Ref.TH.2805-CERN

INFRA-RED AND MASS REGULARIZATION IN AF FIELD THEORIES II : QCD

B. Humpert and W.L. van Neerven
CERN - Geneva

A B S T R A C T

We determine the non-leading QCD corrections to deep inelastic (DI) scattering and to the Drell-Yan (DY) process by analysing the order g^2 parton cross-sections. Using the insight and the techniques developed in the more simple environment of ϕ_6^3 , we regularize and subsequently remove the mass $\phi_6^{(M)}$, infra-red (IR), and ultra-violet (UV) divergences of the (double) cut graphs with the methods exposed in Paper I. The QCD calculations are technically more involved. New M divergences are found, IR singularities occur, and overlapping M and IR singularities exist leading to double poles if n dimensional regularization is used. Gluon gauge invariance must be satisfied. Although the off-shell method leads to gauge dependent parton cross-sections, the correction terms extracted via M factorization remain gauge invariant. The off-shell mass assignment was ill-defined in the past and did not satisfy the double cut rule ; we explain the implications. The correction terms are explicitly shown to be independent of any regularization method. Defining the scale dependent momentum distribution to consist of $\log Q^2$ terms only, their influence in DI scattering is moderate but considerable in DY massive $\mu\mu$ pair production for $Q^2 \leq 100 \text{ GeV}^2$.

Ref.TH.2805-CERN

31 January 1980

1. INTRODUCTION

This paper is the continuation of an earlier model-study (paper I)¹⁾ in ϕ_6^3 field theory concerning the order $\bar{g}^2$ correction to the coefficient functions of deep inelastic (DI) scattering and the Drell-Yan (DY) process.

The theoretical justification of perturbative strong interaction corrections to the parton model and their summation by renormalization group (RG) techniques in the framework of QCD, innovated a wealth of leading-log predictions²⁾. They were recently improved by evaluating the next-to-leading correction terms³⁻¹²⁾. They arise from the higher-order contributions of the anomalous dimensions and the β -function, and from the coefficient functions. The first correction type was analysed in DI scattering and found to be of moderate size³⁾. The $\bar{g}^2$ correction to the DY coefficient function, after M factorization¹³⁾ has been worked out, is however substantial⁴⁻¹¹⁾. The scale-dependent momentum distributions were assumed to contain no constant terms but logs only.

The $\bar{g}^2$ corrections of the coefficient functions are formed by the fully evaluated DI/DY parton cross-sections which are composed of UV, M and, in particular, of IR singular contributions. Their characteristics and the methods of their regularization were studied in the preceding paper I¹⁾ in the more simple framework of ϕ_6^3 field theory with no IR infinities. The analysis was mostly concerned with the consistent regularization of the M singularities using the connection between the M-finite, the single-cut-, and the double-cut-graphs. The cross-sections were determined for different mass assignments (on-shell, off-shell, massless n-dimensional), and the regularization independence of the order $\bar{g}^2$ correction term could be established.

This paper exposes the results of an analogous analysis in QCD. We note several complications. New M singularities appear. There are IR divergences which in ϕ_6^3 were suppressed by phase-space numerator terms. Overlapping M and IR divergences occur, causing double poles if dimensional regularization is used. Parton spin imposes trace calculations leading to numerator polynomials of the kinematical sub-invariants and the parton masses. It further induces mixing between different cross-section moments. Gluon (besides photon) gauge invariance must be observed.

Although the parton cross-sections, calculated with off-shell masses, do not satisfy this constraint, the correction terms emerging from M factorization are gauge-independent¹¹⁾. The wrongly defined off-shell mass assignment lead in ϕ_6^3 to the correct, regularization-independent correction terms Δ^{DI} , Δ^{DY} owing to the IR-finite characteristics of this theory¹⁾. In QCD the off-shell mass assignment must be determined consistently in agreement with the double-cut-rule¹⁴⁾, which guarantees that the M/IR singularities in the DI and DY cross-sections are prevented in the same way. Inconsistencies would otherwise appear¹⁵⁾. The above points are illustrated by explicit calculation of the parton cross-sections using on-shell, off-shell, and massless (dimensional) regularization with the individual contributions being exposed. For the strong coupling constant we use $\alpha_s = g^2/4\pi$.

This paper is organized as follows: In the subsequent two sections we determine the DI structure functions (Section 2) and the DY cross-section (Section 3) on the parton level using on- and off-shell M/IR regularization. We expose the form of each contribution and discuss the cancellation of all final-state M/IR divergences. In Section 4 the same cross-sections are evaluated using n-dimensional mass regularization^{6,9)}. We expose the formal implications of the incorrect off-shell assignment in Section 5, and expose the regularization independence of the DI/DY correction terms in Section 6. Here we also point to the theoretical and phenomenological implications of our results. Our conclusion is, as usual, placed in Section 7, and the calculation details are exposed in Appendices A-C.

2. DI SCATTERING

We explicitly evaluate the DI structure functions on the parton level. This study in the framework of perturbative QCD, with spin- $\frac{1}{2}$ quarks and spin-1 gluons, joins perfectly with our earlier presentation in the framework of ϕ_6^3 . For the definitions, conventions, and our notation, we refer to paper I¹⁾, where also the mass assignments on-shell, off-shell, and massless n-dimensional were defined. This section is sub-divided into three parts, where in the first two the structure functions with off-shell and subsequently on-shell mass assignments are evaluated. The third part is reserved for a discussion of the results and further insights.

We carry out this investigation assuming that a weak (electromagnetic) current probes the hadron structure and therefore assume a general coupling $\gamma^\mu(1 + a\gamma_5)$ at the quark-current vertex. We limit ourselves to presenting the results in the main body of the paper, and refer to the Appendices for the calculation details.

The presentation follows the scheme: scaling parton model, virtual gluon corrections, radiative gluon diagrams whose sum leads to the quark structure function, and finally we specify the gluon structure function. The individual contributions are represented by the double-cut-graphs shown in Fig. 1 which were labelled by $a_1, \dots, b_1, b_2, \dots$. In order to distinguish the corresponding structure functions, we use, for example, the notation $\hat{\mathcal{F}}_i^R(b_3)$, meaning graph b_3 of Fig. 1.

2.1 Off-shell regularization

In this subsection we use the off-shell mass assignment as it was defined in Section 2 of paper I. The quark and antiquark graphs emerging from the double-cut-rule are treated symmetrically with the choice: $m_1^2 = \mu_1^2 < 0$, $m_2^2 = \mu_2^2 < 0$, $m_8^2 = \mu_1^2 < 0$, $m_{12}^2 = \mu_2^2 < 0$, and all others vanishing. We therefore can limit ourselves to the quark graphs only.

Assuming the general coupling $\gamma^\mu(1 + a\gamma_5)$ at the electromagnetic (or weak) vertex, the 0-th order scaling parton model leads to the same structure function (in the Bj limit)

$$\hat{\mathcal{F}}_i^{(0)} = K_i \delta(1-z) \quad (2.1)$$

with $K_1 = \frac{1}{2}(1+a^2)$, $K_2 = (1+a^2)$, $K_3 = 2a$, independent of the chosen mass assignment since there are no M and IR divergences.

The virtual gluon corrections consist of the vertex corrections shown in Figs. 1a₁, a₂ and the self-energy diagrams of Figs. 1b₁, b₂, c₁, c₂. We present the final forms, referring to Appendix A for the technical details. The vertex corrections read

$$\begin{aligned} F^{V*}(a_1) = \frac{\alpha_s}{3\pi} \{ & [-\gamma + \ln \frac{M_{res}^2}{-q^2} - 2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - 2 \ln \frac{\mu_1^2}{q^2} - 2 \ln \frac{\mu_2^2}{q^2} \\ & - 4 + 2\pi^2] + \pi i [-2 \ln \frac{\mu_1^2}{q^2} - 2 \ln \frac{\mu_2^2}{q^2} - 4] \} \end{aligned} \quad (2.2)$$

$$F^{V_x}(a_2) = \frac{\alpha_s}{3\pi} \left[-\zeta + \ln \frac{\mu_{reg}^2}{-q^2} - 2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - 2 \ln \frac{\mu_1^2}{q^2} - 2 \ln \frac{\mu_2^2}{q^2} - \frac{2\pi^2}{3} \right] \quad (2.3)$$

In the above expressions we have introduced the strong coupling constant $\alpha_s \equiv g^2/4\pi$, and the colour factor (4/3) is already included. The abbreviation

$$\zeta = \frac{2}{n-4} + \gamma - \ln 4\pi \quad (2.4)$$

was used. Notice the anomalous cut contributions in Eq. (2.2) due to the external mass regular μ_1^2 and the intermediate mass regulator μ_2^2 . The double-logs reflect the overlapping IR and M divergences.

Similar analysis of the self-energy diagrams shown in Figs. 1b₁+b₂ and 1c₁+c₂ leads to

$$\Sigma'_1(b_1+b_2) = \frac{\alpha_s}{3\pi} \left[\zeta - \ln \frac{\mu_{reg}^2}{-p_1^2} - \frac{1}{2} + \pi i \right] \quad (2.5)$$

$$\Sigma'_2(c_1+c_2) = \frac{\alpha_s}{3\pi} \left[\zeta - \ln \frac{\mu_{reg}^2}{-\mu_2^2} - \frac{1}{2} + \pi i \right] \quad (2.6)$$

Here again we notice the appearance of anomalous cuts due to the external regulator mass $\mu_1^2 < 0$ [in Eq. (2.5)] and the intermediate-state regulator mass $\mu_2^2 < 0$ [in Eq. (2.6)]. Each of the terms in Eqs. (2.2)-(2.6) contributes to the structure functions as

$$\hat{F}_i^{V_x}(a_1) = k_i \delta(1-z) F^{V_x}(a_1) \quad (2.7)$$

and similarly for the others. The sum of virtual-gluon graphs forms the renormalized vertex function with

$$2F^V = F^{V_x}(a_1) + F^{V_x}(a_2) + \Sigma'_1(b_1+b_2) + \Sigma'_2(c_1+c_2) \quad (2.8)$$

$$2F^V = 2 \frac{\alpha_s}{3\pi} \left\{ \left[-2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - \frac{3}{2} \ln \frac{\mu_1^2}{q^2} - \frac{3}{2} \ln \frac{\mu_2^2}{q^2} - \frac{5}{2} + \frac{2\pi^2}{3} \right] + \pi i \left[-\ln \frac{\mu_1^2}{q^2} - \ln \frac{\mu_2^2}{q^2} - 1 \right] \right\} \quad (2.9)$$

F^V is called the Sudakov vertex function¹⁶⁾. The factor 2 in front of the double log in Eq. (2.9) is a typical feature of such off-shell vertex correction.

We analyse the real-gluon graphs shown in Figs. 5a₃, a₄, b₃, c₃. The structure functions $\hat{F}_i^R$ of all these contributions are determined in Appendix B. Since the off-shell mass assignment together with the double-cut-rule imposes a different set of masses for each of them, we will find different log-arguments in the expressions corresponding to Figs. 1a₃, a₄, b₃ but no log-function in c₃. Imposing the scaling limit, the structure functions of the real-gluon graphs are

$$\hat{F}_i^R(a_3) = \frac{\alpha_s}{3\pi} K_i \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 \mu_1^2} + 2 - \frac{2}{1-z} - \pi i \left(\frac{2z}{1-z} \right) \right] \quad (i=1,2) \quad (2.10)$$

$$\hat{F}_3^R(a_3) = \frac{\alpha_s}{3\pi} K_3 \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 \mu_1^2} + 3 - \frac{2}{1-z} - \pi i \left(\frac{2z}{1-z} \right) \right] \quad (2.11)$$

$$\hat{F}_i^R(a_4) = \frac{\alpha_s}{3\pi} K_i \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 \mu_1^2} + 2 - \frac{2}{1-z} \right] \quad (i=1,2) \quad (2.12)$$

$$\hat{F}_3^R(a_4) = \frac{\alpha_s}{3\pi} K_3 \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 \mu_1^2} + 3 - \frac{2}{1-z} \right] \quad (2.13)$$

$$\hat{F}_1^R(b_3) = \frac{2\alpha_s}{3\pi} K_1 \left[(1-z) \ln \frac{q^2}{z^2 \mu_1^2} - 1 + z \right] \quad (2.14)$$

$$\hat{F}_2^R(b_3) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln \frac{q^2}{z^2 \mu_1^2} - 1 + 3z \right] \quad (2.15)$$

$$\hat{\mathcal{F}}_3^R(b_3) = \frac{2\alpha_s}{3\pi} K_3 \left[(1-z) \ln \frac{q^2}{z^2 \mu_1^2} - 3 + 2z \right] \quad (2.16)$$

$$\hat{\mathcal{F}}_i^R(c_3) = \frac{2\alpha_s}{3\pi} K_i \left[\frac{1}{2} \frac{1}{1-z} \right] \quad (i=1,2,3) \quad (2.17)$$

Notice that the anomalous cut contribution to $\hat{\mathcal{F}}_i^R(a_3)$ in Eqs. (2.10) and (2.11) is due to the external regulator mass $\mu_1^2 < 0$.

We determine the soft-gluon integrals. Inspecting Eqs. (2.10)-(2.17) we notice that the real-gluon graphs give rise to $z = 1$ singularities. This is a reflection of the M and IR divergences. In the scaling limit all masses drop out. We therefore cannot distinguish between M and IR singularities; both appear as a pole $(1-z)^{-1}$. This also holds for $m^2 = 0$, where the off-shell regulators μ_1^2 prevent both M and IR singularities. We remind the reader of the arguments in Section 2 of paper I. In the $z \rightarrow 1$ limit the intermediate state $|f\rangle$ of an inelastic gluon graph becomes degenerate with the elastic virtual-gluon diagrams. Consequently, if these contributions are added, all intermediate M and IR singularities are expected to disappear. The soft-gluon contribution is defined as (see paper I, Section 3)

$$\hat{\mathcal{F}}_i^{R-\text{soft}} = \delta(\hat{s} - m_f^2) \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s}' \hat{\mathcal{F}}_i^R(\hat{s}', q^2, m^2, \mu_i^2) \quad (2.18)$$

where m_f^2 is the intermediate-state mass of those elastic graphs which are needed to cancel the intermediate M and IR singularities of the inelastic graphs (e.g. Fig. 1a₁ is complementary to 1a₃, therefore $m_f^2 = m_3^2$); $\hat{s}_0$ is the threshold energy of the intermediate state in the inelastic graphs (e.g. $\hat{s}_0 = \mu_2^2$ in Fig. 1a₃). Unlike in the hard-gluon contributions [see Eqs. (2.10)-(2.17)], here we may not neglect the intermediate masses since the integral would diverge at its lower boundary; μ_2^2 , for example, cannot be neglected whilst calculating $\hat{\mathcal{F}}_i^{R-\text{soft}}(a_3)$. A short inspection of Eqs. (2.10)-(2.17) leads us to the conclusion that only Figs. 1a₃, a₄, c₃ have soft-gluon contributions with the results (for calculation details see Appendix B).

$$\hat{F}_i^{R\text{-soft}}(a_3) = \delta(1-z) \frac{\alpha_s}{3\pi} K_i \left\{ \left[2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - 2 \ln \epsilon \ln \frac{\mu_1^2}{q^2} \right. \right. \quad (2.19)$$

$$\left. \left. + 2 \ln \frac{\mu_2^2}{q^2} - 2 \ln \epsilon + 2 - \frac{7\pi^2}{3} \right] + \pi i \left[2 \ln \frac{\mu_1^2}{q^2} + 2 \ln \frac{\mu_2^2}{q^2} - 2 \ln \epsilon + 2 \right] \right\}$$

$$\hat{F}_i^{R\text{-soft}}(a_4) = \delta(1-z) \frac{\alpha_s}{3\pi} K_i \left[2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - 2 \ln \epsilon \ln \frac{\mu_1^2}{q^2} \right. \quad (2.20)$$

$$\left. + 2 \ln \frac{\mu_2^2}{q^2} - 2 \ln \epsilon \right]$$

$$\hat{F}_i^{R\text{-soft}}(c_3) = \delta(1-z) \frac{2\alpha_s}{3\pi} K_i \left[-\frac{1}{2} \ln \frac{\mu_2^2}{q^2} + \frac{1}{2} \ln \epsilon - \frac{1}{4} - \frac{1}{2} \pi i \right] \quad (2.21)$$

Notice that expression (2.19) has anomalous cuts in the external mass regulator $\mu_1^2 < 0$ and in the intermediate mass regulator $\mu_2^2 < 0$, similar to $\hat{F}^{Vx}(a_1)$ in Eq. (2.2). Expression (2.21) has anomalous cuts resulting from the intermediate mass regulator $\mu_2^2 < 0$ only. The soft-gluon contributions are the same for all structure functions, as for the virtual-gluon graphs. If all terms are added, all intermediate mass regulators μ_2^2 [and, with them, all intermediate (state) anomalous cuts] must disappear. This cancellation can be checked by forming the expressions

$$\hat{F}_i^{Vx}(a_1) + \hat{F}_i^{R\text{-soft}}(a_3) ; \hat{F}_i^{Vx}(a_2) + \hat{F}_i^{R\text{-soft}}(a_4) ; \hat{F}_i^{S_2}(c_1+c_2) + \hat{F}_i^{R\text{-soft}}(c_3) \quad (2.22)$$

as was pointed out in Sections 2 and 3 of paper I.

The complete sum of order g^2 contributions forming the parton structure function with a quark target is given by

$$\hat{F}_{i,q} = K_i \left\{ \delta(1-z) + \frac{2\alpha_s}{3\pi} \left[\left\{ \frac{1+z^2}{1-z} \ln \frac{q^2}{z^2 \mu_1^2} - 1 + h_1(z) - \frac{3}{2} \frac{1}{1-z} + \frac{z \ln z}{1-z} \right. \right. \right. \quad (2.23)$$

$$\left. \left. - \pi i \left(\frac{z}{1-z} \right) \right\} + \delta(1-z) \left\{ \left(2 \ln \epsilon + \frac{3}{2} \right) \ln \frac{q^2}{\mu_1^2} - \frac{3}{2} \ln \epsilon - \frac{\pi^2}{2} - \frac{7}{4} - \pi i \left(\ln \epsilon + \frac{1}{2} \right) \right\} \right] \right\}$$

where

$$h_1(z) = 2 + z \quad h_2(z) = 2 + 3z \quad h_3(z) = 1 + 2z \quad (2.24)$$

Exchanging $\mu_1^2 \leftrightarrow \mu_2^2$ leads to the parton structure functions with an antiquark target (Figs. 1a₅, ..., a₈, etc.). In expressions (2.23), only the initial mass regulators and their anomalous cuts are left. The anomalous cuts appear in all parton cross-sections in the same way. This implies that the order $\bar{g}^2$ correction terms to DI scattering and to the DY process (after M factorization has been carried out) will be real.

The initial gluon structure function is based on Figs. 2a₁-c₁. The off-shell mass assignment for this set of graphs, according to Section 2 of paper I, is: $m_5^2 = \lambda^2$, $m_6^2 = q^2$, and $m_i^2 = 0$ otherwise. We include the colour factor (1/2) and obtain (see Appendix B for details):

$$\hat{F}_{i,g} = \frac{\alpha_s}{2\pi} K_i \left[\{z^2 + (1-z)^2\} \ln \frac{q^2}{z^2 \lambda^2} - 1 + g_i(z) \right] \quad (i=1,2) \quad (2.25)$$

$$\hat{F}_{3,g} = 0 \quad (2.26)$$

with $\lambda^2 < 0$ and

$$g_1(z) = 2z(1-z) - 1 \quad g_2(z) = 6z(1-z) - 1 \quad (2.27)$$

The reader is reminded that this result involves contributions from an intermediate state consisting of quarks as well as antiquarks.

2.2 On-shell regularization

In contrast to the off-shell technique which we used in the preceding subsection, the on-shell mass assignment ($m_5^2 = \lambda^2$, $m_6^2 = q^2$, and $m_i^2 = m^2$ otherwise) permits clear distinction between the IR and the M divergences. We reconsider the graphs of Fig. 1 following the same steps and the same notational conventions as above.

The vertex corrections give

$$\begin{aligned} F^{V_x}(a_1) = F^{V_x}(a_2) = & \frac{\alpha_s}{3\pi} \left[-\zeta + \ln \frac{M_{reg}^2}{-q^2} - \ln^2 \frac{m^2}{-q^2} \right. \\ & \left. + 2 \ln \frac{m^2}{-q^2} \ln \frac{m^2}{\lambda^2} - 4 \ln \frac{m^2}{-q^2} + \frac{\pi^2}{3} \right] \end{aligned} \quad (2.28)$$

The product of log-terms $\ln m \cdot \ln \lambda$ is a reflection of the overlapping M and IR divergences. Likewise, the analysis of the self-energy corrections renders

$$\sum_1' (b_1 + b_2) = \sum_2' (c_1 + c_2) = \frac{\alpha_s}{3\pi} \left[\gamma - \ln \frac{M_{\text{res}}^2}{m^2} + 2 \ln \frac{m^2}{\lambda^2} - 4 \right] \quad (2.29)$$

Using Eq. (2.28) we obtain the renormalized vertex function

$$2F^V = \frac{2\alpha_s}{3\pi} \left[-\ln^2 \frac{m^2}{-q^2} + 2 \ln \frac{m^2}{-q^2} \ln \frac{m^2}{\lambda^2} - 3 \ln \frac{m^2}{-q^2} + 2 \ln \frac{m^2}{\lambda^2} + \frac{\pi^2}{3} - 4 \right] \quad (2.30)$$

Notice the coefficient in front of the $\ln^2$ -contribution, which is distinct from the analogous term in Eq. (2.9). This is a characteristic feature of the on-shell vertex function and distinguishes its form from the Sudakov form factor in Eq. (2.9).

The graphs describing real-gluon production (Figs. 1a₃, a₄, b₃, c₃) were evaluated in Appendix B for general mass assignment as indicated in Fig. 1. The complete on-shell result is

$$\hat{F}_i^{\text{R-hard}} = \frac{2\alpha_s}{3\pi} k_i \left[\frac{1+z^2}{1-z} \ln \frac{-q^2}{z(1-z)m^2} + 2 + h_i(z) - \frac{7}{2} \frac{1}{1-z} \right] \quad (2.31)$$

with $h_i(z)$ defined in Eq. (2.24). Note that we have proceeded to the scaling limit where all masses drop out (apart from those which indicate an M singularity). In Eq. (2.31) we notice the appearance of a $z = 1$ singularity which forces us to analyse its origin more carefully by evaluating the soft-gluon integrals. We do not proceed in exactly the same way as that exposed in Section 3 of paper I, but calculate the structure function in the same way as is done in QED. This by now standard method^{4,17)} separates the integral over the gluon momentum in a soft-piece, where $0 < |\vec{k}| \leq \omega_0 \ll m$, and a hard piece with $|\vec{k}| > \omega_0$. In the soft-gluon integral a gluon mass is introduced in order to prevent the IR divergence at $|\vec{k}| = 0$. Since $\omega_0 \ll m$, all terms in the matrix element proportional to k_ν can be neglected. Consequently, all numerator spin- $\frac{1}{2}$ terms disappear, and the "convective" part of the (electromagnetic or weak) current only survives:

$$M_\mu^2 = -ig \bar{u}_2 \lambda^a \gamma_\mu (1 + a\gamma_5) u_1 \left[\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right]_\nu \cdot \epsilon^{\nu*} \quad (2.32)$$

The first (second) term in the square bracket emerges from Fig. 7a (Fig. 7b). The spinor form of Eq. (2.32) is identical with the 0-th order matrix element. The soft-contribution to the structure functions reads,

$$\hat{\mathcal{F}}_i^{\text{R-conv}} \equiv \delta(1-z) k_i \frac{2\alpha_s}{3\pi^2} \int_0^{\omega_0} \frac{d^3k}{2\sqrt{|k|^2 + \lambda^2}} \left[\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right]^2 \quad (2.33)$$

Equation (2.33) is evaluated by forming the square which leads to four contributions: the two squared terms corresponding to Figs. 1b₃, c₃ and the two interference terms represented by Figs. 1a₃, a₄. Each of them is evaluated in the rest frame of the initial quark giving

$$\hat{\mathcal{F}}_i^{\text{R-conv}}(a_3) = \hat{\mathcal{F}}_i^{\text{R-conv}}(a_4) = \delta(1-z) \frac{\alpha_s}{3\pi} k_i \left[-2 \ln^2 \frac{m^2}{-q^2} - 2 \ln \frac{4\omega_0^2}{\lambda^2} \ln \frac{m^2}{-q^2} - \frac{\pi^2}{3} \right] \quad (2.34)$$

$$\hat{\mathcal{F}}_i^{\text{R-conv}}(b_3) = \delta(1-z) \frac{2\alpha_s}{3\pi} k_i \left[-2 \ln \frac{m^2}{-q^2} - \ln \frac{4\omega_0^2}{\lambda^2} \right] \quad (2.35)$$

$$\hat{\mathcal{F}}_i^{\text{R-conv}}(c_3) = \delta(1-z) \frac{2\alpha_s}{3\pi} k_i \left[-\ln \frac{4\omega_0^2}{\lambda^2} + 2 \right] \quad (2.36)$$

These results permit us to check the cancellation of all final-state IR singularities by forming the combinations listed in Eq. (2.22). The additional combination,

$$\hat{\mathcal{F}}_i^{\text{R-conv}}(b_3) + \hat{\mathcal{F}}_i^{S_1}(b_1 + b_2) \quad (2.37)$$

certifies that all initial-state IR singularities drop out. We form these combinations using the above results, and indeed find that all λ -dependence cancels.

However, the intermediate-state m -dependence does not disappear, as was the case with the off-shell results. The reason is clear: we have dropped all k_γ -dependence in the soft-contributions of R-conv., but we have kept it whilst analysing the virtual-gluon graphs.

In order to determine the soft-gluon integrals as determined in Eq. (2.18), we use the connection

$$\begin{aligned}\hat{\mathcal{F}}_i^{\text{R-soft}}(\epsilon) &= \delta(\hat{s}-m^2) \left[\int_{(m+\lambda)^2}^{(m+\omega_0)^2} + \int_{(m+\omega_0)^2}^{-\epsilon q^2} d\hat{s}' \dots \right] \\ &= \hat{\mathcal{F}}_i^{\text{R-conv}}(\omega_0) + \mathbb{L}_i(\omega_0, \epsilon)\end{aligned}\quad (2.38)$$

All λ -dependence can be dropped in the second integral $\mathbb{L}_i$ since the IR divergence is prevented by the cut-off ω_0 . We use the replacement $\hat{s} = -q^2(1-z+z_m)/z$, $z_m = (m^2/-q^2)z$, and re-evaluate the structure functions of all graphs keeping, however, all first-order $(m^2/-q^2)$ correction terms. One finds

$$\hat{\mathcal{F}}_i^{\text{R}}(a_3) = \hat{\mathcal{F}}_i^{\text{R}}(a_4) = \frac{\alpha_s}{3\pi} K_i \left[\frac{2z}{1-z} \text{Log} + a_i - \frac{2}{1-z} \right] \quad (2.39)$$

$$\hat{\mathcal{F}}_i^{\text{R}}(b_3) = \frac{2\alpha_s}{3\pi} K_i \left[(1-z) \text{Log} + b_i - \frac{2}{1-z+z_m} \right] \quad (2.40)$$

$$\hat{\mathcal{F}}_i^{\text{R}}(c_3) = \frac{2\alpha_s}{3\pi} K_i \left[-\frac{z_m}{2(1-z+z_m)^2} + \frac{5}{2(1-z+z_m)} - \frac{2}{1-z} \right] \quad (2.41)$$

where

$$\text{Log} \equiv \text{Ln} \left\{ -\frac{q^2}{m^2} \frac{1}{z(1-z+z_m)} \right\} \quad (2.42)$$

and

$$a_1 = a_2 = 2 \quad ; \quad a_3 = 3 \quad (2.43)$$

$$b_1 = 2+z \quad ; \quad b_2 = 2+3z \quad ; \quad b_3 = 3z \quad (2.44)$$

$1-z$ originates from $\hat{s}-m^2$, and $1-z-z_m$ is traced back to $\hat{s}$. We introduce $\delta \equiv (2\omega_0/m)(m^2/-q^2)$ and explicitly evaluate

$$\mathbb{L}_i = \delta(\hat{s}-m^2) \int_{m^2+2m\omega_0}^{-\epsilon q^2} d\hat{s}' \hat{\mathcal{F}}_i^{\text{R}}(\hat{s}', q^2, m^2) = \delta(1-z) \int_{1-\epsilon}^{1-\delta} d\hat{z} \hat{\mathcal{F}}_i^{\text{R}}(\hat{z}, \dots) \quad (2.45)$$

for each graph. This procedure leads us to the results

$$\hat{\mathcal{F}}_i^{R-SOFT}(a_3) = \hat{\mathcal{F}}_i^{R-SOFT}(a_4) = \delta(1-z) \frac{\alpha_s}{3\pi} K_i \left[\ln^2 \frac{m^2}{-q^2} + 2 \ln \frac{m^2}{-q^2} - 2 \ln \frac{m^2}{\lambda^2} \ln \frac{m^2}{-q^2} - 2 \ln \epsilon \ln \frac{m^2}{-q^2} - \ln^2 \epsilon - 2 \ln \epsilon - \frac{2\pi^2}{3} \right] \quad (2.46)$$

$$\hat{\mathcal{F}}_i^{R-SOFT}(b_3) = \delta(1-z) \frac{2\alpha_s}{3\pi} K_i \left[-\ln \frac{m^2}{\lambda^2} - 2 \ln \epsilon \right] \quad (2.47)$$

$$\hat{\mathcal{F}}_i^{R-SOFT}(c_3) = \delta(1-z) \frac{2\alpha_s}{3\pi} K_i \left[-\frac{1}{2} \ln \frac{m^2}{-q^2} - \ln \frac{m^2}{\lambda^2} + \frac{1}{2} \ln \epsilon + \frac{3}{2} \right] \quad (2.48)$$

which also could have been obtained by directly integrating Eq. (2.18) as was done in Ref. 10. The above results are in complete agreement with those of Ref. 10.

From Eqs. (2.30) and (2.31) and Eqs. (2.46)-(2.48) we obtain the complete on-shell quark structure function^{4,10}:

$$\hat{\mathcal{F}}_{i,q} = K_i \left\{ \delta(1-z) + \frac{2\alpha_s}{3\pi} \left[\left\{ \frac{1+z^2}{1-z} \ln \frac{-q^2}{z(1-z)m^2} + 2 + h_i(z) - \frac{7}{2} \frac{1}{1-z} \right\} + \delta(1-z) \left\{ (2 \ln \epsilon + \frac{3}{2}) \ln \frac{-q^2}{m^2} - \ln^2 \epsilon - \frac{7}{2} \ln \epsilon - \frac{5}{2} - \frac{\pi^2}{3} \right\} \right] \right\} \quad (2.49)$$

with the $h_i(z)$ defined in Eq. (2.24).

The on-shell form of the gluon structure function, using $\lambda^2 = 0$, $m_1^2 = m^2$, is derived in Appendix B, with the result^{4,10}

$$\hat{\mathcal{F}}_{i,g} = \frac{\alpha_s}{2\pi} K_i \left[\{z^2 + (1-z)^2\} \ln \frac{-q^2(1-z)}{z m^2} + 2z(1-z) + g_i(z) \right] \quad (2.50)$$

$$\hat{\mathcal{F}}_{3,g} = 0 \quad (2.51)$$

The $g_i(z)$ are defined in Eq. (2.27).

2.3 Analysis of the parton structure functions

In Section 2 of paper I¹⁾ we discussed the cancellation pattern of the IR and the M singularities as it occurs among the various cut-graphs of Figs. 1 to 4.

Since all structure functions now are known for on-shell and off-shell mass assignments, we are able to illustrate some of the points which emerged from this first study.

The M and IR singularities originating from the intermediate-state lines (indicated by the vertical dashed line B) appear in Figs. 1a₁-a₄, b₃, c₃. The KLN theorem, however, imposes that all intermediate M and IR singularities disappear in the appropriate sum of diagrams. This is checked by forming the subsequent combinations, which are expected to be singularity-free. Using the off-shell results of Section 2.1, we find

$$\hat{\mathcal{F}}_i^{V_x}(a_1) + \hat{\mathcal{F}}_i^{R\text{-soft}}(a_3) = \frac{\alpha_s}{3\pi} K_i \delta(1-z) \left\{ \left[-\gamma + \ln \frac{M_{\text{reg}}^2}{-q^2} - (2\ln\epsilon + 2) \left(\ln \frac{M_i^2}{q^2} + 1 \right) - \frac{\pi^2}{3} \right] - \pi i [2\ln\epsilon + 2] \right\} \quad (2.52)$$

$$\hat{\mathcal{F}}_i^{V_x}(a_2) + \hat{\mathcal{F}}_i^{R\text{-soft}}(a_4) = \frac{\alpha_s}{3\pi} K_i \delta(1-z) \left[\gamma + \ln \frac{M_{\text{reg}}^2}{-q^2} - (2\ln\epsilon + 2) \ln \frac{M_i^2}{q^2} - 2\ln\epsilon - \frac{2\pi^2}{3} \right] \quad (2.53)$$

$$\hat{\mathcal{F}}_i^{S_2}(c_1+c_2) + \hat{\mathcal{F}}_i^{R\text{-soft}}(c_3) = \frac{\alpha_s}{3\pi} K_i \delta(1-z) \left[\gamma - \ln \frac{M_{\text{reg}}^2}{-q^2} + \ln\epsilon - 1 \right] \quad (2.54)$$

μ_2^2 , which regulates the intermediate M and IR singularity, has completely disappeared. Our claim that the intermediate-state infinities cancel is correct.

The analogous expressions for on-shell regularization are

$$\hat{\mathcal{F}}_i^{V_x}(a_1) + \hat{\mathcal{F}}_i^{R\text{-soft}}(a_3) = \hat{\mathcal{F}}_i^{V_x}(a_2) + \hat{\mathcal{F}}_i^{R\text{-soft}}(a_4) = \frac{\alpha_s}{3\pi} K_i \delta(1-z) \left[-\gamma + \ln \frac{M_{\text{reg}}^2}{-q^2} - (2\ln\epsilon + 2) \ln \frac{m_i^2}{-q^2} - \ln\epsilon - 2\ln\epsilon - \frac{\pi^2}{3} \right] \quad (2.55)$$

$$\hat{\mathcal{F}}_i^{S_2}(c_1+c_2) + \hat{\mathcal{F}}_i^{R\text{-soft}}(c_3) = \frac{\alpha_s}{3\pi} K_i \delta(1-z) \left[\gamma - \ln \frac{M_{\text{reg}}^2}{-q^2} + \ln\epsilon - 1 \right] \quad (2.56)$$

The infrared regulator λ has disappeared; therefore all intermediate IR singularities, as expected, are cancelled. The intermediate mass regulators also drop out. This is seen by comparing the coefficient in front of the logarithm $\ln m_i^2/-q^2$ in Eqs. (2.53) and (2.55). The expressions (2.54) and (2.56) are equal and IR/M-finite. This follows from the virtual Compton graph, Fig. 3d₁, which has no initial

M singularities. The Compton graphs in Figs. 3b₁,c₁ instead reveal initial-state M singularities and therefore depend on the chosen mass assignment.

We want to discuss cancellation of the initial-state IR singularities. In paper I we came to the conclusion that they should disappear in the sum of graphs Figs. 1b₁-b₃ or Figs. 3a₂,f₂. This is explicitly checked. We use the off-shell expressions of Section 2.1 and find

$$\hat{\mathcal{F}}_i^{S_1}(b_1+b_2) + \hat{\mathcal{F}}_i^{R\text{-soft}}(b_3) = \frac{\alpha_s}{3\pi} k_i \delta(1-z) \left[\zeta - \ln \frac{M_{reg}^2}{-\mu_i^2} - \frac{1}{2} + \pi i \right] \quad (2.57)$$

Notice that $\hat{\mathcal{F}}_i^{\text{soft}}(b_3) = 0$; there is no $(1-z)^{-1}$ pole for off-shell mass assignment.

The analogous calculation with on-shell mass choice leaves

$$\hat{\mathcal{F}}_i^{S_1}(b_1+b_2) + \hat{\mathcal{F}}_i^{R\text{-soft}}(b_3) = \frac{\alpha_s}{3\pi} k_i \delta(1-z) \left[\zeta - \ln \frac{M_{reg}^2}{m^2} - 4 \ln \epsilon - 4 \right] \quad (2.58)$$

The leftover M singularities originate from the initial lines. The expressions in Eqs. (2.57) and (2.58) differ since the M divergences were regularized in a different way.

We are not limited to the regularization methods (namely mass assignments) which we have discussed in Sections 2.1 and 2.2; many others are possible. Two of them deserve a short comment. The off-shell mass assignment was fixed by $m_1^2 = m^2 + \mu_1^2$, $m_2^2 = m^2 + \mu_2^2$, etc. with $m^2 = 0$. Had we set $m^2 \neq 0$ with $m^2 \gg \mu_i^2$ instead, μ_i^2 would act as an IR regulator and m^2 would prevent the M singularities^{1,15}). Repeating all computations of Section 2.1 in the same way, we find $\hat{\mathcal{F}}_{i,q}$ of Eq. (2.49), which was obtained by using on-shell regularization. This is not surprising since in $\hat{\mathcal{F}}_{i,q}$ only the initial M singularity is left, which in both cases is regularized in the same way. This insight also holds for small q^2 values, which implies that off-shell regularization can equally well be used to overcome QED divergence problems. We emphasize that the above findings apply exclusively to the complete sum of terms contributing to $\hat{\mathcal{F}}_{i,q}$ but not to its individual members such as $\hat{\mathcal{F}}_i^V$ and $\hat{\mathcal{F}}_i^{\text{soft}}$. We realize that the on-/off-shell mass assignments have no physical meaning; they only indicate the way in which the M/IR singularities are regularized. This

fact is often ignored in the literature, and models with arbitrary mass parameters are used for comparison with the data -- a procedure which is certainly not correct. An alternative on-shell technique can be defined. The IR divergence is replaced by dimensional regularization assuming $\lambda^2 = 0$ but $m_1^2 = m^2 \neq 0$. The virtual and bremsstrahlung contributions are modified, and the $\ln \lambda$ term can be seen to appear as an $(n-4)^{-1}$ pole. We compare Eqs. (2.28) and (2.29) and (2.34)-(2.36) with Eqs. (21.8) and (20.13) in Ref. 18, and note that the substitution formula

$$\ln \lambda^2 \longrightarrow \ln M_{\text{reg}}^2 - \zeta \quad (2.59)$$

with ζ defined in Eq. (2.4) will establish the connection.

3. THE DY PROCESS

The analysis of the DY cross-sections in order g^2 QCD perturbation theory proceeds along the same steps and uses the same techniques as those discussed in paper I¹⁾. For an introduction to the Feynman diagrams of this process (Figs. 5 and 6), the basic formalism, and the notation, we refer to our earlier study in ϕ_6^3 . We here slightly generalize the analysis by evaluating the matrix element with a quark-current vertex $\gamma^\mu(1+a\gamma_5)$ which describes the electromagnetic ($a = 0$) and the weak interactions ($a = \pm 1$) at the same time. We will use the off-shell and, subsequently, the on-shell regularization procedure to prevent the IR and M singularities. The specific mass choices were defined in Section 2.

3.1 Off-shell regularization

The 0-th order cross-section in the scaling limit

$$\hat{W}^{(0)} = (1+a^2) \delta(1-z) \quad \left(z \equiv \frac{\tau}{x_1 \cdot x_2} \right) \quad (3.1)$$

is independent of the mass assignments since there are no infinities.

The mass assignment for the order g^2 contributions has been given in Section 2.1. Following the notational convention of Section 2, we analyse the initial quark graphs exposed in Fig. 5. The colour factor (4/3) is included in the subsequent results. The vertex corrections, Figs. 5a₁, a₂, follow from our DI results [Eqs. (22), (23)] by analytic continuation to time-like q^2 values:

$$F^{V_x}(a_1) = \frac{\alpha_s}{3\pi} \left\{ \left[-\gamma + \ln \frac{M_{reg}^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 4 \right] \right. \\ \left. + \pi i \left[-2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 4 \right] + \pi i \left[2 \ln -\frac{\mu_1^2}{Q^2} + 2 \ln -\frac{\mu_2^2}{Q^2} + 3 \right] \right\} \quad (3.2)$$

$$F^{V_x}(a_2) = \frac{\alpha_s}{3\pi} \left\{ \left[-\gamma + \ln \frac{M_{reg}^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} \right. \right. \\ \left. \left. - 2 \ln -\frac{\mu_2^2}{Q^2} + \frac{4\pi^2}{3} \right] + \pi i \left[2 \ln -\frac{\mu_1^2}{Q^2} + 2 \ln -\frac{\mu_2^2}{Q^2} + 3 \right] \right\} \quad (3.3)$$

Considering Eq. (3.2) we notice two cut contributions. One arises from the anomalous cut since $\mu_1^2 < 0$, which is already present in the DI result Eq. (2.2). The second one originates from the real (unitarity) cut which starts at $Q^2 > -(\mu_1 + \mu_2)^2$. It is sometimes called the Coulomb phase. This second cut contribution is also seen in Eq. (3.3). Since the vertex correction in graph a_1 arises from the complex conjugate matrix element in the interference term, we need to determine its form below the unitarity cut. Note that only the Coulomb phase is changed, and not the anomalous cut contribution. One finds

$$F^{V_x^*}(a_1) = \frac{\alpha_s}{3\pi} \left\{ \left[-\gamma + \ln \frac{M_{reg}^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 4 \right] \right. \\ \left. + \pi i \left[-2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 4 \right] - \pi i \left[2 \ln -\frac{\mu_1^2}{Q^2} + 2 \ln -\frac{\mu_2^2}{Q^2} + 3 \right] \right\} \quad (3.4)$$

The self-energy contributions lead to

$$\Sigma'_1(b_1 + b_3) = \frac{\alpha_s}{3\pi} \left[\gamma - \ln \frac{M_{reg}^2}{-\mu_1^2} - \frac{1}{2} + \pi i \right] \quad (3.5)$$

$$\Sigma'_2(c_1 + c_3) = \frac{\alpha_s}{3\pi} \left[\gamma - \ln \frac{M_{reg}^2}{-\mu_2^2} - \frac{1}{2} + \pi i \right] \quad (3.6)$$

We apply the same notational convention as in Eq. (2.7) and set

$$\hat{W}^{V_x}(a_i) = K_2 \delta(1-z) F^{V_x}(a_i) \text{ etc.} \quad (3.7)$$

with K_2 defined as in Eq. (2.1). The renormalized vertex correction is formed

$$2 F^V \equiv F^{V_x^*}(a_1) + F^{V_x}(a_2) + \sum_1 (b_1 + b_2) + \sum_2 (c_1 + c_2) = \quad (3.8)$$

$$\frac{2\alpha_s}{3\pi} \left\{ \left[-2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 2 + \frac{2\pi^2}{3} \right] + \right. \\ \left. \pi i \left[-\ln -\frac{\mu_1^2}{Q^2} - \ln -\frac{\mu_2^2}{Q^2} - 2 \right] \right\} \quad (3.9)$$

The real hard-gluon contributions from the diagrams Figs. 5a₃, a₄, b₂, c₂ result in the expressions

$$\hat{W}^R(a_3) = \frac{\alpha_s}{3\pi} K_2 \left[\frac{2z}{1-z} \ln -\frac{(1-z)Q^2}{z^2\mu_1^2} + \frac{2z}{1-z} \ln -\frac{(1-z)Q^2}{z^2\mu_2^2} - \pi i \left(\frac{2z}{1-z} \right) \right] \quad (3.10)$$

$$\hat{W}^R(a_4) = \frac{\alpha_s}{3\pi} K_2 \left[\frac{2z}{1-z} \ln -\frac{(1-z)Q^2}{z^2\mu_1^2} + \frac{2z}{1-z} \ln -\frac{(1-z)Q^2}{z^2\mu_2^2} - \pi i \left(\frac{2z}{1-z} \right) \right] \quad (3.11)$$

$$\hat{W}^R(b_2) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln -\frac{(1-z)Q^2}{z^2\mu_1^2} - 2 + z \right] \quad (3.12)$$

$$\hat{W}^R(c_2) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln -\frac{(1-z)Q^2}{z^2\mu_2^2} - 2 + z \right] \quad (3.13)$$

Again we find anomalous cuts, appearing in the expressions (3.10) and (3.11), due to the fact that the external regulator masses were chosen as $\mu_1^2 < 0$. There is a $\mu_1^2 \leftrightarrow \mu_2^2$ symmetry between Eqs. (3.10), (3.11) and Eqs. (3.12), (3.13), which reflects the symmetrically chosen off-shell mass assignment in order to arrive at equal quark and antiquark structure functions.

We focus our attention on the soft-gluon contributions. A short inspection of Eqs. (3.10) and (3.11) with their $(1-z)^{-1}$ poles directs us to the graphs a₃, a₄ in Fig. 5. In analogy with Eq. (2.18) for DI scattering, we here use the definition

$$\hat{W}^{R\text{-soft}} = \delta(\hat{s}-Q^2) \int_{Q^2}^{(1+\epsilon)Q^2} d\hat{s}' \hat{W}^R(\hat{s}', Q^2, m^2, \mu_i^2) \quad (3.14)$$

We substitute the general expression for $\hat{W}^R(\hat{s}, Q^2, \dots)$ (as given in Appendix B, without proceeding to the scaling limit) into this integral and find

$$\begin{aligned} \hat{W}^{R\text{-soft}}(a_3) = & \delta(1-z) \frac{\alpha_s}{3\pi} k_2 \left\{ \left[2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln \epsilon \ln -\frac{\mu_1^2}{Q^2} \right. \right. \\ & \left. \left. - 2 \ln \epsilon \ln -\frac{\mu_2^2}{Q^2} + 2 \ln^2 \epsilon - \frac{\pi^2}{3} \right] + \pi i \left[2 \ln -\frac{\mu_2^2}{Q^2} - 2 \ln \epsilon \right] \right\} \end{aligned} \quad (3.15)$$

$$\begin{aligned} \hat{W}^{R\text{-soft}}(a_4) = & \delta(1-z) \frac{\alpha_s}{3\pi} k_2 \left\{ \left[2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln \epsilon \ln -\frac{\mu_1^2}{Q^2} \right. \right. \\ & \left. \left. - 2 \ln \epsilon \ln -\frac{\mu_2^2}{Q^2} + 2 \ln^2 \epsilon - \frac{\pi^2}{3} \right] + \pi i \left[2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln \epsilon \right] \right\} \end{aligned} \quad (3.16)$$

We again notice in Eqs. (3.15) and (3.16) anomalous cuts due to the external regulator masses $\mu_i^2 < 0$. This was anticipated from the hard-gluon results in Eqs. (3.10) and (3.11). Whilst adding all virtual- and soft-gluon contributions, we cancel the double-log terms which reflect the coinciding M and IR divergences (see Section 2 of paper I).

The sum of contributions from virtual-, soft-, and real-gluons brings us to the $q\bar{q}$ cross-section,

$$\begin{aligned} \hat{W}_{q\bar{q}} = & k_2 \left\{ \delta(1-z) + \frac{2\alpha_s}{3\pi} \sum_{i=1}^2 \left[\left\{ \frac{1+z^2}{1-z} \ln -\frac{(1-z)Q^2}{z^2\mu_i^2} - 2 + z + \frac{z \ln z}{1-z} - \pi i \left(\frac{z}{1-z} \right) \right\} \right. \right. \\ & \left. \left. + \delta(1-z) \left\{ \left(2 \ln \epsilon + \frac{3}{2} \right) \ln \frac{Q^2}{\mu_i^2} + \ln^2 \epsilon - \frac{5}{4} + \frac{\pi^2}{6} - \pi i \left(\ln \epsilon + \frac{1}{2} \right) \right\} \right] \right\} \end{aligned} \quad (3.17)$$

The anomalous cut contribution in this expression is exactly the same as was discovered in the DI-structure function $\hat{\mathcal{F}}_{i,q}$ given in Eq. (2.23). M factorization will lead to their cancellation leaving us with a real-valued correction term.

The Feynman graphs with a gluon in the initial state contributing to the DY process are exposed in Fig. 6. We apply the mass assignment as in DI scattering [defined near Eq. (2.25)] and find

$$\hat{W}_{gg} = \frac{\alpha_s}{4\pi} K_2 \left[\{z^2 + (1-z)^2\} \ln \frac{(1-z)Q^2}{z^2 \lambda^2} - \frac{1}{2} + z - \frac{3}{2} z^2 \right] \quad (3.18)$$

with $\lambda^2 < 0$. The colour-factor ($\frac{1}{2}$) is already included.

3.2 On-shell regularization

The above analysis of the DY diagrams shown in Fig. 5 is now repeated using on-shell mass assignment. It is defined at the beginning of Section 2.2 and was extensively discussed in Section 2 of paper I.

The vertex corrections of Figs. 5a₁, a₂ are formed by analytically continuing Eq. (2.28) to time-like Q^2 -values. $F^{Vx*}(a_1)$ chooses $Q^2 \geq 4m^2$ below the cut and $F^{Vx}(a_2)$ above. One finds

$$F^{Vx*}(a_1) = \frac{\alpha_s}{3\pi} \left\{ \left[-\gamma + \ln \frac{M_{reg}^2}{Q^2} - \ln^2 \frac{m^2}{Q^2} + 2 \ln \frac{m^2}{Q^2} \ln \frac{m^2}{\lambda^2} - 4 \ln \frac{m^2}{Q^2} + \frac{4\pi^2}{3} \right] - \pi i \left[-2 \ln \frac{m^2}{\lambda^2} + 2 \ln \frac{m^2}{Q^2} + 3 \right] \right\} \quad (3.19)$$

$$F^{Vx}(a_2) = \frac{\alpha_s}{3\pi} \left\{ \left[-\gamma + \ln \frac{M_{reg}^2}{Q^2} - \ln^2 \frac{m^2}{Q^2} + 2 \ln \frac{m^2}{Q^2} \ln \frac{m^2}{\lambda^2} - 4 \ln \frac{m^2}{Q^2} + \frac{4\pi^2}{3} \right] + \pi i \left[-2 \ln \frac{m^2}{\lambda^2} + 2 \ln \frac{m^2}{Q^2} + 3 \right] \right\} \quad (3.20)$$

The self-energies emerge as

$$\Sigma'_1(b_1 + b_3) = \Sigma'_2(c_1 + c_3) = \frac{\alpha_s}{3\pi} \left[\gamma - \ln \frac{M_{reg}^2}{m^2} + 2 \ln \frac{m^2}{\lambda^2} - 4 \right] \quad (3.21)$$

The sum of all virtual-gluon contributions listed in Eqs. (3.19) to (3.21) gives the renormalized form factor

$$2F^V = F^{Vx*}(a_1) + F^{Vx}(a_2) + \Sigma'_1(b_1 + b_3) + \Sigma'_2(c_1 + c_3) \quad (3.22)$$

$$= \frac{2\alpha_s}{3\pi} \left[-\ln^2 \frac{m^2}{Q^2} + 2 \ln \frac{m^2}{Q^2} \ln \frac{m^2}{\lambda^2} - 3 \ln \frac{m^2}{Q^2} + 2 \ln \frac{m^2}{\lambda^2} - 4 + \frac{4\pi^2}{3} \right] \quad (3.23)$$

The contributions following from the real-gluon graphs in Fig. 8a,b are derived from the general forms in Appendix B. One finds

$$\hat{W}^R(a_3) = \hat{W}^R(a_4) = \frac{2\alpha_s}{3\pi} K_2 \left[\frac{2z}{1-z} \ln \frac{Q^2}{z m^2} \right] \quad (3.24)$$

$$\hat{W}^R(b_2) = \hat{W}^R(c_2) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln \frac{Q^2}{z m^2} - \frac{1+z^2}{1-z} \right] \quad (3.25)$$

The calculation of the soft-gluon contributions proceeds in precisely the same way as in DI scattering^{4,17)}. The convective part of the matrix element is given by

$$M_\mu^a = -i g \bar{v}_2 \lambda^a \gamma_\mu (1 + \alpha \gamma_5) u_1 \left[\frac{p_2}{p_{2,k}} - \frac{p_1}{p_{1,k}} \right]_\nu \epsilon^{\nu*} \quad (3.26)$$

The first (second) term in the square brackets results from Fig. 8a (Fig. 8b).

Since M_μ^a is manifestly a correction to the 0-th order amplitude, we find

$$\hat{W}^{R-conv} \equiv \delta(1-z) \frac{2\alpha_s}{3\pi^2} K_2 \int_0^{\omega_0} \frac{d^3k}{2\sqrt{1k^2 + \lambda^2}} \left[\frac{p_1}{p_{1,k}} - \frac{p_2}{p_{2,k}} \right]^2 \quad (3.27)$$

We evaluate the square and obtain four terms corresponding to Figs. 5a₃, a₄, b₂, c₂.

The d^3k integration is carried out in the c.m. system of the incoming quarks, leaving

$$\hat{W}^{R-conv}(a_3) = \hat{W}^{R-conv}(a_4) = \delta(1-z) \frac{2\alpha_s}{3\pi} K_2 \left[-\frac{1}{2} \ln^2 \frac{m^2}{Q^2} - \ln \frac{4\omega_0^2}{\lambda^2} \ln \frac{m^2}{Q^2} - \frac{\pi^2}{3} \right] \quad (3.28)$$

$$\hat{W}^{R-conv}(b_2) = \hat{W}^{R-conv}(c_2) = \delta(1-z) \frac{2\alpha_s}{3\pi} K_2 \left[-\ln \frac{4\omega_0^2}{\lambda^2} - \ln \frac{m^2}{Q^2} \right] \quad (3.29)$$

The convective [Eqs. (3.28) and (3.29)] and the virtual [Eqs. (3.19)-(3.21)] contributions can now be added in the same way as in DI scattering (see Section 2.2).

We find that the IR infinities drop out but the M singularities instead persist.

By dropping all numerator k_ν dependence in Eq. (3.26), we have neglected the spin- $\frac{1}{2}$ and, partially, the spin-0 contributions. The M cancellation was therefore destroyed.

Proceeding in complete analogy to DI scattering [see Eq. (2.38)], the following connection is established:

$$\begin{aligned}\hat{W}_{(\epsilon)}^{R\text{-soft}} &= \delta(\hat{s} - Q^2) \left[\int \frac{(\sqrt{Q^2} + \omega_0)^2}{(\sqrt{Q^2} + \lambda)^2} + \int \frac{(1+\epsilon)Q^2}{(\sqrt{Q^2} + \omega_0)^2} d\hat{s}' \dots \right] \\ &\equiv \hat{W}_{(\omega_0)}^{R\text{-conv}} + \mathbb{L}(\omega_0, \epsilon)\end{aligned}\quad (3.30)$$

with $\hat{W}^{R\text{-soft}}$ defined in Eq. (3.14). Remember the constraints

$$|\vec{k}| < \omega_0 \rightarrow \hat{s} < (\sqrt{Q^2} + \omega_0)^2 \quad \epsilon Q^2 \gg m^2, \quad \omega_0 \ll m$$

We use $\hat{s} = Q^2/z$, $\delta \equiv (2\omega_0/m)/(m/\sqrt{Q^2})$, drop the λ -dependence in $\mathbb{L}(\omega_0, \epsilon)$, and integrate

$$\mathbb{L}(\omega_0, \epsilon) = \delta(\hat{s} - Q^2) \int \frac{(1+\epsilon)Q^2}{(\sqrt{Q^2} + \omega_0)^2} d\hat{s}' \hat{W}^R(\hat{s}', Q^2, m^2) = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \int_{1-\epsilon}^{1-\delta} d\bar{z} \hat{W}^R(\bar{z}, \dots) \quad (3.31)$$

with $\hat{W}^R$ given in Eqs. (3.24) and (3.25). We find

$$\hat{W}_{(a_3)}^{R\text{-soft}} = \hat{W}_{(a_4)}^{R\text{-soft}} = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left[L_n^2 \frac{m^2}{Q^2} - 2 L_n \frac{m^2}{\lambda^2} L_n \frac{m^2}{Q^2} - 4 L_n \epsilon L_n \frac{m^2}{Q^2} - \frac{2\pi^2}{3} \right] \quad (3.32)$$

$$\hat{W}_{(b_2)}^{R\text{-soft}} = \hat{W}_{(c_2)}^{R\text{-soft}} = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left[-2 L_n \frac{m^2}{\lambda^2} - 4 L_n \epsilon \right] \quad (3.33)$$

We observe that ω_0 has dropped out as expected. The expressions in Eqs. (3.32)

and (3.33) are precisely the same as in Ref. 10. The authors were only concerned with the soft-integral of Eq. (3.14) instead of splitting it up into the convective part plus the correction term, as was done for demonstration purposes in Eq. (3.30).

Using Eqs. (3.19) to (3.33) we form the complete quark-antiquark DY cross-section^{4,10)}

$$\begin{aligned}\hat{W}_{q\bar{q}} &= K_2 \left\{ \delta(1-z) + \frac{\alpha_s}{3\pi} \left[\left\{ \frac{1+z^2}{1-z} L_n \frac{Q^2}{z m^2} - \frac{1+z^2}{1-z} \right\} \right. \right. \\ &\quad \left. \left. + \delta(1-z) \left\{ \left(2 L_n \epsilon + \frac{3}{2} \right) L_n \frac{Q^2}{m^2} - 2 L_n \epsilon - 2 + \frac{\pi^2}{3} \right\} \right] \right\}\end{aligned}\quad (3.34)$$

The cross-section following from the gluon-initiated DY processes, as shown in Fig. 8 with the on-shell mass assignment defined near Eq. (2.50), reads^{4,10)}

$$\hat{W}_{gg} = \frac{\alpha_s}{4\pi} K_2 \left[\{z^2 + (1-z)^2\} \ln \frac{(1-z)^2 Q^2}{z m^2} + \frac{1}{2} + 3z - \frac{7}{2} z^2 \right] \quad (3.35)$$

3.3 Analysis of the parton cross-sections

Our first observation concerns the final-state (indicated by the cut-line B in Figs. 5 and 6) M and IR singularities. There simply are none. Instead, there are initial-state IR singularities in all diagrams which, however, cancel in the sums

$$\begin{aligned} \hat{W}_{(a_1)}^{Vx} + \hat{W}_{(a_3)}^{R\text{-soft}} & ; \quad \hat{W}_{(b_1+b_3)}^{S_1} + \hat{W}_{(b_2)}^{R\text{-soft}} \\ \hat{W}_{(a_2)}^{Vx} + \hat{W}_{(a_4)}^{R\text{-soft}} & ; \quad \hat{W}_{(c_1+c_3)}^{S_2} + \hat{W}_{(c_2)}^{R\text{-soft}} \end{aligned} \quad (3.36)$$

We use the off-shell results of Section 3.2 and check

$$\begin{aligned} \hat{W}_{(a_1)}^{Vx} + \hat{W}_{(a_3)}^{R\text{-soft}} &= \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left\{ [-3 + \ln \frac{M_{reg}^2}{Q^2} - (2\ln\epsilon + 2)(\ln \frac{M_1^2}{Q^2} + \ln \frac{M_2^2}{Q^2}) \right. \\ &\quad \left. + 2\ln^2\epsilon - 4 - \frac{\pi^2}{3} \right] + \pi i \left[-2\ln \frac{M_1^2}{Q^2} - 4 - 2\ln\epsilon \right] - \pi i \left[2\ln \frac{M_1^2}{Q^2} + 2\ln \frac{M_2^2}{Q^2} + 3 \right] \Big\} \end{aligned} \quad (3.37)$$

$$\begin{aligned} \hat{W}_{(a_2)}^{Vx} + \hat{W}_{(a_4)}^{R\text{-soft}} &= \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left\{ [-3 + \ln \frac{M_{reg}^2}{Q^2} - (2\ln\epsilon + 2)(\ln \frac{M_1^2}{Q^2} + \ln \frac{M_2^2}{Q^2}) \right. \\ &\quad \left. + 2\ln^2\epsilon + \pi^2 \right] + \pi i \left[2\ln \frac{M_1^2}{Q^2} - 2\ln\epsilon \right] + \pi i \left[2\ln \frac{M_1^2}{Q^2} + 2\ln \frac{M_2^2}{Q^2} + 3 \right] \Big\} \end{aligned} \quad (3.38)$$

We observe that the double-log terms arising from the IR singularity have completely vanished in Eqs. (2.37) and (2.38). Further, we notice that there are no $z = 1$ poles in expressions (3.12) and (3.13), which implies IR finiteness of Figs. 5b₂, c₂ for off-shell mass assignment. We conclude, via Eqs. (3.36), that the self-energy contributions also must be IR-finite.

Using our on-shell results of subsection 3.2, we discover

$$\hat{W}^{Vx}(a_1) + \hat{W}^{R\text{-soft}}(a_3) = \hat{W}^{Vx}(a_2) + \hat{W}^{R\text{-soft}}(a_4) = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left\{ \left[-\zeta + \ln \frac{M_{reg}^2}{Q^2} - (4\ln\epsilon + 4) \ln \frac{m^2}{Q^2} + \frac{2\pi^2}{3} \right] \mp \pi i \left[-2 \ln \frac{m^2}{\lambda^2} + 2 \ln \frac{m^2}{Q^2} + 3 \right] \right\}, \quad (3.39)$$

where the minus sign refers to the first combination in Eq. 3.39 and the plus sign to the second.

$$\begin{aligned} \hat{W}^{S_1}(b_1+b_3) + \hat{W}^{R\text{-soft}}(b_2) &= \hat{W}^{S_2}(c_1+c_3) + \hat{W}^{R\text{-soft}}(c_2) \\ &= \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left[\zeta - \ln \frac{M_{reg}^2}{m^2} - 4 - 4 \ln\epsilon \right] \end{aligned} \quad (3.40)$$

Ignoring the Coulomb phase, all λ - (or IR-) dependent terms disappear in these expressions. The Coulomb phase-factor is cancelled by adding $\hat{W}^{Vx}(a_1) + \hat{W}^{Vx}(a_2)$.

We make a short comment on other regularization methods. Using the same off-shell assignment but now with $m^2 \neq 0$, $\mu_1^2 \ll m^2$, we re-discover the result of Eq. (3.34) which was obtained with on-shell regularization^{4,10,15}. We emphasize the equivalence between massive on- and off-shell calculation, which applies not only for $Q^2 \gg m^2$ but in the same way holds for smaller Q^2 values. Therefore this off-shell method can also be used in QED. We point to an alternative on-shell method. One sets $\lambda = 0$ and uses dimensional regularization to prevent the IR divergences. We discover the same correspondence rules between these two techniques as were found earlier in Section 2.3.

4. n-DIMENSIONAL MASS REGULARIZATION

In this section we determine the DI structure functions and DY parton cross-sections using dimensional regularization in order to prevent the M and IR singularities^{6,9,19}. The calculation technique was explained in paper I. We here experience additional complications due to spin and the occurrence of overlapping M and IR singularities, the latter manifesting themselves as $(n-4)^{-2}$ double poles. We first determine the form of the DI structure functions and subsequently compute the parton cross-sections for the DY process. For the calculation details we refer to Appendix C.

4.1 DI scattering

We discuss the individual contributions. The scaling "Born" term is given in Eq. (2.1). The unrenormalized form factor of the vertex diagram is readily determined from Appendix A giving

$$F^V = F^{V_K} = \frac{\alpha_s}{3\pi} \left[-\frac{8}{(n-4)^2} + \frac{6-4\gamma}{(n-4)} - 8 + 3\gamma - \gamma^2 + \frac{\pi^2}{6} + \right. \\ \left. \frac{4}{n-4} \text{Ln} \frac{M_{\text{reg}}^2}{-q^2} + (2\gamma-3) \text{Ln} \frac{M_{\text{reg}}^2}{-q^2} - \text{Ln}^2 \frac{M_{\text{reg}}^2}{-q^2} \right] (4\pi)^{2-\frac{n}{2}} \quad (4.1)$$

This form is identical with the renormalized form factor as indicated above, since the self-energy diagrams will not contribute in this case of all massless partons, $Z_2 = 1$. We will now sketch the arguments. Whilst determining the self-energy diagram in the Feynman gauge, one realizes that it is mass singular even if n -dimensional regularization is applied. This follows from the definition of Z_2 in Eq. (4.2), which does not exist in a completely massless theory. In the Landau gauge we have no problem since it vanishes if all masses are set to zero. We renormalize at the mass pole (which is not minimal subtraction!) and therefore can use the renormalized self-energy Σ_f in Eq. (4.2) below in order to establish a one-to-one correspondence between the massive and the massless n -dimensional method to regularize the M singularities. We will again find that the M and UV singularities cancel. We explicitly expose the results. Straightforward calculation of the self-energy diagram for on-shell mass assignment with massive fermion fields gives

$$1-Z_2^{-1} \equiv \frac{\partial \Sigma}{\partial \psi} \Big|_{\psi=m} = \frac{\alpha_s}{3\pi} \left[\underbrace{\left(\zeta - \text{Ln} \frac{m^2}{M_{\text{reg}}^2} \right)}_{\text{U.V.}} + 2 \underbrace{\left(\zeta - 2 - \text{Ln} \frac{M_{\text{reg}}^2}{m^2} \right)}_{\text{I.R.}} \right] \quad (4.2)$$

where the first ζ [see Eq. (2.4)] results from the UV divergence and the second one from the IR singularity. With the above result we aim to determine Z_2 using

the replacement rule $\ln m^2 \rightarrow (n-4)^{-1} + \dots$. The latter is established via the renormalized self-energy

$$\Sigma_f(s) = \Sigma(s) - (\not{p}-m)(1-Z_2^{-1}) - \Sigma(\not{p}=m) \quad (s \equiv p^2) \quad (4.3)$$

with a finite mass m . We repeat the analogous calculation assuming $m=0$ and compare the two results. In both cases we use dimensional regularization. Since spin- $\frac{1}{2}$ is involved, we set

$$\text{Im } \Sigma(p) = \not{p} A(s) + m B(s) \quad (4.4)$$

with

$$A(s) = \text{Tr}[\not{p} \text{Im } \Sigma(p)] \quad , \quad B(s) = \text{Tr}[\text{Im } \Sigma(p)] \quad (4.5)$$

where $\text{Im } \Sigma(p)$ is trivially specified. With these ingredients we form the dispersion integral

$$\Sigma(p) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\not{p} A(s') + m B(s')}{s' - s} \quad (4.6)$$

where the lower integration limit is $s_0 = (m + \lambda)^2$ in order to prevent IR singularities. The formation of $\Sigma(\not{p}=m)$ and $\partial \Sigma / \partial \not{p}|_{(\not{p}=m)}$ brings us to Σ_f . As intermediate results we list

$$A(s) = g^2 M_{\text{reg}}^{4-n} \frac{\pi^{\frac{n}{2}+1}}{(2\pi)^n} \frac{\Gamma(\frac{n}{2})}{\Gamma(n-2)} \left(\frac{s-m^2}{s} \right)^{n-3} \left(\frac{s+m^2}{s} \right) s^{\frac{n}{2}-2} \quad (4.7)$$

$$B(s) = g^2 M_{\text{reg}}^{4-n} \frac{\pi^{\frac{n}{2}+1}}{(2\pi)^n} \frac{\Gamma(\frac{n}{2}-1)}{\Gamma(n-2)} \left(\frac{s-m^2}{s} \right)^{n-3} s^{\frac{n}{2}-2} \quad (4.8)$$

We then find

$$\begin{aligned} \Sigma_f(s, m^2, n) = \not{p} \frac{\alpha_s}{3\pi} \left[2 \left\{ \gamma - 1 + \ln \frac{m^2}{M_{\text{reg}}^2} \right\} - \left(\frac{s-m^2}{s} \right) \left\{ 1 + \left(\frac{s+m^2}{s} \right) \ln \frac{m^2 s}{m^2} \right\} \right] \\ + m \frac{\alpha_s}{3\pi} [\dots] \end{aligned} \quad (4.9)$$

$$\Sigma_f(s, 0, n) = \not{p} \frac{\alpha_s}{3\pi} \left[-\gamma + 1 - \ln \frac{s}{M_{\text{reg}}^2} \right] \quad (4.10)$$

which permits us to establish the replacement rule

$$\ln m^2 = \ln M_{\text{reg}}^2 - \gamma + \frac{4}{3} \quad (m^2 \rightarrow 0, n \rightarrow 4) \quad (4.11)$$

Applied on Eq. (4.2), we immediately find $Z_2 = 1$. We conclude that the UV divergence cancels the M and IR singularities completely. Furthermore we can easily establish $\delta m = 0$.

The calculation of the radiative gluon diagrams follows the pattern sketched earlier in Section 2. The projections are slightly changed owing to the multiplicative n-dependent factors as well as to the additional n-dependence coming from spin. From Appendix C we derive

$$\hat{F}_i^{\text{R-hard}} = \frac{2\alpha_s}{3\pi} K_i \left[\frac{1+z^2}{1-z} \left\{ \ln \frac{-q^2(1-z)}{z M_{\text{reg}}^2} + \gamma \right\} + 1-z + h_i(z) - \frac{3}{2} \frac{1}{1-z} \right] \quad (4.12)$$

with $h_i(z)$ defined in Eq. (2.24).

Whilst carrying out z-integration we notice the $z = 1$ divergence, which we prevent, as explained in paper I, by separation of the z-integral in a soft- and a hard-gluon part. We go back to the defining Eq. (2.18), and after a lengthy but straightforward calculation we find the result

$$\begin{aligned} \hat{F}_i^{\text{R-soft}} = & \frac{2\alpha_s}{3\pi} K_i \left[\frac{8}{(n-4)^2} + \frac{4\gamma-3}{(n-4)} + \gamma^2 - \frac{3}{2}\gamma + \frac{\gamma}{2} - \frac{\pi^2}{2} \right. \\ & \left. + \frac{4}{n-4} \ln \frac{-\epsilon q^2}{M_{\text{reg}}^2} + \ln^2 \frac{-\epsilon q^2}{M_{\text{reg}}^2} + (2\gamma - \frac{3}{2}) \ln \frac{-\epsilon q^2}{M_{\text{reg}}^2} \right] (4\pi)^{2-\frac{n}{2}} \end{aligned} \quad (4.13)$$

The constant K_i is defined as in Eq. (2.1). The n-4 expansion of the last (4π) factor has for presentation reasons not been carried out but should not be forgotten.

We now are in the position to specify the complete quark-structure functions

$$\hat{F}_{i,q} = \hat{F}_i^{(0)} + \hat{F}_i^{\text{R-hard}} + \hat{F}_i^{\text{R-soft}} + \hat{F}_i^{\text{V}} \quad (4.14)$$

which are cast in a simple form^{6,9)}

$$\hat{F}_{i,q} = K_i \left\{ \delta(1-z) + \frac{2\alpha_s}{3\pi} \left[\left\{ \frac{1+z^2}{1-z} \left(\ln \frac{q^2(1-z)}{z M_{\text{reg}}^2} + \gamma \right) + 1 - z + h_i(z) - \frac{3}{2} \frac{1}{1-z} \right\} \right. \right. \\ \left. \left. + \delta(1-z) \left\{ \left(2 \ln \epsilon + \frac{3}{2} \right) \left(\ln \frac{q^2}{M_{\text{reg}}^2} + \gamma \right) + \ln \epsilon - \frac{3}{2} \ln \epsilon - \frac{9}{2} - \frac{\pi^2}{3} \right\} \right] \right\} \quad (4.15)$$

with $h_i(z)$ defined in Eq. (2.24).

The analysis of the gluon structure functions follows precisely the steps outlined earlier in this section. For calculation details we refer to Appendix C, and here simply give our results. We find^{6,9)}

$$\hat{F}_{i,q} = \frac{\alpha_s}{2\pi} K_i \left[\left\{ z^2 + (1-z)^2 \right\} \left(\ln \frac{q^2(1-z)}{z M_{\text{reg}}^2} + \gamma \right) + 1 + g_i(z) \right] \quad (4.16)$$

$$\hat{F}_{3,q} = 0 \quad (4.17)$$

with $g_i(z)$ defined in Eq. (2.27).

4.2 DY process

The evaluation of the DY parton cross-sections for massless partons using dimensional regularization follows the earlier calculation schemes of Section 3. Here we limit ourselves to the results, and find in Appendix C the intermediate steps.

The Born-term of Eq. (3.1) is unchanged and is valid here too. The sum of virtual gluon graphs leads us to the renormalized form factor, as sketched in subsection 5.1, for DI scattering. It is analytically continued to time-like Q^2 values, which introduces an additional π^2 factor. We find

$$F^V = \frac{\alpha_s}{3\pi} \left[-\frac{8}{(n-4)^2} + \frac{6-4\gamma}{n-4} - \gamma + 3\gamma - \gamma^2 + \frac{7\pi^2}{6} + \frac{4}{n-4} \ln \frac{M_{\text{reg}}^2}{Q^2} \right. \\ \left. + (2\gamma-3) \ln \frac{M_{\text{reg}}^2}{Q^2} - \ln^2 \frac{M_{\text{reg}}^2}{Q^2} \right] (4\pi)^{2-\frac{n}{2}} \quad (4.18)$$

with

$$\hat{W}^V = K_2 \delta(1-z) 2 F^V \quad (4.19)$$

The cross-sections resulting from the diagrams describing real-gluon production (Fig. 2a,b) are expressed as linear combinations of the phase-space integrals $J_i(n)$ (see paper I, Appendix B) whose pole extraction leaves

$$\hat{W}^R = \frac{4\alpha_s}{3\pi} k_2 \left[\frac{1+z^2}{1-z} \left\{ \ln \frac{Q^2(1-z)^2}{z M_{reg}^2} + \zeta \right\} \right] \quad (4.20)$$

The z -pole near 1 needs separation of the integral over the momentum fraction in a soft- and a hard-gluon part as defined in Eq. (3.14). Explicit evaluation brings us to

$$\begin{aligned} \hat{W}^{R-soft} = & \delta(1-z) \frac{4\alpha_s}{3\pi} k_2 \left[\frac{8}{(n-4)^2} + \frac{4\gamma}{(n-4)} + \gamma^2 - \frac{\pi^2}{2} + \frac{4}{(n-4)} \ln \frac{Q^2}{M_{reg}^2} + \right. \\ & \left. 2\gamma \ln \frac{Q^2}{M_{reg}^2} + \ln^2 \frac{Q^2}{M_{reg}^2} + \frac{8}{(n-4)} \ln \epsilon + 4\gamma \ln \epsilon + 4\ln^2 \epsilon + 4\ln \epsilon \ln \frac{Q^2}{M_{reg}^2} \right] (4\pi)^{2-\frac{n}{2}} \end{aligned} \quad (4.21)$$

We determine the sum of all $q\bar{q}$ contributions to the DY cross-section and find^{6,9)}

$$\begin{aligned} \hat{W}_{q\bar{q}} = & \hat{W}^{(0)} + \hat{W}^{R-soft} + \hat{W}^V + \hat{W}^{R-hard} = \\ & k_2 \left\{ \delta(1-z) + \frac{4\alpha_s}{3\pi} \left[\left\{ \frac{1+z^2}{1-z} \left(\ln \frac{Q^2(1-z)^2}{z M_{reg}^2} + \zeta \right) \right\} + \right. \right. \\ & \left. \left. \delta(1-z) \left\{ \left(2\ln \epsilon + \frac{3}{2} \right) \left(\ln \frac{Q^2}{M_{reg}^2} + \zeta \right) - 4 + \frac{\pi^2}{3} \right\} \right] \right\} \end{aligned} \quad (4.22)$$

where ζ was defined in Eq. (2.4).

In a completely analogous manner we find the cross-section corresponding to the Feynman diagrams with a gluon in the initial state giving^{6,9)}

$$\hat{W}_{gg} = \frac{\alpha_s}{4\pi} k_2 \left[\{ z^2 + (1-z)^2 \} \left(\ln \frac{Q^2(1-z)^2}{z M_{reg}^2} + \zeta \right) + 1 + \frac{1}{2}(1-z)(1+3z) \right] \quad (4.23)$$

5. INCORRECT OFF-SHELL MASS ASSIGNMENT

In the preceding paper I we discussed the implications of an often-used but incorrect off-shell mass assignment which does not satisfy the double-cut-rule. We learnt that in ϕ_6^3 theory the correct DI/DY correction terms Δ^{DI} , Δ^{DY} are still found, despite the incorrect M regularization method. We analyse the analogous problem in QCD and find Δ^{DY} to depend on the right/wrong off-shell mass assignment; Δ^{DI} is not affected.

The incorrect off-shell assignment chooses all initial momenta (of the DI/DY subprocesses) space-like in order to avoid anomalous cuts. A selected initial line therefore has one and the same off-shell mass in all contributing graphs. The intermediate- (or final-) state M/IR singularities are regularized by auxiliary masses m_0^2 . They are chosen such as to preserve the unitarity relation between the virtual Compton diagrams and their cut-graphs.

We expose the changes if the incorrect off-shell mass assignment is used. In Figs. 1a₁, a₃ we set $p_4^2 = p^2 < 0$, $m_3^2 = m_0^2 < 0$, and use $p_1^2 = p^2 < 0$, $p_2^2 = m_0^2 < 0$ for Figs. 1a₂, a₄. All other masses vanish. We find (see Appendix A)

$$\hat{F}_i^{Vx}(a_1) = \hat{F}_i^{Vx}(a_2) = \delta(1-z) \frac{\alpha_s}{3\pi} K_i \left[-3 + \ln \frac{M_{reg}^2}{-q^2} - 2 \ln \frac{p^2}{q^2} \ln \frac{m_0^2}{q^2} - 2 \ln \frac{p^2}{q^2} - 2 \ln \frac{m_0^2}{q^2} - \frac{2\pi^2}{3} \right] \quad (5.1)$$

The soft-gluon corrections are given by

$$\hat{F}_i^{R-soft}(a_3) = \hat{F}_i^{R-soft}(a_4) = \delta(1-z) \frac{\alpha_s}{3\pi} K_i \left[2 \ln \frac{p^2}{q^2} \ln \frac{m_0^2}{q^2} + 2 \ln \frac{m_0^2}{q^2} - 2 \ln e \ln \frac{p^2}{q^2} - 2 \ln e \right] \quad (5.2)$$

The self-energy corrections of Figs. 1b₁, b₂ with $p_7^2 = p_9^2 = p^2 < 0$ are equal to

$$\hat{F}_i^{S_1}(b_1) = \hat{F}_i^{S_1}(b_2) = \delta(1-z) \frac{\alpha_s}{6\pi} K_i \left[3 - \ln \frac{M_{reg}^2}{-p^2} - 1 \right] \quad (5.3)$$

The self-energy corrections in Figs. 1c₁, c₂ with the masses $p_{11}^2 = p_{13}^2 = m_0^2$ are found as

$$\hat{F}_i^{S_2}(c_1) = \hat{F}_i^{S_2}(c_2) = \delta(1-z) \frac{\alpha_s}{6\pi} K_i \left[3 - \ln \frac{M_{reg}^2}{-m_0^2} + 1 \right] \quad (5.4)$$

and the soft-gluon integral of Fig. 1c₃ with the above masses reads

$$\hat{\mathcal{F}}_i^{R\text{-soft}}(c_3) = \delta(1-z) \frac{\alpha_s}{3\pi} K_i \left[-\ln \frac{m_0^2}{q^2} + \ln \epsilon - 2 \right] \quad (5.4)$$

The real-gluon graphs in the scaling limit are easily evaluated using the results of Appendix B. The complete quark-structure function reads

$$\begin{aligned} \hat{\mathcal{F}}_{i,q} = K_i \bigg\{ & \delta(1-z) + \frac{2\alpha_s}{3\pi} \left[\left\{ \frac{1+z^2}{1-z} \ln \frac{q^2}{z^2 p^2} - 1 + h_i(z) - \frac{3}{2} \frac{1}{1-z} \right\} \right. \\ & \left. + \delta(1-z) \left\{ \left(2 \ln \epsilon + \frac{3}{2} \right) \ln \frac{q^2}{p^2} - \frac{3}{2} \ln \epsilon - 1 - \frac{2\pi^2}{3} \right\} \right] \bigg\} \end{aligned} \quad (5.5)$$

with $h_i(z)$ defined in Eq. (2.24). Expression (5.5) is independent of the auxiliary mass parameter m_0^2 (see also Ref. 11, where another mass assignment was used). In order to make a comparison with Ref. 3, we determine its moments [see Eq. (5.5) and Table 1]. Our calculation is carried out in the Feynman gauge. The two results can be seen to agree, provided the external self-energy contributions Eq. (5.3) are removed from our result Eq. (5.5) since they were not taken into account in Ref. 3. It is easy to check that all DI correction terms $\Delta_{i,q}^{DI} \sim h_i - h_2$ remain unaffected by changing from the incorrect to the correct off-shell assignment. The reason is clear: the $h_i(z)$ remain unaffected by the change of masses.

We determine the DY cross-sections with the incorrect off-shell mass assignment. The vertex corrections of Figs. 5a₁, a₂ are derived from Eq. (5.1) by analytic continuation from space- to time-like q^2 values. This adds the term $2\pi^2$ (Sudakov form factor!). The self-energy diagrams of Figs. 5b₁, b₃, c₁, c₃ are specified by Eq. (5.3). The soft-gluon integrals resulting from Fig. 5a₃ ($p_2^2 = p'^2_2 < 0$, $p_4^2 = p'^2_1 < 0$) and from Fig. 5a₄ ($p_3^2 = p'^2_2 < 0$, $p_4^2 = p'^2_1 < 0$) read

$$\hat{W}^{R\text{-soft}}(a_3) = \hat{W}^{R\text{-soft}}(a_4) = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \sum_{i=1}^2 \left[\ln \frac{p_i'^2}{Q^2} \ln \frac{p_i'^2}{Q^2} - 2 \ln \epsilon \ln \frac{p_i'^2}{Q^2} + \ln^2 \epsilon - \frac{\pi^2}{6} \right] \quad (5.6)$$

Real line p'_1 , p'_2 denote generically the incoming parton momenta. The "hard" real-gluon contributions are easily derived from Appendix B. The complete DY parton cross-section with $q\bar{q}$ initial state then reads

$$\hat{W}_{q\bar{q}} = K_2 \left\{ \delta(1-z) + \frac{2\alpha_s}{3\pi} \sum_{i=1}^2 \left[\left\{ \frac{1+z^2}{1-z} \ln \frac{-(1-z)Q^2}{z^2 p_i'^2} - z + z \right\} \right. \right. \\ \left. \left. + \delta(1-z) \left\{ \left(2 \ln \epsilon + \frac{3}{2} \right) \ln \frac{Q^2}{-k_i^2} + \ln^2 \epsilon + \frac{\pi^2}{2} - \frac{1}{2} \right\} \right] \right\} \quad (5.7)$$

We form the correction term $\Delta_{q\bar{q}}^{DY} = f_{q\bar{q}} - 2f_{q,2}$. Without explicitly exposing the result, we point to the changes if compared with the correct result given in Eq. (6.17) below. The hard-gluon part is unaltered. However, the coefficient of the $\delta(1-z)$ function is changed. Using the incorrect off-shell mass assignment, we here find $[1 + (7\pi^2/3)]$. This result was also found in Refs. 6, 15 and 20. We further analyse the reason for this discrepancy, obviously originating in the off-shell mass choice. The coefficient of the $\delta(1-z)$ function appearing in the DI structure function (DY cross-section) receives contributions from Figs. 1a₁-a₄, c₁-c₃ (Figs. 5a₁-a₄). The self-energy diagrams in Figs. 1b₁, b₂ (DI) cancel the analogous graphs 5b₁, b₃, c₁, c₃ (DY) provided both sets of graphs are M-regularized in the same way. The sum of contributions corresponding to Figs. 1c₁-c₃ remains unaltered, which leads us to conclude that the discrepancy between the right/wrong off-shell assignment originates from the graphs 1a₁-a₄ and 5a₁-a₄. The complexity of these graphs arises from their overlapping M and IR divergences. Finally, we remark that the same incorrect off-shell result, i.e. $1 + (7\pi^2/3)$, is found assuming all quarks to be massive $m^2 \neq 0$. The incorrect off-shell assignment is introduced via $p_i^2 = m^2 + \mu^2$ with $\mu^2 < 0$ for all external lines.

6. CORRECTION TERMS

In the preceding section we have determined the DI structure functions and the DY cross-sections using different methods of M/IR regularization which permits us to form the order $\bar{g}^2$ corrections to the coefficient functions. We first specify its form for DI scattering. Subsequently we calculate the structure function moments, and finally the DY correction term is evaluated. We successively analyse our results and compare them with the literature.

The order α_s parton structure functions $\hat{\mathcal{F}}_{i,\ell}(z, q^2)$, with $\ell = (q, \bar{q}, g)$, have the form

$$\hat{\mathcal{F}}_{i,q} = a_{i,q} \delta(1-z) + \alpha_s \left[a_{i,q} P_q(z) \ln \frac{q^2}{p^2} + f_{i,q}(z) \right] \quad (6.1)$$

$$\hat{\mathcal{F}}_{i,g} = \alpha_s \left[a_{i,g} P_g(z) \ln \frac{q^2}{p^2} + f_{i,g}(z) \right] \quad (6.2)$$

where

$$\hat{\mathcal{F}}_L \equiv \hat{\mathcal{F}}_2 - 2 \hat{\mathcal{F}}_1; \quad a_{2,\ell} = 2 a_{1,\ell} = 1; \quad a_{3,\bar{q}} = -a_{3,q} = 1, \quad a_{3,g} = 0 \quad (6.3)$$

P_ℓ and $f_{i,\ell}$ are obtained from Eqs. (2.15)-(2.17), (2.23)-(2.27), and (2.49)-(2.51).

Notice that $f_{i,\ell}$ is M and IR regularization-dependent. The correction terms to DI scattering are given by (see paper I)

$$\Delta_{i,\ell}^{DI} = \mathcal{F}_{i,\ell} - a_{i,\ell} \mathcal{F}_{2,\ell} \quad (6.4)$$

All regularization schemes leave the same result

$$\Delta_{i,q}^{DI} = a_{i,q} \frac{2}{3\pi} k_2 \left[h_i(z) - h_2(z) \right] \quad (i = 1, 2, 3) \quad (6.5)$$

$$\Delta_{L,q}^{DI} = -2 \Delta_{1,q}^{DI} \quad (6.6)$$

$$\Delta_{i,g}^{DI} = a_{i,g} \frac{1}{4\pi} k_2 \left[g_i(z) - g_2(z) \right] \quad (i = 1, 2) \quad (6.7)$$

$$\Delta_{3,g}^{DI} = 0 \quad (6.8)$$

$$\Delta_{L,g}^{DI} = -2 \Delta_{1,g}^{DI} \quad (6.9)$$

with $h_i(z)$ and $g_i(z)$ defined in Eqs. (2.24) and (2.27). Notice that all $\Delta_{i,\ell}^{DI}$ are free of anomalous cuts.

The regularization independence of the above quantities is easy to understand. They all originate from combinations of $\hat{\mathcal{F}}_i$ which are explicitly M/IR finite. For instance, $\hat{\mathcal{F}}_L$, and with it $\Delta_{L,q}^{DI}$, $\Delta_{L,g}^{DI}$ are formed by the projection $D = p^\mu p^\nu W_{\mu\nu}/m^2$ which manifestly has no IR/M divergences. We therefore can ignore all masses right from the start. Moreover this latter insight leads us to conclude that $\Delta_{i,\ell}^{DI}$ is gauge independent.

The moments of the parton structure functions are defined by

$$\hat{M}_{i,l}^{(n)} = \int_0^1 dz z^{n-1} \hat{F}_{i,l}(z, q^2) \quad (6.10)$$

$$\hat{M}_{i,q}^{(n)} = a_{i,q} + \alpha_s \left[a_{i,q} P_q^{(n)} \ln \frac{q^2}{p^2} + f_{i,q}^{(n)} \right] \quad (6.11)$$

$$\hat{M}_{i,g}^{(n)} = \alpha_s \left[a_{i,g} P_g^{(n)} \ln \frac{q^2}{p^2} + f_{i,g}^{(n)} \right] \quad (6.12)$$

The $f_{i,l}^{(n)}$ as obtained from the three regularization schemes are listed in Table 1.

The $P_\ell^{(n)}$ represent the anomalous dimensions in the operator product expansion.

For all mass assignments we find the same result

$$P_q^{(n)} = \frac{2}{3\pi} \left[\frac{1}{n(n+1)} - 2 \sum_{k=2}^n \frac{1}{k} - \frac{1}{2} \right] \quad (6.13)$$

$$P_g^{(n)} = \frac{1}{4\pi} \left[\frac{n^2+n+2}{n(n+1)(n+2)} \right] \quad (6.14)$$

We inspect Table 1 and notice an imaginary part in $f_{i,q}^{(n)}$ if off-shell M regularization is applied. This is due to the anomalous cuts in the graphs $1a_1, a_3, b_1, b_2$.

Whilst carrying out M factorization they will be absorbed in the operator matrix element and will be cancelled by the higher-order contributions to the anomalous dimension provided the latter are worked out in the same regularization scheme.

$f_{i,q}^{(n)}$ (on-shell) has been calculated with a different method in Ref. 21. There clearly appears a discrepancy between our result and the one of Ref. 21. We disagree in the term $(n^2-2n-1)/n^2(n+1)^2$, which in Ref. 21 reads $(n^2+2n-1)/n^2(n+1)^2$. Our result is in agreement with Ref. 22. Our $f_{i,q}^{(n)}$ (n-dimensional) is in agreement with the analogous calculation of Ref. 9. The coefficient ζ in front of $P_q^{(n)}$ is a typical artifact of this type of regularization method; it represents the mass pole. We point to the double sum $\sum_{k=1}^n (1/k) \sum_{j=1}^n (1/j)$, which originates from the term $(1-z)^{-1} \cdot \ln(1-z)$ and is characteristic for the on-shell and n-dimensional regularization techniques; $f_{i,g}^{(n)}$ (off-shell) is in agreement with Ref. 8 and the n-dimensional expression is also found in Ref. 9.

We focus on the DY process. The order α_s parton cross-sections are parametrized as (see paper I)

$$\hat{W}_{q\bar{q}} = \delta(1-z) + \alpha_s \left[P_q(z) \ln \frac{q^2}{p^2} + P_{\bar{q}}(z) \ln \frac{q^2}{p^2} + F_{q\bar{q}}(z) \right] \quad (6.15)$$

$$\hat{W}_{qg} = \alpha_s \left[P_g(z) \ln \frac{q^2}{p^2} + F_{qg}(z) \right] \quad (6.16)$$

where P_ℓ and $f_{k,\ell}$ are determined by Eqs. (3.17) and (3.18), (3.34) and (3.35), and (4.22) and (4.23). The order $\bar{g}^2$ correction term of the DY coefficient function is in any regularization scheme given by

$$\Delta_{q\bar{q}}^{DY} = \frac{2}{3\pi} K_2 \left[\left\{ 2 \frac{1+z^2}{1-z} \ln(1-z) - 4z - 6 + \frac{3}{1-z} \right\} + \delta(1-z) \left\{ 1 + \frac{4\pi^2}{3} + 2 \ln^2 \epsilon + 3 \ln \epsilon \right\} \right] \quad (6.17)$$

$$\Delta_{qg}^{DY} = \frac{1}{4\pi} K_2 \left[\left\{ z^2 + (1-z)^2 \right\} \ln(1-z) + \frac{3}{2} - 5z + \frac{9}{2} z^2 \right] \quad (6.18)$$

provided the double-cut-rule is respected. We observe that in $\Delta_{q\bar{q}}^{DY}$ all anomalous cut contributions have disappeared.

We add a few final comments: although the off-shell regulator method with $m = 0$ involves anomalous cuts in the cross-sections, they disappear in the correction terms Δ^{DI} and Δ^{DY} . These latter two quantities are physical observables, but not $\hat{F}_{i,\ell}$ and $\hat{W}_{k\ell}$. We comment on the often-misinterpreted term $[1 + (4\pi^2/3)]$ in $\Delta_{q\bar{q}}^{DY}$. In the literature its π^2 factor is frequently attributed to the analytic continuation of the vertex function from space- to time-like q^2 values^{9,20,23}. This applies to on-shell and n-dimensional regularization only, but no longer holds true in the off-shell case. The extra term $2\pi^2$, for example, which follows from the normal-cut (in q^2) is completely cancelled by the anomalous cut terms. The vertex correction therefore does not contribute at all. The correction term here arises from the soft-gluon contributions. We therefore emphasize that the mass parameters must be considered as regulators and not as physical quantities.

We briefly comment on the magnitude of the correction terms. In DI scattering they were found to be relatively small. The same applies for the gluon-correction Δ_{qg}^{DY} in the DY process. However, in the region $Q^2 \lesssim 100 \text{ GeV}^2$ the correction term $\Delta_{q\bar{q}}^{DY}$ is of substantial size and even surmounts the leading-log contribution⁴⁻¹¹.

7. SUMMARY AND CONCLUSION

This paper presents the details of our calculations aiming at the order α_s non-log correction terms in DI scattering and the DY process in the perturbative framework of QCD. We calculated the parton cross-sections using off-shell, on-shell, and dimensional regularization in order to prevent the M and IR singularities.

We have paid particular attention to the off-shell mass assignment since it led to confusion and mistakes in the recent literature^{6,8,11,20}). The masses must be chosen in agreement with the double-cut-rule which follows from unitarity and analyticity. The intuitively motivated belief of the initial partons being "off-shell", since they are bound in the hadron, is incorrect. The perturbative evaluation of the QCD Lagrangian at short distances immediately introduces the hypotheses, rules, and characteristics of Lagrangian field theory. They manifest themselves, for instance, in the M singularities which must be prevented in agreement with the above-mentioned double-cut-rule. Off-shell masses therefore have no physical meaning, in contrast to the on-shell method where the masses can have a physical meaning as, for instance, in QED.

This insight leads to a DY correction term independent of any regularization method. That the incorrect off-shell mass assignment did not spoil the ϕ_6^3 correction terms is explained by the simplicity of such a theory (no IR divergences). Similarly, Δ^{DI} is found to be independent of any mass assignment. It is manifestly M/IR-finite. This no longer applies for the order α_s DY correction term. Here incorrect off-shell mass assignment influences exclusively the $\delta(1-z)$ part of $\Delta_{q\bar{q}}^{\text{DY}}$ where the anomalous cuts no longer properly cancel.

Acknowledgements

We enjoyed fruitful conversations with N. Sakai. One of us (V.N.) thanks Professor D. Yennie, A.N. Schellekens and R. Petronzio for clearing discussions, and the other (B.H.) has profited from helpful comments by J. Ellis, D. Nanopoulos and A. de Rújula.

APPENDIX A

VIRTUAL GLUON CORRECTIONS IN QCD

We present the calculations of the virtual gluon corrections, which appear in the process $q\gamma^* \rightarrow q$ (Figs. 7c,d,e) and $q\bar{q} \rightarrow \gamma^*$ (Figs. 8c,d,e). The computation of the diagrams is carried out for the mass assignments, which are discussed in Sections 2, 3, and 4. Further, we use the Feynman gauge for the gluon field in our calculations. First we compute the electromagnetic (weak) vertex for space-like and time-like q^2 . Unlike in the ϕ_6^3 theory, the vertex will be mass-regularization-dependent. Then we call attention to the self-energy, which also is mass-dependent as it is in ϕ_6^3 . In both virtual corrections we will encounter anomalous cuts for the off-shell mass assignment (see Sections 2 and 3).

THE ELECTROMAGNETIC VERTEX CORRECTION

We remind the reader that the quark-gluon coupling constant is equal to

$$-i g_n = -i g M_{\text{reg}}^{2-\frac{n}{2}} \quad (\text{A.1})$$

The electromagnetic (weak) vertex is given by $-i\gamma_\mu(1+a\gamma_5)$. Straightforward application of the Feynman rules gives the lowest-order vertex correction,

$$\begin{aligned} -i\Gamma_\mu^{(1)} &= -\frac{4i}{3} (-i g_n)^2 \int \frac{d^n k}{(2\pi)^n} \frac{\gamma_\lambda (\not{p}_2 - \not{k} + m_2) \gamma_\mu (1+a\gamma_5) (\not{p}_1 - \not{k} + m_1) \gamma_\lambda i^2(-i)}{(k^2 - \lambda^2) [(k-p_2)^2 - m_2^2] [(k-p_1)^2 - m_1^2]} \\ &= -\frac{4}{3} g_n^2 2 \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx \int_0^1 dy \quad y \cdot \frac{\gamma_\lambda (\dots) \gamma_\mu (1+a\gamma_5) (\dots) \gamma_\lambda}{[D_k]^3} \quad (\text{A.2}) \end{aligned}$$

In the second equality we have introduced Feynman parameters; D_k is equal to

$$D_k \equiv k^2 - 2k \{ yx p_2 + y(1-x)p_1 \} + yx(p_2 - m_2^2) + y(1-x)(p_1^2 - m_1^2) - (1-y)\lambda^2 \quad (\text{A.3})$$

In the asymptotic limit $-q^2 \gg m_i^2$, p_i^2 we can neglect all masses in the numerator.

In this case we can write

$$-i\Gamma_\mu^{(1)} = -i \gamma_\mu (1+a\gamma_5) F(q^2) \quad (q = p_1 - p_2) \quad (\text{A.4})$$

After having performed the integral over k , we observe that the asymptotic form factor is composed of several contributions:

$$F(q^2) = F_a(q^2) + F_b(q^2) + F_c(q^2) \quad (A.5)$$

with

$$F_a = \frac{4}{3} \frac{2\pi^{\frac{n}{2}}}{(2\pi)^n} g_n^2 (n-3) \Gamma(2-\frac{n}{2}) \int_0^1 dx \int_0^1 dy \frac{y}{[D]^{2-\frac{n}{2}}} \quad (A.6)$$

$$F_b = \frac{4}{3} \frac{2\pi^{\frac{n}{2}}}{(2\pi)^n} g_n^2 \Gamma(3-\frac{n}{2}) \int_0^1 dx \int_0^1 dy \frac{y^3 x(1-x)q^2}{[D]^{3-\frac{n}{2}}} \quad (A.7)$$

$$F_c = \frac{4}{3} \frac{2\pi^{\frac{n}{2}}}{(2\pi)^n} g_n^2 \Gamma(3-\frac{n}{2}) \int_0^1 dx \int_0^1 dy \frac{y(1-y)q^2}{[D]^{3-\frac{n}{2}}} \quad (A.8)$$

The denominator function reads

$$D \equiv yx m_2^2 + y(1-x)m_1^2 + (1-y)\lambda^2 - xy(1-y)p_2^2 - (1-x)y(1-y)p_1^2 - y^2x(1-x)q^2 \quad (A.9)$$

If we also take the asymptotic limit in the denominator, we observe that F_a and F_b are M- and IR-finite in contrast with F_c . The latter expression also appears in the ϕ^3 theory in four dimensions. Introducing the strong coupling constant $\alpha_s = g^2/4\pi$, we obtain

$$F_a = \frac{\alpha_s}{3\pi} \left[\ln \frac{\mu_{reg}^2}{-q^2} - 3 + 1 \right] \quad (A.10)$$

$$F_b = - \frac{\alpha_s}{3\pi} \quad (A.11)$$

F_c depends on the chosen mass assignment. In this paper we have chosen off-shell, on-shell, and n-dimensional (massless) regularization.

a) Off-shell mass assignment

Here we have $p_1^2 = p_2^2 = \lambda^2 = 0$, $m_1^2 = \mu_1^2$, $m_2^2 = \mu_2^2$. This corresponds to the mass assignment which is chosen in Fig. 1a₁. The result is

$$F_c = \frac{\alpha_s}{3\pi} \left[-2 \ln \frac{\mu_1^2}{-q^2} \ln \frac{\mu_2^2}{-q^2} - 2 \ln \frac{\mu_1^2}{-q^2} - 2 \ln \frac{\mu_2^2}{-q^2} - 4 \right] \quad (\mu_i^2 > 0) \quad (A.12)$$

For $\mu_1^2 < 0$ we obtain anomalous cuts due to the fact that the internal masses m_{int} and the external masses m_{ext} satisfy the relation $m_{\text{ext}}^2 > \Sigma m_{\text{int}}^2$. In the analytic continuation we adopt the convention $\ln \mu_1^2 \rightarrow \ln(-\mu_1^2) + \pi i$; $F_C(q^2)$ becomes

$$F_C = \frac{\alpha_s}{3\pi} \left\{ \left[-2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - 2 \ln \frac{\mu_1^2}{q^2} - 2 \ln \frac{\mu_2^2}{q^2} - 4 + 2\pi^2 \right] + \pi i \left[-2 \ln \frac{\mu_1^2}{q^2} - 2 \ln \frac{\mu_2^2}{q^2} - 4 \right] \right\} \quad (\text{A.13})$$

Adding expressions (A.10), (A.11) and (A.13), we obtain $F^{VX}(a_1)$ in Eq. (2.2). If we continue F to time-like q^2 we obtain, in addition to the anomalous cut, also a normal cut contribution. These originate from Eq. (A.10) and (A.13). The result for $Q^2 > -(\mu_1 + \mu_2)^2$ is

$$F_2 = \frac{\alpha_s}{3\pi} \left[\ln \frac{M_{\text{reg}}^2}{Q^2} - 3 + 1 - \pi i \right] \quad (\text{A.14})$$

$$F_C = \frac{\alpha_s}{3\pi} \left\{ \left[-2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 4 \right] + \pi i \left[-2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} - 4 \right] + \pi i \left[2 \ln -\frac{\mu_1^2}{Q^2} + 2 \ln -\frac{\mu_2^2}{Q^2} + 4 \right] \right\} \quad (\text{A.15})$$

The second and the third term in Eq. (A.15) are the contributions due to the anomalous and normal cut, respectively. Adding Eq. (A.11), (A.14), and (A.15) gives Eq. (3.2).

The second off-shell mass assignment which corresponds with the one chosen in Fig. 1a2 is $p_1^2 = \mu_1^2 < 0$, $p_2^2 = \mu_2^2 < 0$, $\lambda^2 = m_1^2 = m_2^2 = 0$. The result is

$$F_C = \frac{\alpha_s}{3\pi} \left[-2 \ln \frac{\mu_1^2}{q^2} \ln \frac{\mu_2^2}{q^2} - 2 \ln \frac{\mu_1^2}{q^2} - 2 \ln \frac{\mu_2^2}{q^2} - \frac{2\pi^2}{3} \right] \quad (\text{A.16})$$

and for $Q^2 > 0$

$$F_C = \frac{\alpha_s}{3\pi} \left\{ \left[-2 \ln -\frac{\mu_1^2}{Q^2} \ln -\frac{\mu_2^2}{Q^2} - 2 \ln -\frac{\mu_1^2}{Q^2} - 2 \ln -\frac{\mu_2^2}{Q^2} + \frac{4\pi^2}{3} \right] + \pi i \left[2 \ln -\frac{\mu_1^2}{Q^2} + 2 \ln -\frac{\mu_2^2}{Q^2} + 4 \right] \right\} \quad (\text{A.17})$$

Adding (A.10), (A.11) and (A.16) gives $F^{VX}(a_2)$ in Eq. (2.3). In an analogous way we obtain $F^{VX}(a_2)$ for time-like Q^2 by adding Eqs. (A.11), (A.14) and (A.17) [see Eq. (3.3)].

b) On-shell mass assignment

Here we have $p_1^2 = p_2^2 = m_1^2 = m_2^2 = m^2$, $\lambda^2 \neq 0$. The calculation of F_c is straightforward and is given in many textbooks²⁹). We get

$$F_c = \frac{\alpha_s}{3\pi} \left[-\ln^2 \frac{m^2}{-q^2} + 2 \ln \frac{m^2}{\lambda^2} \ln \frac{m^2}{-q^2} - 4 \ln \frac{m^2}{-q^2} + \frac{\pi^2}{3} \right] \quad (A.18)$$

F^{Vx} [see Eq. (2.28)] can again be obtained from Eqs. (A.10), (A.11), and (A.18).

c) n-dimensional regularization

Here all masses are put equal to zero and n is arbitrary. Computation of (A.8) leads to the well-known gamma functions. The result is

$$\begin{aligned} F_c &= -\frac{4}{3} \cdot \frac{2\pi^{\frac{n}{2}}}{(2\pi)^n} \cdot g_n^2 \frac{\Gamma(3-\frac{n}{2})\Gamma(\frac{n}{2}-2)}{\Gamma(n-2)} (-q^2)^{\frac{n}{2}-2} \\ &= -\frac{4}{3} \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} g_n^2 \left[\frac{8}{(n-4)^2} + \frac{4\gamma-8}{(n-4)} + 8-4\gamma+\gamma^2-\frac{\pi^2}{6} \right] \left(\frac{\mu_{reg}^2}{-q^2} \right)^{2-\frac{n}{2}} \end{aligned} \quad (A.19)$$

By adding (A.10), (A.11), and (A.19) we obtain expression F^{Vx} in Eq. (4.1).

SELF-ENERGY CORRECTION

For general mass assignment, the self-energy contribution is given by (Figs. 7b,e):

$$\begin{aligned} -i\Sigma(p) &= \frac{4}{3} (-i g_n)^2 \int \frac{d^n k}{(2\pi)^n} \frac{\gamma_\lambda (\not{p}-\not{k}+m) \gamma_\lambda (i)(-i)}{[k^2-\lambda^2][(\not{p}-\not{k})^2-m^2]} \\ &= -\frac{4}{3} (i g_n)^2 \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} \left\{ \Gamma(2-\frac{n}{2}) \int_0^1 dx \frac{[-2(1-x)\not{p}+4m]}{[-x(1-x)p^2+xm^2+(1-x)\lambda^2]} \right. \\ &\quad \left. + \Gamma(3-\frac{n}{2}) (\not{p}-2m) \right\} \end{aligned} \quad (A.20)$$

The mass and wave function renormalization constant are defined by

$$\Sigma(p) = \delta m_q + (1-Z_1^{-1})(\not{p}-m_q) + \Sigma_f(p; m_q^2, m_l^2) = (1-Z_2^{-1})(\not{p}-\bar{m}_q) + \Sigma_f(p; \bar{m}_q^2, \bar{m}_l^2) \quad (A.21)$$

where m_q ($\bar{m}_q$) and m_l ($\bar{m}_l$) represent the unrenormalized and renormalized external (internal) masses; $\Sigma_f \sim (\not{p}-\bar{m}_q)^2$ stands for the renormalized part of $\Sigma(p)$. The wave function renormalization constant defined by

$$1 - Z_2^{-1} = \frac{\partial \Sigma}{\partial \not{p}} \Big|_{\not{p} = \bar{m}_q} \equiv \Sigma'(\bar{m}_q^2, \bar{m}_l^2, \dots) \quad (\text{A.22})$$

depends on the chosen mass assignment. The different choices are as follows:

a) Off-shell

$p^2 = 0$, $m^2 = \mu^2$, $\lambda^2 = 0$. From (A.20) and (A.22) we infer

$$1 - Z_2^{-1} = \Sigma'(\mu^2) = - \frac{\alpha_s}{3\pi} \left[\text{Ln} \frac{M_{\text{reg}}^2}{\mu^2} - \gamma + \frac{1}{2} \right] \quad (\mu^2 > 0) \quad (\text{A.23})$$

$$1 - Z_2^{-1} = \Sigma'(-\mu^2) = - \frac{\alpha_s}{3\pi} \left[\text{Ln} \frac{M_{\text{reg}}^2}{-\mu^2} - \gamma + \frac{1}{2} - \pi i \right] \quad (\mu^2 < 0) \quad (\text{A.24})$$

In the last expression we observe an anomalous cut due to the fact that $\mu^2 < 0$.

See also our remark below, Eq. (A.12).

b) On-shell

The on-shell result ($p^2 = m^2$, $\lambda^2 \neq 0$) is given in many textbooks²⁴⁾, see Eq. (2.29).

APPENDIX B

REAL GLUON CONTRIBUTIONS TO THE DI STRUCTURE FUNCTIONS AND DY CROSS-SECTIONS IN QCD

We present the calculation details of the DI structure functions and the DY cross-sections following from real-gluon production in QCD. Unlike in the ϕ_6^3 theory, the numerator terms, which are proportional to m^2 , survive in the scaling limit. Therefore we have to take them into account. Fortunately these mass terms are only contributing if the diagram has two equal propagators provided they contain the angular variable over which one integrates. The n-dimensional regularization method will be treated separately in Appendix C, since extra n-dependent terms appear there which are due to the spin of the quark and the gluon (see also Ref. 19).

The parton structure tensor in DI scattering $\gamma^* + p \rightarrow p_1 \dots p_n$ (Appendix C, paper 7) is given by

$$\hat{W}_{\mu\nu}(p, q) = \frac{1}{4\pi m} \sum_{k=0}^{\infty} \int \prod_{i=1}^k \frac{1}{(2\pi)^{n-1}} \frac{d^4 p_i}{2 p_{i0}} (2\pi)^n \delta^4(p+q-\sum p_i) M_{\mu}(u) M_{\nu}^*(u) \quad (B.1)$$

From Lorentz covariance and time-reversal invariance we infer

$$\begin{aligned} \hat{W}_{\mu\nu}(p, q) = & \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) \hat{W}_1 + \left(p_{\mu} - \frac{p_{\mu}q_{\nu}}{q^2} \right) \left(p_{\nu} - \frac{p_{\nu}q_{\mu}}{q^2} \right) \frac{\hat{W}_2}{m^2} \\ & + i \epsilon_{\mu\nu\alpha\beta} p^{\alpha} p^{\beta} \frac{\hat{W}_3}{m^2} + q_{\mu} q_{\nu} \frac{\hat{W}_4}{m^2} + (p_{\mu} q_{\nu} + p_{\nu} q_{\mu}) \frac{\hat{W}_5}{m^2} \end{aligned} \quad (B.2)$$

$\hat{W}_n$ and $\hat{W}_5$ do not contribute in the Bjorken (Bj) limit and therefore we will omit them from the rest of our paper. $\hat{W}_i$ are determined by the projections $C \equiv \hat{W}_{\mu}^{\mu}$, $D = (p^{\mu} p^{\nu} / m^2) \hat{W}_{\mu\nu}$, and $E = (1/m^2) \epsilon^{\mu\nu\lambda\sigma} p_{\lambda} q_{\sigma} \hat{W}_{\mu\nu}$:

$$\left(\frac{n-2}{4} \right) m \hat{W}_1 = \frac{m}{\beta} [-\beta C + D] \underset{b_j}{\approx} \frac{m}{2} \left[-\frac{1}{2} C + \frac{m^2}{\sqrt{s}} D \right] \quad (B.3)$$

$$\left(\frac{n-2}{4}\right) \sqrt{\hat{W}_2} = -\frac{\sqrt{V}}{4\beta^2} [\beta C - (n-1)D] \approx_{b_j} m_z \left[-\frac{1}{2}C + (n-1)\frac{m_z}{\sqrt{V}}D\right] \quad (B.4)$$

$$\sqrt{\hat{W}_3} = i \frac{m^2}{\sqrt{V}} E \quad (B.5)$$

with

$$\beta \equiv \left(1 - \frac{V^2}{q^2}\right), \quad V = \frac{pq}{m}, \quad z = -\frac{q^2}{2pq}$$

Definitions (B.1) to (B.4) are also valid for the massless case, provided one omits the m (for definition see also Appendix C, paper I).

In the following we determine the contributions of the cut-diagrams in Figs. 1, 2 and 5, 6 for all possible non-zero mass assignments.

We start with the calculation of the DI process in which a gluon and a quark appear in the final state (Figs. 7a,b). The structure tensor becomes

$$\hat{W}_{\mu\nu} = \frac{1}{4m} \frac{1}{(2\pi)^{n-1}} \int d^n p_i d^n p_5 \delta^+(p_i^2 - m_i^2) \delta^+(p_5^2 - m_5^2) \delta^n(p_i + q - p_j - p_5) M_\mu(2) M_\nu^*(2) \quad (B.6)$$

with $M_\mu(2)$ given by:

$$M_\mu^2(2) = -ig^2 \lambda^a \epsilon_\nu^*(p_5) \bar{u}(p_j) \left\{ \gamma_\mu (1 + \gamma_5) \frac{(\not{p}_j - \not{q} + m_{x_1})}{(p_j - q)^2 - m_{x_1}^2} \gamma^\nu + \gamma^\nu \frac{(\not{p}_i + \not{q} + m_{x_2})}{(p_i + q)^2 - m_{x_2}^2} \gamma_\mu (1 + \gamma_5) \right\} u(p_i) \quad (B.7)$$

We define $\hat{s}_i = (p_i + q)^2$ and $\hat{t}_j = (p_j - q)^2$. Since the off-shell mass assignment differs from diagram to diagram in Fig. 1, we split $\hat{W}_{\mu\nu}$ into the separate cut-graph contributions which are indicated by Figs. 1a₃, a₄, b₃, c₃. The projections C, D, and E for these contributions are given by ($n = 4$)

$$C(a_3) = \frac{4}{3} K_C \int d^4 p_5 \frac{1}{(\hat{t}_2 - m_1^2)(\hat{s}_4 - m_3^2)} \left\{ -q^2 \hat{t}_2 - q^2(\hat{s}_4 - q^2) \right\} \quad (B.8)$$

$$C(a_4) = \frac{4}{3} K_C \int d^4 p_5 \frac{1}{(\hat{t}_3 - m_4^2)(\hat{s}_1 - m_2^2)} \left\{ -q^2 \hat{t}_3 - q^2(\hat{s}_1 - q^2) \right\} \quad (B.9)$$

$$C(b_3) = \frac{4}{3} K_C \int d^4 p_S \frac{1}{(\hat{t}_{10} - m_\eta^2)^2} \{ (\hat{t}_{10} - m_\eta^2)(\hat{s}_8 - m_\eta^2) - q^2 \Delta_{8,7} \} \quad (B.10)$$

$$C(c_3) = \frac{4}{3} K_C \int d^4 p_S \frac{1}{(\hat{s}_{14} - m_\eta^2)^2} \{ (\hat{t}_{12} - m_\eta^2)(\hat{s}_{14} - m_\eta^2) - q^2 \Delta_{12,11} \} \quad (B.11)$$

$$D(b_3) = \frac{4}{3} K_D \int d^4 p_S \frac{1}{(\hat{t}_{10} - m_\eta^2)^2} \{ (\hat{t}_{10} - m_\eta^2)^3 + (\hat{s}_8 - q^2)(\hat{t}_{10} - m_\eta^2)^2 \} \quad (B.12)$$

$$D(a_3) = D(a_4) = D(c_3) \underset{b_j}{\approx} 0 \quad (B.13)$$

Notice that Eq. (B.13) does not contribute to the hard as well as the soft-gluon part of the structure function

$$E(a_3) = \frac{4}{3} i K_E \int d^4 p_S \frac{1}{(\hat{t}_2 - m_\eta^2)(\hat{s}_4 - m_\eta^2)} \{ -\hat{t}_2^2 \hat{s}_4 - \hat{t}_2 \hat{s}_4^2 + \hat{t}_2 q^4 - q^2(\hat{s}_4 - q^2)^2 \} \quad (B.14)$$

$$E(a_4) = \frac{4}{3} i K_E \int d^4 p_S \frac{1}{(\hat{t}_3 - m_\eta^2)(\hat{s}_1 - m_\eta^2)} \{ -\hat{t}_3^2 \hat{s}_1 - \hat{t}_3 \hat{s}_1^2 + \hat{t}_3 q^4 - q^2(\hat{s}_1 - q^2)^2 \} \quad (B.15)$$

$$E(b_3) = \frac{4}{3} i K_E \int d^4 p_S \frac{1}{(\hat{t}_{10} - m_\eta^2)^2} \{ 2(\hat{t}_{10} - m_\eta^2)^2 (\hat{s}_8 - m_\eta^2) + (\hat{t}_{10} - m_\eta^2) \hat{s}_8 (\hat{s}_8 - q^2) - q^2 (\hat{s}_8 - q^2) \Delta_{8,7} \} \quad (B.16)$$

$$E(c_3) = \frac{4}{3} i k_E \int d^4 p_S \frac{1}{(\hat{s}_{14} - m_{11}^2)} \left\{ (\hat{t}_{12} - m_{11}^2)(\hat{s}_{14} - m_{11}^2)(\hat{s}_{14} - q^2) + q^4 \Delta_{12,11} \right\} \quad (B.17)$$

With the definitions

$$k_C = \frac{\alpha_s}{\pi^2} \frac{(1+z^2)}{m} \quad ; \quad k_D = \frac{\alpha_s}{2\pi^2} \frac{(1+z^2)}{m^3} \quad ; \quad k_E = \frac{\alpha_s}{\pi^2} \frac{z}{m^3} \quad (B.18)$$

$d^4 p_S$ denotes the phase space of the final states indicated by the dashed line B in Fig. 1:

$$\Delta_{i,j} = 4 m_i m_j - m_i^2 - m_j^2 + m_5^2 \quad (B.19)$$

Note that we have put $m_7^2 = m_9^2$ and $m_{11}^2 = m_{13}^2$ (four-momentum conservation). In the above we have kept all masses in the numerator, which survive in the scaling limit [see, for example, Eq. (B.8)] and in the soft-gluon integral [see, for example, Eq. (B.10)]. The D projections do not contribute to the soft-gluon integrals, as we will see later. Following Appendix C in paper I, we can express C, D, and E in the phase-space integrals $I_i(p_j)$ given in Appendix B (paper I). In the Bjorken limit $I_i(p_j)$ is reduced to its asymptotic form given in Eq. (B.5) (paper I). Performing all integrations and the substitution of Eqs. (B.8) to (B.17) into Eqs. (B.3) to (B.5) leads to the parton structure functions $\hat{F}_i^{\text{R-hard}}(z, q^2)$. For each cut-graph in Fig. 1 they are given by

$$\hat{F}_i^{\text{R-hard}}(z) = \frac{\alpha_s}{3\pi} k_i \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 M_{q,1}^2(z)} + 2 - \frac{2}{1-z} \right] \quad (i=1,2) \quad (B.20)$$

$$\hat{F}_3^{\text{R-hard}}(z) = \frac{\alpha_s}{3\pi} k_3 \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 M_{q,1}^2(z)} + 3 - \frac{2}{1-z} \right] \quad (B.21)$$

$$\hat{F}_i^{R\text{-hard}}(a_4) = \frac{\alpha_s}{3\pi} K_i \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 M_{b_4}^2(z)} + 2 - \frac{2}{1-z} \right] \quad (i=1,2) \quad (B.22)$$

$$\hat{F}_3^{R\text{-hard}}(a_4) = \frac{\alpha_s}{3\pi} K_3 \left[\frac{2z}{1-z} \ln \frac{q^2}{z^2 M_{b_4}^2(z)} + 3 - \frac{2}{1-z} \right] \quad (B.23)$$

$$\hat{F}_1^{R\text{-hard}}(b_3) = \frac{2\alpha_s}{3\pi} K_1 \left[(1-z) \ln \frac{q^2}{z^2 M_{g,\eta}^2(z)} + z + A_{g,\eta}(z) \right] \quad (B.24)$$

$$\hat{F}_2^{R\text{-hard}}(b_3) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln \frac{q^2}{z^2 M_{g,\eta}^2(z)} + 3z + A_{g,\eta}(z) \right] \quad (B.25)$$

$$\hat{F}_3^{R\text{-hard}}(b_3) = \frac{2\alpha_s}{3\pi} K_3 \left[(1-z) \ln \frac{q^2}{z^2 M_{g,\eta}^2(z)} - 2 + 2z + A_{g,\eta}(z) \right] \quad (B.26)$$

$$\hat{F}_i^{R\text{-hard}}(c_3) = \frac{\alpha_s}{3\pi} K_i \left[\frac{1}{2} \frac{1}{1-z} \right] \quad (i=1,2,3) \quad (B.27)$$

K_i is given in Eq. (2.1):

$$M_{i,j}^2 = z^{-1} (1-z)^{-1} \{ z(1-z) m_i^2 - z m_j^2 - (1-z) m_j^2 \} \quad (B.28)$$

$$A_{i,j}(z) = \{ M_{i,j}^2(z) \}^{-1} \Delta_{i,j} \quad (B.29)$$

SOFT-GLUON CONTRIBUTIONS

In Section 3 of paper I we have discussed the origin of the $(1-z)^{-1}$ terms appearing in expressions (B.20) to (B.27). From these equations we infer that the soft-gluon contributions can only come from diagrams $1a_3, a_4, c_3$. For the on-shell case, we have in addition to the former contributions also a soft-gluon contribution from diagram $1b_3$. The soft-gluon structure functions are given by (see Section 3, paper I).

$$\hat{F}_i^{R\text{-soft}} = \delta(\hat{s} - m_j^2) \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s}' \hat{F}_i^R(\hat{s}', q^2, m_j^2) \quad (B.30)$$

Since the $(1-z)^{-1}$ poles are the Bjorken limits of the $(\hat{s} - m_1^2)^{-1}$ terms appearing in expressions (B.8) to (B.17), we conclude that only the projections C and E contribute

to the integral in Eq. (B.30). Therefore we obtain

$$\left(\frac{n-2}{2}\right) \hat{\mathcal{F}}_i^{\text{R-soft}} = -\delta(\hat{s}-m_f^2) \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s} \, m_i C(\hat{s}, q^2, m_j^2) \quad (i=1,2) \quad (\text{B.31})$$

$$\hat{\mathcal{F}}_3^{\text{R-soft}} = -2\delta(\hat{s}-m_f^2) i \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s} \, \frac{m_1^3}{q^2} E(\hat{s}, q^2, m_j^2) \quad (\text{B.32})$$

Since we have taken in the integrand $-q^2 \gg \hat{s}, m_i^2$, we can drop all terms which are non-leading in $-q^2$. Therefore in leading order in $-q^2$ we have the identity

$$E = \frac{k_E}{k_C} \{-q^2 C\} \quad (\text{B.33})$$

For all cut graphs this implies $2\hat{\mathcal{F}}_1^{\text{R-soft}} = \hat{\mathcal{F}}_2^{\text{R-soft}} = (1+a^2)/2a \hat{\mathcal{F}}_3^{\text{R-soft}}$.

Thus we have only to calculate $\hat{\mathcal{F}}_2^{\text{R-soft}}$. Further, in order to compute the soft-gluon contributions we need the full phase-space integral without any approximation (see Appendix B, paper I):

$$\mathcal{I}_i(p_j) = \frac{\pi}{2} \frac{|\vec{p}_f^*|}{\sqrt{\hat{s}}} \int_{-1}^1 dx \frac{1}{[A+Bx]^i} \quad (\text{B.34})$$

$$A = q^2 - 2 p_{f0}^* \cdot q_0^* + m_f^2 - m_j^2 \quad (\text{B.35})$$

$$B = 2 |\vec{p}_f^*| |\vec{q}^*| \quad (\text{B.36})$$

$p_{f0}^*, q_0^*, |\vec{p}_f^*|, |\vec{q}^*|$ are the energy and momenta in the c.m. frame of the incoming photon and quark.

We start with the calculation of diagram $1a_3$. Take first $\mu_1^2 > 0$ (no anomalous cuts).

From (B.34) to (B.36) we derive

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \int_{\mu_i^2}^{-\epsilon q^2} d\hat{s} \left\{ \frac{|\vec{p}_2^*|}{\sqrt{\hat{s}}} \frac{\hat{s}-q^2}{\hat{s}} \frac{2}{B} \ln \frac{A-B}{A+B} - \frac{|\vec{p}_2^*|}{\sqrt{\hat{s}}} \frac{4}{\hat{s}} \right\} \quad (\text{B.37})$$

$$A = q^2 - 2 \vec{p}_{20}^* q_0^* + m_2^2 - m_1^2 = q^2 + \mu_2^2 - \mu_1^2 - \frac{(\hat{s} + \mu_2^2)(\hat{s} + q^2)}{2\hat{s}} \quad (\text{B.38})$$

$$B = 2 |\vec{p}_2^*| |\vec{q}^*| = \frac{(\hat{s} - \mu_2^2)(\hat{s} - q^2)}{2\hat{s}} \quad (\text{B.39})$$

The calculation of the integral in (B.37) is straightforward, and we obtain

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left\{ 2 \ln \frac{\mu_1^2}{-q^2} \ln \frac{\mu_2^2}{-q^2} - 2 \ln \epsilon \ln \frac{\mu_2^2}{-q^2} + 2 \ln \frac{\mu_2^2}{-q^2} - 2 \ln \epsilon + 2 - \frac{\pi^2}{3} \right\} \quad (\text{B.40})$$

For $\mu_1^2 < 0$ we get anomalous cuts. The result is given in Eq. (2.19) (replace $\ln \mu_1^2 \rightarrow \ln(-\mu_1^2) + i\pi$). For diagram 1a, we find the expression ($\mu_1^2 < 0$)

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \int_0^{-\epsilon q^2} d\hat{s} \left\{ \frac{|\vec{p}_3^*|}{\sqrt{\hat{s}}} \frac{\hat{s}-q^2}{\hat{s}-\mu_1^2} \frac{2}{B} \ln \frac{A-B}{A+B} - \frac{|\vec{p}_3^*|}{\sqrt{\hat{s}}} \frac{4}{\hat{s}-\mu_1^2} \right\} \quad (\text{B.41})$$

$$A = q^2 - 2 \vec{p}_{30}^* q_0^* + m_3^2 - m_4^2 = \frac{q^2 - \hat{s} + \mu_1^2}{2} \quad (\text{B.42})$$

$$B = 2 |\vec{p}_3^*| |\vec{q}^*| = \frac{\hat{s}-q^2}{2} - \frac{\mu_1^2}{2} \frac{\hat{s}+q^2}{\hat{s}-q^2} \quad (\text{B.43})$$

Substituting A and B in Eq. (B.41) we get, after a straightforward calculation,

Eq. (2.20).

Finally, we are still left with diagram 1c₃. Take first $\mu_1^2 > 0$ (no anomalous cuts).

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{2\alpha_s}{3\pi} K_2 \int_{\mu_i^2}^{-\epsilon q^2} d\hat{s} \left\{ \frac{|\vec{p}_{12}^*|}{\sqrt{\hat{s}}} \frac{2A}{\hat{s}q^2} + \frac{|\vec{p}_{12}^*|}{\sqrt{\hat{s}}} \frac{2\mu_1^2}{\hat{s}^2} \right\} \quad (\text{B.44})$$

$$A = q^2 - 2 p_{12,0}^* q_0^* + m_{12}^2 - m_{11}^2 = q^2 + \mu_2^2 - \frac{(\hat{s} + \mu_2^2)(\hat{s} + q^2)}{2\hat{s}} + \mu_2^2 \quad (B.45)$$

The calculation is straightforward and the result is equal to

$$\hat{F}_2^{R\text{-soft}} = \delta(1-z) \frac{2\kappa_2}{3\pi} K_2 \left\{ -\frac{1}{2} \ln \frac{\mu_2^2}{-q^2} + \frac{1}{2} \ln \epsilon - \frac{1}{4} \right\} \quad (B.46)$$

For $\mu_2^2 < 0$ we find anomalous cuts. The result is given in Eq. (2.21).

We now call our attention to the DI process with a gluon in the initial state (Figs. 7f,g). Since there are no soft or virtual contributions, it is not necessary to analyse this process, diagram by diagram, as we did in the final gluon case.

The structure tensor reads (for the momentum assignment, see Fig. 7):

$$\hat{W}_{\mu\nu} = \frac{1}{4\lambda} \frac{1}{(2\pi)^3} \int d^4 p_1 d^4 p_2 \delta^+(p_1^2 - m_1^2) \delta^+(p_2^2 - m_2^2) \delta^4(k+q-p_1-p_2) M_\mu(2) M_\nu(2)^* \quad (B.47)$$

The amplitude is given by

$$M_\mu^a = i g \lambda^a \bar{u}(p_1) \left\{ \gamma_\mu (1 + 2\gamma_5) \frac{(p_1 - q + m_X)}{(p_1 - q)^2 - m_X^2} \gamma^\nu - \gamma^\nu \frac{(p_2 - q + m_X)}{(p_2 - q)^2 - m_X^2} \gamma_\mu (1 + 2\gamma_5) \right\} v(p_2) \epsilon_\lambda(\kappa) \quad (B.48)$$

Introduce the invariants $\hat{s} = (k+q)^2$ and $\hat{t}_i = (p_i - k)^2 = (p_i - q)^2$, $i, j = 1, 2$:

$$C = \frac{1}{2} K_C \sum_{i=1}^2 \int d^4 p_S \left\{ \frac{1}{(\hat{t}_i - m_X^2)^2} [(\hat{t}_i - m_X^2)^2 (\hat{s} - q^2) + q^2 \Delta_i] + \frac{1}{(\hat{t}_i - m_X^2)} \frac{2\hat{s} q^2}{\hat{s} - q^2} \right\} \quad (B.49)$$

$$D = \frac{1}{2} K_D \sum_{i=1}^2 \int d^4 p_S \{ \hat{s} \} \quad (B.50)$$

$$E = \frac{1}{2} K_E \sum_{i=1}^2 \int d^4 p_S (-1)^i \left\{ \frac{1}{(\hat{t}_i - m_X^2)^2} [(\hat{t}_i - m_X^2)^2 (\hat{s} - q^2) + (\hat{t}_i - m_X^2) (\hat{s} - q^2)^2 \right. \quad (B.51)$$

$$\left. + q^2 (\hat{s} - q^2) \Delta_i \right] + \frac{2\hat{s} q^2}{(\hat{t}_i - m_X^2)} \right\}$$

$$\Delta_i \equiv 4 m_i m_X - m_X^2 - m_i^2 + \lambda^2 \quad (B.52)$$

After having performed the integrations, we substitute C, D, and E in Eqs. (B.3)

to (B.5). The result is

$$\hat{F}_{1,g} = \frac{\alpha_s}{4\pi} K_1 \sum_{i=1}^2 \left[\{ z^2 + (1-z)^2 \} \ln \frac{q^2}{z^2 M_i^2(z)} - 1 + 2z(1-z) - H_i(z) \right] \quad (B.53)$$

$$\hat{F}_{2,g} = \frac{\alpha_s}{4\pi} K_2 \sum_{i=1}^2 \left[\{z^2 + (1-z)^2\} \ln \frac{q^2}{z^2 M_i^2(z)} - 1 + 6z(1-z) - H_i(z) \right] \quad (B.54)$$

$$\hat{F}_{3,g} = \frac{\alpha_s}{4\pi} K_3 \left[\{z^2 + (1-z)^2\} \ln \frac{M_3^2(z)}{M_1^2(z)} + H_2(z) - H_1(z) \right] \quad (B.55)$$

$$M_i^2(z) = z^{-1}(1-z)^{-1} [z(1-z)\lambda^2 - z m_1^2 - (1-z)m_x^2] \quad (B.56)$$

$$H_i(z) = \{M_i^2(z)\}^{-1} \Delta_i \quad (B.57)$$

THE DRELL-YAN PROCESS

The Drell-Yan cross-section is given by

$$\hat{S} Q^2 \frac{d\hat{\sigma}}{dQ^2} = \frac{4\pi\alpha^2}{9} \hat{W}(\hat{s}, Q^2) \quad (B.58)$$

with

$$\hat{W} = - \sum_{n=0}^{\infty} \int d^n q \delta^+(q^2 - Q^2) \prod_{i=1}^n \frac{k_i}{(2\pi)^{n-1}} \frac{d^{n-1} k_i}{2k_{i0}} \delta^n(p_1 + p_2 - \sum k_i - q) M_\mu(k) M_\mu^*(k) \quad (B.59)$$

As in deep inelastic scattering, we concentrate on the real-gluon contributions.

We study the process with a gluon in the final state (Figs. 8a,b). The cut-graphs are given in Figs. 5a₃, a₄, b₂, c₂. The cross-section up to order g^2 is given by

$$\hat{W}^R = -\frac{1}{4} \frac{1}{(2\pi)^3} \int d^4 q d^4 p_5 \delta^+(q^2 - Q^2) \delta^+(p_5^2 - m_s^2) \delta^4(p_1 + p_j - q - p_5) M_\mu(1) M_\mu(1)^* \quad (B.60)$$

$M_\mu(1)$ is equal to

$$M_\mu^a(1) = -i g \lambda^a \epsilon_{\nu}^*(p_5) \bar{V}(p_i) \left\{ \gamma^\nu \frac{(\not{p}_i - \not{q} - m_{x_1})}{(p_i - q)^2 - m_{x_1}^2} \gamma_\mu (1 + 2\gamma_5) \right. \\ \left. - \gamma_\mu (1 + 2\gamma_5) \frac{(\not{p}_j - \not{q} - m_{x_2})}{(p_j - q)^2 - m_{x_2}^2} \gamma^\nu \right\} u(p_j) \quad (B.61)$$

$\hat{s} = (p_i + p_j)^2 = (p_5 + q)^2$; $\hat{t}_i = (p_i - q)^2$. The contributions to $\hat{W}$ are indicated by Figs. 5a₃, a₄, b₂, c₂. We obtain

$$\hat{W}^R(a_3) = -\frac{4}{3} K_W \int d^4 p_S \frac{1}{(\hat{t}_4 - m_3^2)(\hat{t}_2 - m_1^2)} \{ \hat{s} Q^2 \} \quad (B.62)$$

$$\hat{W}^R(a_4) = -\frac{4}{3} K_W \int d^4 p_S \frac{1}{(\hat{t}_3 - m_4^2)(\hat{t}_1 - m_2^2)} \{ \hat{s} Q^2 \} \quad (B.63)$$

$$\hat{W}^R(b_2) = -\frac{4}{3} K_W \int d^4 p_S \frac{1}{(\hat{t}_{10} - m_7^2)^2} \{ (\hat{t}_{10} - m_7^2)^2 + (\hat{t}_{10} - m_7^2)(\hat{s} - Q^2) + Q^2 \Delta_{8,7} \} \quad (B.64)$$

$$\hat{W}^R(c_2) = -\frac{4}{3} K_W \int d^4 p_S \frac{1}{(\hat{t}_{14} - m_{11}^2)^2} \{ (\hat{t}_{14} - m_{11}^2)^2 + (\hat{t}_{14} - m_{11}^2)(\hat{s} - Q^2) + Q^2 \Delta_{12,11} \} \quad (B.65)$$

$\Delta_{8,7}$, and $\Delta_{11,12}$ are given in Eq. (B.19) $K_W = (\alpha_s/\pi^2)(1+a^2)^{-1}$. Following Appendix C in paper I we can express $\hat{W}$ in the phase-space integrals $J_i(p_j)$ given in Appendix B of paper I. Proceeding to the scaling limit, we get

$$\hat{W}^{R\text{-hard}}(a_3) = \frac{\alpha_s}{3\pi} K_2 \left[\frac{2z}{1-z} \ln \frac{-(1-z)Q^2}{z^2 M_{2,3}^2(z)} + \frac{2z}{1-z} \ln \frac{-(1-z)Q^2}{z^2 M_{4,1}^2(z)} \right] \quad (B.66)$$

$$\hat{W}^{R\text{-hard}}(a_4) = \frac{\alpha_s}{3\pi} K_2 \left[\frac{2z}{1-z} \ln \frac{-(1-z)Q^2}{z^2 M_{1,4}^2(z)} + \frac{2z}{1-z} \ln \frac{-(1-z)Q^2}{z^2 M_{3,2}^2(z)} \right] \quad (B.67)$$

$$\hat{W}^{R\text{-hard}}(b_2) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln \frac{-(1-z)Q^2}{z^2 M_{8,7}^2(z)} - 1 + z + A_{8,7}(z) \right] \quad (B.68)$$

$$\hat{W}^{R\text{-hard}}(c_2) = \frac{2\alpha_s}{3\pi} K_2 \left[(1-z) \ln \frac{-(1-z)Q^2}{z^2 M_{12,11}^2(z)} - 1 + z + A_{12,11}(z) \right] \quad (B.69)$$

with $M_{i,j}^2(z)$ and $A_{i,j}(z)$ defined in (B.28) and (B.29), and K_2 as given in Eq. (2.1).

We call attention to the soft-gluon part. Inspecting formulas (B.66) to (B.69) we conclude that the $(1-z)^{-1}$ poles originate from the graphs 5a₃, a₄. In the on-shell case we have, in addition, also diagrams 5b₂, c₂.

The definition for $\hat{W}^{R\text{-soft}}$ is analogous to the one we have for $\hat{F}_i^{R\text{-soft}}$:

$$\left(\frac{n-2}{2}\right) \hat{W}^{R\text{-soft}} = \delta(\hat{s} - p^2) \int_{\hat{s}_0}^{(1+\epsilon)Q^2} d\hat{s}' \hat{W}^R(\hat{s}', Q^2) \quad (\hat{s}_0 \text{ lowest threshold}) \quad (B.70)$$

As in the DI case, we need the full phase-space integral without any approximation. It is given by

$$J_i(p_j) = \frac{\pi}{2} \frac{|\vec{q}^*|}{\sqrt{s}} \int_{-1}^1 dx \frac{1}{[A+Bx]} i \quad (B.71)$$

$$A = Q^2 - 2 p_{j0}^* q_0^* + m_j^2 - m_i^2 \quad (B.72)$$

$$B = 2 |\vec{p}_j^*| |\vec{q}^*| \quad (B.73)$$

p_{j0}^* , q_0^* , $|\vec{p}_j^*|$, $|\vec{q}^*|$ are the energies and momenta in the c.m. frame of the incoming quarks. We start with the calculation of diagram $5a_3$. Take $\mu_i^2 < 0$ ($i = 1, 2$). We have

$$\hat{W}^{R\text{-soft}} = \delta(1-z) \frac{2\alpha_s}{3\pi} K_2 \int_{Q^2}^{(1+\epsilon)Q^2} d\hat{s} \left\{ \frac{1}{\hat{s} - Q^2 + \mu_1^2 - \mu_2^2} \left[L_n \frac{A_2 - B_2}{A_2 + B_2} + L_n \frac{A_4 - B_4}{A_4 + B_4} \right] \right\} \quad (B.74)$$

$$A_2 = Q^2 - 2 p_{20}^* q_0^* + m_2^2 - m_1^2 = Q^2 + \mu_2^2 - \mu_1^2 - \frac{(\hat{s} + \mu_2^2)(\hat{s} + Q^2)}{2\hat{s}} \quad (B.75)$$

$$B_2 = 2 |\vec{p}_2^*| |\vec{q}^*| = \frac{(\hat{s} - \mu_2^2)(\hat{s} - Q^2)}{2\hat{s}} \quad (B.76)$$

$$A_4 = Q^2 - 2 p_{40}^* q_0^* + m_4^2 - m_3^2 = Q^2 - \frac{(\hat{s} - \mu_2^2)(\hat{s} + Q^2)}{2\hat{s}} \quad (B.77)$$

$$B_4 = 2 |\vec{p}_4^*| |\vec{q}^*| = \frac{(\hat{s} - \mu_2^2)(\hat{s} - Q^2)}{2\hat{s}} \quad (B.78)$$

Substituting A_i and B_i [Eqs. B.75) to (B.78)] in (B.74) we obtain, after a straightforward calculation,

$$\begin{aligned} \hat{W}^{R\text{-soft}} = & \delta(1-z) \frac{\alpha_s}{3\pi} K_2 \left\{ \left[2 L_n - \frac{\mu_1^2}{Q^2} L_n - \frac{\mu_2^2}{Q^2} - 2 L_n \epsilon L_n - \frac{\mu_1^2}{Q^2} - 2 L_n \epsilon L_n - \frac{\mu_2^2}{Q^2} \right. \right. \\ & \left. \left. + 2 L_n^2 \epsilon - \frac{\pi^2}{3} \right] + \pi i \left[2 L_n - \frac{\mu_2^2}{Q^2} - 2 L_n \epsilon \right] \right\} \quad (B.79) \end{aligned}$$

[see Eq. (3.15)]. Notice that we have an anomalous cut due to the fact that $\mu_2^2 < 0$.

The calculation of diagram 5a₄ proceeds in an analogous way. Since we have chosen a symmetric mass assignment, the expression belonging to graph 5a₄ can be obtained by interchanging $\mu_1^2 \leftrightarrow \mu_2^2$. The result is given in Eq. (3.16).

The Drell-Yan cross-section for a process with a gluon in the initial state is given in Figs. 8f,g. Since there are no soft or virtual-gluon contributions, it is not necessary to analyse this process for each diagram separately. The cross-section reads

$$\hat{W}_{qg} = -\frac{1}{4} \frac{1}{(2\pi)^3} \int d^4q d^4p_1 \delta^+(q^2 - q^2) \delta^+(p_1^2 - m_1^2) \delta^4(k + p_2 - q - p_1) M_{\mu(1)} M_{\mu(1)}^* \quad (B.80)$$

The amplitude is given by

$$M_{\mu(1)}^2 = -ig\lambda^2 \bar{u}(p_1) \left\{ \gamma^\nu \frac{(p_2 - q + m_x)}{(p_2 - q)^2 - m_x^2} (1 + \alpha\gamma_5) \gamma_\mu + \gamma_\mu (1 + \alpha\gamma_5) \frac{(p_1 + q + m_x)}{(p_1 + q)^2 - m_x^2} \gamma^\nu \right\} u(p_2) \epsilon_\nu(k) \quad (B.81)$$

Introduce the invariants $\hat{s} = (p_1 + q)^2 = (p_2 + k)^2$, $\hat{t} = (p_2 - q)^2 = (p_1 - k)^2$

$$\hat{W}_{qg} = -\frac{1}{2} k_W \int d^4p_1 \left[\frac{1}{(\hat{t} - m_x^2)^2} \left\{ (\hat{t} - m_x^2)(\hat{s} - m_x^2) - Q^2 \Delta_1 \right\} - \frac{2}{(\hat{t} - m_x^2)(\hat{s} - m_x^2)} \left\{ (\hat{t} - m_x^2) Q^2 + Q^2(\hat{s} - Q^2) \right\} + \frac{1}{(\hat{s} - m_x^2)^2} \left\{ (\hat{t} - m_x^2)(\hat{s} - m_x^2) \right\} \right] \quad (B.82)$$

Δ_1 is defined in Eq. (B.52).

Integration over the phase-space gives

$$\hat{W}_{qg} = \frac{\alpha_s}{4\pi} K_2 \left[\left\{ z^2 + (1-z)^2 \right\} \ln \frac{-(1-z)Q^2}{z^2 M_1^2(z)} + \frac{1}{2} (1-z)(1+3z) - A_1(z) \right] \quad (B.83)$$

with $M_1^2(z)$ and $A_1(z)$ defined in Eq. (B.56), (B.57) respectively.

APPENDIX C

n-DIMENSIONAL REGULARIZATION

Since all masses are equal to zero it is not necessary to distinguish between the separate contributions originating from each cut-diagram. For DI scattering with a gluon in the final state, we start from diagrams 7a,b. The structure tensor and the matrix element are given by Eqs. (B.6) and (B.7), respectively, using the trace identities in the Appendix of Ref. 16:

$$C = \frac{4}{3} k_{c,n} \int d^n p_S \left[\left(\frac{n}{2} - 1 \right)^2 \left(\frac{\hat{s}}{\hat{t}} + \frac{\hat{t}}{\hat{s}} \right) + \left(\frac{n}{2} - 1 \right) \left\{ -\frac{2q^2(\hat{s}-q^2)}{\hat{t}\hat{s}} - \frac{2q^2}{\hat{s}} + (n-4) \right\} \right] \quad (C.1)$$

$$D = \frac{4}{3} k_{D,n} \int d^n p_S \left(\frac{n}{2} - 1 \right) (\hat{t} + \hat{s} - q^2) \quad (C.2)$$

$$E = \frac{4}{3} i k_{E,n} \int d^n p_S \left[\left(\frac{n}{2} - 1 \right) \left\{ (\hat{s}-q^2) \left(\frac{\hat{s}}{\hat{t}} + \frac{\hat{t}}{\hat{s}} \right) + 2\hat{s} - 2\hat{t} \right\} + \frac{2q^4}{\hat{s}} - \frac{2q^4(\hat{s}-q^2)}{\hat{t}\hat{s}} - 2\hat{s} \right] \quad (C.3)$$

$$k_{c,n} = 2 \frac{g_n^2}{(2\pi)^{n-1}} (1+a^2); \quad k_{D,n} = \frac{g_n^2}{(2\pi)^{n-1}} (1+a)^2; \quad k_{E,n} = \frac{g_n^2}{(2\pi)^{n-1}} 2a \quad (C.4)$$

with the invariants $\hat{s} = (p_1 + q)^2$, $\hat{t} = (p_2 - q)^2$; $d^n p_S$ stands for the n-dimensional phase-space. Performing the integrals for arbitrary n [see Appendix (B.8) paper I], and substituting C, D, E into (B.3) to (B.5), we obtain the expressions for $\hat{\mathcal{F}}_i^{\text{R-hard}}$ in Eq. (4.12).

We now call attention to the soft integral. The soft contributions originate from graphs 1a₃, a₄ and c₃. Note that graphs 1a₃ does not give a contribution, in contrast to the on-shell case. Further, we mention that expression (B.30) is unchanged, substitute C [Eq. (C.1)] into the integrand of Eq. (B.30); we obtain

$$\hat{F}_2^{R-soft} = \delta(1-z) \frac{4}{3} \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} g_n^2 k_2 \int_0^{-\epsilon q^2} d\hat{s} \left\{ \left(\frac{n}{2}-1\right) \left(\frac{n}{2}+2\right) \frac{\Gamma(\frac{n}{2}-2)}{\Gamma(n-2)} \right\} \hat{s}^{\frac{n}{2}-3} \quad (C.5)$$

Integration of expression (C.5) gives

$$\hat{F}_2^{R-soft} = \delta(1-z) \frac{4}{3} \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} g_n^2 k_2 \frac{(2n+8) \Gamma(\frac{n}{2})}{(n-4)^2 \Gamma(n-2)} \left\{ \frac{-\epsilon q^2}{M_{reg}^2} \right\}^{\frac{n}{2}-2} \quad (C.6)$$

Expansion around $n = 4$ leads to Eq. (4.13).

We now study the DI process with a gluon in the initial state (Figs. 7f,g). The structure tensor and the matrix element are given in Eqs. (B.47) and (B.48), respectively. The projections are

$$C = \frac{1}{2} k_{c,n} \sum_{i=1}^2 \int d^n p_s \left\{ \left(\frac{n}{2}-1\right)^2 \left(\frac{\hat{s}-q^2}{\hat{t}_i} + 1\right) + \left(\frac{n}{2}-1\right) \left(\frac{2\hat{s}q^2}{(\hat{s}-q^2)\hat{t}_i} + (n-4)\right) \right\} \quad (C.7)$$

$$D = \frac{1}{2} k_{D,n} \sum_{i=1}^2 \int d^n p_s \{ \hat{s} \} \quad (C.8)$$

$$E = \frac{1}{2} i k_{E,n} \sum_{i=1}^2 \int d^n p_s \left\{ (-1)^i \left[\left(\frac{n}{2}-1\right) (\hat{s}-q^2) \left(\frac{\hat{s}-q^2}{\hat{t}_i} + 1\right) + \frac{2\hat{s}q^2}{\hat{t}_i} \right] + (n-4) \frac{(\hat{s}-q^2)}{2} \right\} \quad (C.9)$$

After having performed the integrals (see Appendix B, paper I) we substitute Eqs. (C.7) to (C.9) in Eqs. (B.3) to (B.5). We obtain the expressions in Eqs. (4.15) to (4.17).

We now call attention to the Drell-Yan cross-section (fig. 8). The Born term (Fig. 8, DY) is given by

$$\hat{W}^{(0)} = \delta(1-z) \left(\frac{n}{2}-1\right) (1+a^2) \quad (C.10)$$

where $(\hat{n}/2)-1$ is a spinor factor which we have to divide out in all our expressions which follow hereafter. This implies that in the lowest order we obtain

$\hat{W}^{(0)}(z) = (1+a^2)\delta(1-z)$. Let us start with the process where we have a gluon

in the final state (Figs. 8a,b). The cross-section and the matrix element are given in Eqs. (B.59) and (B.60), respectively; $\hat{W}^R$ becomes

$$\left(\frac{n}{2}-1\right)\hat{W}^R = -\frac{4}{3}K_{c,n}\sum_{i=1}^2 \int d^n p_S \left\{ \left(\frac{n}{2}-1\right)^2 \left(\frac{\hat{s}-q^2}{\hat{t}_i} + 1\right) + \left(\frac{n}{2}-1\right) \left(\frac{2\hat{s}q^2}{(\hat{s}-q^2)\hat{t}_i} + (n-4)\right) \right\} \quad (C.11)$$

Integration over the phase space gives the result in Eq. (4.20).

The soft-gluon contributions are coming from diagrams 5a₃,a₄, in contrast to the on-shell case where also diagrams 5b₂,c₂ are contributing. Expression (B.70) is unchanged, and after substitution of $\hat{W}^R$ (C.11) into the integrand of Eq. (B.70) we get

$$\hat{W}^{R-\text{soft}} = \delta(1-z) \frac{4}{3} \frac{8\pi^{\frac{n}{2}}}{(2\pi)^n} g^2 K_2 \int_{q^2}^{(1+\epsilon)q^2} d\hat{s} \frac{\Gamma(\frac{n}{2}-2)}{\Gamma(n-3)} (\hat{s}-q^2)^{n-5} (q^2)^{2-\frac{n}{2}} \quad (C.12)$$

Integration of expression (C.12) gives

$$\hat{W}^{R-\text{soft}} = \delta(1-z) \frac{4}{3} \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} g^2 K_2 \left\{ \frac{16}{(n-4)^2} \frac{\Gamma(\frac{n}{2}-1)}{\Gamma(n-3)} \right\} \left\{ \frac{q^2}{M_{reg}^2} \right\}^{2-\frac{n}{2}} \left\{ \frac{\epsilon q^2}{M_{reg}^2} \right\}^{n-4} \quad (C.13)$$

Expanding around $(n/2)-2$ we obtain the result in Eq. (4.21).

We calculate the DY process with a gluon in the initial state (Figs. 8f,g). The cross-section and amplitude are given in Eq. (B.80) and (B.81), respectively; $\hat{W}_{qg}$ becomes

$$\left(\frac{n}{2}-1\right)\hat{W}_{qg} = -\frac{1}{2}K_{c,n} \int d^n p_S \left\{ \left(\frac{n}{2}-1\right)^2 \left(\frac{\hat{s}}{\hat{t}} + \frac{\hat{t}}{\hat{s}}\right) + \left(\frac{n}{2}-1\right) \left(\frac{-2q^2(\hat{s}-q^2)}{\hat{t}\hat{s}} - \frac{2q^2}{\hat{s}} + (n-4)\right) \right\} \quad (C.14)$$

after having performed the phase-space integral, we obtain the expression in (4.23).

Table 1

Moments of the DI structure functions with a quark in the initial state
[see Eqs. (6.11) and (6.5), (6.6)]

	$f_{i,q}^{(n)}$	$\Delta_{i,q}^{DI(n)}$	$\Delta_{2,q}^{DI(n)}$	$\Delta_{3,q}^{DI(n)}$	$\Delta_{L,q}^{DI(n)}$
OFF SHELL	$\frac{2}{3\pi} K_1 \left[-\frac{7}{4} - \frac{1}{2n} + \frac{3}{n+1} + \frac{2}{n^2} - \frac{2}{(n+1)^2} + \frac{3}{2} \sum_{k=1}^n \frac{1}{k} - \frac{1}{2} \sum_{k=1}^n \frac{1}{k^2} + \pi_1 \left(\sum_{k=1}^n \frac{1}{k} - \frac{1}{2} \right) \right] + \Delta_{i,q}^{DI(n)}$	$-\frac{2}{3\pi} K_1 \left[\frac{2}{n+1} \right]$	0	$-\frac{2}{3\pi} K_3 \left[\frac{2n+1}{n(n+1)} \right]$	$\frac{2}{3\pi} K_2 \left[\frac{2}{L_n+1} \right]$
ON SHELL	$\frac{2}{3\pi} K_1 \left[-\frac{5}{2} + \frac{1}{2n} - \frac{3}{n+1} - \frac{n^2 - 2n - 1}{n^2(n+1)^2} + \left(\frac{7}{2} + \frac{1}{n(n+1)} \right) \sum_{k=1}^n \frac{1}{k} - \frac{1}{2} \sum_{k=1}^n \frac{1}{k^2} - 2 \sum_{k=1}^n \frac{1}{k} \sum_{j=1}^k \frac{1}{j} \right] + \Delta_{i,q}^{DI(n)}$				
N-DIM	$\frac{2}{3\pi} K_1 \left[\zeta_q^{(n)} - \frac{9}{2} + \frac{3}{2n} + \frac{2}{n+1} + \frac{1}{n^2} - \frac{1}{n(n+1)} \sum_{k=1}^n \frac{1}{k} - \frac{1}{2} \sum_{k=1}^n \frac{1}{k^2} + \frac{3}{2} \sum_{k=1}^n \frac{1}{k} - 2 \sum_{k=1}^n \frac{1}{k} \sum_{j=1}^k \frac{1}{j} \right] + \Delta_{i,q}^{DI(n)}$				
"OFF SHELL" (eq. 5.5)	$\frac{2}{3\pi} K_1 \left[-1 - \frac{1}{2n} + \frac{3}{n+1} + \frac{2}{n^2} - \frac{2}{(n+1)^2} + \frac{3}{2} \sum_{k=1}^n \frac{1}{k} - \frac{1}{2} \sum_{k=1}^n \frac{1}{k^2} \right] + \Delta_{i,q}^{DI(n)}$				

Table 2

Moments of the DI structure functions with a gluon in the initial state
[See Eqs. (6.12) and (6.7-6.9)]

	$f_{i,g}^{(n)}$	$\Delta_{1,g}^{DI(n)}$	$\Delta_{2,g}^{DI(n)}$	$\Delta_{3,g}^{DI(n)}$	$\Delta_{L,g}^{DI(n)}$
OFF SHELL	$\frac{1}{4\pi} K_1 \left[-\frac{2}{n} + \frac{6}{(n+1)(n+2)} + \frac{2}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right] + \Delta_{1,g}^{DI(n)}$	$-\frac{1}{4\pi} K_1 \left[\frac{4}{(n+1)(n+2)} \right]$	0	0	$\frac{1}{4\pi} K_2 \left[\frac{4}{(n+1)(n+2)} \right]$
ON SHELL	$\frac{1}{4\pi} K_1 \left[-\frac{1}{n} + \frac{6}{(n+1)(n+2)} + \frac{1}{n^2} - \frac{n^2 + n + 2}{n(n+1)(n+2)} \sum_{k=1}^n \frac{1}{k} \right] + \Delta_{1,g}^{DI(n)}$				
N-DIM	$\frac{1}{4\pi} K_1 \left[\zeta \cdot p_g^{(n)} + \frac{4}{(n+1)(n+2)} + \frac{1}{n^2} - \frac{n^2 + n + 2}{n(n+1)(n+2)} \sum_{k=1}^n \frac{1}{k} \right] + \Delta_{1,g}^{DI(n)}$				

REFERENCES

- 1) B. Humpert and W.L. van Neerven, Infrared and mass regularization in A.F. field theories. I: ϕ_6^3 , Ref. TH 2785-CERN (1980).
- 2) M.K. Gaillard, Proc. XIIth Rencontre de Moriond, Flaine, 1977 (ed. J. Tran Thanh Van) (Univ. Paris, Orsay, 1977), p. 485.
M. Glück and E. Reya, Phys. Lett. 69B (1977) 77.
A.J. Buras and K. Gaemers, Nucl. Phys. B132 (1978) 249.
- 3) W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D 18 (1978) 3998.
E.G. Floratos, D.A. Ross and C.J. Sachrajda, Nucl. Phys. B129 (1977), 66;
E: Nucl. Phys. B139 (1978) 545; and B152 (1979) 493.
- 4) J. Abad and B. Humpert, Phys. Lett. 77B (1978) 105; 78B (1978) 627;
E: 80B (1979) 433; 80B (1978) 115, 80B (1979) 286.
- 5) J. Abad, B. Humpert and W.L. van Neerven, Phys. Lett. 83B (1979) 371.
- 6) B. Humpert and W.L. van Neerven, Phys. Lett. 84B (1979) 327; E: 85B (1979) 471.
- 7) B. Humpert and W.L. van Neerven, Phys. Lett. 85B (1979) 293.
- 8) G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. B143 (1978) 521;
E: B146 (1978) 544.
- 9) G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. B157 (1979) 461.
- 10) J. Kubar-André and F.E. Paige, Phys. Rev. D 19 (1978) 221.
- 11) K. Harada, T. Kaneko and N. Sakai, Nucl. Phys. B155 (1979) 169 and details in Erratum Ref. TH 2619-CERN (1979).
- 12) A.P. Contogouris and J. Kripfganz, Phys. Rev. D 9 (1979) 2207.
- 13) H.D. Politzer, Nucl. Phys. B129 (1977) 301.
C.T. Sachrajda, Phys. Lett. 73B (1978) 185.
D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54, B146 (1978) 29.
R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Phys. Lett. 78B (1978) 281 and Nucl. Phys. B152 (1979) 285.
S.B. Libby and G. Sterman, Phys. Lett. 78B (1978) 618.
- 14) a. T. Kinoshita, J. Math. Phys. 3 (1962) 650;
b. T.D. Lee and M. Nauenberg, Phys. Rev. 133 (1964) B1549.
c. N. Nakaniski, Progr. Theor. Phys. 19 (1958) 159.
- 15) B. Humpert and W.L. van Neerven, Phys. Lett. 89B (1979) 69.
- 16) V. Sudakov, Soviet Phys.- JETP 3 (1956) 65.
- 17) D.R. Yennie, S.C. Frautschi and H. Suura, Annals of Physics 13 (1961) 379.
- 18) N. Papanicolaou, Phys. Rep. 24C (1976) 4.

- 19) G. 't Hooft and M. Veltman, Nucl. Phys. 44B (1972) 189.
C.G. Bollini and J.J. Giambiagi, Nuovo Cimento 12B (1972) 20.
W.J. Marciano, Phys. Rev. D 12 (1975) 3861.
- 20) J. Kripfganz, Comments on regularization prescription dependence of QCD corrections to the Drell-Yan formula, Mc Gill University preprint (1979).
- 21) M. Calvo, Phys. Rev. D 15 (1977) 730.
- 22) T. Muta, A note on the renormalization prescription dependence of the QCD calculations of structure function moments, Research Institute for Fundamental Physics, Kyoto University preprint RIFP-364 (1979).
- 23) J. Ellis, Status of perturbative QCD, Ref. TH. 2744-CERN (1979).
- 24) A.I. Akhiezer and V.B. Berestetskii, Quantum electrodynamics (Interscience, New York, 1965).
L.D. Landau, E.M. Lifshitz and L.P. Pitaevskii, Relativistic quantum mechanics II, 4th Course of theoretical physics (Pergamon Press, Oxford, 1973).

Figure captions

- Fig. 1 : Double-cut-graphs of DI scattering on a quark target (left) and on an antiquark target (right). The corresponding lines are indicated by numbers.
- Fig. 2 : Double-cut-graphs of DI scattering on a gluon target (left), and their alien graphs (right) which are not considered in the cross-section. The corresponding lines are indicated by numbers.
- Fig. 3 : Order g^2 Feynman diagrams of virtual Compton scattering $\gamma^* q \rightarrow \gamma^* q$ (left), and their cut-graphs contributing to DI scattering (right).
- Fig. 4 : Order g^2 Feynman diagrams of virtual Compton scattering on a gluon target $\gamma^* g \rightarrow \gamma^* g$ (left), and the corresponding cut-graphs contributing to DI scattering (right).
- Fig. 5 : Double-cut-graphs of the $q\bar{q}$ -initiated DY process. Corresponding lines are indicated by numbers.
- Fig. 6 : Double-cut-graphs of the qg - and $\bar{q}g$ - initiated DY process (left of principle dashed lines), and their alien graphs (right of principle dashed lines) which are not considered in the cross-section. The corresponding lines are indicated by numbers.
- Fig. 7 : Order g^2 Feynman diagrams contributing to DI scattering.
- Fig. 8 : Order g^2 Feynman diagrams contributing to DY massive μ -pair production.

Fig. 2

VIRTUAL COMPTON AMPLITUDE

DEEP INELASTIC CROSS SECTION

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8