EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/80-175
25 September 1980

QUADRUPOLE INTERACTIONS IN PIONIC AND MUONIC TANTALUM AND RHENIUM

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(Submitted to Nuclear Physics A)

*) This work is part of the joint research program of FOM and ZWO.

ABSTRACT

The hyperfine splitting of pionic and muonic X-rays in natural Re has been studied using the known ratio (accurate to 1.6 parts in 10^5) of the quadrupole moments of the two naturally occurring ^{185}Re and ^{187}Re isotopes. From the hyperfine splitting of the $5g \rightarrow 4f$ and $4f \rightarrow 3d$ pionic X-rays the effective quadrupole hyperfine constants were determined to be $^{187}\text{A}_2^{\text{eff}}(4f) = 1.163 \pm 0.010$ keV and $^{187}\text{A}_2^{\text{eff}}(3d) = 5.39 \pm 0.63$ keV, giving strong interaction quadrupole shifts $\epsilon_2(4f) = 46 \pm 10$ eV and $\epsilon_2(3d) = 1.3 \pm 0.6$ keV. The strong interaction monopole shifts ϵ_0 and widths Γ_0 of the $5g$, $4f$, and $3d$ levels have also been measured. For the two higher orbits, standard optical potential calculations fit the measured shifts and widths quite well. The observed deeper lying $3d$ state, however, has shifts and widths that differ by a factor of 2 or more from the predictions. From the measured quadrupole hyperfine constants of the $4f$ level we calculate the spectroscopic quadrupole moments to be $^{185}\text{Q}^{\mu} = 2.09 \pm 0.04$ b, $^{185}\text{Q}^{\pi} = 2.07 \pm 0.02$ b, $^{187}\text{Q}^{\mu} = 2.21 \pm 0.04$ b, and $^{187}\text{Q}^{\pi} = 2.18 \pm 0.02$ b. In addition, muonic X-rays from ^{181}Ta were observed; using the same methods for determining the quadrupole moment as above, a value of $^{181}\text{Q}^{\mu} = 3.28 \pm 0.06$ b was obtained, in good agreement with earlier published data.

1. INTRODUCTION

For deformed nuclei ($Z \geq 67$) it has been shown that the spectroscopic quadrupole moment can be determined to a rather high precision (of the order of a few per cent) from the hyperfine splittings (h.f.s.) of the $5g \rightarrow 4f$ mesic X-rays in pionic and muonic atoms. As pointed out by Dey et al.^{1,2)}, in mesic atoms errors reflecting uncertainties in the calculation of multi-electron systems and estimating the electric field inhomogeneity at the nucleus and the effect of the polarization of the electron core do not exist, since we are dealing with essentially a one-particle system. In the intermediate domain of energy states for the pionic and muonic atoms, where the orbit size is large compared with nuclear dimensions and yet small compared with the orbits of atomic electrons, the exotic atom is "hydrogen-like". Therefore, one can measure an accurate value for the electric quadrupole moment, one of the fundamental properties of the nucleus. For deformed nuclei it is then possible to test the predictions of the rotational model by relating Q to the $B(E2)$ values. The h.f.s. of the $5g \rightarrow 4f$ transition in the muonic ^{175}Lu atom has been measured by Dey et al.^{1,2)}, whereas the h.f.s. of pionic atoms has been studied for ^{165}Ho , ^{175}Lu , and ^{181}Ta by several groups³⁻⁸⁾. For pionic atoms one determines the effective electric quadrupole h.f. constant, A_2^{eff} , and applies a correction for the strong interaction effects. As the total correction amounts to about 5% for the $5g \rightarrow 4f$ transitions even a rather large error in the correction allows for an accuracy of about 1% in the final value for the spectroscopic quadrupole moment. Furthermore, the $5g \rightarrow 4f$ transitions involve pionic atom levels that have been shown⁴⁾ to be rather insensitive to the nuclear model and the values c and t used for the nuclear charge density distribution. In addition, the corrections due to electron screening, vacuum polarization, and finite size are small (of the order of 1% or less). In muonic atoms no strong interaction corrections exist. The corrections for finite size, vacuum polarization, magnetic dipole, and electric hexadecapole effects made to the quadrupole h.f.s. are small²⁾ for the $5g_{9/2} \rightarrow 4f_{7/2}$ and $5g_{7/2} \rightarrow 4f_{5/2}$ muonic transitions. They add up to only a few per cent.

Pionic atoms, on the other hand, are a major source of information about the strong pion-nucleus interaction at low energies. The observed shifts and widths of pionic levels due to the strong interaction have been used to investigate the optical potential. Since the strong interaction effects grow very rapidly from one level to the next lower one, there is an increase in the absorption which leads to a fast decrease in intensity of X-rays for subsequent transitions. Therefore, once the influence of the strong interaction on an X-ray transition between two levels is large enough to be directly observable, the subsequent X-ray transition is even broader and weaker. This usually limits the measurement of both the strong shift and the width to only one level for a given target nucleus. To learn more about the pion-nucleus interaction and to get a better understanding of the higher-order terms in the optical potential one would like to measure strong interaction effects for two subsequent, strongly shifted and broadened transitions in a given nucleus. Therefore, it was decided to search for both the $4f \rightarrow 3d$ and the $5g \rightarrow 4f$ pionic transitions. Some results of these measurements have been published in an earlier paper⁹⁾ showing the deeper lying 3d states in nuclei with $Z \geq 70$ to have shifts and widths that differ by a factor of 2 or more from standard optical potential predictions¹⁰⁾.

2. FORMALISM

The formulae for calculating the quadrupole moment from experimental data can be found in papers by Dey et al.^{1,2)}, Ebersold et al.^{7,8)}, and Konijn et al.⁹⁾ for muonic and pionic atoms.

The nuclei considered here have spins $I > 1$ and each mesic level (n, ℓ) is split into a h.f. multiplet. Our analysis of the X-ray transitions for such nuclei follows closely the method presented in refs. 4, 9 and 10, and we refer to these articles for the notation and further details.

3. EXPERIMENT AND DATA ANALYSIS

The experiment was performed at the muon channel of the CERN Synchro-Cyclotron (SC) stopping pions and muons in metallic tantalum and rhenium targets, each

consisting of two plates with total thicknesses of 4.98 g/cm^2 Ta and 2.10 g/cm^2 Re, and dimensions of about $14 \times 14 \text{ cm}^2$ and $10 \times 10 \text{ cm}^2$, respectively. The experimental set-up has already been described in earlier papers^{4,9,11}). Special care was taken to detect weak transitions by reducing the background with a Compton suppression system^{12,13}). When the splitting of the $5g \rightarrow 4f$ h.f. complexes had to be measured, the energy resolution was the most important parameter. Therefore, for the muonic h.f. complexes with energies in the region of 340-370 keV we also used a planar low-energy photon spectrometer (LEPS) detector.

To measure the $5g \rightarrow 4f$ h.f.s. in the pionic atoms with energies occurring at 479 keV in Re, we used a large volume Ge(Li) detector, because of the much higher efficiency of this detector at these energies as compared with the LEPS detector. For the energy calibration of the spectra, standard sources such as ^7Be , ^{60}Co , ^{88}Y , ^{133}Ba , and ^{192}Ir were used. The energy values for the calibration sources were taken from Helmer et al.¹⁴).

When deducing the Lorentzian line widths and the quadrupole splitting from the recorded data, it is extremely important to know the experimental line shape and background fitting used in the analysis of the h.f.s. In fig. 1 the computer-fitted experimental line shape for the 477.605 keV ^7Be γ -ray transition is shown. The spectrum was recorded with an 8192 channel LABEN ADC, Model 821, coupled to an HP-2100 computer. The energy scale is 0.20 keV/channel. The system energy resolution for this ^7Be line is 1.43 keV FWHM. The figure shows that the tailing of the photopeak, starting approximately at 1% of the peak height, is well fitted. The parameters describing the instrumental line shape, obtained from this fitting procedure, were kept fixed during the analysis of the h.f.s. of the pionic X-ray complexes.

3.1 Pionic rhenium

The prompt pionic rhenium spectrum is shown in fig. 2. The areas of the $5g \rightarrow 4f$ and $4f \rightarrow 3d$ pionic h.f. complexes are shown separately at energies 479 keV and 1059 keV in figs. 3 and 4, respectively. In the analysis of these h.f. complexes (the determination of h.f. splittings, line widths, energies, and intensities),

it is important to include the instrumental tailing of the photopeak (see fig. 1), since otherwise the obtained Lorentzian line widths are not adequately represented by the fit. The Re target contains the isotopes ^{185}Re and ^{187}Re with abundances 37.4% and 62.6%, respectively. Both isotopes have a nuclear ground-state spin of $J^\pi = 5/2^+$. The ratio of the quadrupole moments of these isotopes has been determined, first by Semin and Bryukhova¹⁵⁾ and later by Segel¹⁶⁾ with resonance methods, to be $^{185}\text{Q}/^{187}\text{Q} = 1.056709(17)$. Assuming the quadrupole effects to scale in the same way, we have fitted our data with a double complex with one multiplet slightly shifted with respect to the other. The value of this shift was calculated from the above ratio and kept proportional to the h.f. splitting during the whole fitting procedure. Each multiplet consists of 15 members with fixed relative intensities. The intensities and energy difference of the individual members of the $5g \rightarrow 4f$ and $4f \rightarrow 3d$ multiplets relative to the $^{13/2} \rightarrow ^{11/2}$ and $^{11/2} \rightarrow ^{9/2}$ transitions, respectively, are given in table 1. From the h.f.s. of the above transitions, we obtain with high precision the effective electric quadrupole constant, A_2^{eff} , directly proportional to the effective spectroscopic quadrupole moment, Q_{eff} . Assuming the monopole strong interaction shifts and widths to be the same (within experimental errors) for both isotopes, we obtain $^{187}\text{Q}_{\text{eff}}(4f) = 2.19 \pm 0.02$ b and $^{185}\text{Q}_{\text{eff}}(4f) = 2.31 \pm 0.02$ b from our experimental value $^{187}\text{A}_2^{\text{eff}}(4f) = 1.163 \pm 0.010$ keV (for the notation see refs. 9 and 10). The calculated electromagnetic quadrupole constant is $A_2(4f) = 1.117$ keV. From the difference a value for the strong interaction quadrupole shift $\varepsilon_2(4f) = 46 \pm 10$ eV is obtained.

The centroid of the $4f \rightarrow 3d$ pionic h.f. complex in Re occurs at 1059 keV and, therefore, correction has to be made for the influence from the nearby 1040 keV γ -ray transition due to inelastic neutron scattering from ^{70}Ge in the Ge(Li) detector itself. The position and line shape of this background peak are well known and are taken from our experiments with other targets. From this h.f. complex we obtain $A_2^{\text{eff}}(3d) = 5.39 \pm 0.63$ keV for ^{187}Re and $Q_{\text{eff}}^{(3d)} = 1.68 \pm 0.20$ b. The calculated electromagnetic quadrupole constant is slightly model-dependent with a variation of about 5% among different models⁹⁾. From the value of $A_2(3d)$ we

estimate a strong interaction quadrupole shift of $\epsilon_2(3d) = 1.3 \pm 0.6$ keV. The indicated error only accounts for the experimental error in A_2^{eff} and not for the model uncertainty⁹⁾.

From the analysis described above, the centroid positions of the h.f. complexes and the X-ray peaks are determined, yielding the transition energies. Our results are shown in table 2. The quoted electromagnetic values are the point Coulomb values with correction for vacuum polarization, Lamb shift, and electron screening. The remaining difference between the calculated electromagnetic and experimental values is due to finite-size Coulomb and strong interaction effects. In this way we obtain a strong interaction monopole shift of the 4f level, $\epsilon_0(4f) = 0.709 \pm 0.040$ keV. To extract the 3d level shift one has to correct the measured $4f \rightarrow 3d$ transition energy for the $\epsilon_0(4f)$ value and one obtains $\epsilon_0(3d) = 11.1 \pm 1.0$ keV. Table 3 shows our results, along with a compilation of other recent measurements^{4-10,17-20)} in neighbouring nuclei.

The strong interaction broadening of the 4f and 3d levels is extracted from the h.f. components with a Lorentzian shape. Subtracting from the fitted values the radiative widths of initial and final levels, we arrive at $\Gamma_0(4f) = 547 \pm 50$ eV and $\Gamma_0(3d) = 16.8 \pm 2.0$ keV. The radiative widths used are 412.0 eV, 88.2 eV, and 27.1 eV for the 3d, 4f, and 5g pionic levels in Re, respectively. These values for Γ_0 are also shown in table 3 along with other recent measurements.

From the recorded spectra, the X-ray intensities of the pionic cascade are obtained. The relative X-ray intensities have been determined from the measured spectra by correcting for the efficiency of the Ge(Li) detector and for self-absorption in the target. Since the solid angle covered by the target with respect to the detector is small, the variation in angle between the detector and target is neglected. The resulting intensities are included in table 2. Given the relative intensities of the pionic X-rays, we can determine the strong interaction width Γ_0 of the level⁹⁾. In our cases the Auger rates can be neglected as compared with the radiative width of the 5g and 4f levels. As shown in ref. 9, we obtain for Re: $\Gamma_0(5g) = 1.1 \pm 2.3$ eV and $\Gamma_0(4f) = 466 \pm 41$ eV, in good

agreement with the widths determined directly from line shapes and with theoretical predictions.

3.2 Muonic ^{181}Ta and natural Re

As is the case for pionic atoms, we have to deal essentially with a one-particle system in muonic atoms. However, no strong interaction effects are present in the muonic atoms, and the muon will cascade down to the 1s orbit before it is either captured by the nucleus or beta-decays. For low-lying states (1s and 2p), the orbit size is comparable with the nuclear radius, implying that finite nuclear size effects are more important for the quadrupole splittings than the nuclear quadrupole moment. Moreover, for heavy deformed nuclei, the 2p fine structure splitting into a $2p_{1/2}$ and $2p_{3/2}$ orbit has the same order of magnitude as the first excited state of the nucleus and in this case the muon and nucleus have to be treated as one system in which there also appears nuclear excitation.

There is, however, an intermediate region of muonic energy states, for which the orbit sizes are large compared with nuclear dimensions, and yet small compared with the atomic electron orbits. For these orbits 3d, 4f, and 5g the quadrupole splitting is described by a quadrupole coupling constant, A_2 , which is the product of the spectroscopic quadrupole moment and the field gradient of the muon at the nucleus. The study of such a $5g \rightarrow 4f$ muonic X-ray transition was first reported by Dey et al.²⁾, who made a precise determination of the spectroscopic quadrupole moment Q in ^{175}Lu from the quadrupole splitting of this transition. For a detailed description of the formalism and data processing we refer the reader to refs. 1, 2, 4, and 9.

The centroids of the quadrupole split $5g_{9/2} \rightarrow 4f_{7/2}$ and $5g_{7/2} \rightarrow 4f_{5/2}$ muonic X-ray transitions occur at energies 341.05 keV, 345.05 keV, and 360.18 keV, 364.64 keV for ^{181}Ta and natural Re, respectively. The study of these h.f. complexes was carried out using a muon beam of 105 MeV/c with an intensity of about 2×10^4 μ /s. This muon beam was stopped in the same ^{181}Ta and natural Re targets as described above. The lower transition energy and smaller splittings as compared with the pion case made it necessary to record the X-ray spectra with a

planar LEPS detector having an energy resolution of 0.855 keV FWHM for the 356.014 keV γ -ray transition in ^{133}Ba . For the energy calibration the same set of standard sources was applied as for the pionic X-ray measurements, reported above. Since the observed X-ray transitions between h.f. states are close in energy, the conversion gain of the ADCs was set to 0.07 keV/channel. The experimental line shape, used in the analysis of the h.f. complexes, was taken from the X-ray of the above-mentioned ^{133}Ba calibration source (see fig. 5).

The intensities and energy differences of the individual members of the h.f. complexes relative to the most intense transition of each multiplet are given in tables 4 and 5. These data have been used to analyse the $5g_{9/2} \rightarrow 4f_{7/2}$ and $5g_{7/2} \rightarrow 4f_{5/2}$ h.f. complexes. As in the analysis of the pionic data, the ratio of the quadrupole moments of the two Re isotopes was used to scale the quadrupole effects. The data were, therefore, fitted with double complexes, one multiplet slightly shifted with respect to the other.

In the analysis of the $5g_{7/2} \rightarrow 4f_{5/2}$ h.f. complex one has to correct for the presence of the $5f_{7/2} \rightarrow 4d_{5/2}$ multiplet, as they coincide in energy in both the muonic Ta and Re atoms. The total correction for the intensity ratio of these multiplets amounts to 9.3% and 7.3% for Ta and Re, respectively.

For each target two separate experimental runs were performed in the time span of a few months. In figs. 6 and 7 one run for each target is shown, representing the prompt muonic Ta and Re X-ray spectra, respectively, as recorded by the LEPS detector.

In figs. 8 and 9 we show the observed h.f. complexes for ^{181}Ta and natural Re, respectively. From fits to our spectra we obtain the quadrupole moments presented in table 6. In fig. 9 there is a sharp peak at 346 keV resolved from the quadrupole multiplet. Based on the presence of other low-energy transitions excited in the same γ -ray cascade, this γ -ray is interpreted as due to the excitation of ^{180}Hf levels²¹⁾ in the $^{181}\text{Ta}(\mu^-, n)$ reaction. To extract the quadrupole moment from our natural Re target we proceeded in the same way as in the pionic case with respect to the ratios of the spectroscopic quadrupole moments^{15,16)} and the isotopic abundances.

The results for the spectroscopic quadrupole moments deduced from the h.f.s. of the $5g \rightarrow 4f$ pionic transitions, corrected for the strong interaction effects⁴⁾, are also presented in table 6.

4. DISCUSSION

The direct measurement of the strong interaction shifts and widths for the low-lying 3d level in Re was reported in an earlier paper⁹⁾ together with our results on Ta and Bi. We repeat the conclusions here. The calculations using the standard Kisslinger optical potential reproduce the 4f shifts and widths reasonably well (see also table 3). In table 3 the calculated values I, II, and III are those optical-potential parameter sets used in ref. 9. All strong interaction shifts are evaluated with respect to the point nuclear Coulomb value without including the repulsive finite size shift. For the deep-lying 3d level, however, the calculations reproduce neither the shifts ϵ_0 nor the widths Γ_0 for Ta, Re, and Bi. Recently, Batty et al.¹⁸⁾ reported measurements of ϵ_0 and Γ_0 for ^{165}Ho , ^{168}Er , Yb, and ^{181}Ta confirming this discrepancy⁹⁾. The disagreement between theory and experiment for the lower level seems to indicate that the standard optical potentials are inadequate to describe large overlaps between the pion wave function and the tail of the nuclear density.

The nuclear spectroscopic quadrupole moments for ^{181}Ta , ^{185}Re , and ^{187}Re are shown in tables 7 and 8. The values from muonic and pionic atoms from the h.f.s. of the $5g \rightarrow 4f$ complexes are in good agreement. Earlier measured values^{4,18,22-40)} of Q for these isotopes are presented in tables 7 and 8. Thus the results from the present experiment confirm the conclusion that the analysis of the $5g \rightarrow 4f$ pionic or muonic h.f. complexes is an accurate tool for the determination of the spectroscopic quadrupole moments for these nuclei.

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Table 1

Relative intensities and energy displacements of the transitions in the pionic $5g \rightarrow 4f$ and $4f \rightarrow 3d$ natural Re h.f. multiplets

Transition $F_i \rightarrow F_f$	5g $\rightarrow$ 4f multiplet		4f $\rightarrow$ 3d multiplet	
	Relative intensity	ΔE (eV/b)	Relative intensity	ΔE (keV/b)
$1^{3/2} \rightarrow 1^{1/2}$	100	0	-	-
$1^{1/2} \rightarrow 1^{1/2}$	9.7	-269.3	-	-
$1^{1/2} \rightarrow 9/2$	76.0	915.9	100	0
$9/2 \rightarrow 1^{1/2}$	0.5	-301.3	-	-
$9/2 \rightarrow 9/2$	14.8	883.8	15.4	-1.1866
$9/2 \rightarrow 7/2$	56.1	883.8	67.9	7.4626
$7/2 \rightarrow 9/2$	1.0	990.1	1.2	-1.1866
$7/2 \rightarrow 7/2$	16.0	990.1	22.6	7.4626
$7/2 \rightarrow 5/2$	40.1	386.5	42.9	5.2238
$5/2 \rightarrow 7/2$	1.3	1155.0	2.9	8.0671
$5/2 \rightarrow 5/2$	14.0	552.3	23.1	5.8208
$5/2 \rightarrow 3/2$	27.6	-201.5	24.0	-0.5821
$3/2 \rightarrow 5/2$	1.0	714.4	4.0	6.5745
$3/2 \rightarrow 3/2$	9.2	- 39.4	19.0	0.1716
$3/2 \rightarrow 1/2$	18.4	-621.0	10.4	-5.5895
$1/2 \rightarrow 3/2$	-	-	3.7	0.7537
$1/2 \rightarrow 1/2$	-	-	13.0	-5.0074

Table 2

Pionic transitions in natural Re. The energies are calculated without taking the strong interaction effects into account

Transition $n_i \ell_i \rightarrow n_f \ell_f$	Transition energies (keV)			Relative X-ray intensities
	Experimental values	Calculated values for point nucleus	Calculated values for finite-size-corrected nucleus	
4f $\rightarrow$ 3d	1059.1 $\pm$ 1.0	1048.8	1045.0	17.8 $\pm$ 0.9
6g $\rightarrow$ 4f	736	735.47	735.46	11.0 $\pm$ 1.0
5g $\rightarrow$ 4f	479.255 $\pm$ 0.040	478.570	478.558	100
7g $\rightarrow$ 5f	414.8 $\pm$ 0.6		415.44	1.3 $\pm$ 0.3
7h $\rightarrow$ 5g	412.68 $\pm$ 0.16		412.75	11.1 $\pm$ 1.5
9g $\rightarrow$ 6f } 9h $\rightarrow$ 6g }	325.2 $\pm$ 0.6		326.41 } 324.76 }	1.1 $\pm$ 0.5
9i $\rightarrow$ 6h	323.83 $\pm$ 0.20		323.72	4.3 $\pm$ 0.7
6g $\rightarrow$ 5f	260.54 $\pm$ 0.28		260.46	3.6 $\pm$ 0.7
6h $\rightarrow$ 5g	258.27 $\pm$ 0.05		258.27	86 $\pm$ 14
8h $\rightarrow$ 6g	256.64 $\pm$ 0.22		256.10	5.6 $\pm$ 0.8
8i $\rightarrow$ 6h	255.23 $\pm$ 0.12		255.17	12.3 $\pm$ 1.3
9h $\rightarrow$ 7g } 9i $\rightarrow$ 7h }	170.3 $\pm$ 0.4		169.79 } 169.24 }	3.4 $\pm$ 0.7
9j $\rightarrow$ 7i	168.84 $\pm$ 0.12		168.86	14 $\pm$ 2
7h $\rightarrow$ 6g } 7i $\rightarrow$ 6h }	155.03 $\pm$ 0.08		155.86 } 155.13 }	71 $\pm$ 12
10k $\rightarrow$ 8j	117.54 $\pm$ 0.14		117.52	13 $\pm$ 4
8g $\rightarrow$ 7f } 8h $\rightarrow$ 7g }	101.7 $\pm$ 0.6		101.85 } 101.13 }	3.6 $\pm$ 0.9
8i $\rightarrow$ 7h } 8j $\rightarrow$ 7i }	100.39 $\pm$ 0.12		100.69 } 100.40 }	47 $\pm$ 15

Table 3

Experimental and theoretical strong interaction monopole shifts ϵ_0 and widths Γ_0 for pionic 4f and 3d levels with respect to the theoretical point-nucleus value

Nucleus	$\epsilon_0(4f)$ (eV)					$\Gamma_0(4f)$ (eV)					Ref.	
	Exper. value*)	Theoret. values**)				Exper. values from		Theoret. values**)				
		I	II	III	IV	line shape	rel. int.	I	II	III		IV
^{165}Ho	295 $\pm$ 80 285 $\pm$ 8 242 $\pm$ 18 264 $\pm$ 30				258 248	190 $\pm$ 40 148 $\pm$ 25 172 $\pm$ 28	211 $\pm$ $^{107}_{59}$				141	7, 8 6 18 5 10
^{168}Er	360 $\pm$ 25				307	220 $\pm$ 30		132	140	147		18
Yb	400 $\pm$ 30				401	170 $\pm$ 120					214	18
^{175}Lu	597 $\pm$ 70	403	415	434		200 $\pm$ 70		243	256	276		7, 8 10
^{176}Lu	564 $\pm$ 2	407	410	438	455							6 10
^{179}Hf	522 $\pm$ 15				532							6
^{181}Ta	549 $\pm$ 100 578 $\pm$ 15 723 $\pm$ 15 552 $\pm$ 25	524	539	568		255 $\pm$ 57 582 355 $\pm$ 34 362 $\pm$ 25	231 $\pm$ 34 297 $\pm$ $^{63}_{47}$	322	335	378		4 9 6 5 18 10
Re	709 $\pm$ 40	680 682	700 702	743 746		547 $\pm$ 50	477 $\pm$ 42	428 433	442 447	517 522		***) 10
Pb	1710 $\pm$ 20					1174 $\pm$ 45						17, 18
^{209}Bi	1830 $\pm$ 150 1870 $\pm$ 60	1794	1850	1959		1157 $\pm$ 70 1240 $\pm$ 140	1343 $\pm$ 155	1216	1228	1484		4 9 17, 18
^{238}U	5320 $\pm$ 250 5190 $\pm$ 200				1940	4050 $\pm$ 860 3650 $\pm$ 650					1284	19 17 9
		4291	4049	5062				3905	3709	4964		

Nucleus	$\epsilon_0(3d)$ (keV)					$\Gamma_0(3d)$ (keV)					Ref.
	Exper. value*)	Theoret. values**)				Exper. value	Theoret. values**)				
		I	II	III	IV		I	II	III	IV	
^{139}Ba	5.10 $\pm$ 0.27	5.92	5.92	6.79		4.3 $\pm$ 0.9	5.8	5.8	6.8		19
^{140}Ce	6.57 $\pm$ 0.29	7.30	7.46	8.13		5.6 $\pm$ 1.0	7.5	7.5	8.5		19
^{142}Ce	6.75 $\pm$ 0.33	7.34	7.33	8.37		6.5 $\pm$ 0.9	7.6	7.6	8.6		19
^{148}Nd	7.18 $\pm$ 0.32	8.48	8.29	9.75		8.8 $\pm$ 1.2	9.8	9.6	8.6		20
^{150}Nd	7.07 $\pm$ 0.22	8.13	7.60	9.61		9.2 $\pm$ 1.1	10.0	9.6	11.8		20
^{165}Ho	14.0 $\pm$ 3.0				14.4	17.0 $\pm$ 4.0				22.7	18 10
		14.2	12.3	17.3			23.7	22.2	27.5		
^{168}Er	14.2 $\pm$ 1.4				16.4	19.5 $\pm$ 13.0				23.8	18
Yb	13.7 $\pm$ 2.0				18.6	24.4 $\pm$ 9.0				29.0	18
^{181}Ta	10.2 $\pm$ 1.0 14.4 $\pm$ 1.2	21.2 21.6	17.9 18.0	26.9 27.4	22.4	17.7 $\pm$ 3.3 19 $\pm$ 5	41.9 42.8	38.2 38.9	49.9 50.8	40.1	9 17, 18 10
Re	11.1 $\pm$ 1.0	24.3 25.1	20.4 21.2	31.6 32.5		16.8 $\pm$ 2.0	50.2 50.8	45.2 45.7	60.9 61.6		***) 10
^{209}Bi	10.6 $\pm$ 1.3	34.5 34.6	28.2	47.6 47.7		25.4 $\pm$ 5.3	90.5 90.5	79.4	109.8 109.9		9 10

*) All values in this column have been corrected to a shift with respect to the point nuclear value.

**) The calculated values for columns I, II, and III use the set of parameters given in Ref. 9, whereas column IV is taken from the references in the last column.

***)) This paper.

Table 4

Relative intensities of the transitions from the
muonic $5g \rightarrow 4f$ and $5f_{7/2} \rightarrow 4d_{5/2}$ ^{181}Ta multiplets

Transition Centroid energy (keV) Total absolute intensity	$5g_{9/2} \rightarrow 4f_{7/2}$ 341.055 0.3284		$5g_{7/2} \rightarrow 4f_{5/2}$ 345.053 0.2533		$5f_{7/2} \rightarrow 4d_{5/2}$ 344.935 0.0235	
$F_i \rightarrow F_f$	Relative intensity	ΔE (eV/b)	Relative intensity	ΔE (eV/b)	Relative intensity	ΔE (eV/b)
$8 \rightarrow 7$	100	0	-	-	-	-
$7 \rightarrow 6$	79.1	282.5	100	0	100	0
$7 \rightarrow 7$	9.8	- 83.9	-	-	-	-
$6 \rightarrow 7$	0.5	-111.4	-	-	-	-
$6 \rightarrow 6$	16.1	255.0	14.4	- 88.4	14.4	- 86.5
$6 \rightarrow 5$	60.7	342.2	72.9	308.2	72.8	290.4
$5 \rightarrow 6$	1.2	265.5	1.1	-109.5	1.1	-107.1
$5 \rightarrow 5$	19.1	352.6	22.9	287.1	22.9	269.8
$5 \rightarrow 4$	45.0	265.5	50.0	314.7	50.0	295.9
$4 \rightarrow 5$	2.0	385.4	2.9	308.2	2.9	290.4
$4 \rightarrow 4$	19.3	298.4	26.0	335.6	26.0	316.5
$4 \rightarrow 3$	32.0	123.8	31.6	159.5	31.6	149.0
$3 \rightarrow 4$	2.5	340.2	4.8	377.8	4.8	357.7
$3 \rightarrow 3$	17.2	165.7	24.6	201.6	24.6	190.2
$3 \rightarrow 2$	21.6	- 26.2	17.4	- 46.3	17.4	- 45.3
$2 \rightarrow 3$	2.5	206.4	6.4	247.9	6.4	235.5
$2 \rightarrow 2$	13.5	14.4	19.8	0	19.8	0
$2 \rightarrow 1$	13.4	-142.6	7.1	-220.4	7.1	-209.4
$1 \rightarrow 2$	1.7	45.9	7.1	37.9	7.2	37.1
$1 \rightarrow 1$	8.6	-111.2	12.8	-182.4	12.8	-172.3
$1 \rightarrow 0$	7.3	-198.4	-	-	-	-
$0 \rightarrow 1$	-	-	6.6	-161.4	6.6	-151.7

Table 5

Relative intensities of the transitions from the muonic
 $5g \rightarrow 4f$ and $5f_{7/2} \rightarrow 4d_{5/2}$ natural Re h.f. multiplets

Transition Centroid energy (keV) Total absolute intensity	$5g_{9/2} \rightarrow 4f_{7/2}$ 360.18 0.374		$5g_{7/2} \rightarrow 4f_{5/2}$ 364.64 0.289		$5f_{7/2} \rightarrow 4d_{5/2}$ 364.45 0.021	
$F_i \rightarrow F_f$	Relative intensity	ΔE (eV/b)	Relative intensity	ΔE (eV/b)	Relative intensity	ΔE (eV/b)
7 $\rightarrow$ 6	100	0	-	-	-	-
6 $\rightarrow$ 5	79.2	366.1	100	0	100	0
6 $\rightarrow$ 6	8.0	-111.5	-	-	-	-
5 $\rightarrow$ 6	0.3	-129.5	-	-	-	-
5 $\rightarrow$ 5	12.4	348.2	12.1	-115.4	12.1	-112.7
5 $\rightarrow$ 4	61.1	381.5	73.1	388.8	73.0	365.0
4 $\rightarrow$ 5	0.7	388.1	0.7	-123.4	0.7	-120.5
4 $\rightarrow$ 4	13.6	421.0	18.2	380.8	18.2	357.1
4 $\rightarrow$ 3	46.0	208.9	50.7	330.4	50.7	309.4
3 $\rightarrow$ 4	0.8	488.8	1.7	432.0	1.7	407.3
3 $\rightarrow$ 3	11.9	276.6	19.4	381.7	19.4	359.4
3 $\rightarrow$ 2	33.9	- 21.9	33.0	79.2	33.0	72.8
2 $\rightarrow$ 3	0.6	348.4	2.2	453.8	2.2	429.9
2 $\rightarrow$ 2	7.7	49.8	16.5	151.3	16.5	143.3
2 $\rightarrow$ 1	24.9	-215.6	19.7	-176.5	19.7	-167.2
1 $\rightarrow$ 2	-	-	1.8	215.3	1.8	205.9
1 $\rightarrow$ 1	-	-	11.0	-112.4	11.0	-104.6
1 $\rightarrow$ 0	-	-	10.2	-314.1	10.2	-295.6

Table 6

Spectroscopic quadrupole moments, Q , obtained from the h.f.s. of the $5g \rightarrow 4f$ transitions in muonic and pionic Ta and Re isotopes

Nucleus	Q^μ (b)	Q^π *) (b)
^{181}Ta	3.28 ± 0.06	3.30 ± 0.06 **)
^{185}Re	2.09 ± 0.04	2.07 ± 0.02
^{187}Re	2.21 ± 0.04	2.18 ± 0.02

*) Corrected for strong interaction effects.

**) Ref. 4.

Table 7

Spectroscopic quadrupole moments Q (in b) in ^{181}Ta *)

Q (b)	Method	Ref.
3.5 ± 0.2	μ^- atom	22
3.24 ± 0.13	C.E. γ (NaI) (0 → 1)	23
3.31 ± 0.14	C.E. γ (NaI) (0 → 2)	24,25
3.30 ± 0.12	C.E. inel. (0 → 1)	26
3.17 ± 0.16	C.E. inel. (0 → 1)	26
2.87 ± 0.21	$T_{1/2}$ (1 → 0)	23
3.66 ± 0.23	π^- atom, no correction	27
3.26 ± 0.23	π^- atom, with correction	27
3.45 ± 0.10	K^- atom	28
3.45 ± 0.22	K^- atom 8j level	18
3.30 ± 0.12	K^- atom 7i level	18
3.30 ± 0.20	π^- atom 5g level	18
3.35 ± 0.02	π^- atom 4f level	18
3.414 ± 0.005	} μ^- atom, model dependent	29
3.384 ± 0.004		
3.314 ± 0.004		
3.18 ± 0.03	μ^- atom	30
3.30 ± 0.06	π^- atom 4f level	4
3.28 ± 0.06	μ^- atom 4f level	Present result

*) Explanation of methods:

C.E. : Coulomb excitation
 (0 → 1), (0 → 2): to first, second rotational state
 inel. inelastic scattering
 $T_{1/2}$ (1 → 0) : Partial E2 half-life of first rotational state to ground state
 μ^- : Muonic X-ray spectra
 π^- : Pionic X-ray spectra
 K^- : Kaonic X-ray spectra

Table 8

Spectroscopic quadrupole moments Q (in b) in ^{185}Re and ^{187}Re *)

^{185}Re	^{187}Re	Method	Ref.
2.8 ± 0.7	2.6 ± 0.6	h.f.s. optical	31
2.9 ± 0.7	2.9 ± 0.7	h.f.s. optical	32
2.3 ± 0.9	2.2 ± 0.9	h.f.s. optical	33
1.7 ± 0.4	1.5 ± 0.4	C.E. γ (NaI) ($0 \rightarrow 1$)	34
2.7 ± 0.7	2.1 ± 0.5	C.E. e^- ($0 \rightarrow 1$)	35
1.5 ± 0.4		C.E. e^- ($0 \rightarrow 2$)	35
1.5 ± 0.4	1.8 ± 0.5	C.E. γ (NaI) ($0 \rightarrow 1$)	36
1.8 ± 0.5	2.0 ± 0.5	C.E. γ (NaI) ($0 \rightarrow 2$)	36
2.1 ± 0.5	1.9 ± 0.5	C.E. e^- ($0 \rightarrow 1$)	37
2.0 ± 0.2	1.9 ± 0.2	C.E. γ (NaI) ($0 \rightarrow 1$)	38
2.1 ± 0.2	2.0 ± 0.2	C.E. γ (NaI) ($0 \rightarrow 2$)	38
2.24 ± 0.11	2.0 ± 0.1	C.E. γ (NaI) ($0 \rightarrow 1$)	25
2.26 ± 0.11	2.2 ± 0.1	C.E. γ (NaI) ($0 \rightarrow 2$)	25
2.5 ± 0.3	2.1 ± 0.3	C.E. γ (NaI) ($0 \rightarrow 1$)	39
2.4 ± 0.4	2.2 ± 0.4	C.E. γ (NaI) ($0 \rightarrow 2$)	39
1.92 ± 0.10	1.99 ± 0.10	C.E. inel. ($0 \rightarrow 1$)	40
1.99 ± 0.10	2.01 ± 0.10	C.E. inel. ($0 \rightarrow 2$)	40
	1.95 ± 0.17	$T_{1/2}$ ($1 \rightarrow 0$)	23
2.18 ± 0.02	2.07 ± 0.02	π^- atom	Present results
2.21 ± 0.04	2.09 ± 0.04	μ^- atom	Present results

*) Explanation of methods:

- C.E. : Coulomb excitation
 ($0 \rightarrow 1$), ($0 \rightarrow 2$): to first, second rotational state
 inel. inelastic scattering
 $T_{1/2}$ ($1 \rightarrow 0$) : Partial E2 half-life of first rotational state to ground state
 μ^- : Muonic X-ray spectra
 π^- : Pionic X-ray spectra
 h.f.s. : Hyperfine structure optical spectroscopy

Figure captions

- Fig. 1 : The 477.605 keV calibration line from a ^7Be source. This photo peak was used as a calibration standard to extract the instrumental line shape for fitting the $5g \rightarrow 4f$ pionic h.f. complex at 479 keV in Re. The open circles represent the fit to the data points (black dots). The difference in the height of the background between the right-hand and the left-hand side of the photo peak is taken care of by a step-function (crosses).
- Fig. 2 : The prompt γ -ray spectrum from pions stopped in a natural Re target as measured with a large volume Ge(Li) detector.
- Fig. 3 : Part of the prompt γ -ray spectrum presented in fig. 2, showing the $5g \rightarrow 4f$ pionic h.f. complex at 479 keV. The open circles represent the fit of the complex to the data (black dots). The crosses give the background as fitted with the same step function as shown in fig. 1.
- Fig. 4 : Part of the prompt γ -ray spectrum presented in fig. 2, showing the $4f \rightarrow 3d$ pionic h.f. complex at 1059 keV. The solid line represents the fit to the data points (black dots). The background used in the analysis was obtained by examining a large energy region below and above the h.f. complex.
- Fig. 5 : The 356.014 keV calibration line from a ^{133}Ba source. This photo peak was used as a calibration standard to analyse the muonic $5g \rightarrow 4f$ h.f. complexes.
- Fig. 6 : The prompt γ -ray spectrum from muons stopped in a natural Ta target. The data are recorded by a LEPS detector. This spectrum only contains about a half of the data.
- Fig. 7 : The prompt γ -ray spectrum from muons stopped in a natural Re target. The data are recorded by a LEPS detector. This spectrum only contains about a half of the data.

Fig. 8 : The muonic $5g_{9/2} \rightarrow 4f_{7/2}$ and $5g_{7/2} \rightarrow 4f_{5/2}$ X-ray complexes in ^{181}Ta . The crosses represent the fit to the data points (black dots). The corrections due to the background are displayed by the dashed line. The γ -ray at 346 keV is due to a γ -ray transition depopulating²¹⁾ the 1637.1 keV level excited in the $^{181}\text{Ta}(\mu^-, n)^{180}\text{Hf}$ reaction.

Fig. 9 : The muonic $5g_{9/2} \rightarrow 4f_{7/2}$ and $5g_{7/2} \rightarrow 4f_{5/2}$ X-ray h.f. complexes in natural Re.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9