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DIVERGENCE CANCELLATIONS IN GRAVITY-MATTER SYSTEMS
FROM SUPERGRAVITY EMBEDDING

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A B S T R A C T

The absence of otherwise allowed one-loop divergences in gravity-Yang-Mills systems is explained by considering their supersymmetric extensions. In the embedding theories, just those counterterms are forbidden by supersymmetry, so that contributions from diagrams involving the new fermions must cancel the purely gravitational radiative corrections in question. The former are then shown to vanish separately through an effective Yang-Mills duality invariance, which therefore also accounts for the vanishing of the latter. Finiteness of the gravitational radiative correction to the quadrupole moment in vector QED also follows as a corollary.

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1. INTRODUCTION

The non-renormalizability of gravity-matter (of spin ≤ 1) systems has been borne out by explicit one-loop calculations¹⁾⁻³⁾ which confirm the presence of new, higher dimensional counterterms associated with the dimensionality of κ^2 . Yet, of the invariants allowed as possible counterterms by the co-ordinate and gauge symmetries, a number are mysteriously absent, notably in Einstein-Yang-Mills coupling. We shall show that these cancellations may be entirely understood by embedding the systems in a wider, supersymmetric framework and making full use of the additional invariances available there. In the resulting supergravity supermatter couplings, the missing counterterms are excluded by supersymmetry, not being part of any supermultiplet (supersymmetry is certainly preserved at one-loop order). Now this vanishing total radiative correction is just the sum of the purely gravitational one (the finiteness of which is to be explained) and of new contributions not involving virtual bosons, but only the supersymmetrizing fermions (spin $3/2$ and $1/2$ in these models) as internal lines. These new contributions will then be shown to vanish on the basis of an effective Yang-Mills duality invariance (on the external lines) to which they are separately subject. Consequently, the vanishing of the purely gravitational effects will have been established by imbeddability in a supergravity framework and the symmetries of the embedding theories. There are no mysteries left: all infinities are, as usual, either compulsory or forbidden.

Explanation of the infinity cancellations in Einstein-Yang-Mills theory (with arbitrary internal symmetry) will then also account for the finite gravitational correction to the electric quadrupole moment in vector QED, which has been shown by van Proeyen⁴⁾ to follow from the former. The remaining cancellations of gravitational infinities in matter interactions occur in their contributions to the magnetic moments of charged spinors⁵⁾, and charged vectors⁴⁾. All these cancellations presuppose absence of a bare anomalous coupling, which is also required for supersymmetry embedding. Although Pauli terms are known⁶⁾ to be forbidden by supersymmetry, the previous arguments do not apply so immediately here; a brief discussion is given in the third section.

2. EINSTEIN-VECTOR SYSTEMS

The list of independent one-loop counterterms allowed by discrete, gauge and co-ordinate invariances includes a purely non-Abelian one, with no Maxwell counterpart

$$\Delta I \sim g\kappa^2 \text{tr} \int F_{\mu\nu} [F^\nu{}_\alpha, F^{\alpha\mu}]. \quad (1a)$$

It would arise as a graviton radiative correction to the three-point Yang-Mills vertex (hence the factor $g\kappa^2$). Upon explicit calculation, however, neither (1a) nor the following terms which do have Maxwell limits,

$$\kappa^4 \text{tr}(F^a F^b)^2; \kappa^4 (\text{tr} F^2)^2; \kappa^4 \text{tr}(F^4); \kappa^2 R_{\mu\nu\alpha\beta} \text{tr}(F^{\mu\alpha} F^{\nu\beta}); \kappa^2 R \text{tr} F^2 \quad (1b)$$

appear as infinities either on- or off-shell. [Details of the calculation and discussion of the invariants are given in Ref. 3).] All other gauge allowed invariants,

$$R_{\mu\nu}^2; \kappa^2 R_{\mu\nu} T^{\mu\nu}; \kappa^4 T_{\mu\nu}^2; \kappa^2 \text{tr} (D_\mu F^{\mu\nu})^2 \quad (2)$$

do have infinite coefficients, reducing on-shell to the single infinite term $\kappa/\epsilon R_{\mu\nu}^2$. Here we have used various identities to express other combinations such as $(D_\alpha F_{\rho\gamma})^2$ in terms of the above.

It is striking that all the missing terms (1), and none of those present (2), would be forbidden were the theory invariant under

$$\delta F_{\mu\nu} = \epsilon {}^*F_{\mu\nu}, \quad {}^*F^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}. \quad (3)$$

But it is known⁷⁾ that these duality transformations cannot even be defined (let alone form a group) in the non-Abelian case. Nor can all the absent terms (1) be eliminated by appeal to the $g \rightarrow 0$ Abelian limit, which is duality invariant. Thus the list (1) constitutes a mystery in that quantum corrections seem to respect a non-invariance of the tree action.

Motivated by the close relation between supersymmetry and helicity/duality invariances, we enlarge the matter system to be the $N = 1$ supersymmetric multiplet⁸⁾

$$\mathcal{L}_{\text{matter}}(A, \lambda) = -1/4 \text{tr} F_{\mu\nu}^2 - i/2 \bar{\lambda} \not{D} \lambda \quad (4a)$$

where the spinors are in the adjoint representation. This brings in a new, minimal interaction

$$\mathcal{L}_{\text{int}}(A, \lambda) = ig \text{tr} \bar{\lambda} \gamma^\mu [A_\mu, \lambda]. \quad (5a)$$

When we couple (4a) to $N = 1$ supergravity,

$$\begin{aligned} \mathcal{L}_{SG}(e, \psi) = & -\frac{1}{2}\kappa^2 R(e, \omega) - i/2 \bar{\psi}_\mu \gamma_5 \gamma_\nu \not{F}^{\mu\nu} \\ & f_{\mu\nu} \equiv D_\mu \psi_\nu - D_\nu \psi_\mu \end{aligned} \quad (4b)$$

a second relevant vertex is introduced through the Noether coupling

$$\mathcal{L}_{N.M.}(\psi_\mu, \lambda; e, F) = \kappa \bar{\psi}_\mu \text{Tr} \sigma \cdot F \gamma^\mu \lambda \quad (5b)$$

of the spin $3/2$ field to the conserved spinor current associated with the supersymmetry of (4a). For our one-loop purposes, all other vertices, such as quartic contact terms are irrelevant. In this enlarged system, all diagrams which could contribute to (1) form two separate sets. First, there are the purely gravitational radiative corrections which do not involve the new vertices (5) but are due entirely to loops with virtual boson lines. These are the ones to be explained away. The second set involves only fermion triangles or boxes^{*)} from which the vector lines emanate as purely external particles. It is clear by power counting and fermion conservation that there are no "cross-diagrams" to this order. Thus all new contributions to (1) are generated by the vertices (5); they involve at least two non-minimal ones (5b) to provide a κ^2 factor, plus either further minimal ones (5a) to give $\sim \kappa \kappa^2$ (and build up non-linearities in F) or two more (5b) giving κ^4 over all. We emphasize that in all cases the bosons $(e_{\mu\alpha}, A_\mu, F_{\mu\nu})$ are purely external and the fermions (ψ_μ, λ) purely virtual.

We now show that the effective action which generates the new diagrams contains an effective duality symmetry which excludes all contributions to (1). Consider a change of the "integration variable" ψ_μ (or λ) to a chirality rotated value $\psi'_\mu \equiv (1 + \epsilon \gamma_5) \psi_\mu$. This leaves the action $L_{3/2}(\psi)$ form invariant, and hence its propagators and ghosts are unchanged. Indeed the only change is in (5b) which becomes

$$\mathcal{L}_{N.M.}(\psi_\mu, \lambda; e, F) \equiv \mathcal{L}_{N.M.}(\psi'_\mu, \lambda; e, F + \epsilon \not{F}) \quad (6)$$

^{*)} Fermion loops with only two external vector lines are the beginnings of $(DF)^2$ terms, which have been reduced above to the $(D_\mu F^{\mu\nu})^2$ one. External line counting, including the buildup of non-linear terms such as R or $F_{\mu\nu}(A)$, becomes trivial if one imagines a background field formulation for the (external) bosons here.

because of the matrix identity $\gamma_5 \sigma_{\mu\nu} = * \sigma_{\mu\nu}$. This means that any purely bosonic counterterm^{*)} generated by (5) must be invariant (after "integration" over the virtual fermions) under duality rotations $\delta F = e * F$ in their $F_{\mu\nu}$ dependence. This does not apply to $F_{\mu\nu}$ formed from the external A_μ in (5a), but only to those from (5b). So without performing a (forbidden) duality rotation (we have not even altered A_μ), we can still enjoy all the advantages of an effective duality invariance. For example, (1a) is now excluded, since replacement of any two F 's by their duals there (it is symmetric in all three) does not leave it invariant: $F * F * F = - F F F$ for a finite rotation. Likewise, all other terms (1b), being either $\kappa^4 F^4$ or $\kappa^2 F^2 R$, are due entirely to the vertex (5b), and again excluded by duality. The remaining terms, (2), are duality invariant and would receive contributions from (5), which is neither surprising nor especially interesting since we know that supergravity-supermatter coupling in general is also non-renormalizable⁹⁾.

Next, we show that the total contributions to (1) in the embedding theory are all forbidden by supersymmetry, so that the purely gravitational effects must separately vanish as well. One amusing way is precisely to note that supersymmetry implies formal duality invariance, i.e., that only terms symmetric under dual exchange can form part of a supersymmetric invariant¹⁰⁾; indeed this was the motivation for embedding in the first place. One may also verify explicitly that none of the terms (1) mix on-shell with any others, and that they do not transform correctly under supersymmetry. For example, (1a) is like the $\text{tr}(R^{\mu\nu\alpha\beta})^3$ term in two-loop supergravity, which was excluded there for this reason⁹⁾. This concludes the argument in the Einstein-Yang-Mills sector, and could have been used also in the Abelian case, although scarcely necessary there since duality is directly available. In both systems, the net result is that the on-shell counterterm is proportional to $R^2_{\mu\nu}$.

As shown by van Proeyen⁴⁾, absence of (1a) also implies that of infinite gravitational corrections to the quadrupole moment, $\kappa \kappa^2 F_{\mu\nu} G^{\mu\alpha} \bar{G}^\nu_\alpha$, of a charged vector field with field strength $G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and complex potential B_μ . His argument is based on adding a Higgs term to the Einstein-Yang-Mills system, which defines a charged direction without affecting ultra-violet behaviour. Clearly, the F^3 term contains this moment. The calculated result is finite but non-vanishing, which is not in contradiction with our purely ultra-violet considerations. Since soft breaking of supersymmetry is also introduced thereby,

^{*)} Counterterms with external fermions are irrelevant: even on shell, they could only produce allowed terms $\kappa^2 (D_\mu F^{\mu\nu})^2$ or $\kappa^2 R_{\mu\nu} T^{\mu\nu}$.

there is no longer the strictly vanishing requirement of the massless supersymmetric case studied here. It is noteworthy for us that finiteness would be lost if there were a bare Pauli term in the charged vector's QED couplings. For in that case, it would not be possible to supersymmetrize the model in the first place.

3. COMMENTS

We conclude with some remarks on the two cancellations not treated here. The first one to have been found⁵⁾ was the gravitational correction to the electron-photon vertex $\propto (\kappa^2 m^2) e/m \bar{\psi} \sigma \cdot F \psi$, which was finite (again only in the absence of a bare Pauli term). Without gravity, it is a known result of supersymmetric QED⁶⁾ that the usual $\alpha/2\pi$ from minimal coupling must be (and is explicitly) cancelled by the new vertices introduced in supersymmetrizing, although there is no direct way to see this from the new vertices alone. Upon coupling to supergravity, the total moment corrections would still have to vanish. However, the new contributions do not exhibit any obvious symmetry which would show them to vanish separately. In particular, duality arguments alone cannot affect the diagrams in which the external photon is minimally emitted by a virtual charged scalar there.

The final known cancellation is that of the infinity in the magnetic moment correction $\propto \kappa^2 m^2 F_{\mu\nu} \bar{B}^{\mu} B^{\nu}$ of the charged vector. This term was also related⁴⁾ to one involving the Higgs broken Yang-Mills theory, but through an essentially scalar invariant $\propto g \kappa^2 \text{tr}\{F_{\mu\nu} [D_{\mu} \phi, D_{\nu} \phi]\}$. Thus, one would have to consider, in addition, coupling of a massive scalar (Higgs) supermultiplet to apply the argument. It seems probable that both these magnetic moment results could also be understood by more detailed application of the embedding argument.

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REFERENCES

- 1) G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré 20 (1974) 69.
- 2) S. Deser and P. Van Nieuwenhuizen, Phys. Rev. D10 (1974) 401, 411.
- 3) S. Deser, H.-S. Tsao and P. Van Nieuwenhuizen, Phys. Rev. D10 (1974) 3337.
- 4) A. van Proeyen, Nucl. Phys. B165 (1980) 189.
- 5) F.A. Berends and R. Gastmans, Phys. Lett. 55B (1975) 311;
M.T. Grisaru, P. Van Nieuwenhuizen and C.C. Wu, Phys. Rev. D12 (1975) 1813.
- 6) S. Ferrara and E. Remiddi, Phys. Lett. 53B (1974) 347.
- 7) S. Deser and C. Teitelboim, Phys. Rev. D13 (1976) 1592.
- 8) S. Ferrara and B. Zumino, Nucl. Phys. B79 (1979) 413;
A. Salam and J. Strathdee, Phys. Lett. 51B (1974) 353.
- 9) S. Deser, J.H. Kay and K.S. Stelle, Phys. Rev. Letters 38 (1977) 527.
- 10) R.E. Kallosh, Nucl. Phys. B165 (1980) 119; JETP Lett. 29 (1979) 192, 493;
S.M. Christensen, S. Deser, M.J. Duff and M.T. Grisaru, Phys. Lett. 84B (1979) 411.