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ON CHIRAL REALIZATIONS OF CONFINING THEORIES

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ABSTRACT

It is argued that chirality is not necessarily broken in $SU(N)$ colour theories with a chiral flavour symmetry. In this case, massless fermion bound states contribute to anomaly equations, and loop expansions in a $1/N$ approach are invalid. As a hint to understanding which option is realized by the theory, we investigate it in two dimensions for very small quark masses. The light spectrum shows a rich structure not obtained in a $1/N$ expansion. Light flavoured baryons and mesons indicate a partial realization of the chiral symmetry.

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Several arguments have been recently provided in order to infer that a chiral confining theory should undergo a spontaneous symmetry breaking, thus generating Goldstone bosons. An intuitive justification¹⁾ is that a quark, when approaching the confining region (bag) should change its velocity thus behaving as having an effective mass. More precise arguments have been provided on the basis of lattice calculations¹⁾ and, more recently, Coleman and Witten²⁾ have argued that chiral symmetry breaking of a $SU(N)$ colour with a global chiral $SU(n) \times SU(n)$ flavour symmetry is predicted by a $1/N$ expansion. In the loop expansion, the strong $U(1)$ anomaly was also shown to imply chiral breaking³⁾.

The crucial point of this last approach is that due to anomalies, triangular Green functions $\Gamma_{\mu\nu\rho}(k^2)$ must have a $1/k^2$ pole. However, in the planar quark loop expansion generated by $1/N$ approach, the quantities in question are described, at leading order by a single quark loop as that of Fig. 1. It is then apparent that the only possibility of displaying a $1/k^2$ singularity is to generate a massless scalar bound state composed by $q\bar{q}$ and an arbitrary number of gluons. This would be the Goldstone massless boson, a sign of the spontaneous symmetry breaking.

Other ways of saturating anomalies exist in principle. Indeed, it is well known that the $1/k^2$ pole needed to satisfy the anomaly requirement can also be provided by massless fermions⁴⁾ which, due to confinement, should be colour singlets (let us call them baryons).

In this note we point out that chiral realizations may indeed be possible even for large N . We shall first argue that the loop expansion used in Ref. 2) is valid only under the assumption that the chirality is completely broken. This means that if the whole chiral group (or a chiral subgroup of it) is actually realized, the loop expansion is unwarranted and massless fermions would contribute to the saturation of anomaly equations. This agrees with conclusions obtained independently by Veneziano⁵⁾ with an effective Lagrangian approach to anomalies.

Witten has shown⁶⁾ that baryons can be accommodated in a $1/N$ approach, but of course not in a loop expansion. In a non-chiral realization, these baryons would have a large mass (mN where m is the quark mass) and their presence would not be expected to spoil the loop expansion that approximately described the underlying dynamics. However, this is not the case if quarks remain massless ($m = 0$), as demanded by a chiral realization. The baryon contribution to the anomaly would look as in Fig. 2 which is a terribly non-planar diagram not necessarily depressed by $1/N$ factors^{*)}. Massless baryons are therefore evidence of chiral realizations, while massless scalars (Goldstones) are a sign of chiral breaking. The presence of both indicates a partial chiral realization.

^{*)} Arguments⁵⁾ that baryon pairs couple to currents like $\exp - cN$ are also questionable for $m = 0$.

Anomalies imply massless particles and, in order to be saturated, provide stringent conditions on them. 't Hooft⁷⁾ first discussed these conditions for a purely chiral saturation. The recognition and classification of different possible solutions is a group theoretical problem. Which of them is actually realized is instead a purely dynamical question, directly related to which are the massless (colourless) states in the spectrum of the confining theory. This is, unfortunately, a difficult question to answer.

In order to have an insight, we shall investigate the theory in two dimensions where confinement is guaranteed and where the light spectrum can be detected^{*)}.

Let us immediately recognize that at first sight it seems odd to look for massless bound states or Goldstones in two-dimensional models where we know they are forbidden⁸⁾. This difficulty is easy to circumvent. We shall introduce an ad hoc small quark (current) mass m_0 and look for the spectrum for $Nm_0/g \ll 1$ (the charge g has dimension of a mass in two dimensions). These are the states that in four dimensions would be expected to become massless in the chiral $m_0 \rightarrow 0$ limit. The pathology of two dimensions is reflected by the fact that for $m_0 \rightarrow 0$ these states usually decouple and disappear, thereby respecting the Coleman dictate. As we shall see, we shall find light composite fermions which, as we said before, indicate an active chiral protection.

Let us consider a gauge $SU(N_c)$ theory (Latin indices) with a flavour $U(N_f)$ symmetry (Greek indices) in two dimensions (QCD_2). In order to analyse its light spectrum in the strong coupling limit $N_c(m_0/g) \ll 1$, it is convenient to work in a gauge first proposed by Baluni⁹⁾ and subsequently used by Steinhardt¹⁰⁾. Our treatment closely follows that of Ref. 10) and, therefore, we only sketch a few essential steps. The gauge is defined by

$$E_k^i = 0 \quad i \neq k \quad ; \quad A_i^i = 0 \quad (1)$$

The bosonization of the theory is performed by defining

$$\sqrt{\pi} : \bar{q}^{\alpha i} \gamma_0 q_{\alpha i} : = \partial_x \phi_{\alpha i} \quad \sqrt{\pi} : \bar{q}^{\alpha i} \gamma_x q_{\alpha i} : = \pi_{\alpha i} \quad (2)$$

^{*)} We shall not discuss in this note the anomaly structure in two dimensions and how the states we shall identify will contribute to their saturation. This is currently under study in collaboration with S. Elitzur and Y. Frishman.

Then, if m is the bare quark mass and g is the electric charge, the Hamiltonian reads

$$H = N_\mu \left\{ \sum_{i,\alpha} \left[\frac{1}{2} \pi_{\alpha i}^2 + \frac{1}{2} (\partial_x \phi_{\alpha i})^2 + 2 m_\mu (1 - \cos 2 \sqrt{h} \phi_{\alpha i}) \right] + \right. \\ \left. + \frac{g^2}{8\pi N_c} \sum_{i,j} \left[\sum_{\alpha} (\phi_{\alpha i} - \phi_{\alpha j}) \right]^2 + \sum_{i,j} F \left(\sum_{\alpha} (\phi_{\alpha i} - \phi_{\alpha j}) \right) \right\} \quad (3)$$

where μ is an arbitrary mass scale used to normal order and F is a term involving the fields ϕ in the combination indicated explicitly.

The Hamiltonian of Eq. (3) has a $S_{N_c} \times S_{N_f}$ discrete symmetry (S_N being the permutation group of N elements). The first is not surprising due to the fact that the conditions (1) do not completely fix the gauge, leaving a global S_{N_c} and a local $(U(1))^{N_c-1}$ symmetry. The S_{N_f} symmetry seems more surprising due to the fact that the original $(QCD)_2$ theory had a continuous $U(N_f)$ global invariance. The $U(N_f)$ symmetry shall indeed appear as a secret symmetry¹¹⁾ which will manifest itself in the bound state spectrum.

It is convenient to introduce a basis that singles out flavour and colour scalars

$$\chi = (N_c N_f)^{-1/2} \sum_{\alpha i} \phi_{\alpha i} \quad , \quad \chi_{\alpha} = (N_c)^{-1/2} \sum_{\beta i} \phi_{\beta i} \tau_{\beta\alpha}^{\alpha} \\ \chi_i = (N_f)^{-1/2} \sum_{\alpha j} \phi_{\alpha j} \lambda_{ij}^i \quad , \quad \chi_{\alpha i} = \sum_{\beta j} \lambda_{\beta\alpha}^{\alpha} \tau_{ij}^i \phi_{\beta j} \quad (4)$$

where $\lambda(\tau)$ are the $N_c - 1$ ($N_f - 1$) diagonal matrices representing the commuting set of generators of $SU(N_c)$ ($SU(N_f)$). The term proportional to g^2 in Eq. (3) can then be written as $g^2/8\pi \sum_i \chi_i^2$ implying that the χ_i fields have masses proportional to g^2 . As discussed in Ref. 10), the decoupling of these heavy fields is easy. Their only contributions not depressed by powers of m_0^2/g^2 are loops in which χ_i are emitted and reabsorbed by a renormal ordering. This means that the $N_c - 1$ fields χ_i can be set to zero provided the mass (which appears now as the only parameter which gives the mass scale and by which fields are normal ordered) is given by

$$m = \frac{g}{\sqrt{\pi}} \left(\frac{m_0 \sqrt{\pi}}{g} \right)^{\frac{N_c N_f}{N_c(N_f+1)-1}} \quad (5)$$

We also notice that the F term in Eq. (3) vanishes when χ_i are set to zero. Equation (5) implies that $m/g \rightarrow 0$ as $m_0/g \rightarrow 0$ so that m is the only small mass parameter of the theory. In order to analyse the light spectrum, we will rewrite H in terms of the $N_c(N_f-1) + 1$ fields χ , χ_α and $\chi_{\alpha i}$ that describe low-lying excitation.

Let us analyse the $N_f = 2$ case, the extension to the general case being a simple matter. A convenient basis is χ and $\chi_i^- = (\phi_{i1} - \phi_{i2})/\sqrt{2}$ ($i = 1, \dots, N_c$) in terms of which H is written as

$$H = N_m \left\{ \frac{1}{2} \pi^2 + \frac{1}{2} \sum_i \pi_i^2 + \frac{1}{2} (\partial_x \chi)^2 + \frac{1}{2} \left(\partial_x \sum_i \chi_i^- \right)^2 + \right. \\ \left. + 4m^2 \left[N_c - \left(\sum_{i=1}^{N_c} \cos \sqrt{2\pi} \chi_i^- \right) \cos \sqrt{\frac{2\pi}{N_c}} \chi \right] \right\} \quad (6)$$

which describes a generalized sine-Gordon system which we shall analyse with semi-classical methods¹²⁾. The mass term, which breaks chiral symmetry leaves to the Hamiltonian of Eq. (6) a discrete chiral symmetry corresponding to $\phi_{i\alpha} \rightarrow \phi_{i\alpha} + n\sqrt{\pi}$ which allows a classification of different vacua. This discrete symmetry is nevertheless also broken by solitons that interpolate the different vacua at space co-ordinate $-\infty$ and $+\infty$. Excitations will be classified by these different jumps (topological numbers) that are directly related to baryon numbers and the third component of the isospin. Indeed, using Eq. (2) the quark charge B and I_3 are given by

$$B = \int_{-\infty}^{+\infty} dx \sum_{\alpha, i} \bar{q}^{\alpha i} \gamma_0 q_{\alpha i} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dx \sum_{\alpha, i} \partial_x \phi_{\alpha i} = \sqrt{\frac{2N_c}{\pi}} \Delta \chi \\ I_3 = \int_{-\infty}^{+\infty} dx \sum_{\alpha, i} \bar{q}^{\alpha i} \gamma_0 \tau_{\alpha\alpha}^3 q_{\alpha i} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx \sum_i \partial_x \chi_i^- = \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{N_c} \Delta \chi_i^- \quad (7)$$

where

$$\Delta\chi \equiv \chi(x=+\infty) - \chi(x=-\infty)$$

We recall that the mass (energy) of a soliton in a sine-Gordon system

$$H = \frac{1}{2} \pi^2 - \frac{1}{2} (\partial_x \phi)^2 + \alpha (1 - \cos \beta \phi) \quad (8)$$

is given by¹¹⁾

$$M_s = \frac{8\sqrt{\alpha}}{\beta} \quad (9)$$

Let us now start discussing the mesonic ($\Delta\chi = 0$) spectrum. Solitons can be excited in every χ_i^- . The lowest excitations will correspond to a single χ_i^- jumping (by $\pm\sqrt{2\pi}$) leaving all other fields unexcited. From (3) and (9) we evaluate its masses to be $M = 2M_0$, where for later convenience we have defined

$$M_0 \equiv 4\sqrt{\frac{\alpha}{\pi}} m \quad (10)$$

They correspond to mesons ($B = 0$) with $I_3 = \pm 1$. We notice that we should count these states as a single meson for each I_3 . Indeed, the fact that each of the χ_i^- ($i = 1, \dots, N_c$) can jump, is a reflection of the S_{N_c} residual symmetry left by the gauge conditions (1) which must, of course, be disposed of.

The degeneracy of $I_3 = +1$ and $I_3 = -1$ states (soliton and antisoliton) are expected by the discrete S_2 flavour symmetry. As pointed out by Coleman¹¹⁾, the $I_3 = 0$ member of the $I = 1$ triplet emerges from the breather modes as expected from the hidden $SU(2)$ flavour symmetry. Indeed, the soliton-antisoliton states of the sine-Gordon system (8) have masses¹²⁾

$$M_n = 2M_s \sin \frac{\beta^2 n}{1 - \frac{\beta^2}{8\pi}} \quad , \quad n = 1, 2, \dots < \frac{8\pi}{\beta^2} - 1 \quad (11)$$

where M_s is the soliton mass (9). For χ_i^- solitons in (6), $E_s = 2M_0$, $\beta = \sqrt{2\pi}$ and therefore $n = 1, 2$. These are therefore mesons with $I_3 = 0$ and masses

$$M_1 = 2M_0 \quad , \quad M_2 = 2\sqrt{3}M_0 \quad (12)$$

The first breather therefore completes the $I = 1$ triplet while the second, with slightly higher mass, represents an $I = 0$ meson.

Higher mass states are obtained when two fields χ_i^- jump or when one jumps and one breathes or two breath. The lowest form an $I = 2$ multiplet and a degenerate singlet of masses $M = 4M_0$.

Another family of low-lying mesons is provided by breathers of χ . They have of course $I = 0$ if no χ_i^- soliton is excited. Their masses will be again given by Eq. (11), but now

$$M_s = 2N_c M_0, \quad \beta = \sqrt{\frac{2\pi}{N_c}}, \quad n = 1, 2, \dots, 4N_c - 2 \quad (13)$$

so that calling $\hat{M}_n$ the masses of these χ breathers

$$\hat{M}_n = 4M_0 N_c \sin \frac{n\pi}{4N_c - 1} \quad (14)$$

For large N_c the lower χ breather has a mass

$$\hat{M}_1 \sim \frac{\pi}{2} M_0$$

which is even lower than the $I = 1$ meson discussed before.

Let us now turn to the baryon spectrum. We notice that the Hamiltonian of Eq. (6) allows a correlated soliton set for

$$\Delta \chi = \pm \sqrt{\frac{\pi N_c}{2}}, \quad \Delta \chi_i^- = \pm \sqrt{\frac{\pi}{2}} \quad \text{all } i$$

They all have quark numbers $B = \pm N_c$ and are therefore bona fide baryons and antibaryons. We find $N_c + 1$ distinct combinations of χ_i^- solitons and anti-solitons forming $I = N_c/2$ isospin representations for the baryon and anti-baryon. Their mass can be estimated to be $M_B = M_0 N_c$ by substituting $\chi_i^- = \pm \chi / \sqrt{N_c}$ in Eq. (6) thus obtaining an effective Hamiltonian as that of Eq. (8).

A dibaryon with $I = 0$ and mass $2M_0 N_c = 2M_B$ will be obtained for a χ soliton. Likewise, higher excitations including nuclei, can easily be obtained.

We now notice that the $I = 0$ mesons, breathers of χ we found before, can be thought of as soliton-antisoliton (χ soliton of course) bound states. This led Steinhardt¹⁰⁾ to call them baryonium, indeed bound states of an $I = 0$.

dibaryon and its antiparticle. This identification is perhaps misleading. It is nevertheless amusing that baryoniums seem to stem from $U(1)$ anomalies in chiral realizations⁵⁾ by playing the role that the η had in the broken case.

To summarize, for very small m , i.e., $(N_C m/g) \ll 1$ we found a plethora of very light states, including mesons and baryons with a spectrum quite different from the one obtained in the $1/N_C$ expansion¹³⁾. This is not a contradiction, due to the fact that the $1/N_C$ expansion is supposedly valid in the weak coupling limit, i.e., $g^2/m^2 \sim 1/N_C \ll 1$.

Another property, worth mentioning is the striking difference between the $SU(N_C)$ and the $U(N_C)$ case. For $U(N_C)$, there appears in the bosonized Hamiltonian of Eq. (3) an extra term

$$\frac{g^2}{8\pi N_C} \left(\sum_{\alpha,i} \phi_{\alpha,i} \right)^2 = \frac{g^2}{8\pi} N_f \chi^2$$

This means that the χ field receives a high mass and therefore its excitations would not contribute to the light spectrum. This results in the elimination from the light spectrum of all baryons as well as mesons we could have identified with baryoniums. This follows from the fact that the $U(1)$ charge in the decomposition $U(N_C) = U(1) * SU(N_C)$ is confined in two dimensions^{*)}. Thus the $SU(N_C)$ singlet and $U(1)$ non-singlet $\epsilon_{12\dots N_C} q_1 q_2 \dots q_{N_C}$ is confined in two dimensions.

We were also able to witness the pathology of the two-dimensional theory for $m \rightarrow 0$. Indeed m controls a sine-Gordon type interaction and its non-zerosness allows soliton-like solutions and therefore a topological number that disappears for $m = 0$.

We recall that although Goldstone bosons are now allowed in two dimensions, chirally bleached fermions can get a mass mimicking as much as possible in two dimensions dynamical chiral symmetry breaking. Witten¹⁴⁾ has shown that this is indeed the case for the massless $SU(n)$ Thirring model. The baryons in $(QCD)_2$ do not choose this option, they receive masses of the order of the current quark masses. This is the signature of a chiral protection we were looking for.

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*) In four dimensions this could happen if $U(1)$ emerges as a subgroup of a compact group.

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FIG.1

FIG.2