Ref.TH.3012-CERN

A MODEL FOR PION EMISSION FROM NN BOUND STATES *)

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ABSTRACT

Widths for the transition $(p\bar{p})_{\text{ATOMIC}} \rightarrow (NN)_B + \pi$ between initial $1s$ states and final $T = 0$, $S = 1$ natural parity states have been computed within a non-relativistic perturbative model that combines the impulse approximation with p -wave and s -wave pion rescattering terms. The final states are treated as quasi-nuclear NN states bound in a 2-body potential obtained from the Paris NN interaction via G -parity transformation and introduction of a cut-off radius. In the initial atomic states, both annihilation and isospin mixing effects are included. The results indicate that annihilation effects in the initial states, through the suppression of short-range components in the wave functions, are extremely important in diminishing rescattering contributions to the transitions and stabilizing the binding energy dependence of the widths. Without annihilation, the calculated widths are unreasonably large and dominated by the rescattering terms. The effect of shorter range contributions such as multiple rescattering and meson-baryon form factors has also been investigated and found to be unimportant when annihilation effects are included.

*) Supported in part by German Bundesministerium für Forschung und Technologie.

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1. Introduction and Overview

Recently, evidence for the existence of narrow mesonic states lying below the $N\bar{N}$ threshold was obtained in an experiment where antiprotons were stopped in liquid hydrogen targets [1]. In the observed γ -ray spectrum, three narrow structures were detected at energies of 183 ± 17 MeV, 216 ± 9 MeV, and 420 ± 17 MeV respectively, corresponding to states with masses (including recoil corrections) of 1684 MeV, 1646 MeV, and 1415 MeV. All three structures occurred with yields of the order of 10^{-2} per stopped antiproton and with widths less than the instrumental widths (20-35 MeV). Currently, there is some debate concerning the existence of the 420 MeV structure, but the two structures around 200 MeV have survived some checks and remeasurements [2], in marked contrast to the multitude of narrow resonance states above threshold once thought to exist but no longer seen empirically. Note, however, that in another experiment, no narrow $N\bar{N}$ state has been seen below the threshold [3].

The observed structures below threshold have been the subject of several interpretations. On the one hand, invoking explicitly the quark degrees of freedom of the nucleon-antinucleon pair, one expects to observe exotic quark-antiquark configurations, such as $qq\bar{q}\bar{q}$, which may or may not be narrow [4]. On the other hand, the observed structures can be interpreted wholly within the framework of traditional nuclear physics as an $N\bar{N}$ pair deeply bound in some $N\bar{N}$ potential. It should be emphasized that the two approaches are not incompatible. Conceivably, different types of states could exist with characteristics that favor one interpretation or the other, as discussed in ref [5]. There may also be states which manifest a strong overlap between the two models. Given the present experimental status, none of these possibilities can be excluded.

Within either approach one expects to observe not only radiative production of bound states, but also production with pion emission. It is to these latter processes, treated within the nuclear physics point of view, that we will address ourselves in this paper. In particular, we study the transition

$$(p\bar{p})_{\text{Atomic}} \rightarrow (N\bar{N})_B + \pi \quad (1.1)$$

from weakly bound quasi-atomic $p\bar{p}$ states to deeply bound $N\bar{N}$ states. Our aim here is not so much to produce detailed quantitative results, but rather to investigate what can be concluded about the process within a particular approach, to uncover those aspects of the physics which sensitively affect the results, and to obtain some theoretical estimates for comparison with calculations within other approaches and with future experimental results. Concerning the latter comparison, the advent of LEAR within the next couple years should make available new data of much higher quality than exists at present.

At present the experimental data provide no quantum number constraints on the states participating in the transition (1.1). Fortunately, constraints exist within the theory itself arising from meson exchange coherence effects manifested in certain spin-isospin channels of the nucleon-nucleon and nucleon-antinucleon interactions. Such effects have been examined in detail by Buck, Dover and Richard [5], who observed that in the isosinglet, triplet channel with $L = J \pm 1$, the central, quadratic spin-orbit, and tensor components of all meson exchange contributions to the nucleon-antinucleon interaction are attractive. This attractive coherence leads, within any potential model, to a sequence of deeply bound natural parity states, $^{13}P_0$, $^{13}S_1 - D_1$, $^{13}P_2 - F_2$, etc., with $I = 0$ and $S = 1$. In other spin-isospin channels, such coherences either

do not exist or are repulsive ($I = 0, S = 1, J = L$), and hence, the corresponding states are either weakly bound or unbound. Keeping in mind that pion emission in a transition from threshold to a bound state requires at least 140 MeV binding energy (the pion mass), it seems reasonable to suppose that the narrow states observed in the CERN experiment, if they are $N\bar{N}$ states at all, are members of the $I = 0, S = 1$ natural parity band. We adopt this hypothesis in what follows. Furthermore, we restrict our considerations to the three lowest members of the natural parity band, the $^{13}P_0, ^{13}S_1 - D_1$, and $^{13}P_2 - F_2$ states. Higher lying states within this band require considerable short-range attraction to be bound by more than 140 MeV and thus, due to short-range annihilation processes, are less likely to be narrow.

The restriction to $I = 0$ final quasi-nuclear states restricts the number of initial atomic states as well. Since the pion has unit isospin, isospin conservation demands that the atomic states be isotriplet. A further restriction is imposed by experimental considerations: in liquid hydrogen targets, Stark mixing effects tend to depopulate states with orbital angular momentum greater than zero [6] so that only initial s-states need be considered. We also assume that the final quasi-nuclear states are populated exclusively from the lowest lying s-states with principal quantum number $n = 1$. This was done primarily for economy of presentation of the results; our calculations can easily be extended to initial states with $n > 1$ or $L > 0$.

To represent the transition amplitude we adopt a simple, non-relativistic perturbative model analogous to one that has been successfully employed to treat the reaction $pp \rightarrow d\pi^+$ with unpolarized protons [7]. This model consists

of the impulse approximation, represented diagrammatically in fig. 1a, and a rescattering term with two contributions: a p-wave contribution (fig. 1b), where the rescattered meson excites a (3,3) resonance or anti-resonance that subsequently deexcites through emission of the outgoing pion, and an s-wave contribution (fig. 1c), where the absorption of the rescattered pion and the emission of the outgoing pion are incorporated within an effective $\pi\pi NN$ s-wave vertex. Note that pion emission can occur from either the baryon line or the antibaryon line. In fig. 1, only the latter diagrams are depicted, but of course, all six diagrams are included in the full transition amplitude. Note, also, that the diagram analogous to fig. 1b with an intermediate antinucleon line in place of the anti-resonance line is included in the impulse approximation by virtue of the initial state wave function and should not be added explicitly.

In the case of the $pp \rightarrow d\pi^+$ reaction, inclusion of rescattering contributions is necessitated by the reaction kinematics, which require a large momentum transfer between the two nucleons to accommodate the pion mass without upsetting momentum conservation. The situation is somewhat different in the present case: the large binding energies of the quasi-nuclear states permit the pion mass to be absorbed without a large momentum transfer. Nevertheless, rescattering can still make a significant contribution to reaction (1.1) by virtue of the high momentum components of the nuclear wave functions, which offset any momentum imbalance induced by the rescattering. In fact, in the absence of annihilation mechanisms, rescattering contributions to the transition amplitude exceed the impulse approximation amplitude by as much as an order of magnitude. Clearly, such large rescattering contributions make the validity of a perturbative, non-relativistic approach rather questionable. On the other hand, rescattering processes, since they involve high momentum components of the wave functions, are expected to be rather short-ranged and

thus, should be substantially suppressed by short-range annihilation mechanisms. This is indeed the case, as will be discussed in section 4. Also, multiple rescattering processes, which are still more short-ranged, are even more severely suppressed. Hence, the use of a perturbative approach appears to be justified in the present investigation, as long as annihilation mechanisms are included in the treatment.

In the next section the model for the transition amplitude is outlined in somewhat more detail. This is followed in section 3 by a discussion of the $N\bar{N}$ potential employed in the present calculations together with the wave functions of the initial and final states. In section 4 we first summarize the relevant selection rules and then present numerical results, with and without annihilation. Finally, the last section is devoted to conclusions and future perspectives.

2. Contributions to the Transition Amplitude

The various contributions to the transition amplitude for reaction (1.1) can be easily derived by comparison with the corresponding contributions to the $pp \rightarrow d\pi^+$ amplitude and use of the G-parity invariance of the strong interactions. Under G-parity transformations, baryon states are transformed into antibaryon states with the same isospin projection ($p \rightarrow \bar{n}$, for example) and vice versa, and multiple pion states just acquire a factor $(-)^{n_\pi}$, where n_π is the number of pions. G-parity invariance of the strong interactions thus implies that pion-antibaryon-antibaryon and pion-baryon-baryon vertices with the same isospin structure are related by a simple minus sign. Vertices involving pion resonances are in turn, related by a factor $(-)^{n_\pi}$, where n_π is the number of pions that participate in the resonance. With these

G-parity rules in mind, we consider now each of the transition amplitude contributions in some detail, beginning with the impulse approximation.

2.1 The Impulse Approximation

To construct the impulse approximation to the transition amplitude, we employ the lowest order non-relativistic reduction of the elementary πNN Lagrangian,

$$\mathcal{L}_{\pi NN} = -\frac{f}{m_\pi} \vec{\sigma} \cdot \vec{k} \vec{\tau} \cdot \hat{\phi}_\pi, \quad (2.1)$$

where $\vec{k}$ is the pion momentum, $\hat{\phi}_\pi$ is the pion field, and $\vec{\sigma}$ and $\vec{\tau}$ are the Pauli matrices acting in the spin and isospin spaces respectively. Note that this expression does not include recoil terms of the type discussed in ref. [7]. Such terms were found to have a non-negligible effect on the πd absorption cross section, particularly near threshold. However, the treatment employed there is based on a non-relativistic reduction strictly valid only for plane wave states and thus, not appropriate in the present case where both the initial and final states are bound. Also, most of the transitions we study are to states sufficiently far below threshold that recoil terms should not be too important. In any case, no attempt has been made to incorporate recoil effects in the present investigation.

Combining the two contributions to the impulse approximation (with pion emission from either the nucleon or antinucleon line) and applying the G-parity relation between the πNN and πNN vertices, eq. (2.1) yields the amplitude

$$\hat{M}_{\pi A}(k, r) = \frac{f}{m_\pi} \left[e^{ik \cdot r/2} (\vec{\sigma}_1 \cdot \vec{k}) \tau_1^0 - e^{-ik \cdot r/2} (\vec{\sigma}_2 \cdot \vec{k}) \tau_2^0 \right], \quad (2.2)$$

where the indices 1 and 2 refer to the antinucleon and nucleon lines respectively, and charge conservation has been used to restrict the outgoing pion to a π^0 .

2.2 p-wave Rescattering

The p-wave rescattering amplitude is constructed in a similar fashion. We represent the initial meson exchange which excites an $\bar{N}\Delta$ or $\pi\bar{N}\Delta$ state by a transition potential, insert a propagator for the intermediate resonance or anti-resonance, and employ G-parity invariance to relate the external $\pi\bar{N}\Delta$ and $\pi N\Delta$ vertices. This yields for the two contributions to the p-wave rescattering amplitude

$$\hat{M}_p(k, r) = - \frac{f_{\text{eff}}^*(s)}{m_\pi} \left[e^{ik \cdot r/2} V_{12}(r) D_{\bar{\Delta}}(k, s) (\vec{S}_1^+ \cdot \vec{k}) T_1^{0+} - e^{-ik \cdot r/2} V_{21}(r) D_{\Delta}(k, s) (\vec{S}_2^+ \cdot \vec{k}) T_2^{0+} \right], \quad (2.3)$$

where $\vec{k}$ is now the πN (or $\pi\bar{N}$) c.m. momentum, $\sqrt{s}$ is the $\pi N(\pi\bar{N})$ c.m. energy, and $\vec{S}^+$ and $\vec{T}^+$ are the transition spin and isospin operators that act between spin, isospin 1/2 and spin, isospin 3/2 wave functions. Since all contributions to the (3.3) resonance width involve an even number of π -baryon-baryon vertices, the two propagators $D_{\bar{\Delta}}$ and D_{Δ} are identical. We represent them by the expression

$$D_{\bar{\Delta}}^{-1} \equiv D_{\Delta}^{-1} = M_{\Delta} - \sqrt{s} - \frac{1}{2} i \Gamma_{\Delta}(s, k) \quad (2.4)$$

with

$$\Gamma_{\Delta} = \frac{2}{3} \frac{f_{\text{eff}}^{*2}(S)}{4\pi} \frac{k^3}{m_{\pi}^2} \quad (2.5)$$

for the (3.3) resonance width and

$$\frac{f_{\text{eff}}^{*2}(S)}{4\pi} = \left(\frac{f_{\pi N \Delta}^2}{4\pi} = 0.32 \right) \frac{2M_{\Delta}}{M_{\Delta} + \sqrt{S}} \quad (2.6)$$

for the effective coupling at the external $\pi N \Delta$ and $\pi \bar{N} \bar{\Delta}$ vertices. These expressions are discussed in more detail in ref. [7].

The longest range contribution to the transition potential, $V_{12}(r)$, arises from one pion exchange. In momentum space this is given by the standard non-relativistic expression with an additional minus sign originating from the odd number of π -baryon-baryon vertices:

$$V_{12}^{\pi}(\vec{q}) = \frac{f_{\pi N \Delta} f_{\pi N N}}{m_{\pi}^2} \frac{S_1 \cdot \vec{q} \quad S_2 \cdot \vec{q}}{q_0^2 - \vec{q}^2 - m_{\pi}^2} \vec{T}_1 \cdot \vec{T}_2 \quad (2.7)$$

Here q is the momentum transferred by the exchanged pion, and q_0 is the corresponding energy transfer, which, as in ref. [7], we fix equal to one-half the outgoing pion energy.

To obtain a good description of πd production in pp scattering, it was necessary to supplement this relatively long-range exchange by shorter range mechanisms, such as multiple rescattering and processes associated with the off-shell structure of the pion vertices. These short-range mechanisms require the introduction of a number of parameters, which were varied so as to fit the empirical integrated and differential cross sections. In the present case one expects such short-range processes to be largely suppressed by annihilation mechanisms, so that the full p -wave rescattering amplitude will be well approxi-

mated by the one pion exchange contribution alone.

We have investigated this hypothesis in detail by adding to eq. (2.7) the same short-range contributions to the transition potential that were employed in the analysis of the $pp \rightarrow d\pi^+$ reaction in ref. [7]. These contributions are of two types, summarized below:

- (1) To take account of finite range effects in the πNN and $\pi N\Delta$ vertices, the on-shell couplings were extrapolated off-shell using monopole form factors with a common mass parameter, Λ_π . In particular, we used

$$f_{\pi N\Delta}(q^2) = f_{\pi N\Delta} \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2}$$

and

$$f_{\pi NN}(q^2) = f_{\pi NN} \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2}$$

(2.8)

in place of $f_{\pi N\Delta}$ and $f_{\pi NN}$ in eq. (2.7), where $q^2 = q_0^2 - \vec{q}^2$. Note that the form factors associated with antibaryon vertices are identical to these since the microscopic processes contributing to the form factors always involve an even number of π -baryon-baryon vertices (regarding pionic resonances as multiple pion states).

- (2) Second order rescattering contributions were approximated by a distributed mass two pion exchange with isovector, vector structure. In momentum space this takes the form

$$V_{12}^p(\vec{q}) = \int_{4m_\pi^2}^{\infty} dt \rho^*(t) \frac{(\vec{S}_1 \times \vec{q}) \cdot (\vec{S}_2 \times \vec{q})}{q^2 - t} \vec{T}_1 \cdot \vec{T}_2, \quad (2.9)$$

where again $q^2 = q_0^2 - \underline{q}^2$. Note that for two pion exchange there is no minus sign associated with G-parity invariance, as in eq. (2.7). We assume that the mass distribution function, $\rho^*(t)$, is related to the Born subtracted square of the $N\bar{N} \rightarrow 2\pi$, $J^\pi = 1^-$ helicity amplitude, $f_-^1(t)$, by threshold and kinematic factors and a scaling parameter, α_ρ , equal to the ratio of the effective $\rho N\Delta$ and ρNN couplings:

$$\rho^*(t) = (\alpha_\rho \equiv \frac{f_{\rho N\Delta}}{f_{\rho NN}}) \frac{3(t - 4m_\pi^2)^{3/2}}{32 M_N^2 \sqrt{t}} (|f_-^1(t)|^2 - |f_B|^2) \quad (2.10)$$

For the squared helicity amplitude, the results of a recent analysis by Gauron [8] were used with the Born term, f_B , subtracted (to avoid double counting) outside the square so as to preserve certain second order contributions with $N\Delta$ intermediate states that would otherwise be omitted (see ref. [7] for details).

2.3 s-wave Rescattering

The remaining contribution to the transition amplitude, s-wave rescattering, involves pion emission from an effective s-wave $\pi\pi NN$ vertex with one pion off-shell. To represent this vertex, we employ the phenomenological contact Lagrangian of Koltun and Reitan [9],

$$L_s = -4\pi \left[\frac{\lambda_1(t)}{m_\pi} \bar{\psi} \not{\underline{q}} \cdot \not{\underline{q}} \psi + \frac{\lambda_2(t)}{m_\pi^2} \bar{\psi} \not{\underline{q}} \cdot \not{\underline{q}} \times \underline{\pi} \psi \right] , \quad (2.11)$$

where $t = (\omega_\pi - q_0)^2 - (\vec{k} - \vec{q})^2$ is the squared 4-momentum in the πN t-channel, ψ and ϕ are the nucleon and pion fields, and $\vec{\pi}$ is the momentum conjugate to ϕ . The two couplings, λ_1 and λ_2 are given on-shell ($t=0$) by linear combinations of the πN s-wave scattering lengths and have been extrapolated off-shell to finite values of t using a simple dynamical model described in ref. [7]. Since the $\pi\pi NN$ vertex is a two pion vertex, the corresponding antibaryon couplings are identical.

Combining the Koltun-Reitan s-wave vertex with the p-wave pion vertices (including form factors) and the propagator of the exchanged pion, the s-wave rescattering amplitude in momentum space becomes

$$\begin{aligned} \hat{M}_S(\vec{k}, \vec{q}) = & -4\pi \left(\frac{f}{m_\pi}\right) \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2}\right) \frac{1}{q^2 - m_\pi^2} \left\{ e^{i\vec{k}\cdot\vec{r}/2} (\vec{\sigma}_2 \cdot \vec{q}) \right. \\ & \times \left[2 \frac{\lambda_1(t)}{m_\pi} \tau_2^0 - i \frac{\lambda_2(t)}{m_\pi^2} (\omega_\pi + q_0) (\vec{\tau}_1 \times \vec{\tau}_2)^0 \right] + e^{-i\vec{k}\cdot\vec{r}/2} \\ & \times (\vec{\sigma}_1 \cdot \vec{q}) \left[2 \frac{\lambda_1(t)}{m_\pi} \tau_1^0 + i \frac{\lambda_2(t)}{m_\pi^2} (\omega_\pi + q_0) (\vec{\tau}_1 \times \vec{\tau}_2)^0 \right] \left. \right\} \quad (2.12) \end{aligned}$$

Fourier transformation of this expression can be easily accomplished by means of partial fraction techniques.

3. Initial and Final State Wave functions

To obtain wave functions for the initial and final nucleon-antinucleon states within a non-relativistic formulation, an $N\bar{N}$ potential is required. The long and medium range parts of such a potential present no difficulty. They can

be easily extracted from a microscopic model for the nucleon-nucleon interaction, such as the dispersion theoretic Paris interaction [10], using the G-parity invariance of strong interactions discussed in the previous section. One cannot obtain the short-range part of the $N\bar{N}$ potential in this way, however, since the short-range parts of NN interaction models are usually phenomenological rather than microscopic. Unfortunately, there is also very little empirical information available concerning the short-range $N\bar{N}$ interaction.

Given this dearth of information, we adopt a procedure employed in previous investigations of $N\bar{N}$ bound states [5,11] that essentially circumvents the difficulty: the G-parity transformed microscopic interaction is cut off at some radius r_0 and replaced by a square-well potential, i.e.

$$\begin{aligned} V(r) &= V_G(r) \quad \text{for } r > r_0 \\ &= V_G(r_0) \quad \text{for } r \leq r_0 \end{aligned} \quad (3.1)$$

For $V_G(r)$ we use the G-parity transformed potential [11] and vary r_0 to obtain particular binding energies for the various quasi-nuclear states considered. Unfortunately, small variations in r_0 lead to rather large variations in the binding energies [5] so that this procedure is not entirely satisfactory.

Even if one could G-parity transform the nucleon-nucleon interaction over its whole range, one would obtain only a part of the full nucleon-antinucleon interaction, in particular, that part arising from meson exchanges in the t-channel. In addition there are s-channel annihilation processes that contribute to the $N\bar{N}$ potential and which make it complex. From analyticity arguments applied to pointlike nucleons and antinucleons, these s-channel processes are expected to be very short-ranged, of the order of $(2M)^{-1} \sim 0.1$ fm [12]. However, the nucleon is now generally believed to be a composite and extended

object, and, indeed, from the distribution of decay products [14], $N\bar{N}$ annihilation appears to be a quite complicated process. At present there is no realistic treatment of annihilation processes based on a microscopic point of view. On a phenomenological level, Bryan and Phillips [14] proposed several years ago that annihilation processes be parametrized by an imaginary, local potential of Woods-Saxon form. More recently, Dover and Richard [15] combined a complex annihilation potential of the form

$$V_{ANN}(r) = \frac{U_0 + iW_0}{1 + e^{r/a}} \quad (3.2)$$

with the G-parity transformed Paris potential to fit $p\bar{p}$ elastic, charge exchange, and inelastic integrated cross sections up to a laboratory energy of 450 MeV.

The trouble with these phenomenological potentials is that fits to the $p\bar{p}$ inelastic cross section require such large values of W_0 that narrow bound $N\bar{N}$ states, such as those apparently seen in the CERN experiment [1], could not possibly exist. At least part of this inconsistency stems from the local, energy-independent character of the potentials, which is clearly at variance with expectations based on microscopic considerations. According to Shapiro [16], proper inclusion of energy dependence in the annihilation potential would appreciably reduce the annihilation strength effective below threshold, thereby restoring the possibility of narrow bound states.

Recently, a simple model that accomplishes this explicitly has been proposed by Green, Sainio, and Wycech [17]. These authors observe that in addition to the usual one-step multi-pion annihilation processes of short-range, there are two step annihilation processes, such as

which proceed through an intermediate bound state and are relatively long-ranged. When simulated in the $N\bar{N}$ channel, reactions (3.3) contribute an imaginary component to the $N\bar{N}$ potential of the form (suppressing spin and isospin)

$$\begin{aligned}
 \text{Im } V_{\text{ANN}}^{(2)}(r, r') &\propto - \sum_B \Gamma(E+B) \psi_B(r) \psi_B(r') \\
 &\times \left\{ j_0\left(\frac{q(B)}{2}|r-r'|\right) + j_0\left(\frac{q(B)}{2}|r+r'|\right) \right\},
 \end{aligned}
 \tag{3.4}$$

where E is the c.m. energy relative to the $N\bar{N}$ threshold and B is the binding energy in the intermediate bound states. The range of this potential is governed by the bound state wave functions and thus, is of the order of 0.8 - 1.0 fm. The factor $\Gamma(E+B)$, given by

$$\begin{aligned}
 \Gamma(E+B) &\propto q^3(B) = [(E+B)^2 - m_\pi^2]^{3/2} \quad \text{for } E+B > m_\pi \\
 &= 0 \quad \text{for } E+B \leq m_\pi,
 \end{aligned}
 \tag{3.5}$$

largely determines the energy dependence. Note that the number of bound states contributing to $V_{\text{ANN}}^{(2)}$ decreases rapidly as E decreases, since these states must lie at least 140 MeV below E . For sufficiently small E , corresponding to deeply bound $N\bar{N}$ states, $V_{\text{ANN}}^{(2)}$ vanishes altogether. Thus, only the short-range "true" annihilation processes contribute to the width of deeply bound $N\bar{N}$ states; whereas, additional long-range processes, characterized by a strong energy dependence, affect states at threshold and above. Since the

latter processes are long-ranged, they can contribute significantly to the $p\bar{p}$ inelastic cross section, thereby reducing the strength required of the short-ranged contributions.

This model may or may not be realistic, but it does demonstrate in a simple manner how energy dependence in the annihilation potential can reduce the apparent inconsistency between large $p\bar{p}$ inelastic cross sections on the one hand and the observation of narrow bound $N\bar{N}$ states on the other. Clearly, if the existence of narrow states below threshold is confirmed in future experiments, some mechanism like the type proposed in ref. [17] must be operable.

Having selected a potential to represent the nucleon-antinucleon interaction, we can solve the Schrödinger equation to obtain the initial and final state wave functions. For the final quasi-nuclear states we assume, on the basis of the preceding discussion, that annihilation contributions to the wave function are sufficiently short-ranged that they may be safely ignored. The potential inserted into the Schrödinger equation is then just that defined by eq. (3.1), with V_G equal to the G-parity transformed Paris potential [11]. Of course the resulting wavefunctions are not reliable at short range, but this probably has little effect on the transition amplitudes since short-range contributions to these amplitudes are largely suppressed by annihilation mechanisms in the initial states. Note that the tensor component of V_G strongly mixes orbital angular momentum states leading to coupled equations.

For the initial atomic states, the relevant potential is the sum of the strong $N\bar{N}$ potential and the electromagnetic Coulomb potential. Both interactions must be retained in the Schrödinger equation, as emphasized by Dover, Richard, and Zabeck [18], the Coulomb interaction to guarantee the correct normalization of the atomic wavefunctions, and the strong interaction to maintain orthogonality between the initial and final states. Since the initial state

energies are near zero on a nuclear scale, the tensor force is weak, and orbital angular momentum mixing can be ignored. However, the Coulomb interaction mixes isospin states, leading to equations coupled in the isospin space. In practice, since the asymptotic wave functions are just the Coulomb ones, it is more convenient to work in the $|p\bar{p}\rangle$, $|n\bar{n}\rangle$ basis where the Coulomb interaction is diagonal and mixing occurs through the strong interaction. Such isospin mixing effects in $p\bar{p}$ atomic systems were first treated by Kaufmann and Pilkuhn [19] and subsequently, in connection with transitions to quasi-nuclear $N\bar{N}$ states, by Dover, Richard, and Zabeck [18].

Since the atomic states lie near threshold, annihilation effects are expected to be important and cannot be realistically ignored. We have included such effects in the initial wave functions using the phenomenological potential of Dover and Richard [15], eq.(3.2), with $U_0 = -21$ GeV, $W_0 = -20$ GeV, and $a = 0.2$ fm. With annihilation included, the effect of isospin mixing in the atomic states is mainly to renormalize the atomic wave functions.

4. Transition Widths

To obtain the matrix elements of the transition amplitude, discussed in section 2, we employ the partial wave expansion techniques outlined in appendix B of ref [7]. The resulting expressions manifest a number of selection rules that restrict the possible transitions. In particular, parity conservation and the symmetry structure of the transition operator require respectively

$$L_i + L_f + L_\pi = \text{odd} \quad (4.1)$$

and

$$S_i + L_\pi = \text{even} , \quad (4.2)$$

where L_i , L_f , and L_π are the initial $p\bar{p}$, final $N\bar{N}$, and pion orbital angular momentum in the c.m. system, and S_i is the initial state spin. Combining these two expressions with the relation

$$L_f = J_f \pm 1 \quad (4.3)$$

for $S = 1$ natural parity states, yields the restriction

$$L_i + S_i + J_f = \text{even} , \quad (4.4)$$

where J_f is the total angular momentum of the final $N\bar{N}$ state. Thus, only three transitions are allowed from initial s-states:

$$\begin{aligned} {}^{31}S_0 &\longrightarrow {}^{13}P_0 \\ {}^{33}S_1 &\longrightarrow {}^{13}S_1 - D_1 \\ {}^{31}S_0 &\longrightarrow {}^{13}P_2 - F_2 \end{aligned} \quad (4.5)$$

In terms of the matrix elements, the transition width for the reaction (1.1) is given by

$$\Gamma_{fi} = \frac{\hbar c}{2\pi} \frac{kB}{2M_N} \frac{4}{3} \sum_{\text{spins}} \sum_{\ell_\pi} \left| \langle f | \hat{M}_{\ell_\pi} | i \rangle \right|^2 , \quad (4.6)$$

where k is the c.m. momentum, B is the binding energy of the quasi-nuclear state, and ℓ_π is the pion orbital angular momentum relative to the quasi-nuclear state. Note that contributions to the transition amplitude from different pion partial waves add incoherently; i.e., there are no cancellations

arising from interference effects. Since the quasi-nuclear wave functions are short-ranged, so that the relevant range of kr values in the pion wave functions is restricted to $kr \lesssim 1$, this has the consequence that only the lowest pion partial waves contributing to a particular transition are important. Moreover, there are no interference terms between the impulse approximation plus p-wave rescattering contribution on the one hand and the s-wave rescattering contribution on the other, because these two contributions always involve different partial waves. Hence, s-wave rescattering is important only when it acts through a lower pion partial wave than does the impulse approximation plus p-wave rescattering.

Numerical results with annihilation effects included in the atomic wave functions, as discussed in the previous section, are presented in tables 1 and 2. Table 1 contains widths for the three transitions (4.5) calculated with the simplest prescription for the $N\bar{N} \rightarrow \bar{N}\Delta$ (or $\bar{N}N \rightarrow \bar{N}\Delta$) transition potential, i.e., with just the elementary one pion exchange term (OPE), eq. (2.7), without form factors. Each nuclear wave function here was constructed with a strong interaction cut-off chosen to yield the binding energy indicated, while the atomic states were generated with one r_0 fixed at $r_0 = 0.82\text{fm}$. For each transition three widths are exhibited, obtained respectively with just the impulse approximation (IA), the impulse approximation plus p-wave rescattering, and the impulse approximation plus both rescattering terms included in the transition amplitude.

Comparing the impulse approximation results for the three transitions reveals that the $^{13}S_1-D_1$ width exceeds by an order of magnitude the $^{13}P_0$ and $^{13}P_2-F_2$ widths, which are roughly equal within the impulse approximation. This result can be easily understood in the light of the previous discussion. In particular, for transitions to the $^{13}S_1-D_1$ state, the impulse approximation involves the partial wave $\ell_\pi = 0$; whereas, for the other two transitions the lowest partial wave contributing to the impulse approximation width is $\ell_\pi = 1$.

Thus, one expects the $^{13}\text{S}_1\text{-D}_1$ transition width calculated within the impulse approximation to substantially exceed the other two transition widths, as observed.

Similar considerations account for the relative magnitudes of the impulse approximation plus p-wave rescattering amplitude and the s-wave rescattering amplitude observed in the three transitions. For transitions to the $^{13}\text{P}_0$ state, s-wave rescattering acts through the partial wave $\ell_\pi = 0$; whereas, the impulse approximation plus p-wave rescattering requires $\ell_\pi = 1$ and 3. Hence, s-wave rescattering dominates this transition, as manifested in table 1. For the other two transitions, however, s-wave rescattering acts through higher partial waves than can contribute to the impulse approximation plus p-wave rescattering amplitude. Consequently, the s-wave rescattering contribution to these transitions is negligible.

Of particular interest in table 1 is the sizeable contribution of the impulse approximation to the transition amplitudes as compared with the rescattering contributions; for the $^{13}\text{S}_1 - \text{D}_1$ and $^{13}\text{P}_2 - \text{F}_2$ transitions, in fact, the impulse approximation dominates. Annihilation effects in the initial atomic states play a crucial role here; without annihilation, the rescattering terms completely overwhelm the impulse approximation, as will be demonstrated explicitly later in this section.

The effect of short-range contributions to the $\text{NN} \rightarrow \text{N}\bar{\text{A}}$ transition potential of the type discussed following eq. (2.7) appears to be very small with annihilation included. We have, indeed, collected results for several values of the ratio of the effective $\rho\text{N}\bar{\text{A}}$ and ρNN couplings, α_ρ , between 1.2 and 2.0 and the pion form factor mass parameter, Λ_π , between 1.0 and 1.5 GeV, using the same wave functions as employed for the results in table 1. The calculated widths never differ from those of table 1 by more than 12 % for any choice of parameters within this physically reasonable range.

Clearly, short-range contributions to the $N\bar{N} \rightarrow N\bar{\Delta}$ transition potential do not significantly affect the transition widths for the present choice of wave functions; i.e.; with annihilation effects included in the initial states, the rescattering series converges rapidly, and a perturbative approach to the transition (1.1) is justified.

Annihilation effects also stabilize the widths with respect to variations in the bound state energies. This can be seen in table 2, where we exhibit the widths for transitions to the $^{13}P_0$ and $^{13}S_1-D_1$ states for several binding energies and the parameters $\alpha_\rho = 1.7$, $\Lambda_\pi = 1.2$ GeV. Most of the energy dependence manifested here is kinematic in origin. In particular, as the binding energy increases, the phase space available to the outgoing pion increases roughly as (neglecting the bound state recoil energy)

$$k^3 \sim (B^2 - m_\pi^2)^{3/2}, \quad (4.7)$$

where k is the c.m. momentum. Removing this factor k^3 from the widths in table 2 leaves an energy dependence that is quite weak.

When annihilation effects are left out, the results reported above are drastically altered. Table 3 illustrates this dramatically. Here we have collected widths for several values of the parameters α_ρ and Λ_π using initial state wave functions generated from an interaction without annihilation. With annihilation effects omitted, we found it necessary to introduce additional ρ -meson baryon form factors to regularize the high t behaviour of $f_-^1(t)$ in eq. (2.10), which is not accomplished in the dispersion theoretic calculations of ref. [8]. In particular, monopole forms analogous to eqs. (2.8) were employed with $m_\rho = 760$ MeV and Λ_ρ fixed at the relatively large value of 2 GeV

in order not to disturb the dispersion theoretic $f_{-}^1(t)$ at low t . Note that the widths appearing in table 3 considerably exceed those in table 2 and that this enhancement is associated almost exclusively with the rescattering terms. The impulse approximation widths are affected quite moderately by the exclusion of annihilation effects and are now small compared with the total widths. Clearly, short-range components in the wave functions, which are no longer suppressed by annihilation effects, make the difference here, since, as discussed previously, rescattering is effective only in conjunction with large momentum components, i.e., short-range components, in the wave functions. By contrast, the relatively long-ranged impulse approximation is not affected much by such short-ranged components.

The non-negligible parameter dependence manifested in table 3 provides further support for these conclusions. Such parameter dependence indicates significant contributions to the transition amplitudes from higher order processes, which are even shorter-ranged than the single rescattering processes, and which, through their importance, make both the convergence of the rescattering series and the validity of a non-relativistic approach rather uncertain in the absence of annihilation effects.

The restoration of short-range contributions to the transition amplitude through the omission of annihilation effects in the initial states influences the transition widths in two other respects worth noting. First, it introduces extra binding energy dependence into the widths in addition to the kinematic energy dependence displayed in table 2, leading sometimes to physically unacceptable values $\Gamma > 1$ keV. Second, it makes the widths sensitive to the strong interaction cut-off used to generate the atomic wave functions. This latter sensitivity is completely unphysical and underscores the importance of including annihilation effects in the initial states.

5. Summary and Conclusions

This investigation has been based on the so-called "nuclear" approach to the $N\bar{N}$ bound system. We have pointed out the significance of the $(N\bar{N})_{\text{atomic}} \rightarrow \pi^0 +$ baryonium process. It lies in the fact that, if strongly bound narrow $p\bar{p}$ states do exist, then pion emission processes of the type considered here should in fact be observable. Their transition widths, for deeply bound baryonium states, are predicted to be sizeable fractions of the total inelastic widths of 1S atomic $p\bar{p}$ states, estimated to be of the order of 1 keV [20].

A significant outcome of this study is the crucial role of annihilation in the initial $p\bar{p}$ atomic state. Once annihilation is included, the dependence on model parameters associated with short range reaction mechanisms in the $p\bar{p} \rightarrow B + \pi^0$ process becomes very weak. If annihilation is not included, the transition rates to deeply bound states are unreasonably large, and rescattering contributions as well as short-range cut-offs at meson baryon vertices become overwhelmingly dominant. Clearly, annihilation acts to suppress rescattering terms in comparison with impulse approximation processes. In a sense, the presence of annihilation justifies, a posteriori, a perturbative approach. In ref. [18], such a perturbative approach was adopted, but annihilation effects in the wave functions were not treated. In ref. [17], only the impulse approximation was included in the driving mechanism.

Inclusion of annihilation in the quasi-nuclear bound states will probably not alter these results qualitatively, as long as the widths are small enough that these states can be observed at all.

The description presented here will have to be confronted with further developments which should eventually permit a calculation of this same process within an alternative quark model framework. In such a model, the $p\bar{p} \rightarrow$ baryonium + π process might be interpreted as a dynamical recombination of three quarks

and three antiquarks into a $q\bar{q}$ pair and a narrow $qq\bar{q}\bar{q}$ state of the type discussed in ref. [4]. It is hoped that, if narrow deeply bound $N\bar{N}$ states really do exist, experimental measurements of the single pion production transition rates discussed here might help clarify the relationship between "potential" and quark model oriented approaches to baryon-baryon dynamics at short and intermediate distances.

NUCLEAR STATE	WIDTH (KEV)		
	I.A. ONLY	IA + OPE P-RESC.	IA + OPE P-RESC. + S-RESC.
3P_0 (179 MeV)	.0029	.0043	.0079
${}^3S_1 - D_1$ (182 MeV)	.0325	.0481	.0481
${}^3P_2 - F_2$ (185 MeV)	.0031	.0041	.0041

Table 1

NUCLEAR STATE	BINDING ENERGY (MEV)	WIDTH (KEV)
3P_0	179	.009
	189	.013
	349	.231
	571	.351
${}^3S_1 - D_1$	140	5.7 E-5
	182	.052
	271	.233
	463	.333

Table 2

NUCLEAR STATE	PARAMETERS		IMPULSE APPROX.	I.A. + p-resc.	I.A. + p + s-resc.
	α_ρ	Λ_π			
3P_0 (179 MeV)	1.7	1.0	.0016	.0031	.527
	↓	1.5		.0072	.385
	1.2	1.2		.0023	.403
	2.0	↓	↓	.0065	.409
${}^3S_1-D_1$ (182 MeV)	1.7	1.0	.024	.241	.241
	↓	1.5		.376	.376
	1.2	1.2		.258	.258
	2.0	↓	↓	.322	.322
${}^3P_2-F_2$ (185 MeV)	1.7	1.0	.00073	.0045	.0045
	↓	1.5		.0089	.0089
	1.2	1.2		.0037	.0037
	2.0	↓	↓	.0082	.0082

Table 3

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Figure Caption

Figure 1: Contributions to the transition amplitude with pion emission from the antibaryon line.

Table Captions

Table 1: Contributions to the transition widths using eq. (2.7) for the $N\bar{N} \rightarrow N\bar{\Delta}$ transition potential and including annihilation effects in the initial states. Each nuclear wave function was obtained with a value of r_0 chosen to give the binding energy in parentheses.

Table 2: Transition widths including annihilation effects in the initial states for the parameters $\alpha_\rho = 1.7$, $\Lambda_\pi = 1.2$ GeV, and various binding energies.

Table 3: Contributions to the transition widths (keV) for various values of the parameters α_ρ and Λ_π without annihilation effects in the initial states.

Figure 1