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PARTON DISTRIBUTIONS IN BARYONS AND MESONS
CONSISTENT WITH THE NAIVE QUARK MODEL AND QCD

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A B S T R A C T

A prescription is presented to construct parton x distributions in baryons and mesons. The momentum fraction carried by a valence quark is given by its contribution to the hadron mass, as estimated in the non-relativistic quark model renormalized by QCD. Flavour and spin-spin interaction mass breaking determine the local $SU(6)$ breaking of parton distributions. In the case of the proton one of the u quarks is fast and carries the spin of the proton while the other u quark and the d quark are slower, average their spins to zero and have identical x distributions. Recent polarization data (large asymmetry at $x > 0$) and similarities between proton and neutron (charge particle ratios) at small x are well understood. While strong breaking in valence quark distributions is predicted (for instance s quark being much faster than $\bar{u}$ quark in the kaon) the fully QCD generated gluon and quark sea distributions are expected to carry Q^2 dependent fractions of momentum which are roughly universal. Some examples of hyperon parton distributions, which may become useful in the future, are shown.

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1. - INTRODUCTION

Quarks and gluons seem to be the basic ingredients required in understanding strong interaction physics at present energies and ranges of momentum transfer. It is, however, not always clear that when quarks and gluons are introduced in different contexts one is talking about the same objects. Quarks and gluons are, at least, used in non-relativistic quark model phenomenology as well as in deep inelastic, e^+e^- and, in general, large Q^2 processes. One, of course, knows that the constituent quarks of the non-relativistic quark model cannot be the current quarks of deep inelastic scattering, and that the confining and interacting gluons of the interquark potential have not the same properties as the radiated gluons at large Q^2 . The problem, however, remains : the quarks in the non-relativistic quark model and in the large Q^2 parton model, as far as quantum numbers are concerned (colour, flavour, spin) are indeed the same in both cases.

In order to relate the quarks of the non-relativistic quark model to the quarks of large Q^2 processes a better knowledge of the theory, QCD, at low Q^2 is required. In particular, the understanding of long-distance and collective effects and of the rôle of the different mass scales involved does not seem yet sufficient to allow the rigorous treatment of this problem in the framework of the theory.

In this situation a more intuitive approach, as the one followed here, which takes into account some essential features of QCD ¹⁾ and incorporates some of the correct results of both the quark model ²⁾ and the parton model ³⁾ may be justified.

The starting point of our discussion is very simple. In the most naïve non-relativistic quark model the hadron is made up of n quarks, which are almost free and practically at rest in the hadron rest frame $M_H \simeq n \bar{m}_q$, where M_H is the mass of the hadron and $\bar{m}_q$ the effective mass of the bound quark. If one boosts the hadron to the infinite momentum frame, $P_H = \gamma M_H$, each quark will acquire a momentum $p_q = \gamma \bar{m}_q$ and the quark distribution function is essentially a δ function at $x = p_q/P_H \sim 1/n$. In the case of the proton one would obtain a distribution peaked at $x \sim 1/3$ while the experimental distribution, at $Q^2 \simeq 2 \text{ GeV}^2$, peaks at $x \sim 0.2$. On the other hand, in the non-relativistic quark model, the gluons, which may be responsible for the potential and for some of the physical properties of

the quarks, never appear as separate objects with interacting properties. Thus, in this picture, the momentum carried by the valence quarks must add to one.

Unfortunately, it is difficult to go beyond the over-simplified δ function quark distribution and consistently boost a quark model energy distribution into an x distribution at infinite momentum ^{4),5)}. In our approach we take the attitude of accepting that the shape of the x distribution is related to the parameters (essentially the quark masses, radii) of the non-relativistic quark model and follow the δ function limit suggestion that $\bar{m}_i/M = \langle x_i \rangle$. As a next step in trying to go beyond the δ function limit we give the distribution, via a convenient ansatz, a width controlled by a parameter K measuring the quark energy fluctuations ($1/K \rightarrow 0$ reproduces the δ function limit) and related to the quark momentum distribution.

Two intriguing features of the quark distribution functions in the proton are of interest to us ^{*}). One is the break of $SU(6)$ locally in x . The other is the fact that more than 50% of the momentum at $Q_C^2 \sim 2 \text{ GeV}^2$ is not carried by the quarks. Regarding the first question we relate the difference between the up quark $u(x)$ and down quark $d(x)$ distributions in the proton to the difference between the effective quark masses of the interacting u and d quarks, namely $\langle x_u \rangle - \langle x_d \rangle \sim (M_\Delta - M_N)$. Concerning the second question, we show that a QCD calculation naturally explains the loss of momentum by gluons and quark sea radiation as the scale is changed from the typical hadronic scale, $Q_h^2 \sim 1/L^2 \sim 0.2 \text{ GeV}^2$ to the typical deep inelastic scale, $Q_o^2 \sim 2 \text{ GeV}^2$. At this point we agree with the results of Refs. 7) and 8).

The picture we obtain for the valence quarks in the proton is the following. As in the non-relativistic quark model the two u quarks have different effective masses, one is heavier, u_f (fast) and the other is lighter, u_s (slow), with a mass equal to the mass of the d quark; when one boosts the hadron to the infinite momentum frame the u_f distribution, which carries the spin of the proton, gets a larger $\langle x \rangle$, $\langle x_{u_f} \rangle \gg \langle x_{u_s} \rangle$, while the u_s and d distributions which average their spins to zero will carry a smaller fraction of the momentum. The

^{*}) For a general and recent discussion of theoretical and phenomenological aspects of parton distributions, see Ref. 6).

break of $SU(6)$ is thus explained as a mass breaking effect. With such a model one easily understands recent results from polarization data ⁹⁾ and similarities in charged particle production between the proton and the neutron ¹⁰⁾.

Our scheme for treating mass breakings is extended, without difficulty, to any hadron, at least as far as average x is concerned, and estimates of parton distributions in hyperons and kaons are given. For instance, we predict that the s quark in the kaon carries about the double of the momentum of the non-strange quark. These distributions will be useful in future, with the development of hyperon and kaon beams for Drell-Yan and large p_T processes.

The paper is organized as follows. In Section 2 we rederive quark model results for the masses of low-lying baryons and mesons, and re-interpret them in terms of effective quark masses. In Section 3 we introduce our ansatz for the valence parton distribution function, thus generalizing the δ function distribution of the non-relativistic quark model with quarks at rest. In Section 4 we discuss the QCD relation between the x distribution of valence quarks at the hadron scale $Q_h^2 \sim 1/L_h^2 \sim 0.2 \text{ GeV}^2$ and the distributions of quarks and gluons at the typical deep inelastic scale, $Q_0^2 \sim 2 \text{ GeV}^2$. In Section 5 we consider some applications of our model to the proton. In Section 6 our considerations are extended to other baryons and mesons. In Section 7 we finally draw our conclusions.

2. - THE QUARK MODEL AND THE VALENCE QUARK MOMENTUM DISTRIBUTION FUNCTIONS

The non-relativistic quark model, in which hadrons are exclusively made up of valence quarks bounded in a confining potential, is remarkably successful in describing the spectroscopy of hadrons, their classification in $SU(N)_F \times SU(2)_S \times O(3)$ multiplets (flavour $\times$ spin $\times$ orbital angular momentum) in calculating the spin-spin and orbit-spin mass splittings, magnetic moments and, in general, coupling constants ²⁾. The evaluation of hadron mass differences within an $SU(N)_F$ multiplet has been very much refined and simplified in recent years ¹¹⁾. In our discussion we shall follow such calculations, and accurately fit the masses of low-lying light mesons and baryons.

In a non-relativistic model one expects the mass of the composite object to be, in first approximation, given by the sum of the effective masses or energy contributions of the constituents (quarks) :

$$M_H = \sum_{i=1}^n \bar{m}_{i/H} \quad (2.1)$$

where n is the number of quarks and $\bar{m}_{i/H}$ the effective mass of quark i in the binding potential of hadron H . In a confining potential the $\bar{m}_i$'s are not the masses of free quarks but effective masses or energy contributions to the hadron mass which, in fact, depend on hadron H . Equation (2.1) thus defines what we call effective masses.

In order to see how this effective quark mass is built up, let us, step by step, switch on the various terms in the interaction leading to the hadron bound state of mass M_H . In the presence of the over-all scalar colour confining potential, the quarks have universal mass m . In this limit all the low-lying mesons (M) have the mass M_M and all the low-lying baryons (B) have the same mass M_B ($M_\pi = M_K = M_\rho = M_{K^*}$; $M_N = M_\Delta$) with $M_B/M_N = \frac{3}{2}$. Flavour breaking (strangeness,...) introduces additional mass terms, $m_s = m + \delta$ and, for instance, $M_P \neq M_\Lambda$ with $M_\Lambda - M_P = \delta$. Next, the shorter range QCD interaction terms of Coulomb type $(\lambda_i \cdot \lambda_j)$ and Breit-like terms $(\lambda_i \cdot \lambda_j)(S_i \cdot S_j)$ are switched on. The Coulomb term $M_0(n)$ (and with it probably other residual interactions) is supposed to be the same in a given $SU(N)_F \times SU(2)_S$ multiplet, $M_\pi = M_\rho = M_0(2) + 2m$, $M_N = M_\Delta = M_0(3) + 3m$, but will depend on the number of quarks (as the average quark-quark potential energy, and average size of hadrons are both different in mesons and baryons), $M_0(2) \neq M_0(3)$. Finally colour spin-spin terms strongly break $SU(N)_F \times SU(2)_S$ multiplets, $m_\rho > m_\pi$ and $m_\Delta > m_N$, giving contributions of the form $(1/m_i m_j) S(n)$, where S again is a parameter depending on the number of quarks. Putting all these terms together the mass of the hadron H composed of n quarks, k being strange, is given by :

$$M_H = M_0(n) + nm + k\delta + \left\langle \sum_{i,j} \frac{S(n)}{m_i m_j} S_i \cdot S_j \right\rangle_H \quad (2.2)$$

Comparing (2.2) to (2.1) we obtain for the effective mass $\bar{m}_{i/H}$ of quark i in hadron H ,

$$\bar{m}_{i/H} = \frac{M_0(n)}{n} + m_i + \frac{1}{2} \left\langle \sum_j \frac{S(n)}{m_i m_j} S_i \cdot S_j \right\rangle_H \quad (2.3)$$

where $m_i = m$, for u, d quarks and $m_i = m(1 + \delta/m)$ for s quarks.

By fixing the six parameters m , δ/m , $M_0(2)$, $M_0(3)$, $S(2)$, $S(3)$ we can compute the various effective masses $\bar{m}_{i/H}$ and the masses of the low-lying mesons and baryons. The result is shown in Table 1. One should note that m and δ/m are the same quantities entering into the calculation of ratios of the magnetic moments and the electromagnetic radii, and that the values used here are quite consistent with such calculations ^{11),12)}. A modification of the colour Breit term, as suggested by Lipkin ¹¹⁾, should not appreciably change our values of $\bar{m}_{i/H}$ but may become important when computing charmed hadron masses.

Looking back to Table 1 one notices that the two u quarks in the proton have different effective masses. The reason is the following. In the proton (uud) we can always select a u and a d quark in the state of spin zero. The spin-spin interaction contribution to the mass arises exclusively from this u,d pair and is negative ($S_u \cdot S_d = -3/4$). This means that the u and d quarks with over-all spin zero have an effective smaller mass than the u quark which carries the proton spin. We shall denote the lighter u quark as u_s (slow) and the heavier u quark as u_f (fast).

Next we shall boost the hadron and the quarks from the hadron rest frame to the infinite momentum frame, neglecting completely the three-momentum of the quarks

$$M_H \longrightarrow P_H = \gamma M_H, \quad \bar{m}_{i/H} \longrightarrow P_{i/H} = \gamma \bar{m}_{i/H} \quad (2.4)$$

with

$$\langle x_{i/H} \rangle = \frac{\bar{m}_{i/H}}{M_H} \quad (2.5)$$

Clearly, the x distribution function for quark i/H is, in this rough limit, not very realistic : a δ function at $\langle x_{i/H} \rangle$ with $\sum \langle x_{i/H} \rangle = 1$.

Our assumption is now, in the spirit of the conventional quark model, that relation (2.5) remains valid even when we have a more realistic situation with a distribution in energy in the rest frame and inclusion of three-momentum of quarks. More specifically we expect, for quarks i and j in hadron H , the relation

$$\frac{\bar{m}_{i/H}}{\bar{m}_{j/H}} = \frac{\langle x_{i/H} \rangle}{\langle x_{j/H} \rangle} \quad (2.6)$$

still to be true. Taking the proton case, we obtain, using Table 1,

$$\frac{\bar{m}_{u_s} + \bar{m}_{u_f}}{\bar{m}_d} = \frac{650}{288} = 2.25 \quad (2.7)$$

to be compared with the ratio of the average momentum carried by the u valence quarks relative to the d valence quarks in deep inelastic scattering¹³⁾ :

$$\frac{\langle x_u \rangle}{\langle x_d \rangle} = \frac{.270}{.124} = 2.18 \quad (2.8)$$

The agreement between (2.7) and (2.8) is, we believe, suggestive and we shall take it as evidence in favour of our attempt to relate the share of the hadron mass by the quarks (in the rest frame non-relativistic model) and the share of the hadron momentum by the quark-partons in large Q^2 processes.

So far, with the simple boost (2.4), we have not generated gluons and the parton sea required by deep inelastic scattering data. In the non-relativistic quark model there is no need for explicit introduction of additional entities, as gluons and quark-antiquark pairs, and it is this fact that makes the quark model successful and with a predictive power. We must then take the attitude that the gluon and the quark sea distributions are both fully generated by QCD bremsstrahlung. This we shall discuss later after introduction of an ansatz for the valence quark parton distributions.

3. - AN ANSATZ FOR VALENCE QUARK DISTRIBUTIONS

We have argued, in the previous section, that the effective quark masses, as determined in the non-relativistic quark model, should affect the quark momentum distribution functions measured in deep inelastic scattering. This possible relation seems perhaps quite acceptable in principle and should be hopefully derivable in the theory. However, we shall not attempt to derive the relation between masses and distribution functions from first principles. The theory at this stage (with long-distance and collective effects being not yet well understood) does not seem to be able to predict constituent masses, hadron masses, momentum distribution functions. Thus we here just extract from the previous section the result (2.5), $\langle x_i \rangle = m_i/M$, and suggest an ansatz for the valence quark distribution function which makes transparent the rôle of the effective constituent masses in the shape of the distribution functions and has some convenient properties.

We write the probability distribution function for the valence quark i with momentum fraction x in a conventional form

$$x p_i(x) = \frac{1}{B(\alpha_i, \beta_i + 1)} x^{\alpha_i} (1-x)^{\beta_i} \quad (3.1)$$

with

$$\alpha_i \equiv K \bar{m}_i \quad (3.2)$$

and

$$\beta_i \equiv K (M - \bar{m}_i) - 1 \quad (3.3)$$

where K is a constant (presumably universal for light hadrons) with dimensions $(\text{mass})^{-1}$ and $\bar{m}_i$ and M are the quark and hadron masses (as given in Table 1). $B(\alpha_i, \beta_i + 1)$ is the beta function and $x p_i(x)$ is normalized to 1. Equations (3.2) and (3.3) represent our main assumptions in this paper, and they are supposed to be valid for $Q_h^2 \sim 1/L^2$ of the order of some hadronic mass scale.

The ansatz (3.1) with (3.2) and (3.3) has the following properties.

i) From Eqs. (3.1), (3.2) and (3.3) we obtain

$$\langle p_i \rangle_2 \equiv \langle x_i \rangle = \frac{\alpha_i}{\alpha_i + \beta_i + 1} = \frac{\bar{m}_i}{M} \quad (3.4)$$

independently of the value of K . The non-relativistic quark model suggestion that $\langle x_i \rangle = \bar{m}_i/M$ is here exactly satisfied and relation (2.5) is thus incorporated in our ansatz. In general, for the higher moments of the distribution one has

$$\langle p_i \rangle_n = \langle p_i \rangle_{n-1} \frac{K \bar{m}_i + (n-2)}{K M + (n-2)}, \quad n \geq 2 \quad (3.5)$$

From (3.5) it can be seen that K is a measure of the width of the x distribution. In the limit $KM \rightarrow \infty$ the distribution becomes a zero width δ function at $x_i = \bar{m}_i/M$ (quarks at rest in hadron rest frame). In the limit $KM \rightarrow 0$ the width is maximal, with the distribution peaking at $x=0$ and 1 (the quark has some probability of carrying zero energy and some probability of carrying the full hadron mass).

ii) In the absence of flavour breaking and quark-quark spin interaction, Eq. (3.1) with (3.3) is compatible, in the $x \rightarrow 1$ limit, with the counting rules¹⁴⁾. In fact, in that limit, $M = nm$, where n is the number of quarks and $N = (n-1)$ the number of spectators and (3.3) becomes

$$\beta_i \longrightarrow Km(n-1)-1 = 2N-1 \quad (3.6)$$

with $Km = 2$. Flavour breaking, spin-spin interaction and QCD bremsstrahlung naturally spoil the naïve counting rules.

iii) Equation (3.1) is written in a form close to Regge-parton model ideas^{3),15)}. At small x , the x dependence of constituent i distribution should be determined by the trajectory containing it, $x p_i \xrightarrow{x \rightarrow 0} x^{1-\alpha_{i\bar{i}}}$, while in the limit $x \rightarrow 1$ it should depend on the trajectory containing the spectator quarks, $x p_i \xrightarrow{x \rightarrow 1} f(1-x) (1-x)^{1-\alpha_{jk\dots j\bar{k}\dots}}$.

In order to illustrate this point we compare the u(d) quark and s quark distribution functions in the $x \rightarrow 0$ limit. In the Regge-parton model we have

$$\frac{x P_u(x)}{x P_s(x)} \xrightarrow{x \rightarrow 0} x^{(1-\alpha_p)-(1-\alpha_\varphi)} \sim x^{\alpha(m_p^2 - m_\varphi^2)} \sim x^{8\alpha' m(m_s - m)} \quad (3.7)$$

With $K = 8\alpha' m$, Eq. (3.7) would have been the result obtained using Eqs. (3.1) and (3.2). A similar conclusion is reached by comparing the ratios of distributions in the $x \rightarrow 1$ limit in the two approaches. In the proton case, the difference between the u and d distribution at $x \rightarrow 1$ is, in the Regge-parton model given by $\alpha_{uuu\bar{u}} - \alpha_{ud\bar{u}\bar{d}}$ which is, in our case, related to $\bar{m}_{u_F} - \bar{m}_{u_S}$. One should, however, note that while our expression has exponents linear in the masses in the Regge-parton model one usually uses the square of the masses.

iv) Equation (3.1) is in a form appropriate to asymptotic freedom leading order calculations ^{1), 16)} for moderate variation of Q^2 , and hopefully appropriate also to higher order calculations. The values of α_i and β_i , (3.2) and (3.3), are valid at the hadronic scale Q_h^2 . The renormalization group calculation of the moments of the distribution as a function of Q^2 leads, for the n^{th} moment, to a relation in the form

$$\langle P_i(Q^2) \rangle_n = \langle P_i(Q_0^2) \rangle_n F_n(Q_0^2, Q^2) \quad (3.8)$$

where the F_n 's are universal functions of the theory. The ratios $\langle x_i \rangle / \langle x_j \rangle$ thus continue, at all Q^2 , to be given by the quark model estimates, $\bar{m}_i / \bar{m}_j$, even with $\langle x_i \rangle$ and $\langle x_j \rangle$ decreasing with Q^2 (due to gluon emission and pair creation).

v) The distribution (3.1) can be obtained from a joint quark distribution where the quarks are independent (uncorrelated) except for the momentum conservation constraint. In the baryon case we have

$$x_1 x_2 x_3 P(x_1, x_2, x_3) \sim x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} \delta(1 - x_1 - x_2 - x_3) \quad (3.9)$$

with α_1 given by (3.2), and

$$x_1 p(x_1) \equiv \int \frac{dx_2}{x_2} \frac{dx_3}{x_3} x_1 x_2 x_3 P(x_1, x_2, x_3) \quad (3.10)$$

is then given by (3.1). The evidence from parton model, as well as from quark model, is that quarks behave as if they were independent inside hadrons. Confinement and interactions renormalize masses but keep the quarks behaving as if almost free without additional structures being generated, and that would be the reason why Bjorken scaling "works" at relatively small values of Q^2 . We thus consider Eq. (3.9) to be rather acceptable and at least consistent with our treatment of rest frame masses.

4. - FROM THE HADRONIC SCALE TO LARGE Q^2

We have now to face the problem of relating the quark distribution functions, Eqs. (3.1)-(3.3), defined at some hadronic scale parameter $Q_h^2 = 1/L^2$, to the large Q^2 quark distributions extracted from deep inelastic structure functions. The hadronic scale parameter should be of the order $1/L^2 \sim 0.20 \text{ GeV}^2$ and the QCD running coupling constant is greater than 1 at such small value of Q^2 . In general, in order to relate different Q^2 distributions, we shall simply use, for the valence quark distributions, the renormalization group result

$$\langle p_i(Q^2) \rangle_n = \langle p_i(Q_0^2) \rangle_n \exp(-\bar{\gamma}^{(n)} G s) \quad (4.1)$$

written in the familiar form of the leading order calculation, with

$$\bar{\gamma}^{(n)} = 1 - \frac{2}{n(n+1)} + 4 \sum_{k=2}^n \frac{1}{k}$$

and $G = 4/(33 - 2N_f)$. Regarding s , the Q^2 dependent parameter, at large Q^2 it will be given by the leading order expression $s = \ln(\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2))$ with a fixed Λ , but at low Q^2 one may have a different dependence on Q^2 or have n dependent Λ 's, to account for higher order corrections. In going from the standard $Q_0^2 \simeq 2 \text{ GeV}^2$ value to $Q_h^2 \sim 1/L^2 \sim 0.20$ we only need the quantity (Gs) and so the exact relation between s and Q^2, Λ, n is not required. This procedure is close to the proposal of Ref. 8).

The G_s dependence of the parameters α_i and β_i , following Ref. 16), will be taken linear and adjusted to agree with (4.1). We thus write

$$\alpha_i(Q^2) = \alpha_i(Q_0^2) [1 + a G_s] \quad (4.2)$$

$$\alpha_i(Q^2) + \beta_i(Q^2) + 1 = (\alpha_i(Q_0^2) + \beta_i(Q_0^2) + 1) [1 + b G_s] \quad (4.3)$$

and then compare (4.2) and (4.3) to (4.1). By using just the second moment and expanding (4.1) - $\bar{\gamma}^{(2)} G_s$ is always smaller than 1 for our applications - we obtain the approximate relation ^{*})

$$-a \simeq b \simeq \frac{1}{2} \bar{\gamma}^{(2)} \simeq 1.33 \quad (4.4)$$

These values roughly agree with the ones of Ref. 16). As it is mainly the Q^2 dependence of the coupling constant that determines the Q^2 dependence of the moments, the coefficients $\bar{\gamma}^{(n)}$ are universal, and the same is approximately true for the slope parameters a and b .

As input we take a conventional d quark distribution at $Q_0^2 = 2 \text{ GeV}^2$, with parameters

$$\alpha_d(Q_0^2) = 0.65 \quad \text{and} \quad \beta_d(Q_0^2) = 3.50 \quad (4.5)$$

These values of α and β give $\langle x_d \rangle = \langle x_{uS} \rangle = 0.126$, which is a compromise between F.F. ¹³⁾ ($\langle x_d \rangle = 0.124$) and B.G. ¹⁶⁾ ($\langle x_d \rangle = 0.163$). The curve has a maximum at 0.157 (0.15 in F.F. and 0.20 in B.G.). The distribution (3.1) with (4.5) at Q_0^2 may be made to agree with (3.1), (3.2) and (3.3) at $Q_h^2 \sim 1/L^2$ using the Q^2 dependences determined by (4.2) and (4.3). In this way one fixes $(G_{s_h}) = -0.314$ ^{**)} and $K = 3.20 \text{ GeV}^{-1}$.

^{*}) For a rigorous treatment, a and b should be fitted as in Ref. 16). Here only the second moment is correctly treated.

^{**)} If we use the leading order definition of s , $s = \ln(\ln Q_h^2 / \Lambda^2) / (\ln Q_0^2 / \Lambda^2)$, and take $G = 4/25$ and $\Lambda = 0.3 \text{ GeV}$ we obtain $Q_h^2 \sim 1/L^2 \sim 0.14 \text{ GeV}^2$ for the magnitude of the hadronic basic scale.

Once K and the scale factor G_s relating Q_h^2 to Q_0^2 are known it is straightforward to compute, using again (3.1), (3.2), (3.3), (4.2) and (4.3), the fast quark u_f distribution at $Q_0^2 = 2 \text{ GeV}^2$. One obtains

$$\alpha_{u_f}(Q_0^2) = 0.82 \quad \text{and} \quad \beta_{u_f}(Q_0^2) = 3.33 \quad (4.6)$$

and $\langle u_f \rangle = 0.1592$. The total fraction of momentum carried by the valence quarks is 41%, to be compared with 39% in F.F. and 49% in B.G.

We note that having started at Q_h^2 with a distribution with 100% of the momentum being carried by the valence quarks we generated at $Q_0^2 \simeq 2 \text{ GeV}^2$ a distribution where 59% of the momentum is not carried by them. A calculation simulating the leading order QCD of production of gluons and sea-quarks when s varies from $s(Q_h^2)$ to $s(Q_0^2)$ gives, for the share of the missing 59% of the momentum, 45% for the gluons and 14% for the quark sea. These fractions agree with experiment and previous QCD calculations ^{7),8)}.

5. - APPLICATIONS

Our purpose in this paper is mainly to try to relate pieces of information from different sources, quark model spectroscopy and deep inelastic parton model, using the QCD framework. We do not expect detailed agreement with data as the Q^2 dependence of the higher ($j > 2$) moments of the parton distributions is treated in an approximate way. However, we should check that our u_f curve is reasonable. In the not too small x region, away from sea contamination, u_f is related to the difference of p, n structure functions

$$\nu W_2^p - \nu W_2^n = \frac{1}{3} u_f \quad (5.1)$$

A comparison with data is shown in Fig. 1.

We would now like to comment on two recently reported deep inelastic electron scattering results ^{8),9)} and their interpretation in our picture of the proton.

Because of spin-spin interaction the two u quarks in the proton do not carry the same energy, one is fast and the other is slow, the latter has an energy distribution identical to the d quark. The u_f carries the spin, u_s and d average their spins to zero.

Two simple results follow from such a picture.

- i) The parton distribution measured with net helicity parallel to the proton is simply u_f . Experimentally ⁹⁾, using polarized electrons on polarized protons one measures $A_1 = (\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}) / (\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}})$ and determines distributions weighted by charges. In terms of u_f and u_s we have :

$$2x g_1^p \equiv v W_2^p A_1 = \frac{4}{9} u_f(x, Q^2) + \frac{1}{9} u_s(x, Q^2) \quad (5.2)$$

In Fig. 2 we show our curves, using Eqs. (3.1), (4.5), (4.6), (4.2) and (4.3), for two values of Q^2 , $Q_0^2 = 2 \text{ GeV}^2$ and $Q^2 = 60 \text{ GeV}^2$. The data points are in the region $1 < Q^2 \leq 4 \text{ GeV}^2$. The disagreement with data in the low x region is probably the result of the interaction of the valence quarks with gluons ¹⁸⁾ (necessarily lowering the net spin of the valence quarks) as is reflected in the QCD violation of Bjorken sum rule in 15-20% ¹⁹⁾.

- ii) As u_s and d have identical distribution functions in the low x region the deep inelastic physics of the proton and the neutron should be identical in that region even when the distributions are weighted by the squares of the charges. An analysis of the data at $Q^2 \simeq 2 \text{ GeV}^2$ in the $\omega = 1/x \lesssim 40$ region below the dominance of the neutral sea and $\omega \gtrsim 5$, above the dominance of u_f , should reveal the similarities of proton and neutron distributions. Such an effect is indeed observed in charged hadron production data ¹⁰⁾. If one computes the charge ratios $R_H(x) \equiv N_{h+} / N_{h-}$ as functions of x for proton and neutron these functions can be written as :

$$R_p = \frac{4Z + u_s(x)/u_f(x) [4Z+1]}{4Z + u_s(x)/u_f(x) [4 + \tilde{Z}]} \quad , \quad (5.3)$$

$$R_n = \frac{1 + u_s(x)/u_f(x) [4Z+1]}{\tilde{Z} + u_s(x)/u_f(x) [4 + \tilde{Z}]} \quad (5.4)$$

where

$$Z = \int_{z_0}^1 (D_u^{\pi^+} + D_u^{K^+}) dz / \int_{z_0}^1 D_u^{\pi^-} dz \quad (5.5)$$

and

$$\tilde{Z} = \int_{z_0}^1 D_u^{\pi^+} dz / \int_{z_0}^1 D_u^{\pi^-} dz \quad (5.6)$$

Equations (5.3) and (5.4) are easily obtained in the framework of the parton model. The approximation is made that, apart from π^+ and π^- , data on charged particles may contain K^+ from u quark fragmentation. Other contaminations are known to be much less important in the fragmentation region ($z_0 = 0.3$ in experiment). For charged pion data $Z = \tilde{Z}$ in Eqs. (5.5) and (5.6).

From Eqs. (5.3) and (5.4) one sees that as

$$x \rightarrow 1, R_p \rightarrow Z \text{ and } R_n \rightarrow \frac{1}{\tilde{Z}} \quad (5.7)$$

with $\pi^\pm$ data, $R_p R_n \xrightarrow{x \rightarrow 1} 1$. As

$$x \rightarrow 0, R_p = R_n \rightarrow \frac{4Z+1}{4+\tilde{Z}} \quad (5.8)$$

(finite)

It is interesting that the contamination of data by K^+ does not spoil the equality (5.8). On the other hand, this equality does not result from equal quark seas in n and p¹⁰⁾ because $(4Z+1)/(4+\tilde{Z}) \neq 1$.

We have estimated Z and $\tilde{Z}$ from F.F. to be $Z \simeq 2.6$ and $\tilde{Z} \simeq 1.6$ which gives $R_p = R_n \rightarrow 2$. For very large ω , due to the sea, the equality $R_p = R_n$ remains but will tend to one. For the neutron R_n will thus show a maximum $\simeq 2$ in the region where u_s dominates. These features are clearly seen in data (see Fig. 2). In Fig. 2 we also present the curves of Eqs. (5.3) and (5.4) with u_s and u_f given by Eqs. (4.5) and (4.6), respectively, in the region $\omega < 40$ for $z_0 = 0.3$ and $z_0 = 0.6$.

6. - STRUCTURE FUNCTIONS OF BARYONS AND MESONS

We saw in the previous sections that with the ansatz (3.1) with (3.2) and (3.3) we obtain parton distribution functions for valence quarks which are reasonable, agree, in some limits, with familiar ideas in hadron physics (counting rules, Regge behaviour) and allow for a qualitative and even quantitative understanding of several aspects of the data.

The next step is to ask how to extend the ansatz (3.1), (3.2), (3.3) to other hadrons. We first remind that according to our interpretation the sea and the gluons are fully generated by QCD bremsstrahlung ; for a given Q^2 their share of momentum is roughly the same for any hadron, and their distributions can be calculated knowing the valence distribution functions ^{*}). So we just need to specify the valence distribution to be able to construct structure functions. Even without specifying the parameter K in (3.2) and (3.3), the ansatz (3.1) with the QCD approximation relations (4.2) and (4.3) makes it possible to calculate the average fractions of momentum carried by the valence quarks $\langle x_i \rangle$ in hadron H , starting from the quark model ratios m_i/m_H (Table 1). The resulting $\langle x_i \rangle_H$ for $Q_0^2 \simeq 2 \text{ GeV}^2$ are shown in Table 2 for low-lying baryons and mesons. For baryons in the decuplet the momentum, except for flavour breaking, is equally shared by the quarks. As expected, strange quarks tend to be faster. It is interesting to note that in the K meson the strange quark (antiquark) carries about the double of the momentum of the non-strange antiquark (quark). This result, which is rather natural in our approach, may be of phenomenological interest when comparing K beams to π beams (in Drell-Yan, large p_T processes).

The construction of the full distribution functions stands on a less firm basis. The simplest possibility is to take, at least for baryons,

^{*}) Taking, for the purpose of gluon and sea production, a δ function approximation to the valence quark distribution and neglecting effective quark mass effects, the sea and gluon distributions are well-approximated by

$$x D_i(x, Q^2) = a_i(Q^2) N_q (1-x)^{b_i(Q^2) N_q - 1}$$

where $a_i(Q^2)$, $b_i(Q^2)$ are computable QCD coefficients and N_q is the number of valence quarks.

K to be exactly the same as for the proton. In Table 2 we present the calculated exponents $\alpha_{i/B}$ and $\beta_{i/B}$ at $Q_0^2 \sim 2 \text{ GeV}^2$. In Fig. 4 we show the valence distribution functions for Σ^+ and Ξ^0 . These distributions may become of some use with the development of higher intensity hyperon beams.

Extending the assumption that K is the same also for mesons, we obtain the values of $\alpha_{i/M}$ and $\beta_{i/M}$ also shown in Table 2. One should immediately notice that $\beta_{q/\pi}$ is negative, i.e., the pion distribution diverges as $x \rightarrow 1$. This is theoretically allowed but should rather be taken as a weakness of our approach coupled with the strong assumptions we made. The reason why this happens only for the pion arises from the smallness of the pion mass. An increase of K_M by a factor 1.6 relative to $K \equiv K_B$ makes the pion distribution flat as $x \rightarrow 1$.

7. - CONCLUSIONS

In this paper we suggested an ansatz for valence quark distributions in hadrons which is consistent with the non-relativistic quark model calculation of hadron masses, with the breaking of $SU(6)_{F \times S}$ symmetry locally in the variable x , with QCD calculus for the loss of momentum by radiation of gluons and sea quarks from the valence quarks, and which is in at least qualitative agreement with deep inelastic data on protons and neutrons. The approximate universality of quark model calculations was carried over to the calculation of parton distributions of hadrons and a general prescription was given to construct Q^2 dependent parton distributions for low-lying baryons and, perhaps, mesons. Some examples were presented of hyperon and kaon distribution functions which may become useful in future.

It should be interesting to obtain better insight on our parameter K and relate it in a more precise manner to the quark distribution in the rest frame of the hadron. Its physical meaning as a parameter measuring the energy fluctuation or Fermi motion of the quarks inside the hadron seems rather acceptable.

While preparing this paper we received a preprint ²⁰⁾ in which the rôle of instantons in generating the $SU(6)$ breaking is emphasized and the connection of such a break with the break in the distribution functions is pointed out. As the exact physical origin of the spin-spin interaction is never required in our work, such an interpretation is quite compatible with our proposal here.

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	Baryons m_Q (MeV)	$M_{calc.}$	$M_{exp.}$
p	$m_{uf} = 362$ $m_{us} = 288$ $m_d = 288$	938	Input
Ξ^0	$m_{sf} = 537$ $m_{ss} = 491$ $m_u = 315$	1343	1314
Σ^0	$m_u = 355$ $m_d = 355$ $m_s = 476$	1186	1189
Λ	$m_u = 288$ $m_d = 288$ $m_s = 537$	1113	1114
Δ^{++}	$3 \times m_u = 3 \times 410$	1230	Input
Ξ^{*0}	$m_u = 391$ $2 \times m_s = 2 \times 570$	1531	1532
Σ^{*+}	$2 \times m_u = 2 \times 401$ $m_s = 567$	1369	1382
Ω^-	$3 \times m_s = 3 \times 556$	1668	1672

- Table 1a -

	Mesons m_Q (MeV)	$M_{calc.}$	$M_{exp.}$
π^+	$m_u = 70$ $m_d = 70$	140	Input
K^+	$m_u = 160$ $m_s = 335$	495	Input
K^{*+}	$m_u = 358$ $m_s = 534$	892	Input
ρ	$2 \times m_u = 2 \times 388$	776	Input
ω	$2 \times m_u = 2 \times 388$	776	783
ϕ	$2 \times m_s = 2 \times 515$	1030	1020

- Table 1b -

Table 1 - Effective quark masses and hadron masses, determined using Eqs. (2.1) and (2.3). Input parameters : $m = 292$ MeV ; $\phi/m = 0.6$; $M_0(2) = 34$ MeV ; $M(3) = 209$ MeV ; $S(2)/m^2 = 636$ MeV ; $S(3)/m^2 = 196$ MeV.
a) Baryons ; b) Mesons.

Baryons			
Hadron	$\langle x_i \rangle$	α_i	β_i
p	$\langle x_{uf} \rangle = 0.16$	0.82	3.33
	$\langle x_{us} \rangle = 0.13$	0.65	3.50
	$\langle x_d \rangle = 0.13$	0.65	3.50
Ξ^0	$\langle x_{sf} \rangle = 0.16$	1.21	5.18
	$\langle x_{ss} \rangle = 0.15$	1.10	5.29
	$\langle x_u \rangle = 0.10$	0.71	5.68
Σ^+	$\langle x_s \rangle = 0.20$	1.07	4.45
	$2 \times \langle x_u \rangle = 2 \times 0.11$	0.80	4.72
Λ	$\langle x_s \rangle = 0.20$	1.21	4.72
	$\langle x_u \rangle = 2 \times 0.11$	0.65	4.47
Δ^{++}	$3 \times \langle x_u \rangle = 3 \times 0.14$	0.92	4.85
Ξ^*	$2 \times \langle x_s \rangle = 2 \times 0.15$	1.28	6.14
	$\langle x_u \rangle = 0.10$	0.88	6.54
Σ^*	$\langle x_s \rangle = 0.17$	1.28	5.25
	$2 \times \langle x_u \rangle = 2 \times 0.12$	0.90	5.63
Ω^-	$3 \times \langle x_s \rangle = 3 \times 0.14$	1.25	6.92

- Table 2a -

Mesons			
Hadron	$\langle x_i \rangle$	α_i	β_i
π	$2 \times \langle x_u \rangle = 2 \times 0.21$	0.16	-0.39
K	$\langle x_s \rangle = 0.28$	0.75	0.97
	$\langle x_u \rangle = 0.13$	0.36	1.36
K^*	$\langle x_s \rangle = 0.24$	1.20	2.71
	$\langle x_u \rangle = 0.16$	0.81	3.10
ρ	$2 \times \langle x_u \rangle = 2 \times 0.21$	0.87	2.40
ω	$2 \times \langle x_u \rangle = 2 \times 0.21$	0.87	2.40
φ	$2 \times \langle x_s \rangle = 2 \times 0.21$	1.16	3.51

- Table 2b -

Table 2 - Average momentum fraction carried by valence quarks at $Q_0^2 = 2 \text{ GeV}^2$.
The parameters α_i and β_i of the distribution functions,
Eq. (3.1), are also shown.
a) Baryons ; b) Mesons.

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FIGURE CAPTIONS

- Figure 1 Comparison of the difference $\nu W_2^p - \nu W_2^n$ with the determined $u_F(x)$ distribution. Data from Ref. 17).
- Figure 2 $A_1 \nu W_2$ as a function of x . The curves are from Eq. (5.2) at two values of Q^2 , 2 GeV^2 and 60 GeV^2 . Data points from Ref. 9).
- Figure 3 Comparison of charge ratios, N_{h+}/N_{h-} in proton and neutron. The curves are from Eq. (5.3) and (5.4) for $z_0 = 0.3$ (full lines) and $z_0 = 0.6$ (dashed-dot lines). Data, for $z_0 = 0.3$, from Ref. 10).
- Figure 4 Predicted valence quark distribution functions at $Q_0^2 = 2 \text{ GeV}^2$:
- a) Σ^+ (uus) ;
 - b) Ξ^0 (us_ss_f). The full strange quark distribution is $s(x) = s_s(x) + s_f(x)$.

FIG.1

FIG.2

FIG.3

FIG. 4a

FIG. 4 b