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To Jens Vigen <Jens.Vigen@cern.ch>

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Kindest regards

Carmen Vasini

Scattering Problems in Field Theory.

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It is well known that the expression for the solution of an S -matrix scattering problem: $S = \sum g^n S_n$ in terms of the powers of the coupling constant (perturbation series) cannot be applied for large values of g

It therefore seems interesting to investigate an approximation method in which the term $g^n S_n$ is replaced by

$$\sum_{\varrho=0}^n x_{\varrho}^{(n)} S_{n-\varrho},$$

where the constants $x_{\varrho}^{(n)}$ are functions of g . This may be done by a reformulation of Fredholm's method [1]. Let us consider the linear equation

$$(1) \quad U = U_0 + \lambda \mathcal{K} U,$$

where U_0 and $\mathcal{K}$ are known operators, U an unknown operator, and λ a c -number which is not an eigenvalue. Its solution can be obtained by means of the resolvent R

$$(2a) \quad U = (1 + \lambda R) U_0,$$

$$(2b) \quad R = \mathcal{K} + \lambda \mathcal{K} R$$

We write down the resolvent R as the ratio of two polynomials in the coupling

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constant g of degree N , plus a rest term

$$(3) \quad R = \frac{\sum_0^N [(-\lambda)^n/n!] \mathcal{D}_{n+1}}{\sum_0^N [(-\lambda)^n/n!] D_n} + R'$$

The $\mathcal{D}_n$ are operators, and the D_n c -numbers; the relations between $\mathcal{D}_n$ and D_n have to be determined by introducing (3) in the (2b). One then obtains the recurrence formulae

$$(4a) \quad \begin{cases} \mathcal{D}_1 = \mathcal{K} \\ \mathcal{D}_{N+1} = D_N \mathcal{K} - N \mathcal{K} \mathcal{D}_N, \end{cases}$$

where the D_n are still undetermined, and the equation for R' :

$$(4b) \quad R' = - \frac{(-\lambda)^{N+1}}{N!} \frac{\mathcal{K} \mathcal{D}_{N+1}}{\sum_0^N [(-\lambda)^n/n!] D_n} + \lambda \mathcal{K} R'$$

Let us now consider a matrix element of R between the states $|I\rangle$ and $\langle II|$. For a given N we have to choose the still undetermined c -numbers D_n such that $\langle III|R'|I\rangle$ is small enough in some sense.

One may prove that a convenient choice of D_n is obtained if they are chosen in such a way that the length of the vector (*)

$$\lambda^{N+1} \frac{\sum_{q=0}^N (-1)^q (D_q/q!) \mathcal{K}^{N+2-q}}{\sum_0^N [(-\lambda)^n/n!] D_n} |I\rangle,$$

is a minimum. This condition gives a system of N quadratical equations which determine $D_1 \dots D_N$ ($D_0 = 1$).

As a first simple application we may consider the scattering of particles the energies of which are below the threshold of production of new particles, and $N=2$. In this case the S matrix can be written as:

$$\langle II|S|I\rangle = \delta_{II} + \frac{II [g^2 S_2 + g^4 (S_4 + x_2 S_2)] I}{1 + g^2 x_2},$$

(*) This vector is obtained by applying the inhomogeneous term in (4b) to the vector $|I\rangle$.

with

$$x_2 = - \frac{\langle \mathbf{I} | S_4(S_2 - g^2 S_4) | \mathbf{I} \rangle}{\langle \mathbf{I} | S_2(S_2 - g^2 S_4) | \mathbf{I} \rangle},$$

Other applications, such as the application to Heitler's integral equation, to nuclear potentials and others, will be given in a more detailed paper to be published shortly

REFERENCE

- [1] A. VISCONTI: *Journ de Phys*, **16**, 1 (1955)