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TUMBLING AND THE STRONG ANOMALY

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A B S T R A C T

A non-trivial strong anomaly is shown to trigger the "tumbling" of gauge theories and to help in determining the possible pattern of consequent symmetry breakings. Solutions with partial conservation of chirality can be exhibited even for QCD-like theories.

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1. - In an effort to avoid "unnatural" theories with fundamental Higgs scalars, considerable attention has recently been given to gauge theories coupled to just spin $\frac{1}{2}$ fermions. It has been argued ^{1),2)} that, when the resulting theory is asymptotically free and the fermion content is not real, a composite Lorentz scalar (typically bilinear in the fermions) may acquire a vacuum expectation value (VEV) which breaks the gauge and possibly also the global (flavour) symmetry of the original Lagrangian. This phenomenon, which may involve several "successive" steps (i.e., several energy scales), has been termed "tumbling" by Dimopoulos, Raby and Susskind ¹⁾ (DRS).

In Ref. 1), DRS have also proposed a maximally attractive channel (MAC) criterion for deciding which composite scalar gets a VEV at each step. Another approach to the same question has been proposed in Ref. 3).

A further interesting aspect of tumbling is that it provides ⁴⁾, when combined with a complementarity principle ⁵⁾, an elegant way of generating (perhaps all the) solutions to 't Hooft's consistency equations ⁶⁾ for the appearance of massless composite fermions. Those equations express the requirement that the effective low energy theory has the same flavour current anomalies as the original fundamental theory.

In the above arguments, the flavour currents possessing a strong (gauge) anomaly were disregarded because of their lack of conservation. It is the purpose of this note to point out that the strong anomaly may, in fact, play an important rôle in tumbling, firstly by triggering it whenever the topological charge is a non-trivial operator and, secondly, by restricting its possible patterns.

The basic observation, already made in Ref. 7) in the context of QCD, is that the assumption of a non-trivial topological charge in the pure Yang-Mills theory without fermions implies, through standard Ward identities, a massless scalar particle pole in a two-point function at the one-fermion-loop level (gauge boson loops being summed over). If the fermion loop expansion is not misleading ^{*}), one can use this information to construct a low energy effective Lagrangian along the lines followed recently ⁸⁾ for quantum chromodynamics (QCD). This allows a simple rephrasing of an argument ^{7),9)} for the occurrence of chiral symmetry breaking in QCD, but also shows how that argument can be avoided if some of the assumptions are relaxed.

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^{*}) This would be the case, for instance, if there were so many fermions that the theory is not asymptotically free although it looks so at each order of the fermion loop expansion (not to speak of supersymmetric theories, where the expansion itself breaks supersymmetry).

We shall illustrate our arguments in a few typical (although unfortunately unrealistic) examples, but we will first write down some general definitions and formulae.

2. - Denoting the gauge group by G_c , the theory is specified by its fermion content F , i.e., by assigning the left-handed fermions to representations of G_c . We shall write :

$$\underline{F} = \sum_i f_i \underline{r}_i \quad (1)$$

where f_i is the number of times the irreducible representation $\underline{r}_i$ of G_c occurs. Having excluded mass terms, the classical Lagrangian has also a global (flavour) symmetry $\tilde{G}_F$ given by

$$\tilde{G}_F = \prod_i U(f_i) \quad (2)$$

up to possible generators already included in G_c . Some of the $U(1)$ currents of $\tilde{G}_F$ have a strong (i.e., G_c) Adler-Bell-Jackiw anomaly, proportional to the topological charge density operator :

$$Q_c(x) \equiv \sum_a \frac{g_c^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x) \quad (3)$$

In order for the strong anomaly to effectively break the anomalous subgroup G_{an} of $\tilde{G}_F$ we require, in analogy with the QCD example ⁸⁾, Kogut-Susskind poles in the pure gauge theory, i.e. ^{*)},

$$A_c \equiv -i \int d^4x \, T \left\langle 0 \left| [Q_c(x) Q_c(0)] \right| 0 \right\rangle \Big|_{\substack{\text{no} \\ \text{fermions}}} > 0 \quad (4)$$

The remaining global symmetry is then :

$$G_F = \tilde{G}_F / G_{an} \quad (5)$$

We recall that the consistency conditions of Ref. 6) apply to the currents of G_F , not to those of $\tilde{G}_F$.

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^{*)} Some evidence in this direction has recently been obtained from lattice calculations (P. Di Vecchia, K. Fabricius, G.C. Rossi and G. Veneziano, in preparation).

3. - We now proceed with a few examples.

$$(i) \quad G_c = SU(5); \quad \underline{F} = 1 \cdot \underline{5} + 1 \cdot \overline{10} = (\psi_\alpha, \psi^{\alpha\beta} = -\psi^{\beta\alpha})$$

$$\alpha, \beta = 1, \dots, 5$$

This example was discussed extensively by DRS ^{1),3),4)}, hence we can compare our results with theirs. Obviously :

$$\tilde{G}_F = U_{\underline{5}}^{(1)} \otimes U_{\overline{10}}^{(1)} : \quad \psi_\alpha \rightarrow e^{i\lambda_5} \psi_\alpha, \quad \psi^{\alpha\beta} \rightarrow e^{i\lambda_{\overline{10}}} \psi^{\alpha\beta} \quad (6)$$

The ABJ anomaly implies that, under the transformation (6) :

$$\mathcal{L} \rightarrow \mathcal{L} + (\lambda_5 + 3\lambda_{\overline{10}}) Q \quad ; \quad Q = Q_{SU(5)} \quad (7)$$

Following the one-fermion-loop Ward identity argument of Ref. 7) (as outlined above), we are led to construct an effective Lagrangian possessing the correct symmetries and anomalies in analogy with the known ⁸⁾ QCD case. The lowest fermion loop level suggests that we limit ourselves to Lorentz scalar composites which are bilinear in the fundamental fermions.

As discussed in Ref. 1), they consist of a Lorentz-antisymmetrized product ^{*)} of two left (or right) handed fields. In the case at hand, they can be of three types according to whether we combine two $\underline{5}$, two $\overline{10}$ or one $\underline{5}$ and one $\overline{10}$. These composites transform under (6) with phases $2\lambda_5$, $2\lambda_{\overline{10}}$ and $(\lambda_5 + \lambda_{\overline{10}})$, respectively. It is then clear that more than one of these composite scalars must be used in order to obtain the phase combination $(\lambda_5 + 3\lambda_{\overline{10}})$ appearing in (7). By far the simplest choice consists of putting together a $(\overline{10} \times \overline{10})$ and a $(5 \times \overline{10})$ scalar in a $\underline{5}$ and a $\overline{5}$, respectively. Defining :

$$\varphi_\alpha \equiv \varepsilon_{\alpha\beta\gamma\delta\epsilon} \psi^{\beta\gamma} \psi^{\delta\epsilon} \quad ; \quad \chi^\alpha \equiv \psi_\beta \psi^{\alpha\beta} \quad ; \quad M \equiv \varphi_\alpha \chi^\alpha$$

$$M^+ \equiv \varphi^{*\alpha} \chi_\alpha^* \quad (8)$$

it is obvious that M transforms under (7) with phase $(\lambda_5 + 3\lambda_{\overline{10}})$, and that it is a singlet of both $SU(5)_c$ and of the anomaly free $G_F = U_q(1)$ generated by :

$$q = 3q_5 - q_{\overline{10}} \quad (9)$$

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^{*)} As in Ref. 1), Lorentz indices are not explicitly shown in the following.

An effective Lagrangian with the required properties is then

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\Psi} i \not{D} \Psi + |D_\mu \varphi_\alpha|^2 + |D_\mu \chi^\alpha|^2 + \frac{Q^2}{2A} + \frac{i}{2} Q \not{D} \frac{M}{M^\dagger} - V \quad (10)$$

where the terms quadratic and linear in Q enforce Eqs. (4) and (7), respectively, and $V(\phi, \chi, \psi)$ is a $G_C \otimes \tilde{G}_F$ invariant potential. As in the QCD example, $Q(x)$ can be eliminated⁸⁾ and gives an extra contribution to the potential :

$$\mathcal{V}_{\text{anomaly}} = -\frac{1}{8} A \log^2 (M/M^\dagger) \geq 0 \quad (11)$$

Obviously, such a term requires $M, M^\dagger$ to have a VEV in order to be well defined. Recalling the definition of M , Eq. (8), this requires aligned Vev's for ϕ_α and χ^α , e.g.,

$$\langle \varphi_\alpha \rangle_0 = F \delta_{\alpha 5} \quad ; \quad \langle \chi^\alpha \rangle_0 = F' \delta_{\alpha 5} \quad (12)$$

As a result, $SU(5)$ breaks down to $SU(4)$, while the global $U_q(1)$ symmetry is replaced¹⁾ by a new global $U(1)$ invariance. This is confirmed by a check of the massless scalar spectrum : of the 20 real bosons ϕ_α, χ^α only 10 are possible Goldstones. However, nine are eaten up à la Higgs and one acquires a mass of order $A^{\frac{1}{2}} F^{-1}$ from the anomaly. Also, fermions acquire masses of order F or F' with the exception of one fermion (ψ_5) which does not enter in the condensates (12). Such a massless fermion saturates⁴⁾ the anomaly equation⁶⁾ for the $U_q(1)$ unbroken flavour symmetry.

In the complementary symmetric picture, this state can be interpreted as an $SU(5)$ singlet massless "baryon" saturating the $U_q(1)$ anomaly conditions. The strong anomaly equation is instead saturated by the four-fermi scalar M .

In conclusion, our results agree, in this example, with those of DRS. The new information brought in by our method is, apparently, the inevitability of tumbling once $A > 0$, the necessity of the two VEV's to align and (perhaps) the lack of any reason for F and F' to be very different from each other and from $A^{\frac{1}{4}}$. Although we have not used any single-gauge boson exchange approximation or (a fortiori) any MAC criterion, the MAC and second MAC choices turned out to be by far the simplest ones for constructing a correct strong anomaly term.

(ii) QCD : $G_c = SU(N)$

$$\underline{F} = f \underline{N} + f \underline{\bar{N}} = (\psi_{\alpha,i}, \psi_a^\alpha); \begin{matrix} \alpha=1 \dots N \\ i,a=1 \dots f \end{matrix}$$

Clearly

$$\hat{G}_F = U(f) \otimes U(f) \quad (13)$$

However, under

$$\psi_{\alpha,i} \rightarrow e^{i\lambda} \psi_{\alpha,i}; \quad \psi_a^\alpha \rightarrow e^{i\bar{\lambda}} \psi_a^\alpha$$

$$\mathcal{L} \rightarrow \mathcal{L} + f(\lambda + \bar{\lambda}) Q(x); \quad G_F = SU(f) \otimes SU(f) \otimes U(1) \quad (14)$$

If we want to construct an f_{eff} by using colour singlet composites :

$$M_{ia} = \psi_{\alpha,i} \psi_a^\alpha \quad (15)$$

the correct Lagrangian is ⁸⁾ :

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + \frac{Q^2}{2A} + \frac{i}{2} Q \log(\det M / \det M^\dagger) \quad (16)$$

This implies that $\det \langle M \rangle \neq 0$ and selects as a stable choice :

$$\langle M_{ia} \rangle_0 = F_\pi \delta_{ia} \quad (17)$$

i.e., $G_F \supset SU_V(f) \otimes U(1)$ in agreement with other arguments ¹⁰⁾.

It thus appears that we can offer the above elegant rederivation of the results of Refs. 7), 9) and 10). We can, however, ask if a different pattern can be chosen by using colour-non-singlet composites, bearing in mind a possible symmetric reinterpretation at the end. We now show how this is possible for any odd N and $f = 2$.

Consider the scalar coloured composites :

$$M_{\alpha_1 \alpha_2} = -M_{\alpha_2 \alpha_1} = \epsilon_{ij} \psi_{\alpha_1, i} \psi_{\alpha_2, j}; \quad \bar{M}^{\alpha_1 \alpha_2} = \epsilon_{ij} \psi_i^{\alpha_1} \psi_j^{\alpha_2} \quad (18)$$

Both M and $\bar{M}$ are $SU(2) \otimes SU(2)$ invariant, but transform under $U_V(1)$ and $U_A(1)$. A term with the correct $U_A(1)$ transformations (14) can be constructed as :

$$\mathcal{L}_{anom} = \frac{i}{2} Q \log (M_{\alpha_1 \alpha_2} \bar{M}^{\alpha_1 \alpha_2} / h.c.) \quad (19)$$

Equation (19) requires $\langle M \rangle, \langle \bar{M} \rangle \neq 0$, e.g.,

$$\langle M_{12} \rangle = - \langle M_{21} \rangle = \langle \bar{M}^{12} \rangle = - \langle \bar{M}^{21} \rangle = F \quad (20)$$

The VEV (20) breaks $SU(N)$ to $SU(N-2) \otimes SU(2)$ and $G_F = SU(2) \otimes SU(2) \otimes U_V(1)$ to $SU(2) \otimes SU(2) \otimes U_V(1)'$ where the generator of $U_V(1)'$ is a suitable combination of baryon number and a generator of $SU(N)$. The process can be continued until

$$G_c \supset SU(2) \otimes SU(2) \otimes \dots \otimes SU(2) \quad \left(\frac{N-1}{2} \text{ factors} \right)$$

$$G_F \supset SU(2) \otimes SU(2) \otimes U_q(1) ; \quad q = B\text{-diag} (1, 1, \dots, 1, -N) \quad (21)$$

The final spectrum consists of just two massless doublets $\psi_{N,i}, \psi_j^N$ which satisfy 't Hooft's equations for the $SU(2) \otimes SU(2) \otimes U_q(1)$ anomalies. The model can be reinterpreted in a symmetric picture where the spectrum consists of a $(p,n), (\bar{p}, \bar{n})$ system with :

$$(p, n) = B_i = \psi_{i,N} \varepsilon^{N \alpha_1 \alpha_2 \dots \alpha_N} M_{\alpha_1 \alpha_2} M_{\alpha_3 \alpha_4} \dots M_{\alpha_{N-1} \alpha_N} \quad i = 1, 2 \quad (22)$$

It is easy to see that $B_i, \bar{B}_i$ saturate the anomaly equations of the original $G_F = SU(2) \otimes SU(2) \otimes U_V(1)$. It is amusing to observe that, on the other hand, the anomalous Ward identity ⁷⁾ is saturated by a massless "baryonium" state of the type ¹¹⁾ :

$$\varepsilon^{N \alpha_1 \dots \alpha_N} \varepsilon_{N \beta_1 \dots \beta_N} M_{\alpha_1 \alpha_2} \dots M_{\alpha_{N-1} \alpha_N} \bar{M}^{\beta_1 \beta_2} \dots \bar{M}^{\beta_{N-1} \beta_N} \quad (23)$$

i.e., an $SU(2) \otimes SU(2)$ singlet made of $(N-1)$ quarks and $(N-1)$ antiquarks ^{*}). This state gets a mass of order $\frac{1}{A^2 F}$ from the $U(1)$ anomaly.

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^{*}) Light states of this type accompanying light fermions have been shown to occur in two-dimensional QCD with a small quark mass ¹²⁾.

It seems that, for $f = 2$ and any N , we have been able to avoid the "results" of Refs. 10) and 7). The first of these results has been avoided by allowing baryons to contribute to the anomaly at any N [in fact with coupling N to the $U_V(1)$ charge]. As for the argument of Ref. 7), based on the fermion loop expansion, it was assuming a well-defined meaning for the flavour content of a quark loop. Both in the Higgs and in the symmetric picture, our VEV implies a mixing of the two flavours and this can be shown to avoid the necessity of a set of massless Goldstones (out of which all but one remain massless). In the chiral invariant solution discussed above, only one singlet would-be-Goldstone is required at the lowest fermion loop level and acquires a mass at higher orders like the $U(1)$ particle of QCD ⁸⁾.

(iii) QCD ; $G_c = SU(3)$, $f = 3$

with partial breaking of chirality

In this case ^{*)}, unlike the $f = 2$ case, we are unable to construct a $G_F = SU(3) \otimes SU(3) \otimes U(1)$ invariant solution in agreement with Ref. 6). It turns out, however, that, besides the standard Goldstone solution, one can construct other possible (although dynamically very unlikely) breaking patterns based on the composite scalars

$$M^{\gamma,k} = \varepsilon^{\alpha\beta} \varepsilon^{kij} \psi_{\alpha,i} \psi_{\beta,j} ; M_{\alpha\beta,ij} = (\psi_{\alpha,i} \psi_{\beta,j} + i \leftrightarrow j)$$

$$M \equiv M^{\alpha,i} M^{\beta,j} M_{\alpha\beta,ij} ; \int \mathcal{L} = i Q \log M \quad (24)$$

In the Higgs picture, taking $\langle M^{3,3} \rangle$, $\langle M_{33,33} \rangle \neq 0$, one breaks $SU(3)_C$ to $SU(2)_C$ and G_F to $H_F = SU(2) \otimes SU(2) \otimes U(1)'$. Two fermion doublets, $\psi_{3,k}$ and $\psi_{3,\ell}$ ($k, \ell=1,2$), remain massless and saturate the H_F anomaly equations. In the symmetric picture, two corresponding massless "baryon" doublets saturate the anomalies of the $SU(2) \otimes SU(2) \otimes U_V(1)$ subgroup of G_F . One can see, however, that other G_F anomalies require Goldstone boson contributions in order to be saturated, with the amplitudes for $\eta_8 \rightarrow 2\gamma$, etc. altered by the fermionic contributions. In other words, we have an amusing (though unrealistic) example of a chiral symmetry which is realized partly à la Goldstone and partly à la Wigner (i.e., with massless fermions). Amusingly, such types of realizations have been seen to occur in two-dimensional QCD ¹²⁾.

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^{*)} Most of the material presented in this paragraph has been worked out in collaboration with R. Barbieri.

4. - We have worked out several other examples of tumbling as induced by the anomaly, unfortunately without finding realistic examples. In several cases, the anomaly resolves ambiguities inherent in the MAC criterion. In other cases, the strong anomaly seems to suggest a different tumbling pattern than the one of MAC. An example of this type is ⁴⁾ :

$$G_c = SU(7)_c, \quad \tilde{F} = 3 [3]_7 + 2 [5]_7 \quad (25)$$

in the notation of Refs. 1), 2) for representations. DRS break directly $SU(7)_c$ to $SU(4)_c$ whereas we can preserve up to the very end an $SU(5)_c$. We have found, however, that the final low energy spectrum is consistent with that of Ref. 4), yet the two schemes may differ at intermediate energy scales. Another amusing point concerns secondary mass generation of otherwise massless fermions. In Ref. 4), one such mechanism was presented which goes via broken (heavy) generators. The resulting fermion mass was of order

$$M_F^{DRS} \approx v/M^2 \approx F'^3 F^{-2} \quad (26)$$

for two expectation values F, F' with $F' \ll F$. In our scheme, we find the possibility of a different secondary mass generation by the fact that the anomaly occasionally requires the direct mixing of light and heavy fermions, i.e.,

$$\langle \psi_L \psi_H \rangle = F' ; \quad \langle \psi_H \psi_H \rangle = F \gg F' ; \quad \langle \psi_L \psi_L \rangle = 0 \quad (27)$$

In the one-MAC-at-a-time approach of DRS, ψ_H gets a mass $O(F)$ and is disregarded at the next stage. On the other hand, if the anomaly requires Eqs. (27) to be valid, this immediately gives

$$M_{F_L} \approx F'^2/F \gg F'^3 F^{-2} \quad (28)$$

In the $SU(7)_c$ example mentioned above, we find this mechanism to be necessary for consistency with the anomalies at all scales.

Our main conclusions can be summarized as follows.

- A) - A pure gauge theory (defined by the group G_c) possessing a non-trivial topological charge reacts non-trivially to the introduction of fermions. Various possibilities emerge according to the way the fermions transform under G_c .
- B) - Insisting on fermi-bilinear composite scalars, the tumbling of G_c necessarily emerges if the fermion content is not real.
- C) - Simplicity in fulfilling the strong anomaly can be used as a criterion (alternative or complementary to MAC) for choosing among various possible breaking patterns.
- D) - A symmetric (i.e., G_c preserving) reinterpretation is often possible at the expense of introducing composites of more than two fermions, and solutions to 't Hooft's conditions are thus generated.

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