SUPERSYMMETRIC THEORIES OF FUNDAMENTAL INTERACTIONS

S. Ferrara
CERN, Geneva, Switzerland

CONTENTS

1. Introduction
 2. Supersymmetry algebras and particle supermultiplets
 3. Supersymmetric Lagrangian field theories
 4. Supergravity and tensor calculus
for local supersymmetry
 5. Spontaneous symmetry-breaking
and super-Higgs mechanism
 6. Extended supergravity and
models unifying electrodynamics and chromodynamics
with gravitation
 7. Conclusions
- References

Invited lectures at the
International School of Subnuclear Physics,
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1. INTRODUCTION

Supersymmetry¹⁾ is a new symmetry of local quantum field theory which extends in a non-trivial way the relativity group of space-time. The new symmetry operations are carried by spinorial charges which, according to the spin-statistic theorem, obey anticommutation relations. Particles are classified, in any supersymmetric field theory, according to representations of the symmetry group, and irreducible representations contain particle states of different spin, both integer and half-integer. Supersymmetry is therefore the first example of a genuine relativistic spin containing symmetry.

Previous no-go theorems which prevented possible relativistic generalizations of $SU_w(6)$ are now circumvented.

The key point is that the algebraic structure related to supersymmetry is not an ordinary Lie algebra, to which the above-mentioned theorems applied, but rather a graded Lie algebra (GLA).

Graded Lie algebras contain generators obeying both commutation and anticommutation relations. The spinorial charges obeying anticommutation relations are called the odd elements of the GLA, while

even generators obeying commutation relations form an ordinary Lie algebra which contains, as part, the Poincaré Lie algebra.

Investigations of several model field theories in the past years and the detailed study of their renormalizability properties have shown that not only is supersymmetry a consistent symmetry at the quantum level but usually it improves the divergences of perturbation theory. This general feature of any supersymmetric model gave us the great hope that a supersymmetric version²⁾ of the Einstein theory of gravitation, now called supergravity, could once and for all overcome the well-known difficulties related to the quantization of gravity.

This program³⁾ has been partially successful, and there are existing models which have finite one- and two-loop quantum corrections in scattering processes due to gravitational forces. It is at present an open problem whether this trend will persist at higher loops and will lead to a renormalizable theory of gravity.

One of the main goals of supersymmetric theories of particles is to provide a new framework for unifications of fundamental interactions. The two main directions that have been pursued have been, from one side, a supersymmetric extension of chromodynamics unified with weak and electromagnetic interactions⁴⁾; from the other side, in a more ambitious way, the construction of the so-called extended supergravity theories in which all fundamental particles of nature sit in the same irreducible multiplet²⁾.

A third scheme that is worth while mentioning is the construction of superconformal gravity theories which can be regarded as supersymmetric extensions of Weyl gravity³⁾.

The first class of theories⁴⁾ seem at present to give the most realistic examples because they are flexible enough to accommodate present particle phenomenology. The exciting feature of these supersymmetric extensions of strong, weak, and electromagnetic interactions is that they imply the existence of a class of new

particles having a new quantum number R , the old known particles having $R = 0$. The spinorial charge of the supersymmetry algebra is the carrier (one unity) of this new quantum number.

In these theories there are two colourless, massless spin $\frac{1}{2}$ particles beyond the usual electron and muon neutrino; they are the photino (the spin $\frac{1}{2}$ partner of the photon) and the goldstino, the Goldstone fermion of spontaneously broken supersymmetry. This last massless excitation is eaten by the gravitino, the spin $\frac{3}{2}$ partner of the graviton, when supersymmetry is promoted to a local invariance and gravity is switched on.

As a consequence of the super-Higgs effect, an intimate mixing between weak and gravitational interactions is expected through the massive gravitino. Needless to say, possible detectable effects⁵⁾ of the new particles implied by these supersymmetric unified theories are crucial in order to give further support to this scheme of unification. The unsatisfactory point of this approach is that it does not avoid the old renormalizability problems of gravitation. This is mainly due to the fact that the generators of the local gauge group of strong, weak and electromagnetic interactions commute with the spinorial charge, and therefore a true unification is lacking. The second scheme of unification, based on extended supergravity theories⁶⁾, looks the most appealing because all the fundamental particles of nature sit in the same irreducible multiplet of the underlying symmetry. It is clear that the spinorial charges no longer commute with the generators of the gauge group because, for example, they make the transition between coloured (gluons, quarks) and colourless (leptons, weak bosons, Higgs scalars, graviton) states. Symmetry breaking in these theories is expected to occur dynamically, with possible elimination of the cosmological term which is induced, at the classical level, by the Yang-Mills coupling. These models have finite one- and two-loop quantum corrections³⁾. They seem to be the best candidates for renormalizable theories of gravitation. The drawback of these unified theories is that they

are so constrained that both the local gauge group and the representations to which the lower spin particles belong are entirely fixed by the symmetry. The Yang-Mills invariance is restricted to be $SO(N)$, and N cannot exceed 8 under the requirement that all fundamental particles have maximum spin 2, which is the case for the graviton. The $SO(8)$ model contains exact colour and e.m. symmetry but accommodates only a $U(1)$ subgroup of the weak isospin group $SU(2)$, which is believed to be the minimal group of weak interactions.

The third scheme, relying on conformal theories^{3,7)} in order to have contact with Nature, must exhibit both conformal and supersymmetry breaking. It has the nice property of having one coupling constant only and a vanishing cosmological constant, but it is plagued with the problems of instability and non-unitarity of higher derivative Lagrangian field theories.

To summarize, although at present all the unification schemes based on supersymmetric interactions seem to have their drawbacks, they nevertheless seem fascinating. At present they appear to offer an almost unique possibility of having a unified theory of all particle interactions including gravitation. They seem also to offer a solution to the challenging problem of a consistent formulation of a quantum theory of gravitation.

Finally, it is worth while mentioning that supersymmetric theories of particles provide a theoretical explanation for the existence of fermions and bosons, the building blocks of the universe (matter and radiation) through a symmetry principle which is ultimately related to the underlying geometrical structure of space and time.

2. SUPERSYMMETRY ALGEBRAS AND PARTICLE SUPERMULTIPLETS

Supersymmetry algebras⁸⁾ provide an enlargement of the basic space-time symmetry algebras through an operation which is often called the graduation of a given Lie algebra.

The basic supersymmetry algebra consists of N spinorial charges Q_α^i , $\alpha = 1, \dots, 4$, $i = 1, \dots, N$, which transform according to the fundamental self-conjugate representation of the Lorentz group

$$i [M_{\mu\nu}, Q_\alpha^i] = (\sigma_{\mu\nu})_\alpha^\beta Q_\beta^i \quad (2.1)$$

These $4N$ charges are the odd elements or the grading representation of the graded Poincaré algebra. The even part consists of the Poincaré generators P_μ , $M_{\mu\nu}$, and of possible central charges Z^{ij} , Z'^{ij} .

The basic commutators are

$$\begin{aligned} [Q_\alpha^i, P_\mu] &= 0 \\ \{Q_\alpha^i, \bar{Q}_\beta^j\} &= -2\delta_{\alpha\beta}^\mu P_\mu \delta^{ij} + \delta_{\alpha\beta} Z^{ij} + \gamma_{\alpha\beta}^5 Z'^{ij} \\ [Z^{ij}, Q_\alpha^i] &= [Z'^{ij}, Q_\alpha^i] = 0 \\ [Z^{ij}, Z^{\ell m}] &= [Z'^{ij}, Z^{\ell m}] = [Z^{ij}, Z'^{\ell m}] = 0 \end{aligned} \quad (2.2)$$

The operators Z, Z' are real and antisymmetric. In Eq. (2.2) we have used a Majorana (real) representation for the γ -metrics with metric $(-+++)$. Moreover, $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$, $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$.

The algebra given by Eq. (2.2) is called extended supersymmetry for $N > 1$ and simple supersymmetry for $N = 1$. In the latter case, $Z = Z' = 0$.

All irreducible representations of the algebra given by Eq. (2.2) acting on single-particle states of given momentum have been classified⁹⁾.

Each irreducible representation decomposes into a finite direct sum of irreducible Poincaré representations, all degenerate in mass

and with spin shifted by one-half unity. Of particular relevance are the zero mass representations. These representations are suitable for describing unified gauge theories in which gauge particles of different helicity, such as the graviton and the photon, sit in the same irreducible representation. The multiplicity and the helicity content of one-particle states of a massless irreducible multiplet of N -extended supersymmetry can easily be obtained by means of the Wigner method of induced representations. Starting from a singlet state of helicity $|\lambda|$ and then successively applying those spinorial charges Q_i^- ($i = 1, \dots, N$) which act as $\frac{1}{2}$ unit helicity-lowering operators, one gets 2^N states of helicity $|\lambda|, |\lambda| - \frac{1}{2}, \dots, |\lambda| - (K/2), \dots$, up to $|\lambda| - (N/2)$, the multiplicity of a state of given helicity $|\lambda| - (K/2)$ being $N!/[K!(N-K)!]$.

To get acceptable representations consistent with local field theory, CPT conjugate states of reversed helicities must be added. These are obtained by starting with a singlet state of helicity $-|\lambda|$ and by repeatedly applying those spinorial charges Q_i^+ ($i = 1, \dots, N$) which act as helicity-raising operators. Then one gets a doubling of the states, which are therefore 2^{N+1} with the exception of those representations for which $|\lambda| = N/4$, i.e. $|\lambda| = N/4$. These representations are self-conjugate and contain only 2^N states.

From the previous analysis, it is very easy to make the complete list of all basic massless representations of N -extended supersymmetry with a given helicity content. In the construction of supersymmetric gauge-field theories, the interest is in those multiplets which correspond to gauge fields and matter fields of fundamental interactions. This physical requirement limits us to considering representations with $|\lambda|_{\max} = 2$. With this restriction in mind it is easy to show that only a limited number of extended supersymmetries, precisely up to $N = 8$, can be realized in conventional Lagrangian field theories. In particular let us call, by convention, scalar, vector, spinor, and tensor multiplets those representations having $|\lambda|_{\max} = \frac{1}{2}, 1, \frac{3}{2}$, and 2, respectively. They

Table 1

Basic massless irreducible representations of
extended supersymmetry

Isospin \ Helicity	$\lambda = 2$	$\lambda = \frac{3}{2}$	$\lambda = 1$	$\lambda = \frac{1}{2}$	$\lambda = 0$
N = 1	1	1			
		1	1		
			1	1	
				1	2
SO(2)	1	2	1		
		1	2	1	
			1	2	2
				2	4
SO(3)	1	3	3	1	
		1	3	3	$2 = 1 \oplus 1$
			1	$3 \oplus 1$	$3 \oplus 3$
SO(4)	1	4	6	4	$2 = 1 \oplus 1$
		1	4	$6 \oplus 1$	$4 \oplus 4$
			1	4	$6 = 3 \oplus 3$
SO(5)	1	5	10	$10 \oplus 1$	$5 \oplus 5$
		1	$5 \oplus 1$	$10 \oplus 5$	$10 \oplus 10$
SO(6)	1	6	$15 \oplus 1$	$20 \oplus 6$	$15 \oplus 15$
		1	6	15	20
SO(7)	1	$7 \oplus 1$	$21 \oplus 7$	$35 \oplus 21$	$35 \oplus 35$
SO(8)	1	8	28	56	$70 = 35 \oplus 35$

exist in extended supersymmetric theories up to $N = 2, 4, 6$, and 8 spinor charges, respectively. Because of the anticommuting properties of the spinor charges, it is easily seen that particle states of helicity $|\lambda_{\max}| - (K/2)$ transform according to the Kravtsov antisymmetric tensor representation of the $SO(N)$ orthogonal group. In Table 1 we have listed all possible representations of extended supersymmetry which can describe basic particles of fundamental interactions.

Massive representations exist as well. Basic irreducible representations contain 2^{2N} -particle states. More general representations have dimension $2^{2N} \cdot (2J+1) \cdot d_N$ where J, d_N is the spin and the dimension of the "isospin representation" to which the ground state belongs. The latter state is, by definition, that state of the representation which is annihilated by the $2N$ spinorial charges which act as destruction operators in the 2^{2N} dimensional Clifford algebra spanned by the $4N$ Majorana charges Q_{α}^i . In Table 2 we list the massive representations for basic multiplets which can occur in a given Lagrangian field theory. So far we have considered the graded version of the Poincaré algebra. Also of physical interest are the gradings of the $O(4, 2)$ conformal and $O(3, 2)$ de Sitter algebras. The former is relevant for Yang-Mills theories and for

Table 2

Basic massive irreducible
representations of extended supersymmetry

Mass $M \neq 0$	Spin	$J = 1$	$J = \frac{1}{2}$	$J = 0$
$N = 1$		1	2	1
			1	2
$N = 2$		1	4	5
			2	4

supersymmetric versions of Weyl gravity, while the latter is appropriate for supergravity theories with a cosmological constant. The grading of the $O(4, 2) \sim SU(2, 2)$ conformal algebra is obtained as follows: for a given N there are $8N$ odd generators: Q_α^i, S_α^i ($\alpha = 1, \dots, 4; i = 1, \dots, N$). They behave as $2N$ self-conjugate $SL(2, C)$ 4-spinors or as N self-conjugate $SU(2, 2)$ 8-spinors. The even part of the algebra contains the 15 generators of the conformal group $M_{\mu\nu}, P_\nu, D, K_\nu$, and the N^2 generators of the unitary group $U(N)$. The de Sitter grading can be obtained trivially as a graded subalgebra of the conformal algebra. The odd elements are the N spinors $R_\alpha^i = \frac{1}{2}(Q_\alpha^i + S_\alpha^i)$, while the even elements are the 10 generators $M_{\mu\nu}, L_\mu = \frac{1}{2}(P_\mu - K_\mu)$ of $O(3, 2) \subset O(4, 2)$ and the $\frac{1}{2}N(N-1)$ internal symmetry generators of $SO(N) \subset U(N)$.

3. SUPERSYMMETRIC LAGRANGIAN FIELD THEORIES

A great performance of supersymmetry in the physics of particles and fields has certainly been its successful application to Lagrangian field theories. In the past years it has been shown that all renormalizable theories, such as the Yukawa theory, the ϕ^4 theory, and the Yang-Mills theories, can be suitably extended in order to manifest supersymmetry. Supersymmetry usually severely restricts the couplings and the masses in renormalizable interactions. With great surprise it has been realized that the Ward identities related to this new symmetry not only are preserved by the renormalization procedure but usually improve renormalization. In fact quantum divergences in supersymmetric field theories have the tendency to cancel. In many model field theories the quantum infinities which are absorbed in the renormalization constants are less than one would expect from naive power counting arguments. In spite of this success, no example of non-renormalizable field theory which becomes renormalizable after imposing supersymmetry has yet been found. It is at present an open question whether supergravity could be just the first example of this kind of theory.

In order to apply supersymmetry in Lagrangian field theory, representations of this symmetry on local fields must be described.

The simplest of these representations was found by Wess and Zumino⁸⁾ in the following form:

$$\begin{aligned}
 \delta A(x) &= i \bar{\epsilon} \chi(x) \\
 \delta B(x) &= i \bar{\epsilon} \gamma_5 \chi(x) \\
 \delta \chi(x) &= \not{\partial} (A(x) + \gamma_5 B(x)) \epsilon + (F(x) + \gamma_5 G(x)) \epsilon \\
 \delta F(x) &= i \bar{\epsilon} \not{\partial} \chi(x) \\
 \delta G(x) &= i \bar{\epsilon} \gamma_5 \not{\partial} \chi(x)
 \end{aligned} \tag{3.1}$$

This multiplet is often called the scalar multiplet. Here $A(x)$, $F(x)$ are scalars, $B(x)$, $G(x)$ pseudoscalars, and $\chi(x)$ a Majorana spinor; $\delta A(x)$ stands for $[\bar{\epsilon} Q, A(x)]$, etc. ϵ is a constant anti-commuting spinorial parameter which may be regarded as an odd element of a Grassmann algebra, the group manifold of a graded Lie algebra.

One can indeed verify that (3.1) is a representation of the algebra given by Eq. (2.2) with $N = 1$. In fact the commutator of two such variations gives, when applied to any field, an infinitesimal translation:

$$[\delta_1, \delta_2] A(x) = -2i \bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial_\mu A(x), \text{ etc.} \tag{3.2}$$

We note, *en passant*, that only a finite number of local fields are needed to close the representation.

If we impose the mass-shell conditions,

$$(\square - m^2) A(x) = (\square - m^2) B(x) = (\not{\partial} + m) \chi(x) = 0 \tag{3.3}$$

They in turn imply

$$\delta(mA + F) = \delta(mB + G) = 0$$

which have the solution $F = -mA$, $G = -mB$.

For the first time a new phenomenon happens in the representation space of the symmetry.

The number of fields which are needed to close the algebra off-mass shell is greater than the number of fields which realize the symmetry on particle states. The fields F and G play the role of auxiliary fields and they can be eliminated using their field equations. Note that the particle fields A, B, χ just give rise to a representation on single-particle states, which can be identified in Tables 1 and 2 as the lowest spin representation of supersymmetry with $N = 1$.

It is worth while mentioning that the auxiliary fields, like F and G for the example under consideration, play a crucial role in supersymmetric theories. Although they can be eliminated from any given Lagrangian because the equations of motion are purely algebraic, it is much more convenient to retain them in order to have linear transformation laws and closure of the algebra. This last property is very important in order to have a tensor calculus of general validity, as in any other symmetry, and then simple rules for constructing invariants.

In order to have all the ingredients for the formulation of supersymmetric interaction, we need another representation, called the vector multiplet, which is realized as follows:

$$\begin{aligned}
 \delta C &= i \bar{\epsilon} \gamma_5 \mathcal{S} \\
 \delta \mathcal{S} &= \gamma^\mu \mathcal{V}_\mu \epsilon - \partial_\mu C \gamma_5 \gamma^\mu \epsilon + (M + \gamma_5 N) \epsilon \\
 \delta M &= i \bar{\epsilon} \lambda + i \bar{\epsilon} \not{\partial} \mathcal{S} \\
 \delta N &= i \bar{\epsilon} \gamma_5 \lambda + i \bar{\epsilon} \gamma_5 \not{\partial} \mathcal{S} \\
 \delta \mathcal{V}_\mu &= i \bar{\epsilon} \gamma_\mu \lambda + i \bar{\epsilon} \partial_\mu \mathcal{S} \\
 \delta \lambda &= -\frac{1}{2} (\partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu) \gamma^\mu \gamma^\nu \epsilon + \gamma_5 D \epsilon \\
 \delta D &= i \bar{\epsilon} \gamma_5 \not{\partial} \lambda
 \end{aligned} \tag{3.4}$$

It consists of three pseudoscalars (scalars) D , C , N , a scalar (pseudoscalar) M , an (axial) vector V_μ , and two Majorana spinors ζ , λ .

The multiplets defined by the transformation laws in (3.1) and (3.4) have been first obtained by looking at the minimal set of fields needed to close the algebra.

A complete development of the tensor calculus of global supersymmetry has been possible only after the introduction of the superfield concept¹⁰⁾. This technique proved extremely powerful for treating supersymmetric theories in a simple way, and at the same time provided a geometrical meaning to supersymmetry transformations by enlarging the Minkowski space-time to a superspace whose points are labelled by extra anticommuting coordinates.

The rules of tensor calculus can be summarized as follows: Two scalar multiplets S_1 , S_2 can be multiplied in two different ways in order to give either a new scalar multiplet $S = S_1 S_2$ or a (complex) vector multiplet $S_1 \bar{S}_2$. The real and imaginary part of the latter multiplet $\frac{1}{2}(S_1 \bar{S}_2 + \bar{S}_1 S_2)$, $\frac{1}{2}i(S_2 \bar{S}_1 - S_1 \bar{S}_2)$ are two real vector multiplets which are respectively symmetric and antisymmetric in the exchange $1 \leftrightarrow 2$. Two vector multiplets V_1 , V_2 can be multiplied to give a new vector multiplet $V = V_1 \cdot V_2$. To any scalar multiplet S of components A , B , χ , F , G , one can associate a new scalar multiplet of components F , G , $\not{\partial}\chi$, $\square A$, $\square B$, to be called the kinetic multiplet $T(S)$. To any vector multiplet V of components C , ζ , M , N , V_μ , λ , D , one can associate a scalar multiplet (with a spinor index) $W_\alpha(V)$ of components λ , $\sigma^{\mu\nu} F_{\mu\nu} + \gamma_5 D$, $\not{\partial}\lambda$, where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$.

From the transformation laws in Eqs. (3.1) and (3.4) we further observe that the F (G) component of a scalar multiplet and the D component of a vector multiplet transform as total divergences under supersymmetry. This provides the rule for the construction of invariant actions! The F (G) component of any (composite) scalar multiplet or the D component of any (composite) vector multiplet is the candidate for globally supersymmetric Lagrangian densities.

The simplest non-trivial model field theory with global supersymmetry is the self-interaction of a scalar multiplet S , the so-called Wess-Zumino model⁸⁾. This is obtained by adding the following three supersymmetric invariant terms:

$$\begin{aligned}\mathcal{L}_{\text{KIN}} &= (S\bar{S})_D = -\frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}(\partial_\mu B)^2 - \frac{i}{2}\bar{\chi}\not{\partial}\chi + \frac{1}{2}F^2 + \frac{1}{2}G^2 \\ \mathcal{L}_{\text{HASS}} &= m(S^2)_F = m(FA + GB - \frac{i}{2}\bar{\chi}\chi)\end{aligned}\quad (3.5)$$

$$\mathcal{L}_{\text{INT}} = g(S^3)_F = g(FA^2 - FB^2 + 2GAB - i\bar{\chi}(A - \gamma_5 B)\chi)$$

The total Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{KIN}} + \mathcal{L}_{\text{HASS}} + \mathcal{L}_{\text{INT}} \quad (3.6)$$

The current related to the invariance under supersymmetry of the theory can be derived via the standard Noether procedure:

$$J_\mu = \not{\partial}(A - \gamma_5 B)\gamma_\mu\chi - (F + \gamma_5 G)\gamma_\mu\chi$$

and is conserved as a consequence of the field equations.

After elimination of the auxiliary fields F and G by means of their field equations,

$$\begin{aligned}F + mA + g(A^2 - B^2) &= 0 \\ G + mB + 2gAB &= 0\end{aligned}\quad (3.7)$$

we get

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}(\partial_\mu B)^2 - \frac{1}{2}m^2(A^2 + B^2) - \frac{i}{2}\bar{\chi}(\not{\partial} + m)\chi \\ &\quad - gmA(A^2 + B^2) - \frac{g^2}{2}(A^2 + B^2)^2 - ig\bar{\chi}(A - \gamma_5 B)\chi\end{aligned}\quad (3.8)$$

This Lagrangian could be equally well called the supersymmetric extension of the ϕ^3 , ϕ^4 or Yukawa theory. The resulting supersymmetry of the Lagrangian not only implies equality of masses for the particle fields A, B, and χ but also a precise relationship between the interaction terms.

The amazing fact of this theory is that not only are the relations between masses and couplings preserved by renormalization, but they allow the theory to be renormalized with only one basic infinite renormalization constant, a (logarithmically divergent) wave function renormalization Z common to all fields¹¹⁾. For instance, the renormalized mass and coupling constant are given, to each order of perturbation theory, by the following relations:

$$g_2 = Z^{3/2} g_0 \quad , \quad m_2 = Z m_0$$

These relations are reflected in an important relation between the functions $\beta(g)$, $\gamma(g)$ which appear in the renormalization equations of the theory.

The greatest achievement of supersymmetric Lagrangian field theories is perhaps the construction of supersymmetric extensions of Yang-Mills theories¹²⁾. These theories are the building blocks of any supersymmetric unified theory of weak, strong, and electromagnetic interactions which at present have a chance of being realistic.

The gauge multiplet, which is the multiplet to which the gauge mesons belong, is given by a vector multiplet as described by the transformation laws (3.4). The gauge function, in a supersymmetric theory, is also enlarged to an entire multiplet. The additional gauge freedom coming from this bigger gauge invariance, present in any supersymmetric gauge theory, can be used to reduce the vector multiplet to have $C = \zeta = M = N = 0$. This is the so-called Wess-Zumino gauge. In this gauge there is still the freedom of making ordinary gauge transformations on the vector potential V_μ . In the

Wess-Zumino gauge the remaining fields λ , V_μ , and D transform as follows under supersymmetry transformations:

$$\begin{aligned}\delta \lambda &= -G_{\mu\nu} \sigma^{\mu\nu} \epsilon \\ \delta V_\mu &= i \bar{\epsilon} \gamma_\mu \lambda \\ \delta D &= i \bar{\epsilon} \not{D} \lambda\end{aligned}\tag{3.9}$$

The fields λ , V_μ , D are Lie algebra valued fields, $G_{\mu\nu}$ is the Yang-Mills field strength $G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]$, and D_μ is the covariant derivative $D_\mu \lambda = \partial_\mu \lambda + ig[V_\mu, \lambda]$. The commutator of two transformations as in (3.9) gives a combined translation and Yang-Mills gauge transformation with field-dependent parameter $\omega = 2i \bar{\epsilon}_1 \gamma^\mu \epsilon_2 V_\mu$.

The invariant Yang-Mills Lagrangian is given by $\text{Tr}(W_\alpha W^\alpha)_F$, in which W_α is the scalar multiplet previously introduced. It reduces to

$$\text{Tr} \left(-\frac{1}{4} G_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \not{D} \lambda + \frac{1}{2} D^2 \right)\tag{3.10}$$

For pure supersymmetric Yang-Mills theories the equation of motion of the auxiliary field D is simply $D = 0$ and one gets the ordinary Yang-Mills coupling of a massless Majorana spinor in the adjoint representation of the gauge group. The conserved Noether current of supersymmetry is given by

$$J_\mu = -\frac{1}{2} \text{Tr} (G_{\nu\rho} \gamma^\nu \gamma^\rho \gamma_\mu \lambda)$$

Because of superconformal invariance (see Section 2) there exists in this theory a second conserved current $I_\mu = -\gamma \cdot x J_\mu$ related to the second charge S_α of the graded conformal algebra. One could add to the previous Lagrangian a supersymmetric and gauge-invariant

interaction with n flavour scalar multiplets S^i . The resulting field equations for the auxiliary field D are no longer trivial. For instance, if one adds a scalar multiplet of mass m in the adjoint representation, one gets the additional term:

$$\begin{aligned} \mathcal{L}_{\text{MATTER}} = & \text{Tr} \left(-\frac{1}{2} (D_\mu A)^2 - \frac{1}{2} (D_\mu B)^2 - \frac{i}{2} \bar{\chi} \not{D} \chi + \frac{1}{2} (F^2 + G^2) \right. \\ & \left. + g \bar{\lambda} [A + \gamma_5 B, \chi] + i g D [A, B] + m (FA + GB - \frac{i}{2} \bar{\chi} \chi) \right) \end{aligned} \quad (3.11)$$

Elimination of the auxiliary fields F , G , and D gives the final Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{MATTER}} = & \text{Tr} \left(-\frac{1}{4} G_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \not{D} \lambda - \frac{i}{2} \bar{\chi} \not{D} \chi - \frac{1}{2} m^2 (A^2 + B^2) \right. \\ & \left. - \frac{i}{2} m \bar{\chi} \chi + g \bar{\lambda} [A + \gamma_5 B, \chi] + \frac{g^2}{2} [A, B]^2 \right) \end{aligned} \quad (3.12)$$

Supersymmetric Yang-Mills theories offer examples of theories with scalars which are asymptotically free. For the previous theory with a gauge group $SU(N)$ and n flavours, the Callan-Symanzik function is given, in the one-loop approximation, by

$$\beta(g) = - \frac{g^3}{16\pi^2} (3 - n) N \quad (3.13)$$

The theory is asymptotically free if $n < 3$. If $n = 3$, $\beta(g) = 0$. A remarkable phenomenon happens if, for $n = 3$, one adds an additional self-coupling to the scalar multiplet to make the theory $SU(4)$ symmetric and therefore invariant under an extended $N = 4$ supersymmetry algebra¹³⁾. For that theory, $\beta(g)$ has been shown to vanish also at the second loop¹⁴⁾, and it is not impossible that such a theory could provide the first example of an exactly scale (conformal) invariant interaction in four dimensions.

4. SUPERGRAVITY AND TENSOR CALCULUS FOR LOCAL SUPERSYMMETRY

Supergravity²⁾ is the supersymmetric extension of general relativity; it enlarges the gauge group of Einstein transformations to a graded gauge group in which a new fermionic invariance is present. This new fermionic symmetry can be regarded as the curve-space generalization of the flat-space symmetry of the massless Rarita-Schwinger equation. It is the appropriate gauge principle for describing, in a consistent way, interacting massless spin $3/2$ particles. From a more general point of view, supergravity can be regarded as the gauge theory of a graded Lie algebra, the supersymmetry algebra, in much the same way as the Einstein theory can be viewed as the gauge theory of the Poincaré group, and Yang-Mills theories as gauge theories of an internal symmetry group. From a more physical point of view, the basic quantum mechanical concepts of fermionic spin and the connection between spin and statistics enter into the formulation of gravitational theories for the first time. A tensor calculus which provides the construction of multiplets and invariant actions in local symmetry is now available. An obvious, but invaluable, consequence is that one can now obtain actions for complicated systems by simply adding invariant subactions, without the tiresome order-by-order Noether constructions which were needed previously. Heart-warming calculations involving complicated Fierz identities, Bianchi identities, high-precision γ -matrix algebra, possible only to supergravity practitioners, are finally circumvented.

The simplest version of a supergravity theory is the gauging of the $N = 1$ supersymmetry algebra which is given by Eq. (2.2) for $i = 1$.

The fundamental Lagrangian of pure supergravity (without coupling to sources), as was originally obtained¹⁵⁾, is

$$\mathcal{L}_{SG} = -\frac{e}{2\kappa^2} R(e, \hat{\omega}) - \frac{1}{2} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma \epsilon^{\mu\nu\rho\sigma} \quad (4.1)$$

The covariant derivative is

$$\mathcal{D}_\rho \psi_\sigma = \partial_\rho \psi_\sigma + \frac{1}{2} \hat{\omega}_{\rho ab}(e, \psi) \sigma^{ab} \psi_\sigma - \Gamma_{\rho\sigma}^\nu(g) \psi_\nu \quad (4.2)$$

$\hat{\omega}_{\rho ab}$ is the spin-connection with spin $3/2$ torsion:

$$\hat{\omega}_{\rho ab}(e, \psi) = \omega_{\rho ab}(e) + K_{\rho ab}(e, \psi) \quad (4.3)$$

where

$$\omega_{\rho ab}(e) = \frac{1}{2} \left[e_a^\sigma (\partial_\rho e_{b\sigma} - \partial_\sigma e_{b\rho}) + e_a^\mu e_b^\nu (\partial_\nu e_{\mu\rho}) e_\rho^\sigma \right] - (a \leftrightarrow b) \quad (4.4)$$

$$K_{\rho ab}(e, \psi) = \frac{\kappa^2}{4} (\bar{\psi}_\rho \gamma_a \psi_b - \bar{\psi}_\rho \gamma_b \psi_a + \bar{\psi}_a \gamma_\rho \psi_b)$$

$K_{\rho ab}$ being the contorsion tensor.

$\mathcal{L}_{SG}$ describes the (locally) supersymmetric interaction of the massless graviton of helicity states $\lambda = \pm 2$ with the massless gravitino of helicity states $\lambda = \pm 3/2$. From the point of view of the symmetry, the Lagrangian (4.1) describes the self-interaction of the $(2, 3/2)$ massless multiplet of global supersymmetry (see Table 1).

Coordinate invariance and local supersymmetry are necessary requirements in order that the fields $e_{a\mu}$ and ψ_μ describe the interaction of physical degrees of freedom only. The Lagrangian $\mathcal{L}_{SG}$ in Eq. (4.1) is invariant (up to a 4-divergence) under the following field variations^{*)}:

*) From now on we use a different γ -matrix convention than in the previous sections, namely $g_{\mu\mu} = (1, 1, 1, 1)$ and 0 otherwise, $\gamma_\mu^2 = 1$, $\gamma_5^2 = 1$.

$$\delta e_{a\mu} = \kappa \bar{\epsilon} \gamma_a \psi_\mu$$

$$\delta \psi_\mu = \frac{2}{\kappa} D_\mu \epsilon \quad \left(D_\mu = \partial_\mu + \frac{1}{2} \hat{\omega}_{\mu ab} \sigma^{ab} \right) \quad (4.5)$$

This is unsatisfactory because one would then expect that

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_G + \delta_L + \delta_S \quad (4.6)$$

where δ_G , δ_L , and δ_S are general coordinate transformations, local Lorentz transformations, and local supersymmetry transformations. Instead, from (4.5) one gets that the commutator of the local transformations of parameters $\epsilon_1(x)$, $\epsilon_2(x)$ gives, when acting on ψ_μ ,

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \psi_\mu = (\delta_G + \delta_L + \delta_S) \psi_\mu + \text{"eqs. of motion terms"}, \quad (4.7)$$

where "eqs. of motion terms" vanish only on the spin- $\frac{3}{2}$ shell, i.e. if

$$\epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu D_\rho \psi_\sigma = 0 \quad (4.8)$$

This phenomenon already occurred in global supersymmetry when fields such as F and G in Eq. (3.1) and D in Eq. (3.9) played the role of auxiliary fields. "*La seule raison d'être*" of these fields is to make the algebra closed. The same phenomenon takes place in supergravity. The complete set of potentials needed to close the local algebra is given by the following multiplet¹⁶⁾:

$$(e_{a\mu}, \psi_\mu, S, P, A_\mu) \quad (4.9)$$

where S , P , A_μ are a scalar, a pseudoscalar, and an axial vector field, respectively. These fields are the auxiliary fields of supergravity.

The local supersymmetry variations for the gauge potentials with a closed algebra are the following ($R^\mu = e^{-1} \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu D_\rho \psi_\sigma$)

$$\begin{aligned}
\delta e_a{}_\mu &= \kappa \bar{\epsilon} \gamma_a \psi_\mu \\
\delta \psi_\mu &= \frac{2}{\kappa} D_\mu \epsilon + \frac{1}{3} \gamma_\mu (S - i \gamma_5 P) \epsilon + i (A_\mu - \frac{1}{3} \gamma_\mu \not{A}) \gamma_5 \epsilon \\
\delta P &= -\frac{i}{2} \bar{\epsilon} \gamma_5 \gamma \cdot R - \frac{1}{2} \kappa \bar{\epsilon} \gamma \cdot \psi P + \frac{i}{2} \kappa \bar{\epsilon} \gamma_5 \gamma \cdot \psi S + \frac{1}{2} \kappa \bar{\epsilon} A \cdot \psi \\
\delta S &= \frac{1}{2} \bar{\epsilon} \gamma \cdot R - \frac{1}{2} \kappa \bar{\epsilon} \gamma \cdot \psi S - \frac{i}{2} \kappa \bar{\epsilon} \gamma_5 \gamma \cdot \psi P + \frac{i}{2} \kappa \bar{\epsilon} \gamma_5 A \cdot \psi \\
\delta A_\mu &= \frac{3}{2} i \bar{\epsilon} \gamma_5 (R_\mu - \frac{1}{3} \gamma_\mu \gamma \cdot R) + \frac{\kappa}{2} \bar{\epsilon} \psi_\mu P + \frac{i}{2} \kappa \bar{\epsilon} \gamma_5 \psi_\mu S - \\
&\quad - \frac{\kappa}{2} \bar{\epsilon} \gamma \cdot \psi A_\mu + \kappa \bar{\epsilon} \not{A} \psi_\mu - \frac{\kappa}{4} e \epsilon_{\mu\nu\rho\sigma} A^\nu \bar{\epsilon} \gamma_5 \gamma^\rho \psi^\sigma
\end{aligned} \tag{4.10}$$

The Lagrangian which is invariant under (4.10) is

$$\begin{aligned}
\mathcal{L}_{SG} &= -\frac{e}{2\kappa^2} R(e, \hat{\omega}) - \frac{1}{2} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \epsilon^{\mu\nu\rho\sigma} + \\
&\quad + \frac{e}{3} (g^{\mu\nu} A_\mu A_\nu - S^2 - P^2)
\end{aligned} \tag{4.11}$$

From (4.10) one indeed gets a closed algebra

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_S(\epsilon') + \delta_G(\xi^\mu) + \delta_L(\omega^{ab}) \tag{4.12}$$

with

$$\begin{aligned}
\epsilon' &= -\kappa \bar{\epsilon}_2 \gamma^\mu \epsilon_1 \psi_\mu, \quad \xi^\mu = 2 \bar{\epsilon}_2 \gamma^\mu \epsilon_1, \quad \omega^{ab} = 2 \bar{\epsilon}_2 \gamma^\rho \epsilon_1 \hat{\omega}_\rho^{ab} - \\
&\quad - \frac{2}{3} i \kappa \bar{\epsilon}_2 \gamma_\rho \epsilon_1 A^\rho \epsilon^{abcd} - \frac{4}{3} \kappa \bar{\epsilon}_1 \sigma^{ab} \epsilon_2 S - \frac{4}{3} i \kappa \bar{\epsilon}_2 \sigma^{ab} \gamma_5 \epsilon_1 P
\end{aligned}$$

In pure supergravity the field equations for the auxiliary fields are $S = P = A_\mu = 0$, and one re-obtains the original Lagrangian (4.1) and the "Stony Brook algebra", which no longer closes unless $R_\mu = 0$.

We make the following observation. Inspection of the basic group property (4.12) shows that the commutator algebra contains both vector $\epsilon_1 \gamma^\mu \epsilon_2$ and tensor $\epsilon_1 \sigma^{ab} \epsilon_2$ structures. Tensor terms $\epsilon_1 \sigma^{ab} \epsilon_2$ are proportional to the S, P fields. If S and P cannot be eliminated, the commutator algebra that we have introduced seems not to be equivalent to the Breitenlohner commutator algebra¹⁷⁾ (at least in the present formulation) because of the lack, in the latter, of any tensor terms whatsoever.

It is useful to make a comparison between the different algebraic schemes that have been constructed in order to give a geometrical structure to supergravity.

In the Breitenlohner scheme there are $126 = 9 \times 14$ potentials in the basic multiplet. These potentials are just the field content of a Lie algebra valued vector multiplet

$$V = V_a P^a + \bar{V}_\alpha Q_\alpha + V_{ab} M^{ab} \quad (4.13)$$

over the 14-dimensional graded Poincaré algebra. It is clear that (4.13) implies a first-order formulation of supergravity because the spin-connection is treated as an independent potential. De Wit and Grisaru¹⁸⁾ have been able to reduce the 126 potentials to 54 by making a second-order formulation of the Breitenlohner scheme. There are still 22 auxiliary fields in this formulation. The formulation that we have been discussing in this section makes use of only 38 potentials (4.9), of which only 6 (S, P, A_μ) are auxiliary. Of course our formulation is also of the second order in the sense that the spin-connection is not an independent dynamical variable.

It is worth while mentioning that the Breitenlohner potentials are related to the superspace formulation that has been pursued by Brink, Gell-Mann, Ramond and Schwarz¹⁹⁾, while the present formulation is related to the superspace formulation of Wess and Zumino²⁰⁾ and Volkov, Akulov and Soroka²¹⁾. These formulations certainly overlap whenever the over-all theory is superconformal, because in that case S and P can be gauged away, or if supergravity is coupled to conformal matter, because in that case $S = P = 0$ are consistent constraints.

With the completed gauge potential multiplet, one can now formulate a tensor calculus for local supersymmetry²²⁾ in much the same way as it was formulated in global supersymmetry (see Section 3).

The transformation laws of a scalar multiplet with components $Z = A + iB$, $\chi_L = \frac{1}{2}(1 + \gamma_5)\chi$, $H = F + iG$, becomes (we put $k = 1$ from now on):

$$\begin{aligned}\delta Z &= 2\bar{\epsilon}_L \chi_L \\ \delta \chi_L &= \not{D} Z \epsilon_R + H \epsilon_L \\ \delta H &= 2\bar{\epsilon}_R \not{D} \chi_L - \frac{2}{3}\bar{\epsilon}_L \chi_L u - \frac{i}{3}\bar{\epsilon}_R \not{A} \chi_L\end{aligned}\tag{4.14}$$

For a vector multiplet in the "Wess-Zumino" gauge we have:

$$\begin{aligned}\delta V_\mu &= -\bar{\epsilon} \gamma_\mu \lambda \\ \delta \lambda &= \sigma_{\mu\nu} \hat{F}^{\mu\nu} \epsilon + i\gamma_5 \not{D} \epsilon \\ \delta \not{D} &= i\bar{\epsilon} \gamma_5 \not{D} \lambda - \frac{i}{2}\bar{\epsilon} \not{A} \lambda\end{aligned}\tag{4.15}$$

where $u = S - iP$ and curly derivatives $\mathcal{D}_\mu$ are supercovariant derivatives:

$$\begin{aligned}
 \mathcal{D}_\mu z &= \partial_\mu z - \bar{\Psi}_\mu \chi_L \\
 \mathcal{D}_\mu \chi_L &= D_\mu \chi_L - \frac{1}{2} \not{\partial} z \Psi_{\mu R} - \frac{1}{2} H \Psi_{\mu L} \\
 \hat{F}_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu + \frac{1}{2} \bar{\Psi}_\mu \gamma_\nu \lambda - \frac{1}{2} \bar{\Psi}_\nu \gamma_\mu \lambda \\
 \mathcal{D}_\mu \lambda &= D_\mu \lambda - \frac{1}{2} \hat{F}_{ab} \sigma^{ab} \Psi_\mu - \frac{i}{2} \gamma_5 D \Psi_\mu
 \end{aligned} \tag{4.16}$$

Equations (4.14) and (4.15) reduce to the global case (3.1) and (3.9) for vanishing potentials.

Supercovariant derivatives are constructed in the same way as ordinary covariant derivatives in Yang-Mills theories or in any other gauge theory. Moreover, if the vector multiplet is a Yang-Mills multiplet, the supercovariant derivatives contain also Yang-Mills connections.

Scalar and pseudoscalar densities are given by

$$\begin{aligned}
 e F + \frac{e}{2} \bar{\Psi} \cdot \gamma \chi + \frac{e}{2} \bar{\Psi}_\mu \sigma^{\mu\nu} (A - i \gamma_5 B) \Psi_\nu + e (AS + PB) \\
 e G - i \frac{e}{2} \bar{\Psi} \cdot \gamma \gamma_5 \chi + i \frac{e}{2} \bar{\Psi}_\mu \sigma^{\mu\nu} (B + i \gamma_5 A) \Psi_\nu + e (SB - PA)
 \end{aligned} \tag{4.17}$$

Again Eq. (4.17) reduces to the flat density $F(G)$ for vanishing potential.

The kinetic multiplet $T(A, B, \chi, F, G)$ has components

$$A_T = F + \frac{1}{3} (SA + BP), \quad B_T = -G - \frac{1}{3} (BS - AP)$$

$$\begin{aligned}
\chi_T &= \cancel{2} \chi - \frac{i}{6} \cancel{A} \gamma_5 \chi - \frac{1}{3} (A - i \gamma_5 B) \eta \\
F_T &= \tilde{\square} A \\
G_T &= -\tilde{\square} B
\end{aligned} \tag{4.18}$$

where $\tilde{\square}$ is a supercovariant d'Alembertian and η is defined below.

One can also construct scalar multiplets which are functions of the gauge potentials only.

These multiplets vanish in the flat limit but they are crucial for describing the pure supergravity part of the action. Of particular relevance is the scalar curvature multiplet $\tilde{R}$ with components

$$\begin{aligned}
A_{\tilde{R}} &= S, \quad B_{\tilde{R}} = \mathbb{P} \\
\chi_{\tilde{R}} &= \eta = \frac{1}{2} (\gamma \cdot R - \gamma \cdot \psi S - i \gamma_5 \gamma \cdot \psi \mathbb{P} + i \gamma_5 A \cdot \psi) \\
F_{\tilde{R}} &= \frac{1}{2} R + \frac{1}{2} \bar{\Psi}_\mu \gamma^\nu \gamma^\mu R_\nu + \frac{1}{4} \bar{\Psi}_\mu (S - i \gamma_5 \mathbb{P}) \Psi^\mu \\
&\quad + \frac{i}{4} \bar{\Psi} \cdot A \gamma_5 \gamma \cdot \psi - \frac{2}{3} (S^2 + \mathbb{P}^2 + \frac{1}{2} A_\mu^2) \\
G_{\tilde{R}} &= -e^\mu \partial_\mu (e A^\nu) + \frac{i}{2} \bar{\Psi}_\mu \gamma_5 R^\mu - \frac{i}{2} \bar{\Psi}_\mu \sigma^{\mu\nu} \gamma_5 R_\nu \\
&\quad + \frac{i}{4} \bar{\Psi}_\mu \gamma_5 (S - i \gamma_5 \mathbb{P}) \Psi^\mu + \frac{1}{4} \bar{\Psi} \cdot A \gamma \cdot \psi
\end{aligned} \tag{4.19}$$

The supergravity Lagrangian $\mathcal{L}_{\text{SG}}$ given by Eq. (4.11) is the scalar density constructed with $-\tilde{R}$. The locally supersymmetric action for a scalar multiplet in the improved form is the scalar density constructed with $\frac{1}{2} \Sigma \otimes T(\Sigma)$.

The Yang-Mills locally supersymmetric action is the scalar density constructed with $\bar{W}^\alpha(V) \otimes W_\alpha(V)$. A useful identity is $T(I) = \frac{1}{3} \tilde{R}$. Here $I = (1, 0, 0, 0, 0)$ is the identity multiplet.

5. SPONTANEOUS SYMMETRY-BREAKING AND SUPER-HIGGS MECHANISM

Spontaneous symmetry-breaking is one of the most outstanding problems encountered in building models for unified interactions in the framework of supersymmetry invariance.

Exact supersymmetry assigns equal masses to bosons and fermions and, apart from some massless particles such as the photon, graviton, and neutrino, Nature does not look at all supersymmetric. In order not to spoil the nice renormalizability properties of supersymmetric theories, the breaking mechanism has to be spontaneous. When global supersymmetry is spontaneously broken, according to general arguments, a massless Goldstone fermion arises in the mass spectrum^{11,23)}. For spontaneously broken N-extended supersymmetry, one would expect N massless fermions.

In a unified theory with global supersymmetry a necessary condition for spontaneous breaking is that one of the auxiliary fields F, G, D of the basic scalar and vector multiplets takes a non-vanishing value at the minimum of the potential²³⁾. This is because then there is a spin $\frac{1}{2}$ Majorana field for which

$$\delta\lambda = \frac{1}{a} \epsilon + \text{q-number terms} . \quad (5.1)$$

This transformation law in turn implies that the symmetry charge Q_α does not annihilate the vacuum.

An example of Lagrangian with spontaneous breaking of supersymmetry is the Volkov-Akulov⁸⁾ Lagrangian

$$\mathcal{L}_{VA} = -\frac{1}{2a^2} \det(\mathcal{F}_\mu^\nu + i a^2 \bar{\lambda} \gamma^\nu \partial_\mu \lambda) \quad (5.2)$$

in which the Goldstone fermion λ transforms non-linearly under supersymmetry

$$\delta\lambda = \frac{1}{a}\epsilon + ia\bar{\epsilon}\gamma^\mu\lambda\partial_\mu\lambda \quad (5.3)$$

If supersymmetry is promoted to a local invariance a super-Higgs effect can occur, namely the goldstino (Goldstone fermion) can be eaten by the massless gravitino, the gauge fermion of local supersymmetry, after which banquet it becomes massive. This is nothing but the supersymmetric version of the Higgs-Kibble mechanism of ordinary gauge theories.

Deser and Zumino²⁴⁾ have studied the coupling of the Volkov-Akulov Lagrangian to de Sitter supergravity and have shown that, under the assumption of a vanishing cosmological term, there is a relationship between the mass of the gravitino and the constant a which measures the breaking of the symmetry:

$$m_\psi^2 = \frac{1}{6a^2} \quad (5.4)$$

Using the established tensor calculus²²⁾ for supergravity, the super-Higgs effect has recently been investigated²⁵⁾ for general interactions of the scalar multiplet $\Sigma = (A, B, \chi, F, G)$ coupled to supergravity.

The most general interaction for a scalar multiplet Σ with second-order derivative Lagrangian is given by the following expression:

$$\sum_{nm} a_{nm} \Sigma^n \otimes T(\Sigma^m) + \sum_n b_n \Sigma^n \quad (5.5)$$

If we denote by $\Phi = \sum_{nm} a_{nm} z^n \bar{z}^m$, $g = \sum_n b_n z^n$ ($z = A + iB$), this interaction depends on two arbitrary functions $\Phi(z, \bar{z})$ and $g(z)$.

If $\mathcal{L}$ is assumed to be parity conserving, then a_{nm} , b_n are real and, moreover, a_{nm} are symmetric.

The Lagrangian, in its form (5.5), has the following starting terms:

$$\begin{aligned} \mathcal{L} = & \frac{e}{6} \Phi R - e \Phi_{,z\bar{z}} \partial_\mu z \partial_\mu \bar{z} - e \Phi_{,z\bar{z}} \bar{\chi} \not{\partial} \chi \\ & + \frac{1}{6} \Phi \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\rho \varepsilon^{\mu\nu\rho} + \dots \end{aligned} \quad (5.6)$$

Under certain regularity conditions, namely

$$\Phi(z, \bar{z}) < 0, \quad (\log -\Phi)_{,z\bar{z}} < 0 \quad (5.7)$$

it is possible to perform a Weyl rescaling on the vierbein field $e_{a\mu}$ and a field redefinition on the fermion fields ψ_μ, χ such that the final Lagrangian takes a simple and compact form.

These field redefinitions are

$$-\Phi/3 = e^{-2\lambda}, \quad g = \log(-\Phi/3)^3 / |g|^2, \quad g \neq 0$$

$$e_{a\mu}^N = e_{a\mu} e^{-\lambda}$$

$$z^N = z$$

$$\chi_L^N = \left(\frac{g}{g^*}\right)^{1/4} \chi_L \sqrt{-2g_{,z\bar{z}}} e^{\lambda/2} \quad (5.8)$$

$$\psi_{\mu L}^N = \left(\frac{g}{g^*}\right)^{-1/4} \psi_{\mu L} e^{-\lambda/2} - \frac{2}{\sqrt{-2g_{,z\bar{z}}}} \gamma_\mu^N \chi_L^N \lambda_{,z\bar{z}}$$

In terms of the new variables and after elimination of the auxiliary fields S, P, A_μ, F, G , the Lagrangian (5.5) takes the form

$$\mathcal{L} = \mathcal{L}_{SG} + \mathcal{L}_{MATTER} \quad (5.9)$$

$\mathcal{L}_{\text{SG}}$ is the supergravity Lagrangian (4.1) and

$$\mathcal{L}_{\text{MATTER}} = \mathcal{L}_B + \mathcal{L}_F$$

where

$$\mathcal{L}_B = e \, g_{,z\bar{z}} \partial_\mu z \partial_\mu \bar{z} + e^{-g} \left(3 + \frac{|g_{,z}|^2}{g_{,z\bar{z}}} \right) \quad (5.10)$$

$$\begin{aligned} \mathcal{L}_F = & -\frac{e}{2} \bar{\chi} \not{D} \chi + e e^{-g/2} \left[\bar{\Psi}_\mu \sigma^{\mu\nu} \Psi_\nu - \right. \\ & - (-2g_{,z\bar{z}})^{-1/2} \bar{\Psi} \cdot \gamma \hat{g}_{,z} \chi + \\ & \left. + (2g_{,z\bar{z}})^{-1} \bar{\chi} \left(\frac{\hat{g}_{,zz\bar{z}} \hat{g}_{,z}}{g_{,z\bar{z}}} + \hat{g}_{,z}^2 - \hat{g}_{,zz} \right) \chi \right] \quad (5.11) \end{aligned}$$

+ four-fermion terms

+ two-fermion terms containing $\partial_\mu z, \partial_\mu \bar{z}$.

In Eq. (5.11) the terms which have not been written explicitly are not relevant for the super-Higgs effect.

The Lagrangian (5.9) as well as the transformation laws of the new fields contain only the function G . This means that if we re-define the two functions ϕ and g by means of the transformation

$$g(z) \rightarrow g(z) h(z), \quad \Phi(z, \bar{z}) \rightarrow \Phi(z, \bar{z}) (h(z) h(\bar{z}))^{1/3}$$

we do not change the Lagrangian.

The conditions for the occurrence of spontaneous symmetry breaking are

$$\begin{aligned}
 V_{,z}(z_0) &= 0 \\
 V(z_0) &= 0 \\
 e^{-G/2} G_{,z} &\neq 0 \quad \text{at } z = z_0 .
 \end{aligned}
 \quad \begin{array}{l} \text{in which } z_0 \text{ is an} \\ \text{absolute minimum} \end{array}
 \quad (5.12)$$

The first condition implies the existence of a minimum, the second condition the absence of a cosmological term, the third condition the presence of a constant term in the transformation law of χ . By observing that

$$-V(z, \bar{z}) = e^{-G} \left(3 + \frac{|G_{,z}|^2}{G_{,z\bar{z}}} \right) \quad (5.13)$$

from the first two conditions we get

$$\begin{aligned}
 |G_{,z}|^2 &= -3 G_{,z\bar{z}} \\
 G_{,z\bar{z}} (G_{,zz} G_{,\bar{z}} + G_{,z\bar{z}} G_{,z}) - G_{,z} G_{,\bar{z}} G_{,zz\bar{z}} &= 0 \quad \text{at } z = z_0 .
 \end{aligned}
 \quad (5.14)$$

It is easy to see that at $z = z_0$, after a chiral transformation

$$\tilde{\chi} = \left(\frac{\hat{G}_{,z}}{\hat{G}_{,\bar{z}}} \right)^{1/2} \chi$$

the bilinear fermion terms appear in a universal form

$$\mathcal{L}_F^{\text{HASS}} = e^{-G_0/2} \left[\bar{\Psi}_\mu \sigma^{\mu\nu} \Psi_\nu - \sqrt{\frac{3}{2}} \bar{\Psi} \cdot \gamma \tilde{\chi} - \tilde{\bar{\chi}} \tilde{\chi} \right] \quad (5.15)$$

Finally the substitution

$$\hat{\Psi}_\mu = \Psi_\mu - \frac{1}{\sqrt{6}} \gamma_\mu \tilde{\chi} - \sqrt{\frac{2}{3}} e^{G_0/2} \partial_\mu \tilde{\chi} \quad (5.16)$$

eliminates $\tilde{\chi}$ altogether and leaves only the kinetic and mass term for $\hat{\Psi}_\mu$ with mass

$$m_\Psi = e^{-G_0/2} \quad G_0 = G|_{z=z_0}$$

In the class of models for which $G_{z\bar{z}} = -\frac{1}{2}$ everywhere [$\phi = -3e^{-1/6 z\bar{z}}$, $g(z)$ arbitrary], one gets a universal mass formula

$$m_A^2 + m_B^2 = 4m_\Psi^2 \quad (5.17)$$

Formula (5.17) is independent of the input potential $g(z)$ which originated the spontaneous symmetry breaking. An explicit example in which the conditions (5.12) are fulfilled is given by $g(z) = \lambda(z + b)$ (λ arbitrary and $b = 2\sqrt{2} - \sqrt{6}$). Then $z_0 = \sqrt{6} - \sqrt{2}$ and $m_A^2 = 2\sqrt{3} m_\Psi^2$, $m_B^2 = 2(2 - \sqrt{3})m_\Psi^2$.

The analysis carried out in this section shows the very important fact that the super-Higgs effect is a real phenomenon which can occur in local supersymmetry. Moreover, the mass formula that we have derived gives a bound on the mass of the gravitino in terms of the mass of the scalar particles.

Formula (5.17) is interesting because it could be of real interest in more complicated models describing unified theories of fundamental interactions.

6. EXTENDED SUPERGRAVITY AND MODELS UNIFYING ELECTRODYNAMICS AND CHROMODYNAMICS WITH GRAVITATION

Extended supergravity theories²⁾ arise from the gauging of an extended supersymmetry algebra described by Eqs. (2.2). The basic gauge multiplet has a particle content dictated by Table 1. These theories have the property of unifying gravity with interactions of particles with lower spin. If $N \geq 4$, scalar, spin $\frac{1}{2}$, and spin 1 particles appear in the same multiplet of the graviton.

The simplest example of these theories is the gauging of $N = 2$ extended supergravity²⁶⁾. This theory can be regarded as the unification of Maxwell theory with gravity. The photon sits in the same multiplet as the graviton, together with two spin $\frac{3}{2}$ Majorana fields.

It is amazing to observe that this theory improves the usual Maxwell-Einstein theory because it gives finite one- and two-loop corrections to scattering processes. For example, in the non-supersymmetric Maxwell-Einstein system, photon-photon scattering is one-loop divergent, but it turns out to be finite in the supersymmetric version^{2,3)}.

Theories²⁷⁾ with $N \geq 3$ have a global non-Abelian symmetry $SO(N)$. However, in the presence of the gravitational constant, only, the $SO(N)$ group is not gauged. The $N(N-1)/2$ spin 1 gauge potentials gauge an Abelian group $U(1)^{N(N-1)/2}$, the group of the central charges. The $SO(N)$ group can be gauged²⁸⁾ provided one adds to the Lagrangian a mass-like term for the gravitino with mass $m_\psi = \sqrt{2}(g/k)$ and a cosmological term with strength $6(g/k^2)^2$. Here g is the $SO(N)$ Yang-Mills coupling constant.

The reason for this mechanism is easily understood by noting that with a cosmological term present, the graded group is no longer $IO(3, 1)$ but rather the de Sitter group $O(3, 2)$. As discussed in Section 2, the internal symmetry part of an extended graded $O(3, 2)$ algebra is precisely $SO(N)$.

Apart from the problem of interpretation of the cosmological term which one could think to be absent in a realistic situation in which spontaneous symmetry-breaking occurs, these theories have the appearance of truly unified theories of particle interactions in which an underlying Yang-Mills gauge group $SO(N)$ is present. It is not clear at present whether these theories can be used for current particle phenomenology. The main difficulty lies in the fact that the biggest gauge group which can be realized in a Lagrangian field theory is $SO(8)$, which is too small to accommodate the minimal gauge group of unified weak, strong and electromagnetic interactions which is $SU(3) \otimes U(2)$.

Nevertheless, the $SO(8)$ group contains the $SU(3)$ group, and it can be regarded as a unified theory of gravity with quantum chromodynamics.

Let us now take a closer look at the Gell-Mann classification⁶⁾ of the particle states of $SO(8)$ supergravity with respect to their $SU(3)$ -colour content.

The eight spin $\frac{3}{2}$ gravitinos split into $3 \oplus \bar{3} \oplus 1 \oplus 1$ of $SU_c(3)$. Electric charges are assigned to be $Q = -\frac{1}{3}, \frac{1}{3}, 0, 0$, and the rest of the analysis follows without any further assumption. The adjoint representation of $SO(8)$ to which the spin 1 gauge bosons belong is split into: $8(0) \oplus 1(0) \oplus 1(0) \oplus 3(-\frac{1}{3}) \oplus 3(-\frac{1}{3}) \oplus 3(\frac{2}{3}) \oplus \bar{3}(\frac{1}{3}) \oplus \bar{3}(\frac{1}{3}) \oplus \bar{3}(-\frac{2}{3})$. The charge assignment is given in brackets.

The particles can be identified with the gluons, photons, Z , heavy, and superheavy vector bosons which already occur in other attempts to unify non-gravitational interactions. The 56 spin $\frac{1}{2}$ Majorana particles break up into Dirac spinors as follows:

$$28 = 3(\frac{2}{3}) \oplus 3(-\frac{1}{3}) \oplus 3(-\frac{1}{3}) \oplus 3(\frac{2}{3}) \oplus 6(\frac{1}{3}) \oplus 8(0) \oplus 1(-1) \oplus 1(0) .$$

These particles can be identified with the u, d, s, c quarks, a fifth quark flavour, a neutral octet (heavy leptons), and the electron and its neutrino. As one can see, the missing particles are at least the charged vector bosons $W^\pm$, the muon and its neutrino, and the τ .

7. CONCLUSIONS

In the present survey of supersymmetry and its application to Lagrangian field theory and to fundamental particle interactions, we have tried to cover the classification of particle states according to this new symmetry, the construction of the basic Lagrangians, and the description of a new gauge principle -- local supersymmetry -- which appears to be the first convincing framework for unifying all the interactions, including gravitation.

We have also discussed, in some detail, some of the recent technical developments of the field, such as the tensor calculus for local supersymmetry and the study of the super-Higgs effect. Because of lack of space and time we have not, however, been able to cover other important results and to talk about the progress which has been made in the fields, including the superspace formulation of globally and locally supersymmetric theories and the renormalization situation in supergravity.

Another important development has been in the study of local supersymmetry from the point of view of differential geometry, using the fibre bundle approach²⁹⁾. It is worth while mentioning also the study of the anomalies, such as the axial anomaly and the stress tensor anomaly in several supergravity models³⁰⁾.

Recent progress in the tensor calculus for supergravity, with the extension of the density formulae to vector multiplets, has also been made³¹⁾.

Finally, we would like also to mention the construction of superconformal theories³⁾, which arise from the gauging of the graded conformal algebra and which can be regarded as the supersymmetric extension of Weyl gravity. Very recently, more general (sometimes non-local) supersymmetries have been proposed in the context of source theory³²⁾.

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