remains to be clarified and completely understood.

Because of the very small rate of RMC, nuclei are better candidates than protons for its observation; for this reason we will be concerned with the nuclear process. This, of course, adds the problems connected with the nuclear physics calculation to that of the basic process.

Two main cases can be distinguished: exclusive transitions, i.e., the ones to definite final nuclear states and inclusive ones in which final states are summed over. The first class has the advantage that the exact energy transfer is known, however at the price of an even smaller rate.

Here we will be concerned with the prediction of the observables for the high energy part of the photon spectrum in inclusive RMC, exemplified by ^{40}Ca since experiments only exist for this nucleus.

The main problem of a nuclear calculation consists in the sum over all final accessible nuclear states of the relevant matrix elements squared. In all previous calculations, the closure approximation was used, i.e., a mean excitation energy was assumed and the evaluation of the transition matrix elements reduced to the corresponding expectation value over the ground state. The assumption

that all the strength is concentrated to one single energy is, of course, unphysical and must be particularly questioned in the present case. The reason is that the closure sum includes states of the nuclear excitation spectrum which are energetically forbidden to contribute to the high energy photons. Moreover, the higher the photon energy, the smaller the allowed part of the excitation spectrum. It is then clear that the photon spectrum cannot be determined by a single average quantity; at best a mean energy could be defined for every photon momentum.

The influence on the results of the shape of the excitation spectrum has already been discussed semiquantitatively by Christillin et al.³⁾. The present work is a more quantitative study.

The essential ingredient consists in the use of an excitation spectrum dependent on both energy and momentum transfer (response function). As will be explained later, we will be guided by educated guesses when establishing its form. The summation over all final nuclear states is then replaced by a conceptually simpler energy integration. This technique has already been successfully applied in the calculation of total muon capture rates⁴⁾, double radiative pion capture⁵⁾ and in the predictions of RMC¹⁾ in medium heavy nuclei. Since the calculation is model dependent (because of the choice of the response function) its reliability and the comparison with other calculations is an important issue which will be discussed later.

The plan of the paper is the following:

Section 2 briefly recalls the observables;

Section 3 is concerned with the discussion of the shape of the nuclear excitation spectrum;

Section 4 presents results;

Section 5 is devoted to the comparison with other work;

Section 6 draws conclusions.

2. OBSERVABLES

The spectrum for the nuclear process

$$\mu^- + a \rightarrow b + \nu(\nu) + \gamma(k) \quad (1)$$

a standing for the initial ground state and b for any accessible final state, is given by⁶⁾

$$N(k)dk = \frac{e^2}{2\pi^2} |\phi_\mu^2|_{av} \frac{1}{m_\mu^2} \int \frac{d\mathbf{k}}{4\pi} \int \frac{d\nu}{4\pi} \sum_b \sum_{\lambda} G_{\lambda}^2 |M_{ba}|^2 \nu^2 k dk \quad (2)$$

where

$$\sum_b \sum_{\lambda} G_{\lambda}^2 |M_{ba}|^2 = \left(A_\lambda(k, \gamma) + \hat{\mathbf{k}} \cdot \sum_{\lambda} \hat{\boldsymbol{\zeta}} B_\lambda(k, \gamma) \right) \sum_b | \langle b | \sum_{i=1}^A \tau_i^- e^{-i(\nu+k) \cdot \mathbf{r}_i} | a \rangle |^2 \quad (3)$$

λ refers to the two helicity states ± 1 , $\gamma = \cos\theta \hat{\mathbf{k}} \cdot \hat{\boldsymbol{\nu}}$; $\mathbf{k}$, $\boldsymbol{\nu}$ and $\boldsymbol{\zeta}$ are respectively the photon and neutrino momentum and the muon polarization vector. A_λ and B_λ are appropriate combinations of coupling constants explicitly given in Appendix A of Ref. 1), obtained by summing over lepton spin and final nuclear states in the SU(4) limit and $|\phi_\mu^2|_{av}$ is the muon wave function averaged over the nuclear volume.

To better handle Eq. (3) an isospin rotation can be made following standard lines^{7),8)} so that

$$\sum_b |M_{ba}|^2 = \sum_b | \langle b | \sum_i \tau_i^- e^{-i(\nu+k) \cdot \mathbf{r}_i} | a \rangle |^2 \rightarrow \frac{1}{2} \sum_{b'} | \langle b' | \sum_i \tau_i^3 e^{-i(\nu+k) \cdot \mathbf{r}_i} | a \rangle |^2 \quad (4)$$

b' being the analogue states in the parent nucleus, i.e., those excited in photo-reactions, with an excitation energy E' connected to the energy E of the daughter nucleus by $E' = E + E_c - (M_n - M_p)$ (E_c being the Coulomb energy difference of corresponding isobaric states in the initial and final nucleus, and $M_n - M_p$ the neutron-proton mass difference). The usual relation $m_\mu - \epsilon_b + E - (M_n - M_p) \equiv \nu_\mu = \nu + k + E'$ holds among ν , k and E' , ν and k always representing the actual neutrino and photon energy; m_μ and ϵ_b stand for the muon mass and the muon binding energy in the 1S orbit. For ^{40}Ca $\nu_\mu = 111$ MeV.

The following observables of interest are expressed in terms of the previous quantities:

i) photon spectrum $N(k)$

$$N(k)dk = \frac{e^2}{2\pi^2} |\phi_\mu^2|_{av} \frac{1}{m_\mu^2} \int \frac{d\mathbf{k}}{4\pi} \int \frac{d\nu}{4\pi} \left(A_+(k, \gamma) + A_-(k, \gamma) \right) \frac{1}{2} \sum_b |M_{ba}|^2 \nu^2 k dk \quad (5)$$

ii) circular polarization

$$P(k) = \frac{\int \frac{d\mathbf{k}}{4\pi} \int \frac{d\mathbf{v}}{4\pi} (A_+(k, \gamma) - A_-(k, \gamma)) \sum_b |M_{b'a}|^2 v^2 k dk}{\int \frac{d\mathbf{k}}{4\pi} \int \frac{d\mathbf{v}}{4\pi} (A_+(k, \gamma) + A_-(k, \gamma)) \sum_b |M_{b'a}|^2 v^2 k dk} \quad (6)$$

iii) muon spin photon angular correlation

$$\Gamma(k) = \frac{\int \frac{d\mathbf{v}}{4\pi} (B_+(k, \gamma) + B_-(k, \gamma)) \sum_b |M_{b'a}|^2 v^2 k dk}{\int \frac{d\mathbf{v}}{4\pi} (A_+(k, \gamma) + A_-(k, \gamma)) \sum_b |M_{b'a}|^2 v^2 k dk} \quad (7)$$

3. THE NUCLEAR EXCITATION SPECTRUM

The essential point lies in the evaluation of the excitation spectrum

$$R(E', S) = \sum_{b'} | \langle b' | \sum_i \tau_i^3 e^{-i \mathbf{S} \cdot \mathbf{r}_i} | a \rangle |^2 \delta(E_{b'a} - E') \quad (8)$$

This is a function of both energy and momentum transfer. The Fermi gas model spectrum being inadequate for $N = Z$ nuclei, and the harmonic oscillator model suffering from even more severe drawbacks, a phenomenological approach will be used instead. The strength will be divided between GDR and higher states.

For ^{40}Ca the former is approximately reproduced by a Lorentz curve of position $E'_1 = 20$ MeV and $\Gamma_1 = 5$ MeV, multiplied by the square of the elastic form factor of ^{40}Ca ground state, $|F_{e1}(s^2)|^2$ which incorporates the quenching effect of retardation on the "unretarded" photoabsorption giant resonance. This assumption⁷⁾ constitutes a strong anchor point and has been shown to hold true quite generally.

As regards higher resonances, a Lorentz curve with $E'_2 = 35$, $\Gamma_2 = 12$ MeV will be used to account for the isovector quadrupole monopole excitation. This second resonance will be assumed to be affected by a corresponding factor $1 - |F_{e1}(s^2)|^2$. Such a position for the spectrum has also been made in a recent work on $(\pi^-, \gamma\gamma)$ on ^{12}C , and it has been confirmed from the comparison with experiment^{9),10)} of the predicted photon spectra. There being no counterpart of Foldy-Walecka theorem for higher resonances (and their calculations being unreliable) their strength will be adjusted to reproduce the total ordinary muon capture rate.

The family of curves $R(E', s)$ is plotted for illustration purposes in Fig. 1. The physical region for RMC is bounded by the straight line $E' = \nu_\mu - 2k + s$, determining a smaller domain with increasing k . This implies, with respect to ordinary μ capture (line $E' = \nu_\mu - s$), a bigger rôle of the GDR, or in other terms, a smaller value of the average excitation energy.

The neutrino not being observed, all information and checks on the excitation function are necessarily indirect. One such test, for the case of ordinary muon capture, has often consisted in the prediction of the neutron multiplicity in the statistical theory. In such a theory, one assumes that the excitation energy is shared among nucleons to produce a thermally excited nucleus. Neutrons are then boiled off with an energy spectrum of the form $N(\epsilon) \approx \epsilon \exp(-\epsilon/\theta)$, ϵ being the neutron kinetic energy and θ the nuclear temperature. The probability that at least j neutrons be emitted from a nucleus excited to an energy E is¹¹⁾

$$N_j(E) = 1 - \exp\left(-\frac{E - B_j}{\theta}\right) \cdot \sum_{n=0}^{2j-3} \left(\frac{E - B_j}{\theta}\right)^n \cdot \frac{1}{n!} \quad (9)$$

where B_j are the appropriate binding energies.

The meaning of Eq. (9) is simply that, apart from terms proportional to the nuclear temperature ($\approx$ kinetic energy of emitted neutrons), if $E \leq B_1$ there is no neutron emission; if $B_1 \leq E \leq B_2$ one neutron is emitted, etc. The probable number of emitted neutrons is plotted at the bottom of Fig. 2 along with the differential ordinary capture rate obtained from our nuclear excitation spectrum. The experimental average neutron multiplicity in ^{40}Ca is 0.746 ± 0.032 ¹²⁾. It would therefore appear that $\Lambda_\mu(E)$ of Fig. 2, predicting a much bigger multiplicity of order 2, is completely ruled out.

The statistical theory has encountered, so far, a reasonable success in this connection^{13), 14)} in medium heavy nuclei. However, in spite of having been actually employed¹²⁾, its extension to light nuclei is questionable. As a matter of fact, its main assumption, namely that there, as well, neutrons are the only emitted particles is wrong. For calcium in particular, in the region of the GDR, i.e., up to 28 MeV, photoabsorption data¹⁵⁾ show a contribution of roughly six to one for proton versus neutron emission. The presence of p emission therefore completely inhibits the opening of the $2n$ channel up to very high energies, well above quadrupole excitation¹⁶⁾.

This is totally confirmed by neutron multiplicity measurements in radiative pion capture. There the spectrum extends to even higher energies¹⁷⁾ in line with our previous arguments. Still, the observed multiplicity is 0.810 ± 0.024 ¹⁸⁾. We must therefore conclude that the statistical theory of neutron emission cannot be applied to test the proposed excitation function. In this connection, let us also remember that the observed radiative pion capture spectrum, in addition to disproving the statistical model, confirms at the same time the gross structure of the nuclear response function we have used.

As regards finer details, SU(4) invariance has been assumed, i.e., spin independence of nuclear forces. With this position the operators $\sum_i \tau_i^a e^{-is \cdot r_i}$ and $1/\sqrt{3} \sum_i \sigma_i \tau_i^a e^{-is \cdot r_i}$ are degenerate. They have the same strength and they excite the same levels. In reality, such equalities do not hold and various splittings and strength rearrangements are expected. Even if no definite conclusions have been achieved, still the over-all gross structure seems to be in accord with the SU(4) requirements. In the absence of a reliable microscopic theory, this seems to be a reasonable assumption, although it is extremely difficult to assess its degree of accuracy. For this reason, at this stage, we think it more than adequate to make use of a Lorentz curve to reproduce approximately the GDR photoabsorption cross-section.

4. RESULTS FOR ^{40}Ca

Our results are reported in the Table for the spectrum, the photon polarization and the angular correlation. They are given for various values of the photon energy k , as a function of $\Delta = -1, 0, 1, 2$ defined by $g_p = \Delta(2M_\mu/m_\pi^2 + q^2)g_A$ to allow for a possible violation of the Goldberger-Treiman relation ($\Delta = 1$).

For illustration purposes, in Fig. 3 the relative photon spectrum is also plotted for the two cases $\Delta = \pm 1$ (solid lines) using the excitation spectrum explained in the text. In the same figure the spectrum for the case $\Delta = -1$ is reported using an excitation spectrum with all the strength concentrated at the GDR energy (dotted line). In all cases the strength has been adjusted to reproduce the ordinary capture rate. It is self-evident from the figure, as already noted in Ref. 3), that possible nuclear renormalization effects may be counterfeited by an unrealistic excitation spectrum. This also shows the lack of foundation for the widespread statement that the model dependence of the result is washed out when predicting the relative photon spectrum. As a matter of fact, one has a roughly constant ratio of about 2/3 for $\Delta = -1$ in the two mentioned cases.

Figure 1 is helpful in visualizing the reason. In essence it shows that in the former case quadrupole strength is absent in the allowed excitation region for the high energy photon spectrum, while it contributes to ordinary capture. This constitutes the fundamental point, borne out by explicit calculations which show that shape changes for the second resonance are irrelevant (e.g., use for the higher part of a Fermi gas model response function).

In comparing our results to experiments, some comments are in order. First, there is a reasonable agreement in the literature concerning the ordinary muon capture rate, namely $\Lambda_{\mu} = 2.45 \cdot 10^6 \text{ sec}^{-1}$ from Eckhause et al.¹⁹⁾ (to which we have kept for the fit to the quadrupole strength) and $\Lambda_{\mu} = 2.3 \cdot 10^6 \text{ sec}^{-1}$ from Hart et al.²⁰⁾.

As regards radiative spectra, only the high energy part will be considered, namely the ratio R of the integrated spectrum for $k \geq 57 \text{ MeV}$ to ordinary capture. Then, apart from the experiment by Conversi et al.²¹⁾ presumably influenced by neutron contamination, the existing data are $R_{k \geq 57} = 15.4 \cdot 10^{-6}$ from Di Lella et al.²²⁾ and $R = (21.1 \pm 1.4)10^{-6}$ from Hart et al.²⁰⁾. Finally, preliminary data from a S.I.N. experiment²³⁾ seem to suggest a rate somewhat higher than the previous ones.

The result of the present calculation with the canonical value of g_p is $R^{\text{th}} = 24 \cdot 10^{-6}$. If we assume the value of Hart et al. for the ordinary capture rate, then $R^{\text{th}} = 26 \cdot 10^{-6}$. While this is in satisfactory agreement with the data, the experimental precision does not yet permit conclusions about the value of g_p . Better experiments are needed for this purpose.

5. COMPARISON WITH OTHER WORKS

In comparing the present results with other theoretical studies, two points must be carefully examined. The first one concerns the nuclear Hamiltonian and the second, the nuclear excitation spectrum.

As regards the former, the same Hamiltonian as in Ref. 1) has been used. This, with minor corrections due to the muon propagator in the extended nuclear Coulomb field, essentially corresponds to the effective Hamiltonian derived by Rood and Tolhoek⁶⁾ in their pioneering work. It was obtained using the principle of minimal electromagnetic coupling from an appropriate set of Feynman diagrams. This approach has been subsequently improved, applying low energy theorems to the amplitude. Adler and Dothan²⁴⁾ and subsequently Christillin and Servadio²⁵⁾ have thus obtained a consistent expansion to the same order in k and $q = \mu - \nu$.

It has further been demonstrated that in the case of high energy photons, where the process is sufficiently local¹⁾ so that the impulse approximation holds, the non-relativistic reduction of this more exact amplitude closely reproduces the results of Rood and Tolhoek. Also, a recent calculation by Sloboda and Fearing²⁶⁾ in which the relativistic amplitude is expanded to powers of $O(1/M^2)$ (where M is the nucleon mass) instead of $O(1/M)$ of Rood and Tolhoek, shows that the new terms $O(1/M^2)$ appearing in the rate alter the previous formulation in the domain of interest by less than 5%.

A completely different approach to determine the amplitude has been used by Hwang and Primakoff²⁾. They expand the hadronic part of the RMC amplitude in terms of the most general set of structures which can be formed from the available vectors. The leptonic part is treated in the standard way. Then each structure is multiplied by a form factor which is a function of the invariants and coupling constants.

The general relations of CVC, PCAC and current conservation are finally imposed on the full amplitude. This generates a set of constraint equations among the general form factors and the weak form factors of the muon radiating diagram. However, these constraints are not sufficient to determine all form factors. Therefore, a so-called "linearity hypothesis" is introduced. The result is at variance with the previous formulation essentially in the sign of the term

$$\frac{2Mg_A}{(q-k)^2 + m_\pi^2} \delta_{\lambda\alpha} \gamma_5$$

as pointed out by Fearing²⁷⁾. Gmitro and Ovchinnikova²⁸⁾, using a more restricted version of the linearity hypothesis recover the usual amplitude. The difference has been shown by Hwang²⁹⁾ to be due to Low's counterterms, necessary to secure the gauge invariances of their amplitude.

The reasons for the disagreement and the definitive resolution to the problem can be seen by the following consideration. In the notation of Ref. 24) the amplitude of RMC on the nucleon is

$$T = \frac{G}{\sqrt{2}} \left\{ - \langle n | J_\alpha^W | p \rangle \bar{u}_2 \gamma_\alpha (1 + \gamma_5) \frac{ie}{i\gamma \cdot (k - K) + m_\mu} \gamma_\lambda^* u_\mu \frac{1}{\sqrt{2k_0}} \right. \\ \left. + \sqrt{\frac{M^2}{2k_0 k_{10} k_{20}}} e \epsilon_\lambda^* M_{\lambda\alpha} \bar{u}_2 \gamma_\alpha (1 + \gamma_5) u_\mu \right\} \quad (10)$$

where the first term corresponds to radiation by the muon and the second by the hadrons. The essential point of low energy theorems is that the knowledge of the divergence (extension of Low's use of current conservation) allows the determination of the amplitude. Here divergences are to be taken both in k (photon momentum) and in $q = \mu - \nu$. For the point we are interested in, take the divergences first with respect to k and q

$$q_{\alpha} k_{\lambda} M_{\lambda\alpha} = - \left(\frac{k_{10} k_{20}}{M^2} \right)^{\frac{1}{2}} q_{\alpha} \langle n | J_{\alpha}^{\mu} | h \rangle \quad (11)$$

and then in the reverse order^{*)}

$$k_{\lambda} q_{\alpha} M_{\lambda\alpha} = - \left(\frac{k_{10} k_{20}}{M^2} \right)^{\frac{1}{2}} k_{\lambda} \langle n | J_{\lambda}^{\mu} | h \rangle + i \frac{\sqrt{2} M g_A}{g_r} \frac{m_{\pi}^2}{q^2 + m_{\pi}^2} k_{\lambda} T_{\pi\lambda}^{\dagger} \quad (12)$$

Equation (11) tells us, focusing only on the above-mentioned term,

$$q_{\alpha} k_{\lambda} M_{\lambda\alpha} = - 2M g_A \frac{q \cdot (q-k)}{(q-k)^2 + m_{\pi}^2} \bar{u}(h_2) \gamma_5 u(h_1) \quad (13)$$

which means that, writing $M_{\lambda\alpha} = \tilde{M}_{\lambda\alpha}(\text{Born}) + \Delta M_{\lambda\alpha}$, we must add to the Born part

$$\Delta M_{\lambda\alpha} = \frac{2M g_A}{(q-k)^2 + m_{\pi}^2} \delta_{\lambda\alpha} \bar{u}(h_2) \gamma_5 u(h_1) \quad (14)$$

which is the correct term.

When we use Eq. (12) instead, the two formulations are in agreement only if the off-shell photoproduction amplitude obeys the correct gauge condition, namely

$$\frac{i\sqrt{2} M g_A}{g_r} \frac{m_{\pi}^2}{q^2 + m_{\pi}^2} k_{\lambda} T_{\pi\lambda}^{\dagger} = \left[k_{\lambda} \langle n | J_{\lambda}^{\mu} | h \rangle - q_{\alpha} \langle n | J_{\alpha}^{\mu} | h \rangle \right] \sqrt{\frac{k_{10} k_{20}}{M^2}} \quad (15a)$$

^{*)} For simplicity we consider a constant form factor $g_r(q^2)$ in the following and only the polar part in the pseudoscalar coupling constant. Corrections are numerically small and irrelevant to our argument.

$$2Mg_A \frac{m_\pi^2}{(q-k)^2 + m_\pi^2} \bar{u}(h_2) \gamma_5 u(h_1) = (k-q)_\alpha \langle n | J_\alpha^\nu | h \rangle \sqrt{\frac{k_0 h_0}{M^2}} \quad (15b)$$

This obtains, for instance, in the case of the standard photoproduction amplitude

$$\begin{aligned} T_{\pi\lambda} = & \bar{u}(h_2) \left\{ i g_\nu \tau^j \gamma_5 \frac{1}{i\gamma(h_2+q)+M} \frac{i}{2} [\gamma_\lambda(1+\gamma^3) - \right. \\ & - \frac{\sigma_{\lambda 3} k_3}{2M} (\mu^S + \mu^V \gamma^3)] + \frac{i}{2} [\gamma_\lambda(1+\gamma^3) - \frac{\sigma_{\lambda 3} k_3}{2M} (\mu^S + \mu^V \gamma^3)] \cdot \\ & \cdot \frac{1}{i\gamma(h_1-q)+M} i g_\nu \tau^j \gamma_5 + i g_\nu [\tau^j \gamma^3] \gamma_5 \frac{\frac{1}{2}(2q-k)_\lambda}{(q-k)^2 + m_\pi^2} \left. \right\} u(h_1) \end{aligned} \quad (16)$$

The critical point is now that the linearity hypothesis introduced by Hwang-Primakoff corresponds to the dynamical assumption that the photoproduction amplitude $\langle n\gamma | J_\pi^+ | \mu \rangle$, has only the nucleon propagator. Therefore the term in which the γ is attached to the pion is explicitly omitted. Although such an amplitude leads to the correct soft-pion limit for radiative pion capture (Kroll-Rudermann), it is not gauge invariant. With this choice a counterterm must be added according to Low's prescription. Despite Huang's statement that their amplitude does not violate seriously current conservation, the pion pole term is easily shown to be essential for the fulfillment of Eq. (15) yielding a contribution $2Mg_A 2q \cdot k / ((q-k)^2 + m_\pi^2)$. Its omission would make only the second term of the left-hand side of Eq.(15b) contribute to the determination of the counterterm we are interested in. Therefore, from

$$k_\lambda q_\alpha M_{\lambda\alpha} = - 2Mg_A \frac{k(q-k)}{(q-k)^2 + m_\pi^2} \bar{u}(h_2) \gamma_5 u(h_1) \quad (17)$$

$\Delta M_{\lambda\alpha}$ of the opposite sign of Eq. (14) would follow.

Although counterterms are specific to and depend on a given set of Feynman diagrams, the total amplitude is uniquely determined to the given order. Within the framework of perturbation theory we are therefore confident that the standard gauge invariant formulation is the correct one.

As regards the nuclear excitation spectra previously used, let us first briefly review the literature.

The first detailed treatment of RMC was made by Rood and Tolhoek⁶⁾. Its main features are shell-model harmonic oscillator wave functions for the initial nucleus, an "average" maximum photon energy k_m and a closure sum over the final states. Essentially the same features have been used in an improved version by Rood-Yano and Yano³⁰⁾.

The second approach³¹⁾ corrects the shell-model results as far as the dipole part is concerned, taking its energy and strength from photoreactions, and carries over the previous calculation for the other multipoles. Also in this case a closure approximation follows naturally since the experimental giant dipole energy is the same as the incorrect monopole quadrupole shell-model energy used for the other multipoles. A refined version of this approach²⁶⁾, using new photoabsorption data, as well as a recalculation along the lines of Rood and Tolhoek with more sophisticated Hartree-Fock wave functions for the ground state, has been given recently. As stated by the authors for both models, "compared with the experiment of Hart et al., all of the absolute rates come out too high by 30% and 50%, respectively, for ordinary rates and by factors of two to three for the radiative rate in the region $k > 60$ MeV".

In the light of our present considerations, these discrepancies do not imply a quenching effect in the coupling constants but only reflect a bad choice of the mean energy, already for ordinary muon capture, in the previous models. The use of essentially the same mean energy for RMC has bigger consequences, first because the rates are respectively proportional to $\Lambda_\mu \sim (m_\mu - \bar{E}')^4$ and $\Lambda_\mu^{\text{rad}} \sim (m_\mu - \bar{E}')^6$, and second, because all of the excitation strength is considered to be energetically accessible, which is not the case (see Section 4).

The closure approximation has also been used in calculations with the unrealistic (for ^{40}Ca) Fermi gas model excitation spectrum³²⁾. The latest modifications of the closure approximation have been presented in Ref. 33).

The main ingredient is a first order expansion of the spectrum around k_m . Besides the criticism common to all closure calculations, that the high ($k > k_m$) energy part of the spectrum is intrinsically unaccounted for, the main point to be questioned in this procedure is the convergence of the expansion, i.e., the possibility to neglect higher order terms.

6. CONCLUSIONS

In the present work, the process of inclusive RMC has been thoroughly investigated. The recent debate on the form of the basic amplitude has been settled and the effective one-body Hamiltonian in use has been confirmed.

As regards the explicit evaluation of the nuclear matrix elements, the closure approximation has been avoided and the summation over final states has been transformed into an integral over the excitation spectrum. Guided by the analogy with radiative pion capture for which the spectrum is experimentally determined and which has similar axial excitation operators and momentum transfer, the excitation strength has been divided into GDR and higher resonances. The contribution of the former has been calculated from photoreactions, according to Foldy and Walecka's theorem, whereas the strength of the latter has been adjusted to reproduce the ordinary capture rate. Results are found to be irrelevant to details in shape of the higher resonances and hence to be practically model independent. A detailed numerical application is made to ^{40}Ca . With respect to earlier calculations, which use a single lower mean excitation energy, we find a smaller radiative rate.

Our predictions for the spectrum tend towards a better agreement with experiments, although the experimental situation is not yet sufficiently settled (see Section 4) so as to allow definite conclusions about the "measured" value of g_p . We therefore believe that time is ripe for more experimental effort in order to get final reliable results. Measures of photon polarization and angular correlations would be complementary in determining the value of g_p .

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k (MeV)	N(k) · sec · MeV				E' (k) MeV	P(k)				Γ(k)			
	Δ = -1	Δ = 0	Δ = 1	Δ = 2		Δ = -1	Δ = 0	Δ = 1	Δ = 2	Δ = -1	Δ = 0	Δ = 1	Δ = 2
57	3.0	2.8	3.9	6.0	21.3	0.69	0.97	0.87	0.65	1.00	0.82	0.52	
60	2.6	2.5	3.5	5.6	21.1	0.67	0.97	0.87	0.63	1.00	0.81	0.51	
63	2.3	2.2	3.1	5.1	20.8	0.64	0.97	0.86	0.62	1.00	0.80	0.50	
66	2.0	1.9	2.7	4.6	20.5	0.61	0.97	0.85	0.61	1.00	0.80	0.50	
69	1.6	1.5	2.3	4.0	20.0	0.58	0.96	0.84	0.60	1.00	0.79	0.49	
72	1.3	1.2	1.9	3.3	19.5	0.54	0.96	0.84	0.58	1.00	0.78	0.49	
75	1.0	0.9	1.5	2.6	19.0	0.51	0.96	0.83	0.57	1.00	0.77	0.48	
78	0.7	0.6	1.1	2.0	18.4	0.47	0.96	0.82	0.56	1.00	0.76	0.47	
81	0.5	0.4	0.7	1.4	17.8	0.43	0.96	0.80	0.54	1.00	0.75	0.46	
84	0.3	0.2	0.4	0.8	17.0	0.40	0.96	0.79	0.52	1.00	0.74	0.45	
87	0.1	0.1	0.2	0.4	15.8	0.37	0.96	0.78	0.51	1.00	0.73	0.43	
90	0.1	0.0	0.1	0.2	14.3	0.34	0.96	0.77	0.50	1.00	0.72	0.42	
93	0.0	0.0	0.0	0.1	12.7	0.31	0.96	0.76	0.49	1.00	0.71	0.41	
$\Lambda_\mu/10^5 \cdot \text{sec}$	32.0	27.7	24.5	22.2	24								

Predictions of RMC spectrum N(k), photon circular polarization P(k) and of muon spin photon angular correlation Γ(k) for ^{40}Ca , as a function of k and for various values of g_p . E'(k) represents the average excitation energy at each value of k, whereas the average excitation energy $\bar{E}'$ of ordinary muon capture is shown at the bottom.

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Figure Captions

- Fig. 1 : Excitation spectrum of RMC in ^{40}Ca function of the excitation energy E' and momentum transfer s .
- Fig. 2 : Differential ordinary muon capture rate $\Lambda_{\mu}(E)$ and probable number of emitted neutrons $P(E)$ function of the excitation energy E of ^{40}K .
- Fig. 3 : Photon spectrum of RMC of ^{40}Ca with the excitation spectrum used in the present calculation for $\Delta = \pm 1$ (solid line) and for $\Delta = -1$ with all the strength concentrated in giant dipole resonance (dotted line).

Fig. 1

Fig. 2

Fig. 3