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CHALLENGES FOR EXTENDED TECHNICOLLOUR THEORIES

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A B S T R A C T

We develop the dialectic between extended technicolour theories and phenomenological constraints on rare processes. The natural suppression of flavour-changing neutral currents and CP violating processes which can occur in models with elementary scalars is not a general feature of naïve extended technicolour (ETC) models. We study the extent to which naïve ETC estimates must in fact be suppressed in order to be compatible with the phenomenology of such rare processes. We emphasize the potential significance of the exchanges of neutral flavour-changing bosons and find that the strongest constraints arise from considering the combination of the D_1^0 - D_2^0 and K_1^0 - K_2^0 systems. CP violating effects in ETC models must be severely suppressed if they are to be compatible with the observational facts. We point to several rare processes whose further experimental study is of particular concern to ETC theories.

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1. - INTRODUCTION

A classic problem in the construction of weak interaction theories has been to understand the observed strong suppression of flavour-changing neutral interactions. A giant step towards the complete solution of this problem was taken by Glashow, Iliopoulos and Maiani (GIM) ¹⁾. The introduction of charm suppressed to phenomenologically acceptable levels all flavour-changing couplings of neutral vector bosons as well as the effective neutral interactions generated by multiple exchanges of charged vector bosons. It was subsequently shown ²⁾ that the GIM mechanism was essentially the only way of suppressing these neutral interactions naturally, i.e., for arbitrary values of the masses and mixing angles of fermions of given charge and helicity, in the absence of any symmetries or conservation laws restricting the form of the mixing matrix. The naturalness conditions for an arbitrary weak gauge group G can be summarized ³⁾ by the requirements :

- 1) all the fermions of a given charge and helicity must occur in equivalent representations of G , and
- 2) they must all have the same group-theoretical weights within their representations.

The specific GIM model ¹⁾ meets these requirements by assigning all left-handed fermions to $SU(2)$ isodoublets and all right-handed fermions to isosinglets, and giving all helicity states of a given charge identical values of I_3 and Y . In principle, flavour-changing neutral weak interactions could also be mediated by exchanges of spinless bosons such as physical Higgs bosons in theories with fundamental scalar fields. In such theories, the natural suppression of such flavour-changing neutral effects can be suppressed naturally ²⁾ by the requirement that

- 3) all fermions of a given charge and helicity acquire their masses from the same Higgs field.

There is much discontentment with gauge theories containing fundamental scalar fields, fuelled by the large number of apparently arbitrary parameters in their couplings, some of which must be adjusted with unnatural precision. It is therefore proposed that the spontaneous symmetry breaking required in gauge theories be generated dynamically ⁴⁾. This suggestion first entails the introduction of a new exact non-Abelian gauge symmetry called technicolour (TC), whose coupling is strong at a momentum scale of about 300 GeV. It is expected that non-perturbative effects in the strong-coupling regime would cause a breakdown of chiral symmetry accompanied by the appearance of light pseudoscalar bound states

analogous to the pion of QCD. Some of these would be massless and "eaten" by the $W^\pm$ and Z^0 gauge bosons becoming their longitudinal helicity states and allowing them to acquire a mass. Others of the pseudoscalars called pseudo-Goldstone bosons (PGBs) acquire masses $\lesssim 300$ GeV from higher order QCD, weak and electromagnetic interactions as well as so-called extended technicolour (ETC) interactions. These ETC interactions are needed ⁵⁾ for giving masses to the observed fermions by interconnecting them with technifermions.

ETC theories contain several possible sources of flavour-changing neutral interactions. The most popular scenario ⁶⁾ for ETC interactions puts all the conventional fermions of a given charge and helicity into a common representation of the ETC group with a multiplet of technifermions irreducible under the unbroken TC subgroup. The full ETC group is then supposed to break down at a sequence of energy scales into the TC subgroup. The ETC group will therefore also contain generators coupling different conventional fermions of the same charge and helicity and hence giving rise to flavour-changing neutral weak interactions. Box and other higher order diagrams involving the multiple exchanges of charged or neutral ETC bosons will have similar effects. So also will the exchanges of the relatively low-mass PGBs ^{7),8)} whose couplings are similar in nature to those of physical Higgs bosons in theories with fundamental Higgs fields.

By their nature, ETC theories do not obey all the requirements ^{2),3)} necessary for the natural suppression of flavour-changing neutral weak interactions, since all the conventional fermions of a given charge and helicity occupy different weights in a common representation of the ETC group. Also, it is not clear that the PGB couplings will be flavour-diagonal and whether there is an analogue to the requirement 3) above on the form of the Higgs fermion interactions.

It is not clear, however, that the full rigour of the three naturalness requirements above should in fact be applied to ETC theories. For one thing, the masses of the ETC bosons are considerably higher than those of the vector bosons previously encountered in weak interactions, and their single and multiple exchanges are suppressed accordingly. For another, the fermion mass matrices in ETC theories are in principle not arbitrary. The conventional fermion masses are believed to be related to the ETC boson masses by

$$m_f \approx (300 \text{ GeV})^3 / m_{\text{ETC}}^2 \quad (1)$$

and all the mixing angles are expected to be calculable in a forthcoming realistic theory. The naturalness requirements above were derived by contemplating arbitrary mass matrices for the fermions of each charge, and it may well be that in the true ETC theory there will be cancellations which now seem to us accidental, but may eventually be seen to follow from presently unsuspected symmetries of the theory.

In this paper we compute the natural orders of magnitude to be expected for various flavour-changing neutral interactions within the present framework of ETC ideas ^{8),9)}. Known fermion masses and the mass formula (1) enable us to compute the ETC boson masses and hence estimate the expected effects. Comparing these directly with the experimental values or upper limits on flavour-changing neutral interactions we determine what extra suppression factors will be necessary if ETC theories are to become consistent with experiment. While we briefly indicate some possible mechanisms for achieving these extra suppressions, many of them still remain as challenges for ETC theories.

Our estimates of these further necessary suppression factors are set out in Table 1. We have considered four classes of diagrams causing flavour-changing neutral interactions, all of which would be naturally suppressed to levels consistent with experiment in weak gauge theories with explicit spinless fields. These include single gauge boson exchange (SGEX), single PGB exchange (SPEX), and higher order exchanges (HOEX) such as double gauge boson exchange (DGEX). In the cases of SGEX and HOEX we have tabulated the further suppressions necessary given our estimates of the vector boson masses. In the case of SPEX, the considerable uncertainties ⁶⁾⁻⁸⁾ in the expected masses of the PGBs have led us to tabulate the lower limits on their masses suggested by upper limits on the effective neutral interactions they induce in the absence of further suppressions. The most significant challenges seem to arise from purely hadronic $\Delta F = 2$ effects, but we also present results on semi- and purely leptonic processes as well.

We would like to mention three aspects of our analysis which may be somewhat novel. One is our emphasis on the possible effects of exchanges of the neutral flavour-changing gauge bosons which must acquire masses at each stage of the breakdown of the ETC group to the exact TC subgroup. The second is our emphasis on CP violating effects in the K_1-K_2 mass matrix. Opinions differ as to whether the Kobayashi-Maskawa (KM) mechanism ¹⁰⁾ explains naturally the small ratio $O(10^{-3})$ of the imaginary and real $\Delta S = 2$ parts of the K_1-K_2 mass matrix, but the smallness of the phenomenon is certainly a challenge to ETC theories ¹¹⁾. In the absence of some as yet unknown symmetry, the coupling matrices of the ETC bosons have phases of order unity relative to the KM matrix.

This means that the ratios of imaginary to real parts of ETC exchange effects are a priori of order one, and therefore that the real as well as the imaginary parts of the ETC exchanges should be suppressed 10^3 more than suggested by considerations of the real part alone - unless some new symmetry can be found. The third point we would emphasize concerns the constraints imposed by the D_1 - D_2 system. Even the slack present limit ¹²⁾ on D^0 - $\bar{D}^0$ mixing ($\leq 0(10)\%$) already tells us that the numerical suppression of the D_1 - D_2 mass difference must be comparable with the GIM expectation of order G_F^2 . This is enough to prevent one from constructing models in which $\Delta F \neq 0$ transitions among the charge $\frac{2}{3}$ quarks are much larger than those among the charge $-\frac{1}{3}$ quarks.

Most of this paper is devoted to developing the challenges for ETC theories set out in Table 1. In Section 2 we review the generic structure of simple ETC models and estimate the masses of the various ETC bosons. In Section 3 we estimate SGEX diagrams and compare them with the experimental magnitudes of different $\Delta F \neq 0$ effects. In Section 4 we do the same for SPEX while Section 5 is devoted to HOEX such as DGEX. Section 6 contains comments on CP violation and uncertainties in the ETC estimate ¹¹⁾ for the neutron electric dipole moment, given the problems of Table 1 for the imaginary part of the K_1 - K_2 mass matrix. Finally, Section 7 discusses the prospects that ETC theories may meet the challenges raised in earlier parts of this paper. We observe that the leading order exchanges of PGBs are flavour-diagonal in so-called "monophagic" theories ¹³⁾ where all the fermions of any given charge all get their masses from couplings to the same technifermion condensate. We also mention some of the problems arising in trying to GIM the ETC interactions by xerox copying the technifermions to have one technigeneration per conventional generation. These problems include an immense proliferation in the number of PGBs and the difficulties encountered in giving them masses consistent with the limits of Table 1, and the related question of breaking the symmetries between generations. ETC theories are not yet out of the wood. Finally, at the end of this section we remind readers of some specific rare processes: D_1 - D_2 mixing, $K_L^0 \rightarrow e\mu$ and $K^+ \rightarrow \pi^+ e\mu$ decays, $\mu N \rightarrow e N$ capture and $\mu \rightarrow e\gamma$ decay, where further experimental studies may either see ETC effects or pose more serious challenges for ETC theories.

2. - GENERAL FEATURES OF ETC THEORIES

The generic structure of simple ETC theories ^{5),6)} is indicated in Fig. 1. To all the classes of conventional fermions, charge $\frac{2}{3}$ quarks, charge $-\frac{1}{3}$ quarks, neutrinos and charged leptons, are associated technifermions (U, D, N and E) which are collected into a technigeneration parallelling the conventional fermion generations. The ETC group is supposed to contain the TC group and to

have generators transforming conventional fermions into technifermions and vice versa. It is suggested that the ETC group breaks down to the TC group in stages, perhaps "spitting out" a generation of fermions at each stage of breaking by a dynamical tumbling mechanism. ETC theories give masses to fermions via the diagram shown in Fig. 2, from which we abstract the property that ETC bosons must have both left- and right-handed couplings. The ETC boson exchange takes place over a distance short by comparison with the rest of the loop, which is cut off by the softness of the technifermion mass X in Fig. 2. This arises from the dynamical breakdown of chiral symmetry in the TC subgroup which has a characteristic scale of order 300 GeV.

To calculate the masses of the ETC bosons we first Fierz transform the exchange in Fig. 2 to get ⁵⁾ (assuming a conventional normalization analogous to that of the $W^\pm$ for the ETC gauge couplings) :

$$m_f = \frac{g_{ETC}^2}{2m_{ETC_f}^2} \langle 0 | \sum_{TC} \bar{F}_{TC} F_{TC} | 0 \rangle \quad (2)$$

To estimate $\langle 0 | \bar{F}F | 0 \rangle \equiv \sum_{TC} \langle 0 | \bar{F}_{TC} F_{TC} | 0 \rangle$ we rescale naively from QCD :

$$\langle 0 | \bar{F}F | 0 \rangle = \sum_c \langle 0 | \bar{q}_c q_c | 0 \rangle \times \left(\frac{F_\pi}{f_\pi} \right)^3 \times \left(\frac{N}{3} \right)^{1/2} \quad (3)$$

where N is the dimension of the technirepresentation, f_π is the conventional pion decay constant ~ 95 MeV and F_π is its technianalogue ~ 125 GeV. To estimate $\langle 0 | \bar{q}q | 0 \rangle \equiv \sum_c \langle 0 | \bar{q}_c q_c | 0 \rangle$ we use the PCAC relation

$$(m_d + m_u) \langle 0 | \bar{q} q | 0 \rangle = f_\pi^2 m_\pi^2 \quad (4)$$

and the conventional values $m_d = 10$ MeV, $m_u = \frac{1}{1.8} m_d$ to get

$$\langle 0 | \bar{q} q | 0 \rangle = 0.0106 \text{ GeV}^3 \quad (5)$$

Equation (3) then gives us

$$\langle 0 | \bar{F}F | 0 \rangle = 0.0241 \text{ TeV}^3 \quad \text{if } N=4 \quad (6)$$

which substituted into Eq. (2) yields

$$m_f \approx \frac{0.012 \text{ TeV}^3}{v_{ETC_f}^2} \quad (7)$$

where $v_{ETC} = m_{ETC}/g_{ETC}$. The ETC boson masses and coupling constants generally appear in this combination : of course $v_{ETC} = m_{ETC}$ if $g_{ETC} \approx 1$ which seems a reasonable estimate. Re-inserting $m_d = 10 \text{ MeV}$ into Eq. (7) finally yields for the boson which makes the dominant contribution to the mass of the down quark

$$v_{ETC_d}^2 \approx 1200 \text{ TeV}^2 \quad (8)$$

This estimate is of course subject to considerable uncertainties. We have neglected various subtleties such as scaling the number of technicolours in Eq. (3) and the logarithmic momentum scale variation of the fermion masses and operator matrix elements, but these are unlikely to change the estimate (8) by more than a factor of 2. The principal uncertainty is in the value of $m_d = 10 \text{ MeV}$ we used : because we also used m_d in making the estimate (5), $m_{ETC_d}^2 \propto 1/m_d^2$ and an order of magnitude uncertainty in $m_{ETC_d}^2$ is easily conceivable. We can use the approximate formula (7) to estimate masses for the ETC bosons which should give masses to other quarks : the results are shown in Table 2 together with the masses of the quarks we assumed. While these estimates are very tentative, they do suggest that the mass of the ETC_t vector bosons in particular may be not much larger than the dynamical scale of the TC interactions.

Before using the masses of Table 2 to estimate various higher-order flavour-changing neutral interactions we should emphasize one other feature of ETC theories. When the ETC group breaks down giving masses to the ETC_f bosons which give a mass to the fermion f , other electrically neutral vector bosons coupling f to itself and to the other generations of conventional fermions of the same charge (the horizontal ETC bosons of Fig. 1) will also acquire masses of similar magnitude to m_{ETC_f} . For example, one expects to encounter both flavour-diagonal and generation-changing bosons with essentially unit coupling strengths transforming $d \leftrightarrow d$, $d \leftrightarrow s$ and $d \rightarrow b$ with $v_{ETC}^2 \approx 300 \text{ TeV}^2$. To be more specific, suppose that the ETC group commutes with the electroweak group $SU(2)_L$, a reasonable assumption since it would be difficult (though not impossible) to understand the difference in strengths of the technicolour and $SU(2)_L$ couplings if the ETC and $SU(2)_L$ interactions were unified, and baryon number conservation at low energies would no longer be automatic. If ETC and $SU(2)_L$ commute, then the

eigenstates of ETC are the same Cabibbo-rotated ^{*)} d_θ^L, S_θ^L eigenstates of $SU(2)_L$. Therefore any diagonal currents that cause the transition $S_\theta \rightarrow S_\theta$ are going to mediate $\Delta S = 2$ amplitudes of order $1/v_{ETCs}^2$, and similarly for $d_\theta \rightarrow d_\theta$. In general, generation-changing $f \leftrightarrow f'$ couplings of unit strength will be mediated by bosons with masses associated via Table 2 with the lighter of the two fermions f, f' . In addition, the existence of non-zero Cabibbo mixing suggests that there will in general also be Cabibbo-suppressed couplings of $f \leftrightarrow f'$ via a boson whose mass is associated via Table 2 with the heavier of the two fermions f, f' . Generally, we expect that $\Delta G = 0$ processes may be mediated by ETC bosons with unit coupling strengths, while $|\Delta G| = 1, 2$ processes always require a Cabibbo suppression factor of order θ, θ^2 , respectively. We will see in the next section that the exchanges of these massive flavour-non-diagonal ETC bosons will raise interesting challenges for ETC theories.

3. - SINGLE GAUGE BOSON EXCHANGE (SGEX)

K_1-K_2 and D_1-D_2

We start off by considering the contributions of the SGEX diagrams of Fig. 3 to various purely hadronic $\Delta F = 2$ effects : the real parts of the K_1-K_2 and D_1-D_2 mass matrices. From the arguments at the end of Section 2 one expects SGEX to generate $|\Delta S| = 2$ and $|\Delta C| = 2$ (and hence $|\Delta G| = 2$) four-fermion effective Lagrangians of the general orders of magnitude

$$\mathcal{L}_{(\bar{S}d)^2}^{\text{eff}} \approx \frac{1}{2} \left\{ (\bar{S}_L \gamma_\mu d_L)^2, (\bar{S}_L \gamma_\mu d_L)(\bar{S}_R \gamma^\mu d_R), (\bar{S}_R \gamma_\mu d_R)^2 \right\} \times \left(\frac{\theta^2}{v_{ETCs_d}^2}, \frac{\theta^2}{v_{ETCs_s}^2} \right) \quad (9)$$

$$\mathcal{L}_{(\bar{C}u)^2}^{\text{eff}} \approx \frac{1}{2} \left\{ (\bar{C}_L \gamma_\mu u_L)^2, (\bar{C}_L \gamma_\mu u_L)(\bar{C}_R \gamma^\mu u_R), (\bar{C}_R \gamma_\mu u_R)^2 \right\} \times \left(\frac{\theta^2}{v_{ETCs_u}^2}, \frac{\theta^2}{v_{ETCs_c}^2} \right) \quad (10)$$

^{*)} Of course, the origin of the Cabibbo angle may lie in the up quark sector, or in the down quark sector, or can be shared between them.

where the subscripts L, R indicate projection operators $\frac{1}{2}(1 \mp \gamma_5)$, and θ^2 indicates mixing angle factors which we take to be $O(\sin^2 \theta_c) = O(\frac{1}{20})$. As a standard of comparison we can contrast the $\mathcal{L}_{(\bar{s}d)2}^{\text{eff}}$ of Eq. (9) with that obtained from the conventional GIM calculation of Gaillard and Lee¹⁴⁾:

$$\mathcal{L}_{(\bar{s}d)2}^{\text{GIM}} = (\bar{s}_L \gamma_\mu d_L)^2 \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \frac{m_c^2}{m_W^2 \sin^2 \theta_W} \sin^2 \theta_c \cos^2 \theta_c \quad (11)$$

Requiring that $\mathcal{L}_{(\bar{s}d)2}^{\text{eff}}$ be no larger than $\mathcal{L}_{(\bar{s}d)2}^{\text{GIM}}$, we need *)

$$\frac{\theta^2}{2V_{\text{ETC}_d}^2} + \frac{\theta^2}{2V_{\text{ETC}_s}^2} \leq \frac{1}{2} \times 10^{-12} \text{ GeV}^{-2} \quad (12)$$

, and hence if $\theta^2 \sim \frac{1}{20}$

$$V_{\text{ETC}_d}^2, V_{\text{ETC}_s}^2 \gtrsim 5 \times 10^4 \text{ TeV}^2 \quad (13)$$

The conflicts between the limits (13) and the previously deduced values of $V_{\text{ETC}_d}^2$ and $V_{\text{ETC}_s}^2$ shown in Table 2 imply the need for an additional suppression factor in ETC theories of up to 1000, as tabulated in the challenging Table 1. (Having made the point that m_{ETC} generally appears in the combination $V_{\text{ETC}} \equiv m_{\text{ETC}}/g_{\text{ETC}}$, we will henceforth implicitly assume that $g_{\text{ETC}} \approx 1$ unless otherwise stated: this assumption can easily be relaxed.)

We now turn to the D_1 - D_2 system. It is known that

$$\frac{\Gamma(D^0 \rightarrow \kappa^+ \pi^-)}{\Gamma(D^0 \rightarrow \kappa^- \pi^+)} \leq O\left(\frac{1}{10}\right) \quad (14)$$

consistent with the absence of D^0 - $\bar{D}^0$ mixing. This means that the D_1 - D_2 mass difference Δm_D must be less than the D^0 decay rate Γ_D (neutral D's decay before they mix):

$$\frac{1}{2} \left(\frac{\Delta m_D}{\Gamma_D} \right)^2 \leq \frac{1}{10} \quad (15)$$

*) We assume that the matrix elements between K^0 and $\bar{K}^0$ of the LL, LR and RR operators of Eqs. (9), (10) are all comparable. This seems a reasonable approximation, though it has sometimes been argued¹⁵⁾ that the matrix element of the LR operator might be somewhat larger.

The D^0 decay rate is believed theoretically to be of order

$$\Gamma_D \approx O(10) \frac{G_F^2 m_c^5}{192\pi^3} \approx 3.5 \times 10^{-12} \text{ GeV} \quad (16)$$

which corresponds to a lifetime of a few $\times 10^{-13}$ seconds and is not grossly different from the present tentative experimental indications. Combining Eqs. (15) and (16) we find

$$\frac{\Delta m_D}{m_D} \lesssim 10^{-12} \quad (17)$$

To see what constraint this imposes on the coefficients C of $(\bar{c}u)^2$ operators we need to be able to estimate their matrix elements. Gaillard and Lée¹⁴⁾ estimated the matrix element of the $(\bar{s}_L \gamma_\mu d_L)^2$ operator by inserting the vacuum intermediate state to get a value $f_K^2 m_K^2$. This approximation is very crude but more sophisticated approaches¹⁶⁾ give a similar order of magnitude. If we use the same vacuum insertion approximation for the D_1-D_2 system which is much more questionable, we find

$$10^{-12} \gtrsim \frac{\Delta m_D}{m_D} \approx f_D^2 C \quad (18)$$

where f_D is the D decay constant analogous to f_K and f_π . Estimating $f_D^2 \sim 3 \times 10^{-2} \text{ GeV}^2$ we conclude from (18) that

$$C \lesssim 3 \times 10^{-11} \text{ GeV}^{-2} \quad (19)$$

Comparing this limit on C with the expected form (10) of $\mathcal{L}_{(\bar{c}u)^2}^{\text{eff}}$ we see that

$$m_{\text{ETC}_u}^2, m_{\text{ETC}_c}^2 \gtrsim 800 \text{ TeV}^2 \quad (20)$$

Comparing the limits (20) and the previous estimates of $m_{\text{ETC}_u}^2$ and $m_{\text{ETC}_c}^2$ in Table 2, we see that there is again a discrepancy by two order of magnitude, in the ETC_c contribution, as tabulated in Table 1. However, the numerical estimates in this case are clearly shakier than for the $K^0-\bar{K}^0$ system, and it could perhaps be argued that the $D^0-\bar{D}^0$ system does not yet present a serious challenge for ETC theories.

$$\underline{K_L^0 \rightarrow e\mu \text{ and } K^+ \rightarrow \pi^+ e\mu}$$

We now turn to semileptonic flavour-changing neutral interactions. For the sake of argument, we assume that the bosons mediating these interactions as in Fig. 3 are the same as those giving masses to the quarks as in Table 2, with lepton and quark generations associated in the usual way : $(e, \nu_e \leftrightarrow u, d ; \mu, \nu_\mu \leftrightarrow c, s ; \tau, \nu_\tau \leftrightarrow t, b)$. We therefore expect a $|\Delta S| = 1$ but $|\Delta S| = \Delta L_\mu$ and hence $\Delta G = 0$ interaction of the form

$$\mathcal{L}_{(\bar{s}d)(\bar{e}\mu)}^{\text{eff}} = \left(\frac{g_{\text{ETC}}^2}{2}\right) \left\{ \bar{s}_L \gamma_\mu d_L, \bar{s}_R \gamma_\mu d_R \right\} \left\{ \bar{e}_L \gamma^\mu \mu_L, \bar{e}_R \gamma^\mu \mu_R \right\} \times \left\{ \frac{1}{m_{\text{ETC}_d}^2}, \frac{\theta^2}{m_{\text{ETC}_s}^2} \right\} \quad (21)$$

Any individual one of these terms would contribute ⁹⁾ to

$$\frac{\Gamma(K_L^0 \rightarrow e^- \mu^+)}{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)} = \frac{\frac{1}{2} \left(\frac{g_{\text{ETC}}^2}{2}\right)^2 \left(\frac{1}{m_{\text{ETC}_d}^2} \text{ or } \frac{\theta^2}{m_{\text{ETC}_s}^2}\right)^2}{\sin^2 \theta_c \left(\frac{g^2}{2m_W^2}\right)^2} \quad (22)$$

From the upper limit ¹⁷⁾ of 0.63×10^{-9} on this ratio we would extract

$$\left. \begin{array}{l} m_{\text{ETC}_d}^2 \gtrsim 3500 \text{ TeV}^2 \\ m_{\text{ETC}_s}^2 \gtrsim 170 \text{ TeV}^2 \end{array} \right\} \quad (23)$$

These limits are somewhat higher than the ETC boson masses quoted in Table 2, but we do not believe that the discrepancy is significant. Furthermore, the limit (23) can easily be avoided by making the ETC quark couplings purely vectorial. A vectorial combination of (21) would, however, show up in $K^+ \rightarrow \pi^+ e^- \mu^+$ decay. Any individual one of the terms in (21) would give a ratio

$$\frac{\Gamma(K^+ \rightarrow \pi^+ e^- \mu^+)}{\Gamma(K^+ \rightarrow \pi^0 \mu^+ \mu^+)} = \frac{2 \left[\frac{g_{\text{ETC}}^2}{2} \left(\frac{1}{m_{\text{ETC}_d}^2} \text{ or } \frac{\theta^2}{m_{\text{ETC}_s}^2} \right) \right]^2}{\sin^2 \theta_c \left(\frac{g^2}{2m_W^2} \right)^2} \quad (24)$$

and this would be a factor of 8 larger if all the terms in (21) were present with equal strength. From the upper limit ¹⁸⁾ of 1.5×10^{-7} on the ratio (24) we deduce

$$\left. \begin{array}{l} m_{ETC_d}^2 \gtrsim 200 \text{ TeV}^2 \\ m_{ETC_s}^2 \gtrsim 10 \text{ TeV}^2 \end{array} \right\} \quad (25)$$

both of which are comfortably consistent with the constraints in Table 2. The decays $K_L^0 \rightarrow e^- \mu^+$ and $K^+ \rightarrow \pi^+ e^- \mu^+$ therefore produce no challenges for ETC theories. It is, however, possible that they may offer the prospect of soon observing a bizarre decay which could be a signature for ETC. If at the quark vertex the ETC interactions are not purely vectorial, as is expected because of the u - d mass splittings, then the decay $K_L^0 \rightarrow e^- \mu^+$ should probably show up within an order of magnitude below the present experimental limit. If the ETC interactions are not purely axial at the quark vertex, then the decay $K^+ \rightarrow \pi^+ e^- \mu^+$ should probably show up within two orders of magnitude of the present upper limit. It is therefore very interesting to look for these decays as tests of ETC theories, though it is possible that whatever mechanism may suppress ETC contributions to other bizarre amplitudes such as K^0 - $\bar{K}^0$ mixing, etc., might also suppress these decay modes.

$$\underline{K_L^0 \rightarrow \mu^+ \mu^- \text{ and } K_L^0 \rightarrow \gamma\gamma}$$

We should perhaps note at this point that the classic $|\Delta S| = 1$ semileptonic neutral interactions $K_L^0 \rightarrow \mu^+ \mu^-$ and $\gamma\gamma$ do not raise any serious problems for SGEX in ETC theories. The rate for the $|\Delta G| = 1$ process $K_L^0 \rightarrow \mu^+ \mu^-$ is expected to be suppressed by $O(\theta^2)$ relative to the $|\Delta G| = 0$ process $K_L^0 \rightarrow e^- \mu^+$, and the experimental limit ¹⁷⁾ on the latter is more stringent. The process $K_L^0 \rightarrow \gamma\gamma$ is expected to be dominated by the low-mass π^0 and η poles calculated as in the conventional GIM model ¹⁴⁾. Even PGB exchange contributions of the type discussed in Section 4 would be negligible compared with these conventional contributions because the PGBs couple to quarks in proportion to the quark masses.

$$\underline{\mu N \rightarrow e N \text{ capture}}$$

As our last semileptonic process we consider the anomalous muon capture process $\mu N \rightarrow e N$ which has $|\Delta G| = 1$. In order to have coherence at the nuclear vertex we need a vector-like quark coupling, so we assume a vector-like effective interaction of the type

$$\mathcal{L}_{(\mu\bar{e})(q\bar{q})}^{\text{eff}} = (\bar{e}\gamma_\mu \mu) \left\{ \bar{d}\gamma^\mu d \propto \bar{u}\gamma^\mu u \right\} \left(\frac{g_{ETC}^2}{2} \right) \left\{ \frac{\theta}{m_{ETC_d}^2} \right\} \quad (26)$$

The calculation of Cahn and Harari ⁹⁾ yields

$$\frac{\sigma(\mu N \rightarrow e N)}{\sigma(\mu N \rightarrow \nu N)} = \frac{6}{Z} \cdot 8 \cdot \frac{9\pi^2 \cdot \theta^2}{24(g_V^2 + 3g_A^2)} \left(\frac{m_W^2}{m_{ETC_d}^2} \right)^2 \left(\frac{g_{ETC}^2}{g^2} \right)^2 \quad (27)$$

and from the upper limit ¹⁹⁾ of 1.5×10^{-10} on this process we find a lower limit

$$m_{ETC_d}^2 \gtrsim 600 \text{ TeV}^2 \quad (28)$$

Comparing this with the value (8) estimated for $m_{ETC_d}^2$ we see that there is again no incompatibility between ETC and experiment, but another challenging process where ETC theories would expect a signal within an order of magnitude or so of the present experimental upper limit.

$\mu \rightarrow e\bar{e}e$

As a final SGEX contribution to an off-diagonal neutral process we consider the $|\Delta G| = 1$ decay $\mu \rightarrow e\bar{e}e$. Here we might expect the relevant effective Lagrangian to take the form

$$\mathcal{L}_{eff}^{(\bar{e}\mu)(\bar{e}e)} = \left(\frac{g_{ETC}^2}{2} \right) \left\{ \bar{e}_L \gamma_\mu \mu_L, \bar{e}_R \gamma_\mu \mu_R \right\} \left\{ \bar{e}_L \gamma_\mu e_L, \bar{e}_R \gamma_\mu e_R \right\} \times \left(\frac{\theta}{m_{ETC_d}^2} \right) \quad (29)$$

where we have put in one of the lightest quark ETC although it is possible that a different boson (heavier because $m_e < m_d$?) may contribute to purely leptonic processes. Using the LL interaction in Eq. (29) we find

$$\frac{\Gamma(\mu \rightarrow e\bar{e}e)}{\Gamma(\mu \rightarrow e\bar{\nu}\nu)} = \frac{2 \left(\frac{g_{ETC}^2}{m_{ETC_d}^2} \right)^2 \theta^2}{\left(\frac{g^2}{m_W^2} \right)^2} \quad (30)$$

where we have included a factor of 2 from the identical electrons in the final state. Taking the upper limit ²⁰⁾ of 1.9×10^{-9} on this ratio, we get

$$m_{ETC_d}^2 \gtrsim 200 \text{ TeV}^2 \quad (31)$$

which is clearly compatible with the estimate in Table 2. Again, ETC theories suggest that an effect may be found within two orders of magnitude of the present experimental upper limit.

4. - SINGLE PSEUDO-GOLDSTONE EXCHANGE (SPEX)

It is generally agreed ⁶⁾⁻⁸⁾ that there should be many light pseudo-Goldstone bosons (PGBs) in ETC theories. Generically, their couplings to fermions are found to be of order

$$g_{P\bar{f}f} = O\left(\frac{m_f}{F_\pi}\right) \quad (32)$$

and we will assume this form, putting in a "Cabibbo-suppression" factor of θ for each change of generation at a vertex. The PGBs include coloured states ^{6),7)} with masses $O(300 \text{ GeV})$ which decay into $q\bar{q}$ or $q\bar{l}$ combinations according as they are octets or triplets. There are also expected to be colour singlet states with masses which are considerably smaller. Just how small is not very clear, but they could ^{7),8)} be as low as a few GeV. The exchanges of these bosons (Fig. 4) may give rise to significant flavour-changing neutral interactions comparable to those induced by Higgs exchange in models with explicit scalar fields.

K_1-K_2 and D_1-D_2

We first consider their contributions to $|\Delta S| = 2$ and $|\Delta C| = 2$ processes, for which we estimate

$$\mathcal{L}_{(\bar{s}d)^2}^{\text{eff}} = \left\{ \bar{s}_L d_R, \bar{s}_R d_L \right\} \left\{ \bar{s}_L d_R, \bar{s}_R d_L \right\} \frac{m_s^2}{F_\pi^2} \frac{\theta^2}{m_P^2} \quad (33)$$

$$\mathcal{L}_{(\bar{c}u)^2}^{\text{eff}} = \left\{ \bar{c}_L u_R, \bar{c}_R u_L \right\} \left\{ \bar{c}_L u_R, \bar{c}_R u_L \right\} \frac{m_c^2}{F_\pi^2} \frac{\theta^2}{m_P^2} \quad (34)$$

If we extend the vacuum insertion approximation of Gaillard and Lee ¹⁴⁾ to the effective interactions (33), (34), then we must replace

$$\langle \bar{k} | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | k \rangle = f_k^2 m_k^2 \quad (35)$$

by

$$\langle \bar{k} | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | k \rangle = \frac{f_k^2 m_k^4}{(m_s + m_d)^2} \quad (36)$$

We then have from each of the terms in (33)

$$\frac{\Delta m_K}{m_K} = \frac{f_K^2}{F_\pi^2} \frac{m_K^2}{m_P^2} \theta^2 \quad (37)$$

Requiring that this be less than the experimental value of 0.7×10^{-14} means that

$$m_P^2 \geq 4 \times 10^7 \theta^2 \text{ GeV}^2 \quad (38)$$

Although we have not been careful about colour factors, it may be that the limit (38) poses a challenge in so far as the exchanges of octet PGBs are concerned. However, it is clear that the flavour-non-diagonal couplings of colour singlet PGBs must be much more suppressed than the canonical $\theta^2 \simeq \sin^2 \theta_C \approx \frac{1}{20}$, by a factor which depends on their masses. If their masses were $O(3) \text{ GeV}$, then the suppression required would be 10^6 . A similar analysis for the D_1 - D_2 system leads to

$$\frac{\Delta m_D}{m_D} = \frac{f_D^2}{F_\pi^2} \frac{m_D^2}{m_P^2} \theta^2 \quad (39)$$

Taking (17) an experimental upper limit of 10^{-12} on this quantity, we find

$$m_P^2 \geq 7 \times 10^6 \theta^2 \text{ GeV}^2 \quad (40)$$

and hence another strong need for strong suppressions in the flavour-changing couplings of the PGBs, particularly for the colour singlet ones.

$K_L^0 \rightarrow e\mu$

As another example of SPEX let us look at the $K_L^0 \rightarrow e^-\mu^+$ decay. We consider an effective Lagrangian of the form

$$\mathcal{L}_{(5d)(\bar{5}e)}^{\text{eff}} = \frac{m_\mu m_s}{F_\pi^2 m_P^2} \theta^2 (\bar{\mu} \gamma_5 e) (\bar{s} \gamma_5 d) \quad (41)$$

An elementary calculation similar to that of Appendix 2 in Ref. 8) yields

$$\frac{\Gamma(K_L^0 \rightarrow \mu^+ e^-)}{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)} = 16 \left(\frac{\theta^2 m_W^2 m_K^2}{g^2 F_\pi^2 m_P^2 \sin \theta_C} \right)^2 \quad (42)$$

Taking the upper limit of 0.63×10^{-9} on this ratio leads to the limit

$$m_p^2 \gtrsim 2 \times 10^5 \Theta^2 \text{ GeV}^2 \quad (43)$$

This limit is clearly less stringent than those we found earlier from considering the $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ systems in Eqs. (38) and (40). This parallels our finding in Section 3 that the constraints on semileptonic flavour-changing neutral interactions were not as severe as those from purely non-leptonic processes [cf. Eqs. (13) and (23)]. Therefore any trick that suppresses the off-diagonal neutral PGB couplings sufficiently to be consistent with the conditions (38) and (40) will also be automatically consistent with (43). As pointed out elsewhere¹³⁾, such a trick is available in the form of the requirement called "monophagy" that all fermions of a given charge acquire their masses from the same technifermion condensate - the analogue of the Higgs naturalness condition (3) outlined in Section 1. The crossed channel exchange of a coloured leptoquark PGB is not automatically suppressed in monophagic models, but one may estimate¹³⁾ that their exchange gives a rate for $K_L^0 \rightarrow e^- \mu^+$ just consistent with the experimental limit. We will not continue in this section to reconsider all the other flavour-changing semileptonic and purely leptonic neutral processes ($K^+ \rightarrow \pi^+ e^- \mu^+$, $\mu N \rightarrow e N$, $\mu \rightarrow e \bar{e} e$) that we considered in the context of SGEX in Section 3. Based on our experience with the previous calculations of this and the previous section, we believe that no significant new challenges would arise from these processes that are not already met by the trick of monophagy¹³⁾.

5. - HIGHER ORDER EXCHANGES (HOEX)

So far we have only been considering second order single boson exchange diagrams. In this section we go on to consider some higher order diagrams - double gauge boson exchange DGEX (cf. Fig. 5) and a vertex correction to single gauge boson emission (cf. Fig. 6). We will see that in cases where both SGEX and DGEX are possible then the latter is considerably smaller and poses no serious challenge to ETC theories. We also find that flavour-changing vertex corrections to Z^0 couplings are not troublesome. However, we do find that the purely leptonic $\mu \rightarrow e \gamma$ decay challenges ETC theories.

K_1-K_2 and D_1-D_2

We compute the diagram of Fig. 5 using an ETC-q-Q vertex of the form

$$\mathcal{L}_I = \frac{g_{ETC}}{\sqrt{2}} \bar{q}_L \gamma_\mu F_L E^\mu \quad (44)$$

where F is a technifermion, which yields an expression ¹⁴⁾ for the box

$$\mathcal{L}_{(\bar{s}d)^2}^{\text{eff}} = \frac{g_{ETC}^4}{8} \frac{\theta^2}{\int \frac{d^4 k}{(2\pi)^4}} \frac{\bar{s}_L \{\gamma^\mu \not{k} \gamma^\mu\} d_L \bar{s}_L \{\gamma^\mu \not{k} \gamma^\mu\} d_L}{(k^2)^2 (k^2 - m_E^2)^2} \quad (45)$$

The technifermion propagator is expected to be modified at small momenta $k = O(\Lambda_{TC})$, but the typical momenta in the $d^4 k$ integration are $O(m_E)$ so that this modification may be neglected. The expression (45) may be evaluated to give

$$\mathcal{L}_{(\bar{s}d)^2}^{\text{eff}} = \frac{g_{ETC}^4 \theta^2}{128 m_{ETC_S}^2 \pi^2} (\bar{s}_L \gamma_\alpha d_L)^2 \quad (46)$$

where we have put in the mass of the ETCs boson, as this is likely to be the lightest which contributes to $\mathcal{L}_{(\bar{s}d)^2}^{\text{eff}}$ in order θ^2 . We expect similarly to get effective LR and RR interactions. The latter will be of the same magnitude as (46), whereas the LR diagrams should be a factor of 4 larger. Let us evaluate (46) : setting $g_{ETC} \sim 1$ and using the same upper limit as in Eq. (12) we see that from the box diagrams

$$\frac{\theta^2}{128 \pi^2 m_{ETC_S}^2} \lesssim \frac{1}{2} \times 10^{-12} \text{ GeV}^{-2} \quad (47)$$

which will give a bound on $m_{ETC_S}^2$ of order 600 times weaker than in Eq. (13) :

$$m_{ETC_S}^2 \gtrsim 80 \text{ TeV}^2 \quad (48)$$

which is almost compatible with the estimate of $m_{ETC_S}^2$ in Table 2. Adding to the contribution of Eq. (46) the corresponding RR and LR contributions, and taking $g_{ETC} \sim \sqrt{2}$ instead of 1, we would find a one order of magnitude problem for the ETC theories in the real part of the K_1-K_2 mass matrix, which is less than the problem found at the SBEX level. One can go through a similar analysis to deduce a box diagram

$$\mathcal{L}_{(\bar{e}u)^2}^{\text{eff}} = \frac{g_{\text{ETC}}^4 \Theta^2}{128\pi^2 m_{\text{ETC}}^2} (\bar{e}_L \gamma_\alpha u_L)^2 \quad (49)$$

from which we would deduce via the limit of Eq. (19) derived from the observed absence of $D^0-\bar{D}^0$ mixing that

$$\frac{\Theta^2}{128\pi^2 m_{\text{ETC}}^2} \lesssim 3 \times 10^{-11} \text{ GeV}^{-2} \quad (50)$$

and hence that

$$m_{\text{ETC}}^2 \gtrsim 1.3 \text{ TeV}^2 \quad (51)$$

This is compatible with the estimated ETC boson mass in Table 2, and we therefore detect no challenge to ETC theories.

The $\bar{s}d\gamma$ vertex

In view of these failures of the showcase processes of Section 3 to pose serious challenges to ETC theories, we will not continue with estimates of box diagrams for all the other processes discussed in Section 3, none of which there posed caused any serious problems for ETC theories. However, there is another class of loop diagrams which we would like to consider. These are the generic vertex correction diagrams illustrated in Fig. 6. They will in general give flavour-changing contributions to the couplings of the Weinberg-Salam Z^0 boson¹⁴⁾. The logarithmic divergences of these diagrams will be absorbed into the definition of the flavour-diagonal lowest order vertex. As for the finite parts, the leading pieces which have no helicity flip on the internal technifermion line are independent of m_{ETC} and will therefore be subject to a GIM subtraction. The leading contributions to flavour-changing Z^0 vertices involve two helicity flips on the technifermion line as in Fig. 4b and give the order of magnitude

$$\left(\frac{g}{2\cos\theta_w} \right) \frac{\alpha_{\text{ETC}}}{2\pi} \Theta (\bar{s} \gamma_\mu^L d) Z^0 \gamma^\mu \left(\frac{N_{\text{TC}}}{m_{\text{ETC}}^2} \right) \quad (52)$$

and similarly for other $|\Delta G| = 1$ neutral interactions. In order for these contributions to be no larger than the typical GIM suppressed $|\Delta G| = 1$ neutral currents we need

$$\frac{\Lambda_{TC}^2}{m_{ETC_s}^2} \lesssim \frac{m_c^2}{m_w^2} \quad (53)$$

Taking $\Lambda_{TC}^2 \approx 0.1 \text{ TeV}^2$, the limit (53) becomes

$$m_{ETC_s}^2 \gtrsim 200 \text{ TeV}^2 \quad (54)$$

It is difficult to be precise about this number because it depends on the strong coupling régime of the TC interactions. The value of m_{ETC_s} shown in Table 2 grazes the limit (54), but there is no acute problem. For example, it is easy to convince oneself that the value of $m_{ETC_s}^2$ in Table 2 gives a Z^0 exchange contribution compatible with the experimental^s value of $\Gamma(K_L^0 \rightarrow \mu^+ \mu^-) / \Gamma(K^+ \rightarrow \mu^+ \nu)$. Also, if we estimate the order of magnitude of $|\Delta S| = 2$ exchange mediated by Z^0 bosons using Eq. (52), we find that the limit (53) comfortably assures compatibility with experiment for the $K^0 - \bar{K}^0$ system. While we have not attempted a detailed comparison of the ETC vertex correction (52) with all $|\Delta S| \neq 0$ neutral interaction values and limits, we do not expect any of them to pose any serious challenges for ETC theories.

$\mu \rightarrow e\gamma$

As a final one-loop calculation we consider the $\mu \rightarrow e\gamma$ decay in ETC theories, as calculated from the diagrams shown in Fig. 7. The decay rate for this process can be written as

$$\Gamma(\mu \rightarrow e + \gamma) \approx \frac{m_\mu}{8\pi} |F|_{LL, LR, RL, RR}^2 \quad (55)$$

where we have indicated explicitly the contributions coming from different combinations of L and R helicities at the two ETC $f\bar{f}$ vertices. The distinction is important here because the orders of magnitude of the graphs are very different. For the LL and RR cases it is found²¹⁾ that

$$F_{LL, RR} = \frac{e m_\mu^2 g_{ETC}^2 \theta}{32\pi^2 m_{ETC_\mu}^2} \times O(1) \quad (56)$$

while

$$F_{LR,RL} = \frac{e m_\mu \Lambda_{TC} g_{ETC}^2 \theta}{32 \pi^2 m_{ETC_\mu}^2} \times O(1) \quad (57)$$

Taking the larger contribution (57) which is only present if the ETC bosons have both L and R couplings, which they must have in order to generate masses, we find

$$\Gamma(\mu \rightarrow e \gamma) \approx \frac{m_\mu^3}{64 \pi^2} \frac{\alpha^2}{32 \pi^2} \frac{g_{ETC}^4 \Lambda_{TC}^2 \theta^2}{m_{ETC_\mu}^4} \quad (58)$$

Taking an experimental upper limit of order (10^{-10}) for the branching ratio of $\mu \rightarrow e \gamma$ relative to $\mu \rightarrow e \bar{\nu}$ we deduce from Eq. (58) using $g_{ETC} \approx 1$, $\Lambda_{TC}^2 \approx 0.1 \text{ TeV}^2$ and $\theta^2 \approx \frac{1}{20}$ as usual that

$$m_{ETC_\mu}^2 \gtrsim 8 \times 10^4 \text{ TeV}^2 \quad (59)$$

This compares with a value of $m_{ETC_\mu}^2 = 2400 \text{ TeV}^2$ that we would have deduced from the arguments of Section 2. We therefore conclude that the experimental upper limit on the $\mu \rightarrow e \gamma$ decay mode presents a challenge for ETC theories if the assumption of naïve Cabibbo-like ETC couplings with mixing angles $\theta \approx \theta_C$ is also applicable to leptons, as we assumed in order to derive the bound (59) from Eq. (58).

6. - CP VIOLATION

There are several possible sources of CP violation in ETC theories. The exchanges of horizontal ETC currents between ordinary quarks in general involve CP violating complex mixing matrices. The phases of these matrices may originate spontaneously at some mass scale and cannot be rotated away by chiral transformations of any of the fermion fields. These phases are a priori expected to be of order unity. Thus processes mediated by horizontal currents may be expected to exhibit maximal CP violation. In view of this and the observed ratio of $O(10^{-3})$ between the imaginary part and real parts of the $K^0 - \bar{K}^0$ mass matrix,

it seems necessary to suppress all horizontal processes mediating $K^0-\bar{K}^0$ mixing a thousand times more strongly than we deduced in earlier sections on the basis of the real part alone.

CP violation in electroweak interactions may arise in the standard Kobayashi-Maskawa¹⁰⁾ way through phases that appear in the charged weak current coupling matrix $U_d^{L\dagger} U_u^L$. These phases may also arise spontaneously at some mass-scale and cannot be removed by allowed chiral rotations of any fermion fields. This K-M CP violation predicts²²⁾ a value of $10^{-30\pm 1}$ e-cm for the neutron's electric dipole moment d_N , and is probably²³⁾ consistent with the small imaginary part of the K_1-K_2 mass matrix which is observed. This source of CP violation poses no challenges for ETC theories.

Recently another "U(1)" type of CP violation has been suggested¹¹⁾, and it has been estimated to lead to $d_N \sim 10^{-24}$ to 10^{-25} e-cm. This would be very exciting because d_N would then be accessible to the forthcoming generation of experiments. Unfortunately, this ELP mechanism for CP violation also suggests that the CP violating part of the K_1-K_2 mass matrix should be comparable with the CP conserving part. It would therefore be desirable to demonstrate that whatever mechanism suppresses CP violation in the K^0 system by perhaps five orders of magnitude does not also suppress d_N by a similar amount.

In fact, it may be that in the absence of any suppression the ELP source of CP violation may actually give a larger value of d_N than has been suggested. To see this, let us briefly review some of the results of Ref. 11). Consider the chirality breaking part of the effective Lagrangian in an ETC theory which describes the interactions of ordinary particles well below ~ 300 GeV. This effective Lagrangian is obtained by integrating out all the heavy non-ordinary degrees of freedom such as technifermions, ETC bosons, horizontal bosons, etc. It consists of a series of operators whose lowest dimensional strongly-interacting representatives are

$$\begin{aligned} \mathcal{L}^{\text{eff}} \ni & A_{lr} \bar{q}_{lL} q_{rR} + B_{lr} \bar{q}_{lL} \sigma_{\mu\nu} \left(\frac{\lambda^a}{2} \right) q_{rR} G_a^{\mu\nu} \\ & + C_{lr,l'r'} \bar{q}_{lL} q_{rR} \bar{q}_{l'L} q_{r'R} + \text{H.C.} \end{aligned} \quad (60)$$

Here the indices l, r label left- and right-handed helicity quark flavours and $G_{\mu\nu}^a$ is the gluon field strength. The coefficients A, B, C are each expected to have phases of order unity which arise spontaneously in the higher energy sectors. The dominant graphs contributing to A, B and C are shown in Fig. 8. Note that the ETC radiative correction to the ETC vertex in Fig. 8b is only necessary if the exchange of horizontal ETC bosons does not already produce an interaction of the form $\bar{q}\sigma_{\mu\nu}q\bar{Q}\sigma_{\mu\nu}Q$. If such an interaction is present then d_N immediately comes out too large, as was noted in Ref. 11). Note also that the gluonic correction to the ETC vertex which was considered in Ref. 11) is smaller than the ETC correction that we consider here. From these graphs one easily estimates A, B and C to be :

$$\left. \begin{aligned} A &= m_q = \frac{\langle 0|\bar{F}F|0\rangle}{V_{ETC}^2} \equiv \frac{\Lambda_{TC}^3}{V_{ETC}^2} \\ B &\approx \frac{\alpha_{ETC}(m_{ETC})}{2\pi} g_c \left(\frac{\Lambda_{TC}}{V_{ETC}^2} \right) \\ C &\approx \frac{1}{V_{ETC}^2} \end{aligned} \right\} \quad (61)$$

Here as before, $V_{ETC} \equiv m_{ETC}/g_{ETC}$ and $\Lambda_{TC} = \langle \bar{F}F \rangle^{\frac{1}{3}} \simeq 300$ GeV is the technicolour scale. A crude estimate of the contributions of A, B and C to the vacuum energy can now be obtained from (60) and (61) by replacing $\bar{q}q \rightarrow \Lambda_c^3$ and $G_{\mu\nu}^a \rightarrow \Lambda_c^2$, where $\Lambda_c \equiv \langle \bar{q}q \rangle^{\frac{1}{3}} \simeq 240$ MeV is the colour scale. This yields

$$\left. \begin{aligned} A \Lambda_c^3 &= \frac{\Lambda_{TC}^3 \Lambda_c^3}{V_{ETC}^2} \equiv a \\ B \Lambda_c^5 &= \left(\frac{\Lambda_{TC}^3 \Lambda_c^3}{V_{ETC}^2} \right) \left(\frac{\Lambda_c}{\Lambda_{TC}} \right)^2 g_c \frac{\alpha_{ETC}(m_{ETC})}{2\pi} \equiv b \\ C \Lambda_c^6 &= \left(\frac{\Lambda_{TC}^3 \Lambda_c^3}{V_{ETC}^2} \right) \left(\frac{\Lambda_c}{\Lambda_{TC}} \right)^3 \equiv c \end{aligned} \right\} \quad (62)$$

In Ref. 11) the order of magnitude of d_N is estimated to be

$$d_N \approx \left(\frac{b}{a} + \frac{c}{a} \right) 10^{-15} \text{ e-cm} \quad (63)$$

which becomes, according to Eq. (62) :

$$d_N \approx \left(g_c \frac{\alpha_{ETC}(m_{ETC})}{2\pi} \left(\frac{\Lambda_c}{\Lambda_{TC}} \right)^2 + \left(\frac{\Lambda_c}{\Lambda_{TC}} \right)^3 \right) 10^{-15} \text{ e-cm} \quad (64)$$

Since $\Lambda_c/\Lambda_{TC} \approx 10^{-3}$ the c/a term in (63) yields a contribution to d_N of order $\sim 10^{-24}$ e-cm, whereas the b/a term gives

$$d_N \approx g_c \frac{\alpha_{ETC}(m_{ETC})}{2\pi} 10^{-21} \text{ e-cm} \quad (65)$$

We guess that $\alpha_{ETC}(m_{ETC}) \approx 1$ since one has in the leading logarithmic approximation

$$\frac{2\pi}{\alpha_{ETC}(m_{ETC})} \approx b_{ETC} \ln \left(\frac{m_{ETC}}{\Lambda_{TC}} \right) : b_{ETC} = O(10) \quad (66)$$

so that for $m_{ETC} \sim 1$ TeV corresponding to a top quark of mass 30 GeV one finds $\alpha_{ETC}(m_{ETC})/2\pi \sim 10^{-1}$. Inserting this value into Eq. (65) we find $d_N \approx 10^{-22}$ e-cm from the ELP mechanism.

This appears to be too large and therefore a challenge for this mechanism¹¹⁾, but it is by no means a fatal one. The neatest solution would simply be to assume that the techniquarks do not carry ordinary colour. Then colour would have to flow through the ETC boson and graphs like that in Fig. 9 are expected to make a very small contribution to B. Therefore in this case $d_N \approx c/a \cdot 10^{-15} \text{ e-cm} \approx 10^{-24} \text{ e-cm}$. The principal challenge in such a scenario would therefore be to construct ETC theories in which the techniquarks do not carry ordinary colour. If no techni fermions carry ordinary colour there will of course be no coloured pseudo-Goldstone bosons with masses of order 300 GeV.

7. - PROSPECTS

What are we to conclude from the order of magnitude estimates presented in this paper and summarized in Table 1 ? As optimists we may hope that the rare processes will once again prove an excellent guide for model-building. It is constructive to adopt this point of view and try to see what one might learn from

the necessity to suppress rare processes. In the rest of this section we will briefly mention some such possible suppression mechanisms whose developments are still in their pre-embryonic stages.

The first such suppression mechanism is "monophagy" which has already been mentioned ¹³⁾. It is an obvious extension of the Glashow-Weinberg mechanism and says that fermions of a given charge get their masses from a single technifermion condensate. Monophagy cures only SPEX but not SGEX.

Another possible mechanism which smells promising is to introduce an $SU(3)_H$ symmetry acting on $\underline{d}$, $\underline{s}$ and $\underline{b}$, or an $SU(2)_H$ symmetry acting on $\underline{d}$ and $\underline{s}$ (where the tildes indicate the corresponding generations or families), which is unbroken at the ETC scale. Such a symmetry would render $\underline{s}$ and $\underline{d}$ quarks indistinguishable at ETC energies where the disastrous SGEX mediated mixings normally occur, and thus these problems would be avoided. Another way of seeing the same thing is to note that strangeness conservation is a subsymmetry of $SU(3)_H$ or $SU(2)_H$ and thus valid at ETC energies. This scenario has to be implemented with some low energy (~ 300 GeV ?) mechanism (spontaneous breaking ?) incorporated in order to break the $SU(3)_H$ or $SU(2)_H$ symmetry and thus account for the $\underline{d}$ - $\underline{s}$ - $\underline{b}$ or $\underline{d}$ - $\underline{s}$ splitting, without simultaneously introducing dangerous strangeness-changing light pseudo-Goldstone bosons and/or gauge bosons. No details of how this might be done have been worked out.

There are other ways of introducing strangeness conservation at high energies which involve the existence of strangeness, charm, etc., carrying techniquarks. They would introduce the naturalness conditions (1) and (2) of Section 1 into ETC theories and open the way to GIM-like suppressions of gauge boson exchanges ^{6),2)}. These models suffer from the proliferation of possibly non-innocent pseudo-Goldstone bosons associated with the replication of techniquarks, and lack aesthetic appeal.

In conclusion, a solution to all the problems addressed in this paper is not yet in sight. The great wealth of potential problems discussed here suggests that the solution, if any, will have to involve a symmetry valid at ETC energies $E > 300$ GeV, or maybe another GIM-like mechanism. In the meantime, there are ways in which our experimental colleagues can challenge ETC theories further by more searching studies of rare processes, in particular D_1 - D_2 mixing, $K_L^0 \rightarrow e\mu$ and $K^+ \rightarrow \pi^+ e\mu$, $\mu N \rightarrow eN$ capture and $\mu \rightarrow e\gamma$ decay. Optimists hope to see ETC effects in these processes.

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Process	Necessary suppression factor		(Mass) ² limit on PGB in absence of suppression of SPEX
	SGEX	HOEX	
$\text{Re}(K_1 - K_2)$	10^3	10	$> 2 \cdot 10^6 \text{ GeV}^2$
$\text{Im}(K_1 - K_2)$	10^6	10^4	$> 2 \cdot 10^9 \text{ GeV}^2$
$(D_1 - D_2)$	10^2	OK	$> 3 \cdot 10^4 \text{ GeV}^2$
$K_L^0 \rightarrow e^- \mu^+$	Borderline	OK	$> 10^4 \text{ GeV}^2$
$K^+ \rightarrow \pi^+ e^- \mu^+$	OK	OK	Less stringent
$K^0 \rightarrow \mu^+ \mu^-, \gamma\gamma$	OK	OK	Less stringent
$\mu N \rightarrow e N$	OK	OK	Less stringent
$\mu \rightarrow e \bar{e} e$	OK	OK	Less stringent
$\mu \rightarrow e \gamma$	-	$10^{1\frac{1}{2}}$	-
d_N	-	10^2	Less stringent

Table 1
Challenges for ETC theories

q	u	d	s	c	b	t
m_q	6 MeV	10 MeV	200 MeV	1.5 GeV	5 GeV	30 GeV
$v_{\text{ETC}q}^2$	2200 TeV ²	1200 TeV ²	60 TeV ²	8 TeV ²	2.4 TeV ²	0.4 TeV ²
Recall that $v_{\text{ETC}}^2 \equiv m_{\text{ETC}}^2 / g_{\text{ETC}}^2 \approx m_{\text{ETC}}^2$ if $g_{\text{ETC}} \approx 1$						

Table 2

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FIGURE CAPTIONS

- Figure 1 Generic structure of ETC theories ^{5),6)}. The arrows indicate interactions mediated by gauge bosons. The crosses $\times$ indicate conventional fermion generations or families with the substructure illustrated beneath. The line --- indicates a technigeneration whose substructure is also indicated.
- Figure 2 The mechanism whereby ETC bosons ⁵⁾ give masses to conventional fermions.
- Figure 3 The single gauge boson exchange (SGEX) contribution to an effective four-fermion Lagrangian.
- Figure 4 The single pseudo-Goldstone boson exchange (SPEX) contribution to an effective four-fermion Lagrangian.
- Figure 5 A double gauge boson exchange (DGEX) box diagram.
- Figure 6 (a) A typical vertex correction diagram, and
(b) a double helicity flip part of it.
- Figure 7 Diagrams contributing to $\mu \rightarrow e\gamma$ in ETC theories.
- Figure 8 The lowest order diagrams in ETC theories contributing
(a) to A, (b) to B and (c) to C.
- Figure 9 A smaller contribution to B.

Fig. 1

Fig.2

Fig.3

Fig.4

Fig.5

Fig. 6

Fig. 7

Fig. 8

Fig. 9