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STRUCTURE OF $N = 2$ SUPERGRAVITY

B. de Wit^{*)} and J.W. van Holten
CERN - Geneva

and

A. Van Proeyen^{**)}
Instituut voor Theoretische Natuurkunde
Kath. Universiteit, Leuven, België

A B S T R A C T

A number of $N = 2$ superconformal multiplets are presented. A corresponding calculus exists which can be exploited for the construction of superconformally invariant actions. Actions that are invariant under Poincaré and de Sitter supersymmetry are then obtained by imposing a suitable set of gauge conditions. It is shown that there exist two inequivalent off-shell formulations of $N = 2$ Poincaré and de Sitter supergravity.

^{*)} Permanent address: NIKHEF-H, P.O. Box 41882, 1009 DB, Amsterdam, The Netherlands.

^{**)} Aspirant N.F.W.O.; address after August 1 1981: Theory Division, CERN, 1211 Geneva 23, Switzerland.

1. INTRODUCTION

Supergravity offers a unique starting point for a unified description of elementary particles and their interactions. One of its main features is that gravity is an intrinsic part of the theory. It is in this sector that supergravity has been most successful, because the supersymmetry considerably improves the quantum mechanical properties of the theory¹⁾. Therefore, there is hope that supergravity could ultimately lead to a consistent quantum theory of gravity. On the other hand, the intrinsic mass scale of the theory is set by the Planck mass, which makes it difficult to understand its possible implications for the phenomenology of elementary particles at lower energies ($\leq 10^2$ GeV). The question of whether supergravity can lead to a viable unification scenario is of vital importance, and has already been the subject of several studies²⁾.

For the construction and subsequent study of a general variety of locally supersymmetric theories a multiplet calculus is indispensable³⁾. Such a calculus defines irreducible representations of a supersymmetry algebra and allows the construction of other ones by addition, multiplication and various other manipulations. Furthermore, it provides a procedure for obtaining invariant densities for some of these multiplets, whence the study of quantum divergences and spontaneous symmetry breaking becomes feasible. It is the purpose of this paper to discuss such a multiplet calculus for $N = 2$ supergravity and to give a variety of locally supersymmetric theories.

There exist two basic types of supersymmetry. One is based on the Poincaré (or de Sitter) group, the other on the conformal group of space-time transformations. Both lead to a corresponding supergravity theory. Though the Poincaré theory seems the more interesting from a phenomenological point of view, the superconformal theory is much easier to work with as a result of its larger symmetry structure. This imposes strong restrictions on the transformation rules of fields which are therefore simpler than in the corresponding Poincaré case. Also the Weyl multiplet, i.e., the underlying field representation of the superconformal gauge theory, contains more gauge fields and less matter (non-gauge) fields. However, because the Poincaré algebra is a subalgebra of the conformal algebra, there exists a close connection between these theories; in fact, the Poincaré results can be obtained from the conformal ones by imposing gauge conditions to break some of the conformal symmetries.

The above considerations motivated us to choose conformal supergravity as the basis for a multiplet calculus. This forms a crucial difference with the approach followed by Breitenlohner and Sohnius⁴⁾ who employ only part of the superconformal algebra. We have already presented the full gauge-field multiplets of $N = 2$ Weyl

and Poincaré supergravity elsewhere⁵⁾, as well as elements of a superconformal calculus⁶⁾⁻⁸⁾. This paper contains a completion of those results and describes how a variety of Poincaré and de Sitter supergravity theories can be obtained within the superconformal framework. For one particular version of Poincaré supergravity, our results are comparable to those obtained by Breitenlohner and Sohnius.

This paper is organized as follows. We introduce the "minimal field representation" in Section 2. This multiplet contains $32 + 32$ components and consists of the Weyl multiplet and a vector multiplet which generates the central charge. It allows the implementation of a gauge condition which breaks the superconformal symmetry to Poincaré supersymmetry combined with a local $SU(2)$ invariance. In Section 3 we present results on superconformal multiplets: the chiral, vector, linear and scalar multiplet, some of them with a non-Abelian gauge invariance or with a central charge. Furthermore, we discuss the tensor multiplet, which is a special case of the linear multiplet, and a new non-linear multiplet. The latter plays an important part in the construction of certain Poincaré supergravity actions. In Section 4 we summarize various relations between multiplets and give formulae for superconformal densities. In addition, we explain the use of compensating fields which are a crucial ingredient in the construction of actions invariant under local Poincaré supersymmetry. We give a number of applications in Section 5, such as the action for non-Abelian vector multiplets and two different off-shell versions of Poincaré and de Sitter supergravity. We present our conclusions in Section 6. We have collected some useful formulae in an Appendix.

2. THE MINIMAL FIELD REPRESENTATION

The field representation of $N = 2$ conformal supergravity consists of the gauge fields of the superconformal symmetries augmented by matter fields. The gauge fields are e_μ^a , ω_μ^{ab} , b_μ , f_μ^a , ψ_μ^i , A_μ , ψ_μ^i and ϕ_μ^i which correspond to general co-ordinate transformations, Lorentz transformations (M), dilations (D), special conformal boosts (K), chiral $SU(2)$ and $U(1)$ transformations and Q and S supersymmetry respectively. The matter fields which complete the multiplet are a Lorentz tensor, a spinor and a scalar field, T_{ab}^{-ij} , χ^i , D *). Some properties of these fields have been listed in Table 1. This multiplet can be understood from the graded $SU(2,2|2)$ algebra in the presence of certain constraints⁵⁾. These constraints express the gauge fields ω_μ^{ab} , f_μ^a and ϕ_μ^i

*) We use the chiral $SU(2)$ notation explained in Ref. 6). The subscripts C have been deleted for the fields χ^i and D .

in terms of the other fields^{*)}. The matter fields are then required for the preservation of a closed commutator algebra. Altogether, the Weyl multiplet of conformal supergravity consists of $24 + 24$ degrees of freedom.

It is advantageous to introduce an extra superconformal vector multiplet which has $8 + 8$ degrees of freedom. The reducible field configuration consisting of the Weyl multiplet and this vector multiplet is called the minimal field representation. It contains $32 + 32$ degrees of freedom, which is the minimal number needed for the description of superconformal multiplets with a central charge, as well as for obtaining a field representation of $N = 2$ Poincaré supergravity by means of a gauge choice. Before discussing this, we present the conformal supersymmetry transformations of the vector multiplet. Its components are denoted by a , ξ_i , S_{ij} and B_{μ}^{ij} where B_{μ}^{ij} is the gauge field of the central charge transformations. Several properties of these fields have been listed in Table 1. Their Q and S supersymmetry transformations are given by

$$\begin{aligned}\delta a &= \bar{\epsilon}^i \xi_i, \\ \delta \xi_i &= 2 \not{D} a \epsilon_i - 2 S_{ij} \epsilon^j - \frac{1}{2} \sigma_{ij} \epsilon^j + 2 a \eta_i, \\ \delta S_{ij} &= -\frac{1}{2} \bar{\epsilon}_{[i} \not{D} \xi_{j]} - \frac{1}{2} \epsilon_{ik} \epsilon_{jl} \bar{\epsilon}^{kl} \not{D} \xi^l, \end{aligned} \quad (2.1)$$

$$\begin{aligned}\delta t_{abij} &= -2 \bar{\epsilon}_{[i} \not{D} \sigma_{ab} \xi_{j]} + 2 \epsilon_{ij} \epsilon_{kl} \bar{\epsilon}^k \sigma_{ab} \not{D} \xi^l + 4 \bar{\eta}_{[i} \sigma_{ab} \xi_{j]}, \\ \delta B_{\mu}^{ij} &= -\frac{1}{2} \sqrt{2} \epsilon^{ij} (\epsilon_{kl} \bar{\epsilon}^k (\gamma_{\mu} \xi^l + 2 a^* \psi_{\mu}^l) + h.c.),\end{aligned}$$

where ϵ^i and η^i denote the parameters of Q and S supersymmetry respectively. The derivatives are covariant with respect to all superconformal transformations. We have introduced a modified field strength:

$$t_{abij}^{\pm} \epsilon^j + t_{ab}^{\pm ij} \epsilon_{ij} = 2\sqrt{2} \hat{F}_{ab}^{ij}(B) + (a T_{abij}^+ \epsilon^j + a^* T_{ab}^{-ij} \epsilon_{ij}), \quad (2.2)$$

where $\hat{F}(B)$ denotes the supercovariant field strength of B_{μ}^{ij} . Notice that the matter fields of superconformal gravity do not occur explicitly in the transformations (2.1).

The commutator algebra closes on the components of the Weyl and the vector multiplet separately^{5),6)}, and most commutators coincide with those of the $SU(2,2|2)$ algebra. The most important deviation is the commutator of two Q supersymmetry

^{*)} The parameter a that occurs in Eq. (2.14) of Ref. 6) has been chosen equal to one. The solutions for ω_{μ}^{ab} , f_{μ}^{μ} and ϕ_{μ}^1 have been listed in the Appendix.

transformations, which now yields a general co-ordinate transformation together with further field-dependent symmetry transformations. The precise form is

$$\begin{aligned} [\delta_Q(\epsilon_i), \delta_Q(\epsilon_j)] = & 2(\bar{\epsilon}_i^i \gamma^a \epsilon_{,i} + h.c.) D_a + \delta_M(\bar{\epsilon}_i^i T_{ij}^{+ab} \epsilon_j^j + h.c.) \\ & + \delta_K(\frac{3}{2} D \bar{\epsilon}_i^i \gamma^a \epsilon_{,i} + \bar{\epsilon}_i^i D_b T_{ba}^+ \epsilon_j^j + h.c.) \\ & + \delta_S(3 \bar{\epsilon}_i^i \epsilon_j^j \chi_j) + \delta_Z(2 \alpha^* \epsilon_{,j} \bar{\epsilon}_i^i \epsilon_j^j + h.c.) , \end{aligned} \quad (2.3)$$

where the supercovariant derivative indicates a covariant general co-ordinate transformation. The field dependent Lorentz, special conformal and S supersymmetry transformations depend on the matter components of the Weyl multiplet, whereas the central charge transformation originates from the vector multiplet. At this point, the central charge acts only on the gauge field B_{μ}^{ij} by means of a gauge transformation

$$\delta_Z B_{\mu}^{ij} = -\sqrt{2} \epsilon^{ij} \partial_{\mu} Z . \quad (2.4)$$

The discussion of a variety of multiplets which are representations of the commutator algebra (2.3) is one of the main subjects of this paper. The (field-dependent) central charge transformation on the right-hand side of (2.3) indicates the possibility of including multiplets with central charges. We have already demonstrated this in Ref. 7).

The presence of the vector multiplet in the minimal field representation allows one to pass to Poincaré supergravity by choosing a gauge. A suitable gauge condition is:

$$a=1 , \quad \xi_i=0 , \quad b_{\mu}=0 . \quad (2.5)$$

This gauge, which we shall refer to as the Poincaré gauge, breaks the invariance under local dilatations, chiral U(1), S supersymmetry and special conformal boosts. The only fermionic symmetry left is a linear combination of Q, S and K transformations. This is expressed by a uniform decomposition rule which defines the Poincaré supersymmetry transformations in terms of superconformal transformations:

$$\begin{aligned} \delta(\epsilon_i) = & \delta_Q(\epsilon_i) + \delta_S((S_{ij} + \frac{1}{4} \sigma \cdot t_{ij}) \epsilon^j - i \not{A} \epsilon_i) \\ & + \delta_K(\frac{1}{2} \bar{\epsilon}_i^i \gamma_a \chi_i + \frac{1}{2} (\hat{R}_{ai} - \frac{1}{3} \gamma_a \gamma \cdot \hat{R}_i) + h.c.) , \end{aligned} \quad (2.6)$$

where $\hat{R}_{ai}$ is the Rarita-Schwinger equation made covariant with respect to Poincaré supersymmetry. The field-dependent S component is required in order to maintain the second condition (2.5), and the K component in order to keep the third condition unaffected. This procedure is precisely analogous to the one used for $N = 1$ supergravity⁹⁾.

On the basis of Eqs (2.5) and (2.6) it is straightforward to obtain the super-Poincaré multiplets from the corresponding superconformal ones. One simply combines the field-dependent transformations according to (2.6) and substitutes the conditions (2.5). The above procedure guarantees that the remaining Poincaré supersymmetry transformations form a closed algebra. Furthermore, one may express derivatives that are covariant with respect to the superconformal transformations in terms of Poincaré supercovariant quantities. This is always possible since the super-Poincaré algebra is a subalgebra of the superconformal algebra. The gauge fields ω_{μ}^{ab} , f_{μ}^a and ϕ_{μ}^i are to be expressed through the corresponding curvature constraints in terms of Poincaré fields.

It is an obvious necessity in Poincaré supergravity to construct actions from multiplets of arbitrary Weyl weight. This can be done by making use of the vector submultiplet of the minimal field representation as a compensating field. This will be discussed in Section 4. The use of compensating fields together with suitable gauge conditions allows us to discuss the complex Poincaré and de Sitter supergravity theories entirely within the simpler superconformal framework.

3. SUPERCONFORMAL MULTIPLETS

Conformal supermultiplets are representations of the superconformal algebra, possibly augmented with extra internal gauge symmetries. The most conspicuous feature of this algebra is the commutator of two Q supersymmetry transformations given in Eq. (2.3). In this section we discuss five kinds of multiplets: chiral, vector, linear, scalar and non-linear multiplets. We exhibit the relationship between some of these multiplets and present various extensions.

3.1 Chiral Multiplets

The chiral multiplet is based on a chiral superfield¹⁰⁾. The components are denoted by A , Ψ_i , B_{ij} , F_{ab}^- , Λ_i , C . The superfield has an arbitrary Weyl and chiral weight factor expressed by a single real number w (see Table 2). This implies that a chiral superfield of positive chirality transforms under Weyl and chiral $U(1)$ transformations according to

$$\Phi(z^{\mu}, \theta^i) \rightarrow e^{w\zeta^*} \Phi(z^{\mu}, e^{-\frac{1}{2}\zeta} \theta^i) \quad , \quad (3.1)$$

where $\zeta = \zeta_W + i\zeta_A$ contains the transformation parameters of Weyl and chiral transformations. [For the notation we refer to Ref. 6).]

Chiral multiplets can be defined by the requirement that the field with the lowest Weyl weight is a complex scalar, transforming under supersymmetry into a doublet of chiral Majorana spinors. Chiral multiplets cannot have a real central charge such as the central charge present in the algebra (2.3). It is, however, possible to have chiral multiplets with a complex central charge. Under this charge the full multiplet $(A, \psi_i, \dots)$ transforms into another equivalent multiplet $(A^{(z)}, \psi_i^{(z)}, \dots)$ multiplied by the complex parameter z of the central charge transformation. The rigid supersymmetry transformations with the complex central charge are

$$\begin{aligned}
 \delta A &= \bar{\epsilon}^i \psi_i, \\
 \delta \psi_i &= 2 \not{\partial} A \epsilon_i + B_{ij} \epsilon^j + \sigma \cdot F^- \epsilon_{ij} \epsilon^j + \epsilon_{ij} A^{(z)} \epsilon^j, \\
 \delta B_{ij} &= \bar{\epsilon}_{(i} \not{\partial} \psi_{j)} - \bar{\epsilon}^k \Lambda_{(i} \epsilon_{j)k}, \\
 \delta F_{ab}^- &= \bar{\epsilon}_i \not{\partial} \sigma_{ab} \psi_j \epsilon^j + \bar{\epsilon}^i \sigma_{ab} \Lambda_i - \bar{\epsilon}^i \sigma_{ab} \psi_i^{(z)}, \\
 \delta \Lambda_i &= C \epsilon_{ij} \epsilon^j - \sigma \cdot F^- \not{\partial} \epsilon_i - \not{\partial} B_{ij} \epsilon^{jk} + B_{ij}^{(z)} \epsilon^j + \not{\partial} A^{(z)} \epsilon_i, \\
 \delta C &= -2 \epsilon^j \bar{\epsilon}_i \not{\partial} \Lambda_j + \bar{\epsilon}^i \Lambda_i^{(z)} + \bar{\epsilon}_i \not{\partial} \psi_j^{(z)} \epsilon^j.
 \end{aligned} \tag{3.2}$$

The corresponding commutator algebra is

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = 2 (\bar{\epsilon}_1^i \gamma^a \epsilon_{2i} + \text{h.c.}) \partial_a + \delta_z (2 \epsilon_{ij} \bar{\epsilon}_1^i \epsilon_2^j). \tag{3.3}$$

However, we disregard this option henceforth, and remain within the context of the algebra (2.3), which has a real central charge. It is also possible to consider chiral multiplets in some representation of non-Abelian gauge groups. This will be discussed in the next subsection.

The superconformal transformations for chiral multiplets have been given in Ref. 6). These transformations are only defined upto field redefinitions of the multiplet components. We have chosen transformations such that the multiplication rule for chiral multiplets coincides with that for rigid chiral superfields. More precisely, if we multiply two chiral multiplets with weight factors w_1 and w_2 according to the rules of rigid supersymmetry, then the components of this product transform among themselves as the components of a chiral multiplet with weight $w = w_1 + w_2$. The explicit multiplication rule has been given in

Ref. 6) and here we present an extension of that result, namely the components of a chiral superfield taken to some power n :

$$\begin{aligned}
 & [\Phi(A, \psi_i, B_{ij}, F_{ab}, \Lambda_i, C)]^n = \\
 & = \left\{ \begin{aligned}
 & A^n, \quad n A^{n-1} \psi_i, \\
 & n A^{n-1} B_{ij} - \frac{1}{4} n(n-1) A^{n-2} \bar{\psi}_i \psi_j, \\
 & n A^{n-1} F_{ab} - \frac{1}{4} n(n-1) A^{n-2} \epsilon^{ij} \bar{\psi}_i \sigma_{ab} \psi_j, \\
 & n A^{n-1} \Lambda_i - \frac{1}{2} n(n-1) A^{n-2} (\epsilon^{kl} B_{ik} \bar{\psi}_l + \sigma \cdot F \bar{\psi}_i) + \\
 & \quad + \frac{1}{12} n(n-1)(n-2) A^{n-3} \sigma_{ab} \bar{\psi}_i \epsilon^{kl} \bar{\psi}_k \sigma_{ab} \psi_l, \\
 & n A^{n-1} C - \frac{1}{4} n(n-1) A^{n-2} (\bar{\epsilon}^{ik} \epsilon^{jl} B_{ij} B_{kl} - 2 F_{ab}^2 + 4 \epsilon^{ij} \bar{\Lambda}_i \psi_j) \\
 & \quad + \frac{1}{4} n(n-1)(n-2) A^{n-3} (\epsilon^{ik} \epsilon^{jl} B_{ij} \bar{\psi}_k \psi_l - \epsilon^{ij} \bar{\psi}_i \sigma \cdot F \bar{\psi}_j) \\
 & \quad + \frac{1}{48} n(n-1)(n-2)(n-3) A^{n-4} (\epsilon^{ij} \bar{\psi}_i \sigma_{ab} \psi_j)^2
 \end{aligned} \right\}. \quad (3.4)
 \end{aligned}$$

For chiral multiplets one can define a "kinetic" multiplet, which is also a chiral multiplet. However, this is only possible for multiplets with $w = 0$. The kinetic chiral multiplet itself has weight $w = 2$ and starts with the components

$$\begin{aligned}
 & \mathcal{T} \Phi(A, \psi_i, B_{ij}, F_{ab}, \Lambda_i, C) = \\
 & \left\{ C^*, \epsilon_{ij} (-2 \not{D} \Lambda^j + 6 B^{jk} \chi^l \epsilon_{kl} - \frac{1}{2} \sigma \cdot T^{-jk} \not{D} \bar{\psi}^l \epsilon_{kl}, \dots \right\} \quad (3.5)
 \end{aligned}$$

The higher $-\theta$ components can be identified by considering successive supersymmetry transformations.

Chiral superfields with $w = 1$ may be subject to a set of reducibility conditions^{11), 6)}:

$$\begin{aligned}
 & B_{ij} \mp \epsilon_{ik} \epsilon_{jl} B^{kl} = 0, \\
 & \not{D} \bar{\psi}^i \mp \epsilon^{ij} \Lambda_j = 0, \\
 & D_b (F_{ab}^+ \mp F_{ab}^- \pm \frac{1}{4} A T_{ab}{}^{ij} \epsilon^{ij} - \frac{1}{4} A^* T_{ab}{}^{ij} \epsilon_{ij}) + \\
 & \quad + \frac{3}{4} (\mp \bar{\chi}_i \gamma_a \psi^i \epsilon^{ij} - \bar{\chi}^i \gamma_a \psi^j \epsilon_{ij}) = 0, \\
 & -2 \not{D}^c A^* - \frac{1}{4} F_{ab}^+ T_{ab}{}^{ij} \epsilon^{ij} - 3 \bar{\chi}_i \psi^i \mp C = 0. \quad (3.6)
 \end{aligned}$$

The two sign choices are related through a redefinition of components corresponding to $\Phi \rightarrow i\Phi$. The conditions (3.6) play a crucial role in exhibiting the relationship between the chiral multiplet and the vector and linear multiplet. In fact, they allow a straightforward derivation of most of the properties of the latter two multiplets from those of the chiral multiplet.

3.2 Vector Multiplets

The third condition of (3.6) has the structure of a Bianchi identity, suggesting that F_{ab} can be expressed as a field strength in terms of a vector potential. Indeed a chiral superfield subject to (3.6) represents precisely a vector multiplet in the Wess-Zumino gauge¹²⁾, with the gauge potential V_μ transforming under supersymmetry as

$$\delta V_\mu = \bar{\epsilon}_i \gamma_\mu \psi_j \epsilon^j + 2 \bar{\epsilon}_i \psi_{\mu j} A \epsilon^j + h.c. \quad (3.7)$$

Notice that our convention is to choose the upper signs in (3.6) for vector multiplets. The components $F_{ab}^\pm$ are the self-dual combinations of a modified field strength

$$F_{ab} = F_{ab}^+ + F_{ab}^- = \hat{F}_{ab}(V) - \frac{i}{4} (A T_{ab}^+ \epsilon^j \epsilon_j + A^* T_{ab}^- \epsilon^j \epsilon_j), \quad (3.8)$$

with $\hat{F}_{ab}(V)$ the supercovariant field strength of V_μ . The Abelian gauge transformation of V_μ occurs as a central charge in the commutator of the Q supersymmetry transformations. The result is

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \dots - 4 \partial_\mu (A^* \bar{\epsilon}_1^i \epsilon_2^j \epsilon_j + h.c.) \quad (3.9)$$

where the dots represent the terms already appearing in Eq. (2.3).

This defines the Abelian vector multiplet and its representation in terms of a chiral superfield. An example of such a multiplet is the vector multiplet contained in the minimal field representation. Its corresponding chiral multiplet, which we denote by F_ζ , has the components

$$\begin{aligned}
 A &= a, \\
 \psi_i &= \xi_i, \\
 B_{ij} &= -2 S_{ij}, \\
 F_{ab} &= -\frac{1}{4} t_{abij} \epsilon^{ij}, \\
 \Lambda_i &= -\epsilon_{ij} \not{D} \xi^j, \\
 C &= -2 \square^c a^* + \frac{1}{8} t_{ab}^{+ij} T_{abij}^+ - 3 \bar{\chi}_i \xi^i.
 \end{aligned} \tag{3.10}$$

The modified field strength t_{abij}^- was already defined in (2.1) in terms of the vector potential B_{ij}^u of the central charge transformations. The precise correspondence to (3.7) follows from

$$B_{\mu}^{ij} = -\frac{1}{2} \sqrt{2} V_{\mu} \epsilon^{ij}.$$

These results can be extended to the non-Abelian vector multiplet. The components of this multiplet are denoted by $W_{\mu}, X, \Omega_i, Y_{ij}$; some of their properties are listed in Table 2. All these fields take values in the Lie algebra of the corresponding gauge group. Therefore, gauge covariant derivatives are written as $(\partial_{\mu} - gW_{\mu})$, with g the coupling constant of the gauge group. The basis of the Lie algebra will always be chosen so that the components of W_{μ} are real. The other fields of the vector multiplet are complex Lie algebra valued quantities. Complex conjugation is defined by replacing the components of the Lie algebra valued quantities by their complex conjugate; complex conjugation is not implied on the generators of the Lie algebra. With this convention we have

$$\begin{aligned}
 W_{\mu}^* &= W_{\mu}, \\
 Y_{ij} &= \epsilon_{ik} \epsilon_{jl} Y^{kl},
 \end{aligned} \tag{3.11}$$

independent of the representation of the gauge group.

For the non-Abelian vector multiplet the central charge (3.9) is replaced by a non-Abelian gauge transformation:

$$\begin{aligned}
 [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] W_{\mu} &= \\
 &= \dots - 4 \left\{ \partial_{\mu} (X^* \bar{\epsilon}_i^j \epsilon_i^j \epsilon_j + X \bar{\epsilon}_{ii} \epsilon_{jj} \epsilon^{ij}) - g [W_{\mu}, (X^* \bar{\epsilon}_i^j \epsilon_i^j \epsilon_j + X \bar{\epsilon}_{ii} \epsilon_{jj} \epsilon^{ij})] \right\} \tag{3.12}
 \end{aligned}$$

The supersymmetry algebra with this non-Abelian transformation must now be implemented on all components of the vector multiplet. This has been done in Ref. 13) and the result is

$$\begin{aligned}\delta W_\mu &= \bar{\epsilon}_i \gamma_\mu \Omega_j \epsilon^j + \bar{\epsilon}^i \gamma_\mu \Omega_j \epsilon_j + 2 \bar{\epsilon}_i \psi_{\mu j} X \epsilon^j + 2 \bar{\epsilon}^i \psi_{\mu}^j X^* \epsilon_j, \\ \delta X &= \bar{\epsilon}^i \Omega_i, \\ \delta \Omega_i &= 2 \not{D} X \epsilon_i + \gamma_{ij} \epsilon^j + \sigma \cdot F(W) \epsilon_j \epsilon^j - 2g [X X^*] \epsilon_j \epsilon^j + 2 X \eta_i, \\ \delta \gamma_{ij} &= \bar{\epsilon}_{(i} \not{D} \Omega_{j)} + \epsilon_{ik} \epsilon_{jl} \bar{\epsilon}^k \not{D} \Omega^l + 2g \epsilon_{ik} (\bar{\epsilon}_{j)} [X, \Omega^k] - \bar{\epsilon}^k [X^*, \Omega_{j)}]),\end{aligned}\quad (3.13)$$

with $F_{ab}(W)$ the modified field strength:

$$F_{ab}(W) = F_{ab}^+(W) + F_{ab}^-(W) = \hat{F}_{ab}(W) - \frac{i}{4} (X T_{ab}^+ \epsilon^j + X^* T_{ab}^- \epsilon_j). \quad (3.14)$$

$\hat{F}(W)$ is the supercovariant field strength of W_μ . Note that the derivatives in (3.13) are now also covariant with respect to the non-Abelian gauge group.

The next step is to implement the superconformal algebra with the non-Abelian transformation (3.12) on general chiral multiplets in some non-trivial representation of the gauge group. However, to implement this algebra on the first component $\underline{A}$ of such a multiplet is not possible. This component transforms only with supersymmetry parameters of positive chirality, namely ϵ^i , whereas the gauge transformation in (3.12) contains terms proportional to $\bar{\epsilon}_{1i} \epsilon_{2j}$. Therefore, these terms cannot be generated in the commutator which, implies that they should cancel on the right-hand side of (3.12) as well. In other words, to implement the algebra on a chiral multiplet is possible only if

$$X \underline{A} = 0, \quad (3.15)$$

where $\underline{A}$ is the first component of the chiral multiplet in some representation of the group and X represents the Lie algebra valued fields of lowest Weyl weight of the vector multiplet in the representation that is appropriate for $\underline{A}$. Of course, the supersymmetry variations of the left-hand side of (3.15) should also vanish. However, both X and $\underline{A}$ are the lowest dimensional components of chiral supermultiplets, since the vector multiplet can again be extended to a reducible chiral multiplet. Hence it is the product of these two multiplets which must vanish:

$$F_{\Omega} \otimes \underline{\Phi} = 0 \quad , \quad (3.16)$$

where F_{Ω} is the Lie algebra valued chiral multiplet starting with X , and $\underline{\Phi}$ is the chiral multiplet starting with $\underline{A}$. Therefore in the presence of a non-Abelian gauge symmetry there exist only chiral multiplets which satisfy (3.16). This condition has two obvious solutions. The first one is that $\underline{\Phi}$ is in a singlet representation of the group, the second one that $\underline{\Phi}$ is proportional to F_{Ω} . For instance, if $\underline{\Phi}$ equals F_{Ω} , then (3.16) reads

$$[F_{\Omega}, F_{\Omega}] = 0 \quad , \quad (3.17)$$

which is trivially satisfied. Conditions such as (3.16) are related to the preservation of representations, and are usually discussed within the context of superspace [see Ref. 14) and references quoted therein].

We have implemented the supersymmetry algebra on a non-Abelian chiral multiplet $(\underline{A}, \underline{\psi}_i, \underline{B}_{ij}, \underline{F}_{ab}, \underline{\Lambda}_i, \underline{C})$ of arbitrary Weyl weight w , and subject to the condition (3.16). Again we have chosen the transformations such that the product rules of rigid supersymmetry remain valid. The resulting supersymmetry transformations are given by

$$\begin{aligned} \delta \underline{A} &= \bar{\epsilon}^i \underline{\psi}_i \\ \delta \underline{\psi}_i &= 2 \underline{\Phi} \underline{A} \epsilon_i + \underline{B}_{ij} \epsilon^j + \sigma \cdot \underline{F} \epsilon_j \epsilon^j + 2g X^* \underline{A} \epsilon_j \epsilon^j + 2w \underline{A} \eta_i , \\ \delta \underline{B}_{ij} &= \bar{\epsilon}_i \underline{\Phi} \underline{\psi}_j - \bar{\epsilon}^k \underline{\Lambda}_{(i} \epsilon_{j)k} - g \epsilon_{(ik} \bar{\epsilon}^k X^* \underline{\psi}_{j)} - 2g \epsilon_{(ik} \bar{\epsilon}_{j)} \Omega^k \underline{A} \\ &\quad + (1-w) \bar{\eta}_{(i} \underline{\psi}_{j)} \\ \delta \underline{F}_{ab} &= \epsilon^j \bar{\epsilon}_i \underline{\Phi} \sigma_{ab} \underline{\psi}_j + \bar{\epsilon}^i \sigma_{ab} \underline{\Lambda}_i - g \bar{\epsilon}^i \sigma_{ab} X^* \underline{\psi}_i - (1+w) \epsilon^j \bar{\eta}_i \sigma_{ab} \underline{\psi}_j \\ \delta \underline{\Lambda}_i &= -\sigma \cdot \underline{F} \underline{\Phi} \epsilon_i - \underline{\Phi} \underline{B}_{ij} \epsilon^{jk} \epsilon_k + \underline{C} \epsilon_j \epsilon^j \\ &\quad + \frac{1}{2} (\underline{\Phi} \underline{A} \sigma \cdot \underline{T}_{ij}^+ \epsilon^{jk} + w \underline{A} \underline{\Phi} \sigma \cdot \underline{T}_{ij}^+ \epsilon^{jk}) \epsilon_k - \frac{3}{2} \gamma_a \epsilon^{jk} \epsilon_k \bar{\chi}_{[i} \gamma^a \underline{\psi}_{j]} \\ &\quad + g X^* \underline{B}_{ij} \epsilon^j - g X^* \underline{F}_{ij} \sigma \epsilon_j \epsilon^j - 2g (\underline{\Phi} X^*) \underline{A} \epsilon_i \\ &\quad + g \gamma^a \Omega^j (\underline{\psi}_j \gamma_a \epsilon_i - \frac{1}{2} \underline{\psi}_i \gamma_a \epsilon_j) - (1+w) \underline{B}_{ij} \epsilon^{jk} \eta_k \\ &\quad + (1-w) \sigma \cdot \underline{F} \bar{\eta}_i - 2g X^* \underline{A} \eta_i , \end{aligned} \quad (3.18)$$

$$\begin{aligned}
 \delta \zeta = & -2\epsilon^{\dot{j}}\bar{\epsilon}_i \not{D}\Lambda_j - 6\bar{\epsilon}_i \chi_j \epsilon^{ik}\epsilon^{jl} B_{kl} \\
 & - \frac{1}{2}\epsilon^{\dot{j}}\epsilon^{kl} \left((w-1)\bar{\epsilon}_i \sigma_{jk} \not{D}\psi_l + \bar{\epsilon}_i \sigma_{jk} \not{D}\psi_l \right) \\
 & - g\epsilon^{\dot{j}}\bar{\epsilon}_i \sigma_{jk} \Omega^k A - 2g\epsilon^{\dot{j}}\bar{\epsilon}_i (\not{D}X^* \psi_j - \Omega^k B_{kj}) + 2g\bar{\epsilon}^i X^* \Lambda_i \\
 & + 2g^2 \bar{\epsilon}_i [X^*, \Omega^i] A + 2w\epsilon^{\dot{j}}\bar{\eta}_i \Lambda_j + 2g\epsilon^{\dot{j}}\bar{\eta}_i X^* \psi_j .
 \end{aligned} \tag{3.18}$$

cont.

Because of these modified transformation rules the chiral multiplet reducibility conditions (3.6) have extra terms. We have found the following result:

$$\begin{aligned}
 B_{ij} + \epsilon_{ik}\epsilon_{jl} B^{kl} &= 0 , \\
 \not{D}\psi^i + g\epsilon^{\dot{j}} X^* \psi_j - 2g\epsilon^{\dot{j}} \Omega_j A^* + \epsilon^{\dot{j}} \Lambda_j &= 0 , \\
 D_b(\bar{F}_{ab} + F_{ab} \pm \frac{1}{4} A T_{ab}^{\dot{j}} \epsilon^{\dot{j}} - \frac{1}{4} A^* T_{ab}^{\dot{j}} \epsilon_{\dot{j}}) \\
 - \frac{3}{4} (\bar{\chi}^i \gamma_a \psi^j \epsilon_{ij} + \bar{\chi}_i \gamma_a \psi_j \epsilon^{\dot{i}}) - g(\bar{\chi} X^* \vec{D}_a A + X \vec{D}_a A^*) \\
 - \frac{1}{2} g (\bar{\Omega}_i \gamma_a \psi^i + \bar{\Omega}^i \gamma_a \psi_i) &= 0 , \\
 -2\Box^c A^* - \frac{1}{4} F_{ab}^* T_{ab}^{\dot{j}} \epsilon^{\dot{j}} - 3\bar{\chi}_i \psi^i - g\epsilon_{ij} \bar{\Omega}^i \psi^j - 4g^2 [X^*, X] A^* \\
 + 2g^2 X^* X A + \zeta &= 0 .
 \end{aligned} \tag{3.19}$$

We recall that this reduction can only be carried out for chiral multiplets with $w = 1$. Furthermore Eqs (3.19) should be compatible with the gauge symmetry. This requires that the chiral multiplet is in a real representation of the gauge group (or in a representation which is equivalent to its own conjugate representation). The restrictions (3.19) can also be applied to the non-Abelian vector multiplet itself. They then identify the higher- θ components of the chiral multiplet F_Ω . Again we choose upper signs in (3.19), according to the convention defined previously. It is easy to verify that the Eqs (3.18) and (3.19) indeed imply (3.13). The third condition (3.19) is again a Bianchi identity whose solution is given by the modified field strength (3.14).

3.3 Linear Multiplets

The chiral multiplet constraints (3.19) are compatible with the superconformal transformations. In other words, the constraints themselves form a supermultiplet, the components of which will be denoted by $(\underline{L}_{ij}, \underline{\varphi}^i, \underline{G}, \underline{E}_a)$ (see Table 2). This multiplet can only be in real representations of the gauge group, as can be seen from the reality condition on $\underline{L}_{ij}$:

$$\underline{L}^{\dot{j}}_{\dot{i}} \equiv (\underline{L}_{ij})^* = \epsilon^{ik} \epsilon^{jl} \underline{L}_{kl} . \quad (3.20)$$

By this procedure we have obtained the non-Abelian version of the so-called linear multiplet⁴⁾. In the singlet representation it coincides with the tensor multiplet described in Ref. 5). A third version with a central charge will be discussed later in this section. By using (3.19) we can now construct a linear multiplet from any chiral superfield with $w = 1$, which is in an admissible group representation. The precise correspondence is [we now choose the lower signs in (3.19)]:

$$\begin{aligned} \underline{L}_{ij} &= \underline{B}_{ij} + \epsilon_{ik} \epsilon_{jl} \underline{B}^{kl} , \\ \underline{\varphi}^i &= \underline{\phi} \underline{\psi}^i + g \epsilon^{\dot{i}j} X^* \underline{\psi}_{\dot{j}} - 2g \epsilon^{\dot{i}j} \underline{\Omega}_{\dot{j}} A^* + \epsilon^{\dot{i}j} \underline{\Lambda}_{\dot{j}} , \\ \underline{G} &= \underline{C} - 2 \square^c \underline{A}^* - \frac{1}{4} \underline{F}_{ab}^+ T_{ab}^+ \epsilon^{\dot{i}j} - 3 \bar{\chi}_i \underline{\psi}^i - g \epsilon_{\dot{i}j} \underline{\Omega}^i \underline{\psi}^{\dot{j}} \\ &\quad - 4g^2 [X^*, X] \underline{A}^* + 2g^2 X^* X^* \underline{A} , \\ \underline{E}_a &= 2 \underline{D}_b (F_{ba}^+ - \frac{1}{4} A T_{ba}^+ \epsilon^{\dot{i}j} + h.c.) + \\ &\quad + (\frac{3}{2} \bar{\chi}^i \gamma_a \underline{\psi}^i \epsilon_{\dot{i}j} - g \bar{\Omega}^i \gamma_a \underline{\psi}_i + h.c.) + 2g (X^* \vec{D}_a \underline{A} + h.c.) . \end{aligned} \quad (3.21)$$

The vector $\underline{E}_a$ satisfies a supersymmetric constraint which follows either from an explicit calculation using (3.14 - 3.19), or from the requirement that the algebra closes on the components of this multiplet:

$$2 D_a \underline{E}_a - g \gamma^{\bar{j}j} \underline{L}_{\bar{j}j} = -2g (X \underline{G} + \bar{\Delta}_i \varphi_i) - 3 \bar{\varphi}^i \chi^j \epsilon_{ij} + h.c. \quad (3.22)$$

The transformation rules of the non-Abelian linear multiplet are as follows:

$$\begin{aligned} \delta \underline{L}_{\bar{j}j} &= \bar{\epsilon}_{(i} \varphi_{j)} + \epsilon_{ik} \epsilon_{jl} \bar{\epsilon}^{(k} \varphi^{l)}, \\ \delta \varphi^i &= \not{D} \underline{L}^{\bar{j}j} \epsilon_j + \not{D} \epsilon^{\bar{j}j} \epsilon_j - \underline{G} \epsilon^i + 2g X^* \underline{L}^{\bar{j}j} \epsilon_{jk} \epsilon^k + 2 \underline{L}^{\bar{j}j} \eta_j, \\ \delta \underline{G} &= -2 \bar{\epsilon}_i \not{D} \varphi^i - \bar{\epsilon}_i (6 \chi_j \underline{L}^{\bar{j}j} + \frac{1}{2} \sigma_{jk} T_{jk}^+ \varphi_l \epsilon^{\bar{j}j} \epsilon^{kl}) \\ &\quad - 2g X^* (\bar{\epsilon}^i \varphi^j \epsilon_{ij} - h.c.) - 2g \bar{\epsilon}_i \Delta^{\bar{j}j} \underline{L}^{ik} \epsilon_{jk} + 2 \bar{\eta}_i \varphi^i, \\ \delta \underline{E}_a &= 2 \bar{\epsilon}^i \sigma_{ab} D_b \varphi^j \epsilon_j + \frac{1}{4} \bar{\epsilon}^i \gamma_a (6 \epsilon_{ij} \chi_k \underline{L}^{jk} - \frac{1}{2} \sigma_{jk} T_{jk}^+ \epsilon^{jk} \varphi_k) \\ &\quad + 2g \bar{\epsilon}^i \gamma_a X^* \varphi_i + g \bar{\epsilon}^i \gamma_a \Delta^{\bar{j}j} \underline{L}_{\bar{j}j} + \frac{3}{2} \bar{\eta}^i \gamma_a \varphi^j \epsilon_{ij} + h.c. \end{aligned} \quad (3.23)$$

Using these transformation rules one can show that the condition (3.22) is supersymmetric.

The condition (3.22) can be solved when the linear multiplet is inert under the gauge transformations, (i.e., $g = 0$). This is already suggested by the representation (3.21) of $\underline{E}_a$, where $\underline{E}_a$ is almost a total derivative. Further inspection shows that $\underline{E}_a$ can be expressed in terms of a tensor gauge field $E_{\mu\nu}$, which transforms under supersymmetry according to

$$\delta E_{\mu\nu} = -2i \bar{\epsilon}^i \sigma_{\mu\nu} \varphi^j \epsilon_j - i \underline{L}_{\bar{j}j} \epsilon^{jk} \bar{\epsilon}^i \gamma_{[\mu} \psi_{\nu]k} + h.c. \quad (3.24)$$

The corresponding field strength is

$$E_a = \frac{i}{2} \epsilon^i \epsilon_a{}^{\nu\rho\sigma} D_\nu E_{\rho\sigma} \quad (3.25)$$

which is invariant under tensor gauge transformations

$$\delta E_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu \quad (3.26)$$

Hence the tensor multiplet is a special case of the linear multiplet. It was first found in Refs 5) and 15). If a tensor multiplet is constructed from a chiral multiplet, then its tensor gauge field has the following explicit representation

$$E_{\mu\nu} = 2i F_{\mu\nu}^+ + i \epsilon_{ij} \bar{\psi}_{[\mu}^i \gamma_{\nu]} \psi^j + 2i A^* \epsilon_{ij} \bar{\psi}_{\mu}^i \psi_{\nu}^j + \frac{i}{2} A^* T_{\mu\nu}^+ \epsilon_{ij} \quad (3.27)$$

+ h.c. .

The supersymmetry transformations of (3.27) coincide with (3.24) modulo a field dependent gauge transformation of type (3.26).

The Eqs (3.21) allow the construction of a linear multiplet from a vector multiplet by using a chiral multiplet as an intermediate step. In particular, this can be done for the vector multiplet of the minimal field representation. Modulo an over-all normalization the components are given by

$$\begin{aligned} L_{ij} &= S_{ij} , \\ \varphi^i &= -\frac{1}{2} \not{D} \xi^i , \\ G &= \square^c \alpha^* - \frac{1}{16} t_{ab}^{+ij} T_{abij}^+ + \frac{3}{2} \bar{\chi}_i \xi^i , \\ E_a &= \frac{1}{8} D_b (t_{ba}^{+ij} \epsilon_{ij} + \alpha T_{baij}^+ \epsilon^{ij}) - \frac{3}{8} \bar{\chi}^i \gamma_a \xi^j \epsilon_{ij} + h.c. . \end{aligned} \quad (3.28)$$

This is precisely the tensor submultiplet of $N = 2$ Poincaré supergravity. The correspondence with the results of Ref. 5) becomes more obvious after imposing the Poincaré gauge condition (2.5).

It is also possible to have linear multiplets with central charges⁴⁾. The superconformal transformations of such multiplets have been presented elsewhere⁷⁾. Here we note that those results are in direct correspondence with Eqs (3.21) and (3.22). Namely all terms of order g in (3.21) and (3.22) take the form of field dependent non-Abelian gauge transformations on various components of the linear multiplet. The field-dependent parameters are proportional to the fields of the non-Abelian vector multiplet. In the same way, the modifications in the transformation rules and constraints caused by the presence of a central charge take the form of field-dependent central charge transformations, with parameters proportional to the fields of the vector multiplet components of the minimal field representation. The precise correspondence follows from replacing gX , $g\Omega$, gY and gW_{μ} by $\frac{1}{2}a$, $\frac{1}{2}\xi$, $-S$ and $-\frac{1}{2}\sqrt{2} \epsilon_{ij} B_{\mu}^{ij}$ respectively.

3.4 The Scalar Multiplet

The superconformal algebra (2.3) exhibits a field-dependent central charge. This fact can be used to implement this algebra on multiplets with central charges, such as the $N = 2$ scalar multiplet¹⁶⁾. This multiplet consists of a spin zero field A^α_i and a Majorana spinor doublet ζ^α (see Table 2). The scalar fields are assigned to doublet representations of the chiral $SU(2)$ group with corresponding indices i, j, k , etc. In addition they transform under a rigid chiral $SU(2)$ group with indices α, β, γ , etc., which acts on the spinors as well. The field A^α_i also satisfies an invariant constraint

$$A_\alpha^i \equiv (A^\alpha_i)^* = \epsilon_{\alpha\beta} \epsilon^{ij} A^\beta_j. \quad (3.29)$$

The scalar multiplet has a central charge which generates a series of equivalent multiplets denoted by $(A^{(z)}, \zeta^{(z)}), (A^{(zz)}, \zeta^{(zz)}),$ etc. It turns out that implementation of the algebra (2.3) leads to simple supersymmetry transformations of the scalar multiplet

$$\begin{aligned} \delta A^\alpha_i &= 2 \bar{\zeta}^\alpha \epsilon_i + 2 \epsilon^{\alpha\beta} \epsilon_{ij} \bar{\zeta}_\beta \epsilon^j, \\ \delta \zeta^\alpha &= \not{D} A^\alpha_i \epsilon^i + \alpha A^{(z)\alpha}_i \epsilon^{ij} \epsilon_j + A^\alpha_i \eta^i. \end{aligned} \quad (3.30)$$

Note that derivatives are also covariant with respect to central-charge transformations. The implementation of the algebra proceeds from the Q variation of A^α_i as the defining property of a scalar multiplet. This leads to the transformations of ζ^α . Subsequently, one finds that the supersymmetry algebra closes if, and only if, the following condition is met:

$$\epsilon^{\alpha\beta} \not{D} \zeta_\beta + \frac{1}{2} A^{(z)\alpha}_i \xi^i + \alpha^* \zeta^{(z)\alpha} + \frac{1}{8} \sigma \cdot T^{ij} \epsilon^{ij} \zeta^\alpha - \frac{3}{2} \epsilon^{ij} \chi_i A^\alpha_j = 0 \quad (3.31)$$

This condition is not supersymmetric and thus implies a second constraint:

$$\begin{aligned} (\not{D}^2 + \frac{3}{2} D) A^\alpha_i + |\alpha|^2 A^{(zz)\alpha}_i - A^{(z)\alpha}_j S^{jk} \epsilon_{ik} + \epsilon^{\alpha\beta} \bar{\xi}_i \zeta^{(z)}_\beta \\ - \epsilon_{ij} \bar{\xi}^j \zeta^{(z)\alpha} = 0. \end{aligned} \quad (3.32)$$

These equations show that the scalar multiplet has $8 + 8$ degrees of freedom, represented by A_i^α , $A_i^{(z)\alpha}$ and ζ^α . All further components of the multiplet that can be generated by applying the central charge are not independent by virtue of (3.31) and (3.32) or of similar conditions obtained by repeated use of the central charge on (3.31) and (3.32).

Products of two scalar multiplets can give a linear multiplet. They can be found in two different ways, depending on the choice of a multiplication tensor $\eta_{\alpha\beta}$, which equals $\delta_{\alpha\beta}$ or $\epsilon_{\alpha\beta}$. The multiplication rules are

$$\begin{aligned} [L_{ij}]_\eta &= (A_i^\alpha A_j'^\beta + A_j^\alpha A_i'^\beta) \eta_{\alpha\beta}, \\ [\varphi^i]_\eta &= 2(A_\alpha^i \zeta_\beta' + \zeta_\alpha A_\beta'^i) \eta^{\alpha\beta}, \\ [G]_\eta &= (4\bar{\zeta}_\alpha \zeta_\beta' + a^* A_\alpha^i A_\beta'^{(z)j} \epsilon_{ij} - a^* A_\alpha^{(z)i} A_\beta'^j \epsilon_{ij}) \eta^{\alpha\beta}, \\ [E_a]_\eta &= A_i^\alpha \bar{D}_a A_j^\beta \epsilon^{ij} \eta_{\alpha\beta} - 2(\epsilon^{\alpha\gamma} \bar{\zeta}_\gamma \gamma_a \zeta^\beta \eta_{\alpha\beta} + h.c.). \end{aligned} \quad (3.33)$$

The antisymmetric product ($\eta_{\alpha\beta} = \epsilon_{\alpha\beta}$) preserves the rigid $SU(2)$ symmetry, whereas the symmetric product ($\eta_{\alpha\beta} = \delta_{\alpha\beta}$) leaves only an $SO(2)$ subgroup unaffected. Of course, the linear multiplet (3.33) has a central charge in this case. Furthermore one can verify that the product (3.33) satisfies the appropriate version of the constraint (3.22) by virtue of Eqs (3.31) and (3.22).

It is possible to assign the scalar multiplet to a representation of a non-Abelian gauge group, or to extend the rigid $SU(2)$ group to a local group. The construction of such extensions is similar to the one used for the derivation of the non-Abelian chiral multiplets in the previous section.

3.5 A Non-Linear Multiplet

All multiplets discussed in the previous subsections have transformation rules that are linear in the components of the multiplet itself. The non-linear part of these transformations is entirely caused by the presence of the fields of the minimal field representation given in Section 2. We will now present a multiplet which does not possess this property. The non-linear transformations are induced by the presence of a bosonic constraint of the type encountered in non-linear sigma models. We were motivated to construct this non-linear multiplet because it seems present in a formulation of $N = 2$ Poincaré supergravity with local $SU(2)$ symmetry¹³⁾. The multiplet contains Lorentz scalar fields ϕ_α^i ($\alpha = 1, 2$) of zero Weyl weight which are parametrized as an element of $SU(2)$. Hence the 2×2 matrix ϕ_α^i satisfies

$$\Phi^i_\alpha \Phi^\alpha_j = \delta^i_j \quad ; \quad \Phi^\alpha_i \Phi^i_\beta = \delta^\alpha_\beta \quad ; \quad \Phi^\alpha_i = \epsilon^{\alpha\beta} \epsilon_{ij} \Phi^j_\beta \quad , \quad (3.34)$$

where Φ^α_i is the Hermitean conjugate of Φ^i_α . Therefore Φ^i_α represents three spin zero degrees of freedom. Under local $SU(2)$, Φ transforms from the left acting on the index i . In addition, an independent rigid $SU(2)$ group acts from the right on the index α . Apart from the non-linear constraint this is precisely the same as for the scalar multiplet component A^α_i , discussed in the previous section. But we emphasize that the rigid $SU(2)$ symmetries of these two multiplets are independent at this point. Under supersymmetry, Φ transforms into a spinor doublet λ^i . The specific form of the transformation is determined by virtue of the constraints (3.34). The remaining multiplet components and their transformations follow then from a complete implementation of the superconformal algebra (2.3). These components are an antisymmetric complex Lorentz scalar field M^{ij} , and a real Lorentz vector V_a (see Table 2). The vector field turns out to have the noteworthy feature that it transforms under special conformal boosts (K). The full Q, S and K transformations of the multiplet read:

$$\begin{aligned} \delta \Phi^i_\alpha &= (2 \bar{\epsilon}^i \lambda_j - \delta^i_j \bar{\epsilon}^k \lambda_k - h.c.) \Phi^j_\alpha \quad , \\ \delta \lambda^i &= -\frac{1}{2} \not{V} \epsilon^i - \frac{1}{2} M^{ij} \epsilon_j - 2 \lambda^i (\bar{\lambda}^j \epsilon_j + \bar{\lambda}_j \epsilon^j) + \gamma_a \epsilon^i (\bar{\lambda}^j \gamma_a \lambda_j) \\ &\quad + 2 \sigma_{ab} \epsilon_j \bar{\lambda}^j \sigma_{ab} \lambda^i + \Phi^i_\alpha \not{\Phi} \Phi^\alpha_j \epsilon^j + \eta^i \quad , \\ \delta M^{ij} &= 6 \bar{\epsilon}^{[i} \chi^{j]} + \bar{\epsilon}^k \sigma \cdot T^{ij} \lambda_k - 2 \bar{\epsilon}^{[i} \not{V} \lambda^{j]} - 2 \bar{\epsilon}^k \lambda_k M^{ij} \\ &\quad + 4 \bar{\epsilon}^{[i} (\not{\Phi} \lambda^{j]} + \Phi^{j]}_\alpha \not{\Phi} \Phi^\alpha_k \lambda^k) \quad , \\ \delta V_a &= \left\{ \frac{3}{2} \bar{\epsilon}^i \gamma_a \chi_i - \frac{1}{4} \bar{\epsilon}^i \gamma_a \sigma \cdot T^{ij} \lambda_j - \bar{\epsilon}^i \gamma_a \not{V} \lambda_i + \bar{\epsilon}^i \gamma_a \lambda^j M_{ij} \right. \\ &\quad \left. + 4 \bar{\epsilon}^i \sigma_{ab} D_b \lambda_i - 2 \bar{\epsilon}_i \gamma_a \Phi^i_\alpha \not{\Phi} \Phi^\alpha_j \lambda^j - \bar{\lambda}_i \gamma_a \eta^i + h.c. \right\} \\ &\quad + 2 \Lambda_{K a} \quad . \end{aligned} \quad (3.35)$$

Notice that these transformations indeed exhibit the non-linear behaviour mentioned above. The supersymmetry algebra closes if, and only if, we have a supersymmetric constraint

$$D_a V_a - 3D - \frac{1}{2} V_a^2 - \frac{1}{4} |M_{ij}|^2 + D_a \Phi^i_\alpha D_a \Phi^\alpha_i + 2(\bar{\lambda}_i (\not{D} \lambda)^i + \frac{3}{2} \chi^i - \frac{1}{4} \sigma \cdot T^{\dot{i}j} \lambda_j) + h.c.) = 0. \quad (3.36)$$

Because this multiplet has no central charge neither (3.35) nor (3.36) depend on the components of the vector submultiplet of the minimal field representation.

For certain applications which we will discuss in Section 5, it is important to have another version of this multiplet at one's disposal, in which the extra SU(2) group (or a subgroup) is extended to a local invariance. This requires the introduction of a non-Abelian vector multiplet for SU(2), of the type discussed in Section 3. The fields of this multiplet $(W_\mu, X, \Omega_i, Y_{ij})$ take values in the Lie algebra of SU(2). Therefore, they are anti-Hermitian traceless 2×2 matrices with indices α, β . Hence we write these fields as $W_\mu^\alpha_\beta, X^\alpha_\beta, \Omega_i^\alpha_\beta$ and $Y_{ij}^\alpha_\beta$, which satisfy relations such as

$$(X^\alpha_\beta)^* = -X^{*\alpha}_\beta, \quad (3.37)$$

$$\epsilon_{\alpha\gamma} \epsilon^{\beta\delta} X^\gamma_\delta = -X^\beta_\alpha.$$

Notice that this notation is consistent with the one used in subsection 3.2. The non-Abelian gauge field will now be contained in the covariant derivatives. Clearly only derivatives of Φ require such a modification; hence for $\partial_\mu \Phi^\alpha_i$ we make the substitution

$$\partial_\mu \Phi^\alpha_i \rightarrow \partial_\mu \Phi^\alpha_i - g W_\mu^\alpha_\beta \Phi^\beta_i, \quad (3.38)$$

with g the SU(2) gauge coupling constant.

We now implement the superconformal algebra on the non-linear multiplet, including the extra local SU(2) transformation given by (3.12). This leads to modifications in the transformation rules (3.35) which have the same structure as those appearing in the non-Abelian version of the chiral and linear multiplets [cf. (3.18) and (3.23)]. Apart from the implicit modifications contained in the covariant derivatives on Φ , the extra terms have the form

$$\begin{aligned}
 \delta_g \lambda^i &= 2g X^\alpha{}_\beta \Phi^i{}_\alpha \Phi^\beta{}_j \epsilon^{jk} \epsilon_k \ , \\
 \delta_g M^{\bar{j}} &= -4g \epsilon^{\bar{j}} \bar{\epsilon}^L (\Omega_k + 2\lambda_k X)^\alpha{}_\beta \Phi^k{}_\alpha \Phi^\beta{}_L \ , \\
 \delta_g V_a &= 2g \epsilon^{\bar{j}} \bar{\epsilon}_i \gamma_a (\Omega_k + 2\lambda_k X)^\alpha{}_\beta \Phi^k{}_\alpha \Phi^\beta{}_j + h.c. \ .
 \end{aligned} \tag{3.39}$$

The presence of the components of the $SU(2)$ vector multiplet also induces extra terms in the constraint (3.36). Instead of (3.36) we have

$$\begin{aligned}
 D_a V_a - 3D - \frac{1}{2} V_a^2 - \frac{1}{4} |M^{\bar{j}}|^2 + D_a \Phi^i{}_\alpha D_a \Phi^\alpha{}_i + g \epsilon^{\bar{j}} \chi_{kj}^\alpha \Phi^k{}_\alpha \Phi^\beta{}_i \\
 - 4g^2 X^{\alpha}{}_\beta X^\beta{}_\alpha + 2 \left(\bar{\lambda}_i (\not{D} \lambda^i + \frac{3}{2} \chi^i - \frac{1}{4} \sigma \cdot T^{\bar{j}} \lambda_j) \right. \\
 \left. - 2g \Phi^i{}_\alpha \Phi^\beta{}_j \epsilon^{jk} (\Omega_k + \lambda_k X)^\alpha{}_\beta \right) + h.c. = 0 \ .
 \end{aligned} \tag{3.40}$$

4. MULTIPLET CALCULUS

In the previous section we have obtained a variety of superconformal multiplets. Our next task is to discuss certain operations for these multiplets, such as multiplications and reductions and to define supersymmetric densities.

By taking various products of these supermultiplets one can find a large number of other supermultiplets. Most of them have not been considered before and are not of practical importance. The multiplications that are most relevant have already been given. Namely the product of two chiral multiplets of the same chirality defines another chiral multiplet with its Weyl weight equal to the sum of the separate Weyl weights

$$C_{w_1} \otimes C_{w_2} = C_{w_1 + w_2} \ . \tag{4.1}$$

The second multiplication, defined by Eqs (3.33), expresses a linear multiplet in terms of a product of two scalar multiplets

$$S_1 \otimes S_2 = L_\eta \ . \tag{4.2}$$

We have two ways of performing such a multiplication, specified by a tensor $\eta_{\alpha\beta}$ which equals either $\epsilon_{\alpha\beta}$ or $\delta_{\alpha\beta}$.

Besides multiplications we have other manipulations that allow the construction of one kind of multiplet from another. For instance, from a vector multiplet one can construct a chiral multiplet with $w = 1$, whereas a chiral multiplet with $w = 1$ allows the construction of a linear multiplet,

$$V \rightarrow C_{w=1} \rightarrow L(C). \quad (4.3)$$

This has been explained in Section 3. Furthermore, it is possible to construct a "kinetic" chiral multiplet from a multiplet with $w = 0$, as has been indicated in (3.5):

$$C_{w=0} \rightarrow T(C). \quad (4.4)$$

However, Eqs (4.3) and (4.4) are only relevant for chiral multiplets with specific Weyl weight, $w = 1$ or 0 respectively. A similar crucial dependence on the Weyl weight factor will be found later in this section, when we discuss superconformal density formulae. In view of possible applications of such results aimed at a construction of Poincaré and de Sitter supergravity theories, it is desirable to find a mechanism which enables one to compensate for discrepancies in Weyl weight factors. Such a mechanism is provided by the use of the vector submultiplet of the minimal field representation as a "compensating" multiplet. For example, in the calculus of chiral superfields one introduces the chiral multiplet F_ξ , which represents the covariant components of the vector multiplet [see Eq. (3.10)]. Since F_ξ has $w = 1$, we can change the Weyl weight of any chiral multiplet by multiplying by the appropriate power of F_ξ . If we choose F_ξ subject to the Poincaré gauge condition (2.3), its first component equals 1. Therefore, multiplication by powers of F_ξ amounts to merely adding non-linear modifications to the original components of the chiral multiplet. Powers of F_ξ can be determined from Eq. (3.4). Hence the constructions described by Eqs (4.3) and (4.4) can now be defined for chiral multiplets with arbitrary weight w :

$$\begin{aligned} C_w &\rightarrow L(F_\xi^{1-w} C_w), \\ C_w &\rightarrow T(F_\xi^{-w} C_w). \end{aligned} \quad (4.5)$$

The compensating fields also play an important role in the construction of supersymmetric densities. Such a density can be constructed for chiral supermultiplets with $w = 2$. The explicit density formula has been presented in Ref. 6). To construct a density from a general chiral multiplet with arbitrary weight w , one first multiplies by F_{ξ}^{2-w} and then applies the density formula to this product. A second density formula can be constructed for linear multiplets. To obtain this result one first calculates the product of a reduced chiral multiplet representing a vector multiplet with a general chiral superfield of weight $w = 1$. This product is a chiral multiplet with $w = 2$ from which we can construct a superconformal density. However, since the vector multiplet corresponds to a reduced chiral multiplet, one expects that it will only couple to a restricted set of components of the general chiral multiplet which must correspond to a linear multiplet. This observation can then be used to construct the density for the product of a vector and a linear multiplet⁷⁾. If one uses the vector submultiplet of the minimal field representation the result reads:

$$\begin{aligned} e^{-1} \mathcal{L} = & a G + \left(\frac{1}{2} S^{\dot{j}} - \frac{1}{2} \bar{\psi}^{\dot{i}} \gamma \psi^{\dot{j}} - a^* \bar{\psi}_{\mu}^{\dot{i}} \sigma^{\mu\nu} \psi_{\nu}^{\dot{j}} \right) L_{\dot{j}} \\ & + \bar{\varphi}^{\dot{i}} (\xi_{\dot{i}} + a \gamma \cdot \psi_{\dot{i}}) - \frac{1}{4} \sqrt{2} \epsilon_{\dot{i}\dot{j}} B_{\dot{a}}^{\dot{i}} (E_{\dot{a}} + 2 \bar{\varphi}^{\dot{i}} \sigma_{\dot{a}\dot{b}} \psi_{\dot{b}}^{\dot{j}} \epsilon_{\dot{i}\dot{j}} \\ & - \frac{1}{2} \epsilon_{\dot{a}\dot{b}\dot{c}\dot{d}} \bar{\psi}_{\dot{b}\dot{i}} \gamma_{\dot{c}} \psi_{\dot{d}}^{\dot{j}} L_{\dot{j}\dot{k}} \epsilon^{\dot{k}\dot{i}}) + h.c. \end{aligned} \quad (4.6)$$

In this construction, the vector multiplet of the minimal field representation clearly exhibits its dual role. On the one hand it acts as the compensating multiplet for the Weyl weight, such as to enable the construction of the invariant action (4.6). On the other hand, it also acts as the gauge multiplet of central charge transformations. One may easily convince oneself that (4.6) is invariant for both linear multiplets with or without a central charge. The density formula (4.6) can be applied to the product of two scalar multiplets defined in Eq. (4.2). The actions which result from this have been discussed in Ref. 7). Equation (4.6) is not applicable for non-Abelian vector and linear multiplets.

5. APPLICATIONS

The preceding sections contain all the necessary material for the construction of a large variety of locally supersymmetric actions. Initially, all these actions will be invariant under superconformal symmetries, but Poincaré and de Sitter supergravity theories can be described as well, after choosing the Poincaré gauge (2.3), possibly supplemented by further gauge conditions for internal symmetries. We now give a systematic discussion of several of these theories.

5.1 Supersymmetric Yang-Mills Theories

To construct the Yang-Mills action is rather straightforward. The non-Abelian vector multiplet can be represented as a reduced chiral multiplet, F_Ω , with $w = 1$. Its explicit form follows from the reducibility conditions (3.19). Taking the square of this multiplet we obtain a chiral multiplet with $w = 2$, from which we construct a superconformal density. The result can be compared with the action of the Abelian theory⁶⁾, and with the results of Ref. 17).

$$\begin{aligned}
 e^{-1} \mathcal{L}_{YM} = \frac{1}{2} \text{Tr} \Big\{ & X \hat{\Delta}^c X^* - \frac{1}{4} (F_{ab}^-(W))^2 - \frac{1}{2} \bar{\Omega}_i \hat{\Delta}^i \Omega^i + \frac{1}{8} |Y_{ij}|^2 \\
 & + \frac{1}{8} X F_{ab}^+(W) T_{abij} \epsilon^{ij} + \frac{3}{2} X \bar{\chi}_i \Omega^i - g \epsilon^{ij} \bar{\Omega}_i [X^*, \Omega_j] \\
 & - g^2 [X, X^*]^2 + \frac{1}{64} X^2 (T_{abij} \epsilon^{ij})^2 + \\
 & + \frac{1}{2} \bar{\psi}_i \gamma (X \hat{\Delta}^i \Omega^i - \frac{1}{2} \epsilon^{ij} \sigma_{ij} F^-(W) \Omega_j) + \\
 & + \frac{1}{2} g \epsilon^{ij} \bar{\psi}_i \gamma [X, X^*] \Omega_j + \frac{1}{8} X \bar{\psi}_{\mu i} \sigma_{jk} \gamma^\mu \Omega_i \epsilon^{jkl} \\
 & - \frac{1}{4} \bar{\psi}_{\mu i} \sigma^{\mu\nu} \psi_{\nu j} \epsilon^{ik} \epsilon^{jl} \bar{\Omega}_k \Omega_l \\
 & - \frac{1}{4} \epsilon^{ij} \bar{\psi}_{ai} \psi_{bj} \left(2 X F_{ab}^-(W) - \frac{1}{2} X^2 T_{abkl} \epsilon^{kl} - \frac{1}{2} \epsilon^{kl} \bar{\Omega}_k \sigma_{ab} \Omega_l \right) \\
 & + \frac{1}{8} \epsilon^{ij} \epsilon^{kl} e^{-1} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_{\mu i} \psi_{\nu j} \left(2 \bar{\psi}_{\rho k} \gamma_\sigma \Omega_l X + \bar{\psi}_{\rho k} \psi_{\sigma l} X^2 \right) \Big\} \\
 & + h.c.
 \end{aligned} \tag{5.1}$$

We have introduced the notation $\hat{\Delta}_a$ to indicate supercovariant derivatives from which terms containing the fields Y_{ij} have been deleted. But apart from that, the derivatives are fully covariant with respect to all relevant symmetries. In particular the superconformal d'Alembertian contains the trace of the gauge field of conformal boosts f_μ^a . This field is not independent and has terms proportional to the Ricci tensor and the conformal matter field D . The explicit form of f_μ^μ has been given in the Appendix. All the bosonic terms of the Lagrangian (5.1) can be written as follows

$$\begin{aligned}
 (e^{-1} \mathcal{L}_{YM})^{\text{bosonic}} = & \text{Tr} \left\{ -\mathcal{D}_\mu X \mathcal{D}_\mu X^* + \frac{1}{4} |Y_{ij}|^2 \right. \\
 & - \frac{1}{8} \left(F_{ab}^-(W)^2 - \frac{1}{2} F_{ab}^+(W) T_{abij}^+ \epsilon^{ij} - \frac{1}{16} (X T_{abij}^+ \epsilon^{ij})^2 + h.c. \right) \\
 & \left. + f_\mu^\mu X X^* - 2g^2 [X, X^*]^2 \right\} . \quad (5.2)
 \end{aligned}$$

The derivatives $\mathcal{D}_\mu$ are covariant with respect to all linearly realized bosonic symmetries.

5.2 The Minimal Field Representation

Using (5.1) and (5.2) it is straightforward to find the invariant action of the vector submultiplet of the minimal field representation. We only give the main terms needed for subsequent discussion

$$\begin{aligned}
 e^{-1} \mathcal{L}_{min} = & |\partial_\mu a|^2 + (D - \frac{1}{6} R + A_a^2) |a|^2 - \frac{1}{2} |S_{ij}|^2 \\
 & - \frac{1}{64} \left\{ \frac{1}{2} (a T_{abij}^+ \epsilon^{ij} - t_{ab}^+{}^{ij} \epsilon_{ij})^2 - (t_{ab}^+{}^{ij} \epsilon_{ij})^2 + h.c. \right\} + \dots \quad (5.3)
 \end{aligned}$$

Notice that this Lagrangian can be viewed as the density of the unit chiral multiplet. This is a chiral multiplet with only the first component non-vanishing, and equal to 1. Because this multiplet has $w = 0$, we must multiply by the square of the compensating chiral multiplet F_ξ to obtain a multiplet with $w = 2$. The invariant density for this multiplet will coincide with (5.3) by construction.

If we now impose the Poincaré gauge (2.3) one notices that (5.3) resembles the Poincaré supergravity Lagrangian^{18),5)}. We note the presence of the Riemann curvature scalar, of the squares of the auxiliary fields A_a , $t_{ab}^+{}^{ij}$ and S_{ij} and of the field strength of B_μ^{ij} , represented by a linear combination of $t_{ab}^+{}^{ij}$ and T_{abij}^+ [cf. Eq. (2.2)]. However, on closer inspection one finds that the relative coefficients of these terms are not in agreement with those appearing in the supergravity Lagrangian. In fact the Lagrangian (5.3) is rather meaningless because of a term linear in the field D . There is also a mismatch in the number of degrees of freedom. The Poincaré supergravity Lagrangian is known to contain at least $40 + 40$ degrees of freedom, whereas (5.3) is only based on

the $32 + 32$ components of the minimal field representation. In the subsequent subsections we will see how this problem can be resolved in two different ways. The first possibility is to use a non-linear multiplet. Theories of that type will be discussed in Sections 5.3 - 5.5. The second option, explained in subsection 5.6, is to introduce a scalar multiplet.

5.3 The Poincaré Supergravity Action

To extend (5.3) to the Poincaré supergravity Lagrangian we use the non-linear multiplet presented in Section 3. This multiplet has a constraint (3.36) which contains a term linear in the field D . Since the constraint is supersymmetric we can use (3.36) as the definition of D in terms of the components of the non-linear multiplet Φ^i_α , λ^i , M^{ij} and V_a . The vector V_a is then unconstrained, and the net effect is, that we have added $8 + 8$ degrees of freedom. In this way we can eliminate the troublesome term in (5.3) linear in D . The Poincaré Lagrangian is then obtained by taking the following linear combination

$$\begin{aligned} \mathcal{L}_{\text{Poincaré}} = \mathcal{L}_{\text{min.}} + e|a|^2 \left(D_a V_a - 3D - \frac{1}{2} V_a^2 - \frac{1}{4} |M_{ij}|^2 + D_\alpha \Phi^i_\alpha D_a \Phi^{\alpha}_i \right) \\ + 2e|a|^2 \left(\bar{\lambda}_i (\not{D} \lambda^i + \frac{3}{2} \chi^i - \frac{1}{4} \sigma \cdot T^{ij} \lambda_j) + h.c. \right). \end{aligned} \quad (5.4)$$

This linear combination has been chosen so that all terms $|a|^2 D$ cancel. To see this one should realize that V_a transforms under K transformations [see (3.35)], so that its divergence contains the gauge fields of conformal boosts f^a_μ . This leads to

$$D_a V_a = \partial \cdot V - 2 f^r_\mu + \dots = \partial \cdot V + 2D - \frac{1}{3} R + \dots \quad (5.5)$$

If we now impose the Poincaré gauge (2.3) on the Lagrangian (5.4), then we obtain a $SU(2)$ invariant formulation of Poincaré supergravity which coincides with the one presented in Ref. 13) modulo a total derivative $\partial_\mu (e V^\mu)$.

The local $SU(2)$ invariance of (5.4) can be broken by an additional gauge choice

$$\Phi^\alpha_i = \delta^\alpha_i. \quad (5.6)$$

Upon this condition one regains the conventional off-shell formulation of $N = 2$ Poincaré supergravity with rigid $SU(2)$ invariance^{18),4),5)}. Because the non-linear multiplet vanishes under the central charge, this charge is only realized on the vector field B_μ^{ij} by means of (2.4). This is a characteristic property of this version of $N = 2$ supergravity. An important effect of the gauge condition (5.6) is that it changes the decomposition rule of Poincaré supersymmetry (2.6). Namely to maintain (5.6) requires combining the supersymmetry variations of ϕ_α^i with a chiral $SU(2)$ transformation which depends on the spinor fields λ^i . This explains why Poincaré supersymmetry contains chiral $SU(2)$ as an extra component as has been observed previously⁵⁾.

The term $D_a \phi_\alpha^i D_a \phi_i^\alpha$ in the Lagrangian is reminiscent of one class of supersymmetric sigma models proposed by Curtwright and Freedman¹⁹⁾. These authors consider a number of $N = 2$ scalar multiplets subject to a bosonic constraint. In the case of one scalar multiplet this constraint corresponds precisely to the constraint of the non-linear multiplet (3.34). It turns out that this type of model cannot be formulated in flat space which led to the conjecture that a coupling to $N = 2$ supergravity was required for consistency. Surprisingly enough, the Lagrangian (5.4) of the $N = 2$ Poincaré theory itself can be viewed as the simplest version of this class of non-linear sigma models.

5.4 Poincaré Supergravity with Local $SO(2)$

The vector multiplet of the minimal field representation can be considered equally well as a vector multiplet of $SO(2)$. The $SO(2)$ gauge group could then act on the components ϕ_α^i of the non-linear multiplet. We have given an extension of the non-linear multiplet to local $SU(2)$ in Section 3. To restrict the gauge group to $SO(2)$ is straightforward. Namely, we substitute for the components of the $SU(2)$ vector multiplet

$$\begin{aligned} X_\beta^\alpha &= \frac{1}{2} \sqrt{2} \epsilon_{\alpha\beta} a, \\ \Omega_i{}^\alpha{}_\beta &= \frac{1}{2} \sqrt{2} \epsilon_{\alpha\beta} \xi_i, \\ Y_{ij}{}^\alpha{}_\beta &= -\sqrt{2} \epsilon_{\alpha\beta} S_{ij}, \\ W_\mu{}^\alpha{}_\beta &= -\epsilon_{\alpha\beta} \epsilon_{ij} B_\mu^{ij}. \end{aligned} \tag{5.7}$$

We then go through the same procedure as in the previous subsection, and substitute in the vector multiplet Lagrangian (5.3) the expression for D as provided by the constraint of the non-linear multiplet, but now in the presence of a $SO(2)$ gauge group. Combining Eqs (3.40) and (5.7) leads to the following Lagrangian, which is a direct extension of (5.4):

$$\begin{aligned}
 \mathcal{L}_{SO(2)} = & \mathcal{L}_{min} + e|a|^2 (D_a V_a - 3D - \frac{1}{2} V_a^2 - \frac{1}{4} |M^{\dot{ij}}|^2 + D_a \Phi^{\dot{i}}_{\alpha} D_a \Phi^{\alpha}_{\dot{i}}) \\
 & + 2e|a|^2 \left(\bar{\lambda}_{\dot{i}} (\not{D} \lambda^{\dot{i}} + \frac{3}{2} \chi^{\dot{i}} - \frac{1}{4} \sigma \cdot T^{\dot{ij}} \lambda_j + g\sqrt{2} \Phi^{\dot{ij}}(\xi_j + a\lambda_j)) + h.c. \right) \\
 & + g\sqrt{2} |a|^2 S_{\dot{ij}} \Phi^{\dot{ij}} + 4g^2 |a|^2.
 \end{aligned}
 \tag{5.8}$$

We have used the notation

$$\Phi^{\dot{ij}} = \Phi^{\dot{i}}_{\alpha} \Phi^j_{\alpha}.
 \tag{5.9}$$

The Lagrangian (5.8) is invariant under local $SO(2) \times U(2)$. If we now impose the Poincaré gauge, together with the $SU(2)$ gauge condition (5.6), then this symmetry is broken to $SO(2)$. This $SO(2)$ is a non-trivial subgroup of $SO(2) \times U(2)$; it is a combination of the $SO(2)$ subgroup of $U(2)$ and the original $SO(2)$, in such a way that the condition (5.6) remains preserved. Therefore, the effective $SO(2)$ gauge invariance will act on all quantities with $SU(2)$ indices $i, j, k, \dots$, and the $SO(2)$ gauge field B_{μ}^{ij} will effectively couple to these quantities through the minimal substitution. (This coupling arises through mixing with the vector field component of the $SU(2)$ gauge field $\mathcal{V}_{\mu j}^i$.) Therefore, the Lagrangian (5.8) describes the $N = 2$ Poincaré theory with gauged $SO(2)$. The on-shell version of this theory is known²⁰⁾ to have a cosmological term. Indeed the Lagrangian (5.8) has a cosmological term as well, which comes from two different sources. First of all, putting $a = 1$ according to (2.5) generates a direct contribution

$$\mathcal{L}_{cosm.}^{(1)} = 4g^2 e.
 \tag{5.10}$$

Secondly, eliminating the auxiliary field S_{ij} by means of its equations of motion leads to

$$\mathcal{L}_{cosm.}^{(2)} = 2g^2 e.
 \tag{5.11}$$

A masslike term for the Rarita-Schwinger field is contained in the supercovariant divergence of V_a . Its explicit form is

$$\mathcal{L}_{\text{masslike}} = -e\sqrt{2}g\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu. \quad (5.12)$$

Equations (5.10 - 5.12) are indeed in agreement with Ref. 20), which shows that (5.8) represents an off-shell version of Poincaré supergravity with gauged $SO(2)$. A different off-shell formulation of the theory will be presented in Section 5.6.

5.5 Poincaré Supergravity with Matter

According to the procedure used above it is straightforward to couple matter to $N = 2$ supergravity. Both the superconformal actions for non-Abelian vector multiplets and scalar multiplets have been given [cf. Eq. (5.1) and Ref. 7)]. The action of the minimal field representation (5.3) is now added to the actions of these multiplets. This leads to the following terms:

$$\begin{aligned} e^{-1}\mathcal{L}_0 = & |\partial_\mu\alpha|^2 + \text{Tr}(\mathcal{D}_\mu X \mathcal{D}_\mu X^*) - |\mathcal{D}_\mu A^\alpha_i|^2 \\ & - (|\alpha|^2 + \text{Tr}XX^* - |A^\alpha_i|^2)f_\mu^\mu + \frac{3}{2}|A^\alpha_i|^2 D + \dots \end{aligned} \quad (5.13)$$

where we have indicated the kinetic terms of the spin-zero fields of the vector multiplets and of a scalar multiplet. To (5.13) we now add the constraint of the non-linear multiplet, multiplied by a function of these scalar fields in such a way that the D -dependent terms cancel. This requires the addition of

$$\begin{aligned} \mathcal{L}_{\text{add}} = & (|\alpha|^2 + \text{Tr}XX^* + \frac{1}{2}|A^\alpha_i|^2)(\mathcal{D}_\alpha V_\alpha - 3D - \frac{1}{2}V_\alpha^2 - \frac{1}{4}|M^{\dot{\gamma}}|^2 + \\ & + \mathcal{D}_\alpha \Phi^\alpha_i \mathcal{D}_\alpha \bar{\Phi}^\alpha_i + 2\bar{\lambda}_i(\not{D}\lambda^i + \frac{3}{2}\chi^i - \frac{1}{4}\sigma\cdot T^{\dot{\gamma}}\lambda_j) \\ & + 2\bar{\lambda}^i(\not{D}\lambda_i + \frac{3}{2}\chi_i - \frac{1}{4}\sigma\cdot T^{\dot{\gamma}}\lambda^i). \end{aligned} \quad (5.14)$$

To observe the cancellation of all D -dependent terms it is again crucial that the gauge field f_μ^α depends on D . We now make the dependence on the Riemann scalar curvature explicit and write the sum of (5.13) and (5.14) ignoring fermionic terms

$$\begin{aligned} \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{add}} = & |\partial_\mu\alpha|^2 + \text{Tr}(\mathcal{D}_\mu X \mathcal{D}_\mu X^*) - |\mathcal{D}_\mu A^\alpha_i|^2 - \frac{1}{2}(|\alpha|^2 + \text{Tr}XX^*)R \\ & + (|\alpha|^2 + \text{Tr}XX^* + \frac{1}{2}|A^\alpha_i|^2)(\mathcal{D}_\alpha V_\alpha - \frac{1}{2}V_\alpha^2 - \frac{1}{4}|M^{\dot{\gamma}}|^2 + \mathcal{D}_\alpha \Phi^\alpha_i \mathcal{D}_\alpha \bar{\Phi}^\alpha_i) + \dots \end{aligned} \quad (5.15)$$

At this point we could again choose the Poincaré gauge (2.6). This would then lead to the analysis of these theories presented in Refs 6) and 7). However, in this case it is more convenient to impose a somewhat different gauge condition on the field a ^{*)}

$$a = \sqrt{1 - \text{Tr}(XX^*)} \quad . \quad (5.16)$$

This choice has the advantage of normalizing the constant in front of the Riemann curvature scalar to its standard value and thus eliminates the necessity for making Weyl redefinitions of the fields afterwards. This is particularly useful for analyzing the scalar field potentials of the theory, for which it is crucial to consider the combined effect of the gravitational and the scalar fields.

Of course, the above method can be applied to a large variety of model field theories. Furthermore, in the presence of vector multiplets with $SU(2)$ or $SO(2)$ symmetry, we can use the extended version of the non-linear multiplet, as we have already demonstrated in the previous subsection. In this way we have analyzed the scalar potentials in several models to search for models in which one can have spontaneously broken supersymmetry in the absence of a cosmological term²¹⁾. However, we have found that this class of models tends to lead to potentials that are unbounded from below. We have not found a suitable model for demonstrating a viable super-Higgs mechanism in $N = 2$ supergravity. The class of models that we have considered contains those of Refs 6), 7) and 17) as a subset.

5.6 An Alternative Formulation of Poincaré Supergravity

In the previous formulation of Poincaré supergravity the non-linear multiplet was used to express the conformal matter field D in terms of other fields while preserving supersymmetry. The non-linear multiplet can thus be viewed as a pre-curvature. As such it is closely related to the precurvature used in the formulation of Breitenlohner and Sohnius¹⁷⁾. However, there is an alternative method for eliminating the troublesome term linear in D from the Lagrangian (5.3), which makes use of a scalar multiplet. Since scalar multiplets have a non-trivial central charge, this off-shell formulation of Poincaré (and de Sitter) supergravity is characterized by the presence of a rather complicated bosonic symmetry which contains the central charge transformations as its main component.

^{*)} Note that our conventions are such that for compact gauge groups $\text{Tr}(XX^*)$ is negative.

The starting point is again the Lagrangian (5.3), to which we now add the Lagrangian of a scalar multiplet. The latter has been given in Ref. 7). Here we show only the bosonic terms of both the kinetic and the masslike invariant, the latter with coupling constant g .

$$\begin{aligned} e^{-1} \mathcal{L}_{\text{scalar}} = & -|\mathcal{D}_\mu A^\alpha_i|^2 + |A^\alpha_i|^2 (f_\mu^\mu + \frac{3}{2}D) \\ & + (|a|^2 + \frac{1}{8}(\epsilon_{ij} B_{\mu}^{ij})^2) (|A^{(z)\alpha}_i|^2 + 2\sqrt{2}g A^\alpha_i A^{(z)\alpha}_j \epsilon^{ij}) \\ & + \sqrt{2}g A^\alpha_i A^\alpha_j S^{ij} + \dots \end{aligned} \quad (5.17)$$

We now consider a linear combination of (5.3) and (5.17), making the Riemann curvature scalar and the field D explicit. Furthermore, we impose the Poincaré gauge (2.5) and break the $SU(2)$ invariance by means of an additional gauge condition

$$A^\alpha_i = e^u \delta^\alpha_i \quad (5.18)$$

This leads to the following result:

$$\begin{aligned} e^{-1}(\mathcal{L}_{\text{min}} - \mathcal{L}_{\text{scalar}}) = & -\frac{1}{6}(1+2e^{2u})R + (1-e^{2u})D - 2e^{2u}(\partial_\mu u)^2 \\ & - \frac{1}{2}|S_{ij}|^2 + A_a^2 - \frac{1}{4}F_{ab}^{ij}(B)^2 + \frac{1}{64}((\epsilon_{ij}t_{ab}^{ij})^2 + h.c.) \\ & - e^{2u}(1 + \frac{1}{8}(\epsilon_{kl}B_{\mu}^{kl})^2)(|G^i_j|^2 - 2\sqrt{2}g \epsilon^{ij}G^i_j) \\ & + \sqrt{2}g e^{2u} S_{ii} + \dots \end{aligned} \quad (5.19)$$

where we have used the definition

$$G^\alpha_i = e^{-u} A^{(z)\alpha}_i \quad (5.20)$$

The terms proportional to D are no longer troublesome in (5.19). This field simply acts as a Lagrange multiplier, putting the field u equal to zero. When we eliminate all auxiliary fields by means of their field equations, the Lagrangian

(5.19) coincides with the corresponding on-shell versions of the Poincaré and de Sitter Lagrangians (5.4) and (5.8). For instance, when we eliminate the auxiliary fields G^i_j and S_i in (5.19) we find the same cosmological constant and masslike Rarita-Schwinger term [cf. (5.10 - 5.12)]. On the other hand, the off-shell formulations that underlie these Lagrangians are clearly inequivalent. The most obvious difference resides in the auxiliary degrees of freedom. Instead of a vector V_a and a complex scalar M^{ij} , the theory of this section has six scalars provided by the fields D , G^i_j and u . But a more crucial aspect concerns the fate of the central-charge transformations. In the previous formulations the central charge acts on the vector field B^{ij}_μ and after a subsequent gauge choice (5.6) central-charge transformations could also be realized as $SO(2)$ transformations. In the present formulation the invariance under central-charge transformations is affected by the gauge choice (5.18). Therefore, only a linear combination of a central-charge transformation and a field-dependent $SU(2)$ transformation is left as an invariance group. This new local symmetry of the bosonic type is specified by the following decomposition rule:

$$\delta(z) = \delta_Z(z) + \delta_{SU(2)}(z(G^i_j - \frac{1}{2}\delta^i_j G^k_k)) \quad (5.21)$$

Only on-shell, where $G^i_j = \sqrt{2} g \epsilon^{ij}$, will this symmetry reduce to the $SO(2)$ invariance of the previous formulation. In any case, even when $g = 0$, the transformations (5.21) are not generated by an off-shell central charge because of the $SU(2)$ component.

Hence we have found that $N = 2$ Poincaré supergravity has two inequivalent off-shell versions. One formulation has a central charge which may be extended to $SO(2)$ for the de Sitter case. The other one has no central charge; this formulation seems to be related to the one advocated by MacDowell²²⁾. The latter version can be coupled to matter multiplets as well. Theories of this type have also been considered in our search for a supersymmetric Higgs mechanism. This did not change our negative results reported in the previous subsection.

It is of interest to truncate this alternative formulation of Poincaré supergravity to $N = 1$ and to compare it to existing auxiliary-field formulations²³⁾. The $40 + 40$ components of the linearized multiplet split into two irreducible multiplets with respect to $N = 1$ supersymmetry. Both these multiplets contain $20 + 20$ degrees of freedom. One is the $(3/2, 1)$ multiplet discovered in Ref. 15). The other one is the $(2, 3/2)$ multiplet of $N = 1$ supergravity. After various field redefinitions, the linearized transformations take the following form:

$$\begin{aligned}
\delta e_\mu^a &= \bar{\epsilon} \gamma^a \psi_\mu - \frac{2}{3} \bar{\epsilon} \gamma^a \gamma_\mu \zeta, \\
\delta \psi_\mu &= 2 \partial_\mu \epsilon - \omega_\mu \cdot \sigma \epsilon + i \gamma_5 A_\mu \epsilon - \frac{1}{3} \gamma_\mu (S + i \gamma_5 P + i A \gamma_5) \epsilon + \frac{2}{3} \gamma_\mu \not{\partial} u \epsilon, \\
\delta A_\mu &= \frac{3}{2} i \bar{\epsilon} \gamma_5 (R_\mu - \frac{1}{3} \gamma_\mu \gamma \cdot R), \\
\delta S &= -\frac{1}{2} \bar{\epsilon} \gamma \cdot R, \\
\delta P &= -\frac{i}{2} \bar{\epsilon} \gamma_5 \gamma \cdot R, \\
\delta u &= \bar{\epsilon} \zeta, \\
\delta \zeta &= \not{\partial} u - (F + i \gamma_5 G) \epsilon - \frac{i}{2} A' \gamma_5 \epsilon, \\
\delta A'_\mu &= 3 i \bar{\epsilon} \gamma_\mu \gamma_5 \chi - 2 i \bar{\epsilon} \gamma_5 \partial_\mu \zeta, \\
\delta F &= -\bar{\epsilon} (\frac{3}{2} \chi + \not{\partial} \zeta), \\
\delta G &= -i \bar{\epsilon} \gamma_5 (\frac{3}{2} \chi + \not{\partial} \zeta), \\
\delta \chi &= D \epsilon + \frac{2}{3} i \gamma_5 \sigma^{\mu\nu} \partial_\mu A'_\nu \epsilon, \\
\delta D &= \bar{\epsilon} \not{\partial} \chi.
\end{aligned} \tag{5.22}$$

The components $u, \zeta, A'_\mu, F, G_\chi$ and D form a real $N = 1$ multiplet which can consistently be set to zero. In that case the minimal auxiliary field formulation of $N = 1$ supergravity remains.

6. CONCLUSIONS

In this paper we have presented an extensive multiplet calculus based on $N = 2$ superconformal gravity. The advantage of this approach has been demonstrated. Superconformal multiplets have simpler transformation rules than the corresponding Poincaré multiplets and the underlying minimal field representation of $32 + 32$ components is reducible as a conformal multiplet. We have shown that the construction of Poincaré and de Sitter supergravity theories is possible in this approach as well by making use of compensating fields and appropriate gauge choices to break some of the superconformal invariances. The advantage of this is not only that the superconformal formalism is more transparent, but perhaps more importantly that one keeps the option of choosing different versions of Poincaré supergravity theories until the end.

The superconformal approach outlined in this paper should be equally applicable to higher N supergravity theories. In this respect it is important to note that a complete version of $N = 4$ conformal supergravity exists²⁴⁾. To extend this to Poincaré and de Sitter supergravity theories requires knowledge of superconformal matter multiplets. A study of such multiplets for higher N is, therefore, of crucial importance.

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APPENDIX

The multiplet calculus, including the construction of invariant actions for $N = 2$ supergravity is most conveniently formulated in terms of superconformal multiplets and fields. However, if one wants to extract from this the corresponding results for Poincaré or de Sitter theories, one must take into account that some of the superconformal gauge fields are not independent, but determined in terms of other fields through constraints^{5),6)}. In order to facilitate the translation of superconformal results we give here the explicit Poincaré expressions for the quantities needed.

$$\begin{aligned} \omega_{\mu ab} = & -\frac{1}{2} (e_{[a}^{\nu} (\partial_{\mu} e_{b]\nu} - \partial_{\nu} e_{b]\mu}) + e_{[a}^{\rho} e_{b]}^{\sigma} (\partial_{\sigma} e_{c\rho}) e_{\mu}^c \\ & - \frac{1}{4} (\bar{\psi}_{\mu}^i \gamma_{[a} \psi_{b]i} + \bar{\psi}_a^i \gamma_{\mu} \psi_{bi} + h.c.) \end{aligned} \quad (A.1)$$

$$\phi_{\mu}^i = \frac{1}{2} \gamma_{\mu} \chi^i + (\sigma^{\rho\sigma} \gamma_{\mu} - \frac{1}{3} \gamma_{\mu} \sigma^{\rho\sigma}) (\mathcal{D}_{\rho} \psi_{\sigma}^i - \frac{1}{8} \sigma \cdot T^{-ij} \gamma_{\rho} \psi_{\sigma j}), \quad (A.2)$$

$$f_{\mu}^{\mu} = \frac{1}{6} R - D + \frac{1}{4} \left\{ \bar{\psi}^i \gamma \chi_i - \frac{1}{3} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_{\mu}^i \gamma_{\nu} \mathcal{D}_{\rho} \psi_{\sigma i} + h.c. \right\}. \quad (A.3)$$

TABLE CAPTIONS

Table 1 : Fields of the minimal field representation. The coefficients w and c denote Weyl and chiral weight factors.

Table 2 : Fields of various superconformal multiplets. The coefficients w and c denote Weyl and chiral weight factors.

Table 1

Field	Type and algebraic restrictions	w	c
Weyl submultiplet			
e_μ^a	boson; P gauge field	- 1	0
ω_μ^{ab}	boson; M gauge field; not independent	0	0
b_μ	boson; D gauge field	0	0
f_μ^a	boson; K gauge field; not independent	1	0
$\mathcal{V}_\mu^{ij}$	boson; SU(2) gauge field; $\mathcal{V}_{\mu i}^j \equiv (\mathcal{V}_\mu^{ij})^* = -\mathcal{V}_\mu^j{}_i$ $\mathcal{V}_\mu^i{}_i = 0$	0	0
A_μ	boson; U(1) gauge field	0	0
ψ_μ^i	fermion; Q gauge field; $\gamma_5 \psi_\mu^i = \psi_\mu^i$	-1/2	-1/2
ϕ_μ^i	fermion; S gauge field; not independent $\gamma_5 \phi_\mu^i = -\phi_\mu^i$	1/2	1/2
T_{ab}^{-ij}	boson; $T_{ab}^{-ij} = -T_{ba}^{-ij} = -T_{ab}^{-ji} = -\tilde{T}_{ab}^{-ij}$	1	- 1
χ^i	fermion; $\gamma_5 \chi^i = \chi^i$	3/2	-1/2
D	boson	2	0
Vector submultiplet			
a	boson; complex	1	- 1
ξ_i	fermion; $\gamma_5 \xi_i = \xi_i$	3/2	-1/2
S_{ij}	boson; $S_{ij} = S_{ji}$; $S^{ij} \equiv (S_{ij})^* = \epsilon^{ik} \epsilon^{jl} S_{kl}$	2	0
B_μ^{ij}	boson; Z gauge field	0	0
t_{abij}^-	boson; modified B_μ^{ij} field strength; cf. Eq. (2.2)	2	0

Table 2

Field	Type and restrictions	W	C
Chiral multiplet			
A	boson; complex	w	-w
ψ_i	fermion; $\gamma_5 \psi_i = \psi_i$	w + 1/2	-w + 1/2
B_{ij}	boson; $B_{ij} = B_{ji}$	w + 1	-w + 1
F_{ab}^-	boson; $\tilde{F}_{ab}^- = -F_{ab}^-$	w + 1	-w + 1
Λ_i	fermion; $\gamma_5 \Lambda_i = \Lambda_i$	w + 3/2	-w + 3/2
C	boson; complex	w + 2	-w + 2
Non-Abelian vector multiplet			
X	boson; complex	1	-1
Ω_i	fermion; $\gamma_5 \Omega_i = \Omega_i$	3/2	-1/2
Y_{ij}	boson; $Y_{ij} = Y_{ji}$; $Y^{ij} \equiv (Y_{ij})^* = \epsilon^{ik} \epsilon^{jl} Y_{kl}$	2	0
W_μ	boson; non-Abelian gauge field	0	0
$F_{ab}^{(W)}$	boson; modified W_μ field strength; cf. Eq. (3.14)	2	0
Linear multiplet			
L_{ij}	boson; $L_{ij} = L_{ji}$; $L^{ij} \equiv (L_{ij})^* = \epsilon^{ik} \epsilon^{jl} L_{kl}$	2	0
φ^i	fermion; $\gamma_5 \varphi^i = \varphi^i$	5/2	1/2
G	boson; complex	3	1
E_a	boson; constrained vector; cf. Eq. (3.22)	3	0
Scalar multiplet			
A_α^i	boson; $A_\alpha^i \equiv (A_\alpha^i)^* = \epsilon^{ij} \epsilon_{\alpha\beta} A_j^\beta$	1	0
ζ^α	fermion; $\gamma_5 \zeta^\alpha = -\zeta^\alpha$	3/2	-1/2
Non-linear multiplet			
ϕ_a^i	boson; element of SU(2)	0	0
λ^i	fermion; $\gamma_5 \lambda^i = -\lambda^i$	1/2	-1/2
M^{ij}	boson; complex; $M^{ij} = -M^{ji}$	1	-1
V_a^i	boson; constrained vector; cf. Eqs. (3.36) and (3.40)	1	0

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