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DIFFRACTIVE PRODUCTION OF 3π STATES AT 63 AND 94 GeV

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ABSTRACT

Diffraction production of the 3π system has been studied at 63 and 94 GeV using a two-magnet spectrometer with high, uniform acceptance. The total number of events used in the analysis is $\sim 600,000$. The A_2 meson is shown to be diffractively produced. The existence of a resonant component in both the 1^+ and 2^- enhancements is established and resonance parameters for the corresponding A_1 and A_3 mesons are given. There are several indications in the data of states which would correspond to radial excitations in the quark model.

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1. INTRODUCTION

In the initial stages of operation of the CERN SPS, secondary beams of up to 200 GeV became available in the West Area. In the field of meson spectroscopy it became possible to study the production mechanisms of known meson states at very large values of $\sqrt{s}$, the total centre-of-mass energy, and to explore the spectrum of states in a kinematically clean environment, extending into the mass region above 2 GeV which had not previously been accessible. To exploit these opportunities, a multiparticle spectrometer was designed by the ACCMOR Collaboration and installed in the West Area (SPS experiment WA3). This spectrometer was equipped with a hydrogen target, two bending magnets to achieve good acceptance and momentum measurement of low- and high-energy particles, five banks of magnetostrictive spark chambers, and two Čerenkov counters for secondary-particle identification. The spectrometer was fed with a tagged beam capable of providing particles up to 100 GeV. The spectrometer proved to be a very flexible device with good multiparticle efficiency for up to 10 secondary particles. Data have been taken on the reactions

- i) $\pi^- p \rightarrow \pi^- \pi^+ n$ at 63 GeV,
- ii) $\pi^- p \rightarrow K^- K^+ n, \bar{p}pn$ at 63 GeV,
- iii) $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ at 63 and 94 GeV,
- iv) $K^- p \rightarrow K^- \pi^- \pi^+ p$ at 63 GeV,
- v) $(\pi^\pm, K^\pm, p, \bar{p})p \rightarrow (K^- K^+, \bar{p}p) + \text{anything}$ at 63 and 94 GeV,
- vi) $(\pi^-, K^-)p \rightarrow \text{anything}$ at 63 GeV.

Data have also been acquired on reactions with a secondary π^0 or η^0 , in particular

- vii) $\pi^- p \rightarrow \pi^- \pi^+ \pi^0 n$ at 63 GeV,
- viii) $K^- p \rightarrow K^- \omega^0 p$ at 63 GeV.

This paper is concerned with reaction (iii):

$$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p \text{ at 63 and 94 GeV .}$$

The reaction proceeds primarily by diffractive production of the 3π system. It has long been known that at low mass, $\sim 0.9\text{--}1.5$ GeV, the 3π system is dominated

by $\rho\pi$ in an S-wave (the A_1 enhancement) produced by natural spin-parity exchange, although there is also production of the $J^P = 2^+ A_2$ meson, also by natural spin-parity exchange. At higher masses diffractive production of $f\pi$ in an S-wave predominates (the A_3 enhancement). The quark model predicts the existence of a $J^P = 1^+$ meson (A_1) in the low-mass region and a 2^- meson (A_3) around 1.7 GeV. Previous experiments have failed to demonstrate the existence of these mesons, or to provide compelling evidence against their existence. The reason is primarily the presence of background Deck amplitudes, in which the incident pion turns into a ρ or an f by emitting a virtual pion then scattered into a real state from the target nucleon.

In this experiment, some 600,000 events have been gathered, an order of magnitude more than obtained in any previous study. The 3π mass resolution was $\sigma \sim 10$ MeV, the over-all acceptance $\geq 50\%$ and only slowly varying as a function of the kinematic variables up to 3π masses in excess of 2 GeV. Because of the high incident energies, diffractive production of the 3π system was kinematically clean.

The existence of resonant components in the A_1 and A_3 enhancements has been established, the $J^P = 1^+ A_1$ meson having a mass ~ 1.28 GeV and a width ~ 300 MeV. The $J^P = 2^- A_3$ meson has a mass ~ 1.7 GeV and a width ~ 300 MeV, and couples to P-wave $\rho\pi$, D-wave $\epsilon\pi$ and D-wave $f\pi$, in addition to the dominant decay mode, S-wave $f\pi$. The A_1 and A_3 enhancements and the A_2 meson have similar production mechanisms: this experiment has confirmed that the A_2 is diffractively produced. Short accounts of these results have already been published [1-3].

There is some evidence among the minor waves for a resonance in the 0^- amplitudes and, near 1.7 GeV, in 1^+ and 2^+ amplitudes. There is also evidence for a second 2^- resonance 300-400 MeV above the A_3 . Such states could represent radial excitations in the quark model.

2. THE APPARATUS

2.1 The spectrometer - general description

A plan view of the spectrometer is shown in fig. 1. The 50 cm liquid-hydrogen target (T) was surrounded by a box of lead-scintillator sandwich counters (F) with a downstream window matched to the aperture of the first bending magnet (M1). These counters were used to veto events with low-energy charged particles or π^0 's from target fragmentation, or to preferentially select events with large four-momentum transfer to the target proton. Immediately following the target box were two planes of proportional wire chambers (P1) used to measure charged-particle multiplicity. Between P1 and the first bending magnet (M1) was the first bank of spark chambers (S1), 24 wire planes with magnetostrictive readout.

The first bending magnet (M1) had a large aperture, 2.5 m (horizontal) $\times$ 0.6 m (vertical) and a bending power $\int B dz = 8.5 \text{ kG}\cdot\text{m}$. The function of this magnet was to provide good acceptance and momentum determination for low-momentum charged particles ($0.5 < p < 5 \text{ GeV}$). The entrance window was matched to the aperture at exit and had dimensions 2 m $\times$ 0.4 m, defined by sandwich counters (G) used in veto in the trigger. The interior faces of the magnet were also lined with veto counters.

Following the first magnet was a double scintillator hodoscope (P2/3) to measure the charged-particle multiplicity. Between P2/3 and the second bending magnet (M2) came the second bank of spark chambers, 20 wire planes (S2). The geometrical acceptance of the spectrometer, defined by requiring all three secondary tracks in S2, was $> 95\%$ for 3π masses less than 2.5 GeV and a beam momentum of 94 GeV.

The second bending magnet, M2, had an aperture 1.5 m $\times$ 0.5 m and a bending power $\int B dz = 20 \text{ kG}\cdot\text{m}$. Veto counters (H) defined an entrance window equal to the aperture, but information from these counters was not used in the study of diffractive processes. The third bank of spark chambers (S3), 16 planes, followed M2 and was separated from the fourth bank (S4), 12 planes, by the first multicell

Čerenkov counter (C1), with a pion threshold of 8 GeV. The wire planes in S3 and S4 served to locate tracks relative to C1 in addition to their role in momentum determination.

The second Čerenkov counter, C2, had a pion threshold of 17 GeV. The last bank of spark chambers (S5), 8 wire planes, contributed little to the momentum or angular resolution, but served to locate tracks relative to C2.

The last element of the spectrometer consisted of a double scintillator hodoscope, the two layers separated by a wall of lead (P4/5). Information from this array was not used in the study of diffractive 3π production, and for the study of events with a secondary π^0 or η^0 it was replaced with a high-resolution γ -ray calorimeter. The material of the spectrometer presented to secondaries an average of 0.07 radiation length and 0.03 nuclear interaction length between the target and the second bending magnet. The values for the full spectrometer, excluding P4/5, are 0.13 radiation length and 0.07 interaction length.

The spectrometer was set up in the H1B beam line in the West Area at CERN, and data taking started in June 1977. The beam line is discussed in section 2.2 below, and individual components of the forward spectrometer in subsequent sections.

2.2 The beam line

The H1B beam provided positive or negative particles with a maximum energy ~ 100 GeV. The beam was unseparated, with a total length of 218 m, and had three vertical and four horizontal foci. Half a radiation length of lead was inserted at the second horizontal focus to remove electrons, which were present in the beam at the 1% level. The maximum angular acceptance was $10.5 \mu\text{sr}$ and the maximum momentum bite was $\sim 10\%$.

The beam particles were identified with two differential Čerenkov counters (CEDARs) and one threshold Čerenkov counter.

In this experiment, the beam was run at intensities between 5×10^4 and 5×10^5 particles each burst, with a spill time of 800 ms. The composition of

the negative beam at 94 GeV was 95% π^- , 1% μ^- , 3.5% K^- , and less than 0.5% $\bar{p}$. The momentum bite of the beam was usually set to $\pm 0.15\%$, using a momentum-defining slit at the first horizontal focus. The beam at the target had dimensions typically 6 mm (horizontal) $\times$ 2.5 mm (vertical) and divergence 1.6 mrad $\times$ 2.5 mrad.

Between the last beam magnet and the target box were ten proportional wire chambers with 1 mm wire spacing, in two sets separated by 7 m. These chambers were used on line to monitor the beam spot size, position, and divergence, and off line to reconstruct the beam track.

A set of five scintillation counters (B1-5) was used to define the beam. The counter B5, located immediately downstream of the last beam magnet, was 20 cm $\times$ 20 cm with a 3 cm $\times$ 6 cm hole. The circular counter B4 had a diameter of 30 cm and a 1.2 cm diameter hole and was at the entrance to the target box. These two counters were used in veto with signals from B1, B2, and B3, of dimensions 10 $\times$ 14, 10 $\times$ 10, and 3 $\times$ 3 cm², respectively, in defining the beam in the trigger logic.

2.3 The liquid-hydrogen target

The target cell was 50 cm long and 3.5 cm in diameter. The walls were of 150 μ m mylar, plus 30 layers of superinsulation of 6 μ m mylar + 1 μ m aluminium. The cell was contained in a vacuum tank with a tapered Rohacell body, 1 cm thick (0.17 g $\cdot$ cm⁻²) at the exit end. The target cell was part of a closed system, fed from a reservoir containing approximately 2 m³ hydrogen gas at ~ 1.8 atm.

2.4 Veto counters

The target was surrounded by a box of lead-scintillator sandwich counters F. An exploded view is shown in fig. 2. The outer F counters each consisted of 6 layers of 3 mm scintillator interleaved with 6 layers of 5 mm thick lead, the 6 layers for each counter feeding a single photomultiplier. A set of inner F counters, of 3 mm thick scintillator, covered all outer F counters except the two at the upstream end of the box and were equipped with separate photomultipliers. The exit window of the box had dimensions 62 cm $\times$ 15 cm, matched to the acceptance,

from the upstream end of the target, of the first spectrometer magnet. There was a circular entrance window of diameter 18 cm, covered by the beam veto counter B4.

This box of sandwich counters was designed to provide a veto system for events with low-energy charged particles or π^0 's emerging from the target. In the study of diffractively produced three-particle states it was used in three different modes:

i) As part of the hardware trigger, with the requirement that no more than one of the outer counters and no more than one of the inner counters fired.

In this mode genuine events are lost if two or more δ electrons reach the F counters, or if a recoil proton and one or more δ electrons reach the F counters. A further possibility is if debris from interaction of a recoil proton sets further F counters. The condition $ST1 = [\sum_i F_{inner}(i) \leq 1] \cdot [\sum_i F_{outer}(i) \leq 1]$ is thus expected (and found) to bias the data as a function of position of the vertex in the target (because of the window at the downstream end) and as a function of four-momentum transfer squared to the proton, t . Recoil protons reach the inner F counters at $|t| \sim 0.045 \text{ GeV}^2$ and penetrate the outer F counters at $|t| \sim 0.13 \text{ GeV}^2$. The effects of this bias were studied by imposing the condition ST1 in software on data taken without the F counters in the trigger, and are shown in fig. 3 as a function of vertex position and of t . Single δ electrons fired the F counters in $\sim 20\%$ of all genuine events and two δ electrons fired the F counters in $\sim 5\%$ of all genuine events, in good accord with prior calculations.

The condition ST1 thus introduced slowly varying biases in vertex position and t , and reduced the number of events with three fast secondaries by $\sim 22\%$. Study of the missing-mass distribution with and without ST1 showed that $\sim 1/3$ of the rejected events had an N^* recoiling from the baryon vertex ($N^*3\pi$ events). The over-all trigger rate was not otherwise significantly affected and in the later data taking these veto counters were used off line.

ii) The condition

$$ST2 = \left[\sum_i F_{outer}(i) \cdot \overline{F_{inner}(i)} = 0 \right] \cdot \left[\sum_i F_{outer}(j) \cdot F_{inner}(j) \leq 1 \right],$$

where i and j refer to the individual F counters, was imposed off line. This alternative was introduced after studies of data taken without the F counters in the trigger had shown that it yielded smoother efficiency as a function of t and an over-all loss of three-prong events of only $\sim 10\%$, of which about half were $N^*3\pi$.

iii) The F counters were used in the trigger to select preferentially events with $|t| > 0.13 \text{ GeV}^2$ (high- t trigger). The condition used was

$$\text{SHT} = \left(F_{\text{inner}} \geq 1 \right) \cdot \left(F_{\text{outer}} = 1 \text{ or } 2 \right).$$

The limitation on the number of F_{outer} counters firing was imposed to reduce the number of $N^*3\pi$ events contaminating the data. The effect of this condition was studied using data taken without the F counter in the trigger. Because the trigger requires the recoil proton to penetrate to the outer F counters, there is a bias as a function of vertex position, t and the azimuthal angle around the beam of the recoil proton. A simple algorithm was written to correct this bias, and represented well the effect of SHT as a function of any single variable or pair of variables (the data were insufficient for tests as a function of all three simultaneously).

Further veto counters were installed on the front and inside faces of the first magnet (G counters). Each of these counters consisted of 6 layers of 5 mm thick lead and 3 mm scintillator, all six layers of scintillator feeding a single photomultiplier. Wide-angle charged particles or γ -rays were caught by four more G counters mounted between the two halves of the first bank of spark chambers ($S1$). The G counters were always used in veto as part of the trigger, and cut the trigger rate by a factor of two. The proportion of genuine $3\pi p$ events vetoed was 8% of the total, with no detectable bias. This loss of good events is attributable to δ electrons, since $\sim 5\%$ of $3\pi p$ events were accompanied by δ electrons produced in the target and escaping through the exit window of the F counter box, and about 5% of $3\pi p$ events produce sufficiently energetic δ electrons between the target and the first spectrometer magnet.

Studies of the distribution of the missing mass squared recoiling against the 3π system have shown that $\sim 15\%$ of diffractive 3π events are produced with an N^* at the baryon vertex. The F and G veto counters reduce this number to $\lesssim 5\%$. (The missing-mass-squared distribution for a sample of 15,000 events, taken with the F counters in the trigger, is shown in fig. 7b.)

2.5 The proportional wire chambers P1

This pair of chambers was installed to provide a high-resolution measure of the number of charged particles emerging from the target veto box. The upstream chamber, P1U, contained 768 wires inclined at 30° to the vertical. The downstream chamber, P1D, had 1024 vertical wires. The sensitive area of the pair was $100 \times 20 \text{ cm}^2$. In both chambers the wire spacing was 1 mm and the sense planes of the two chambers were separated by 5 cm. The chambers were filled with a mixture of argon, isobutane, and freon, and operated at 8 kV.

All wires in each chamber were read out singly and recorded on tape for off-line analysis. In order to form a fast output for use in the trigger the following procedure was adopted. The central 256 wires in each chamber were read singly or in a cluster mode, a cluster being defined as a single wire or as two or more adjacent wires firing. The outer wires in each chamber were read in groups of 16.

2.6 The hodoscope P2/3

This array of scintillation counters measured the multiplicity of charged particles after the first magnet. The P2 and P3 arrays were identical, and in contact. A beam's eye view is shown in fig. 4. The spatial resolution is best in the central region, where the 2 cm wide vertical strips are split in the horizontal plane. The two centre strips of P2 and P3 were 1.5 mm thick; all other elements were of 3 mm scintillator. The detailed dimensions of the array were chosen to provide good acceptance for the two pions produced in the reaction $\pi^- p \rightarrow \pi^- \pi^+ n$, at energies up to 90 GeV and 2π masses in the range 0.5-2.5 GeV. The centre strips of P2/3 were stepped downstream 3 mm so that δ electrons produced

by a beam track could not fire adjacent elements in both P2 and P3. (This was a serious potential source of background in the study of dipion events, but of no importance in studying 3π production.)

The multiplicity yielded by the P2/3 array was taken as the number of elements in which both a P2 counter and the corresponding P3 counter fired. When a P2/3 multiplicity of three was demanded, the limited spatial resolution of the P2/3 array resulted in a loss $\sim 15\%$ of 3π events at a 3π mass ~ 1.2 GeV, dropping to 12% at a mass ~ 1.5 GeV and $\lesssim 8\%$ at masses above 2 GeV.

2.7 The spark chambers

The large wire spark chambers used in the spectrometer were built to a well-proven design [4]. They had magnetostrictive readout on both planes. The wires were made of 120 μm aluminium with a spacing of 1 mm, giving a resolution normal to the wires of ~ 0.25 mm. Details of the chambers are given in table 1. In operation the chambers were flushed slowly with a mixture of 70% neon and 30% helium, saturated with isopropyl alcohol at 14 °C. The chambers were pulsed to 11-15 kV and had a memory time of 1-2 μs and a recharging time of 5 ms.

The efficiencies of the individual wires were monitored using the off-line reconstruction program TENPRO. In the early stages of reconstruction, straight-line tracks were fitted to the sparks in each bank of chambers separately, and the wire efficiencies obtained from those wires lying on a track which had not fired. If two tracks were separated by less than 5 mm in a given plane, usually only a single spark resulted, but a little displaced from one or other track. The single wire efficiencies thus varied from less than 50% in a small region round the spectrometer axis in S1, when an extra beam track was present, to greater than 95% in regions where the track density was low.

2.8 Čerenkov counters

For secondary particle identification, two multicell threshold Čerenkov counters were used. The first, C1, had a fixed length of 4 m and rigid side walls. Fourteen mirrors, in two rows, each focused Čerenkov light on its own

photomultiplier. This array of mirrors covered an area $3.6 \times 1.4 \text{ m}^2$. The counter was filled with a helium-nitrogen mixture with a refractive index of 1.00017 to give a nominal threshold for pions at 8 GeV. The second Čerenkov, C2, had rigid end frames connected by a rubber bellows to allow the length to be varied: the maximum length was 12 m. Sixteen mirrors, covering an area of $4.4 \times 2.2 \text{ m}^2$, each focused Čerenkov light on its own photomultiplier. This counter was filled with helium, refractive index 1.000034, to give a pion threshold at 17 GeV. In both Čerenkov counters, the mirrors were made from aluminized mylar glued on Rohacell backing of thickness $0.1 \text{ g}\cdot\text{cm}^{-2}$ (C1) and $0.2 \text{ g}\cdot\text{cm}^{-2}$ (C2). Both counters were operated at atmospheric pressure. The results of calibration studies of the Čerenkov counters are shown in fig. 5. The Čerenkov counters could be used as part of a trigger, but in these studies of diffractive 3π production they were used off line for particle identification.

To a first approximation, the response of a given photomultiplier was only used if one and only one track, found in the spark chamber banks at both entrance and exit to the Čerenkov, went through an acceptance window frame 5 cm in from the edges of the mirror. The window frame was imposed to eliminate cross-talk between adjacent mirrors. It was found possible to improve the efficiency of the track-identifying algorithm by allowing two particles pointing at one mirror, if one was below the pion threshold of the Čerenkov, and by discarding the window frame and summing the ADC outputs from adjacent mirrors when a track was close to the edge of one mirror and no track pointed at the adjacent mirror. The ADCs on each photomultiplier also allowed pulse-height information to be used in distinguishing between two possible assignments for a particle above the higher corresponding threshold. The response of the Čerenkov counters, used solely in the threshold mode, is shown in fig. 6. The turn-on of these counters as a function of momentum was gradual: for the purpose of track identification a sharp threshold was defined for each class of particle at the 50% efficiency point and any track which did not give Čerenkov light above threshold for a given particle was assigned a higher mass. Corrections for resulting misidentification were made by Monte Carlo methods later in the analysis (see section 4.2).

3. DATA TAKING

3.1 The trigger system

The cross-section for diffractive production of three pions falls rapidly as a function of 3π mass, and with the four-momentum transfer squared to the proton. The events are characterized by three fast secondaries and a relatively low-energy recoil proton. A minimal trigger condition is therefore to require a good beam track, with three particles after the first bending magnet ($P2/3 = 3$). Some events giving only three particles at $P2/3$ will have further charged particles or π^0 's at a wide angle, so the condition $\bar{G}$ was added to the minimal trigger. If the F counters were used in the trigger, a condition F (= ST1 or SHT) was added, and finally a loose multiplicity condition ($P1U > 1 \cdot OR \cdot P1D > 1$) was included. (This condition did not reject any $3\pi p$ events.)

The useful beam signal was

$$UB = B1 \cdot B2 \cdot B3 \cdot \overline{B4} \cdot \overline{B5} \cdot \overline{F25} \cdot \overline{F26} ,$$

F25 and F26 being the counters at the entrance end of the F counter box, with a hole for the beam. The trigger was thus

$$MASTER = UB \cdot F \cdot \bar{G} \cdot (P1U > 1 \cdot OR \cdot P1D > 1) \cdot (P2/3 = 3) .$$

Upon receipt of a MASTER trigger the spark chambers were fired. After a delay of ~ 3 ms the contents of all counter bit pattern units and ADCs, the proportional-wire-chamber information, and the spark-chamber information were read into a NORD 10 computer and subsequently written on to tape (NORD tapes). The NORD computer was also used to monitor the performance of the apparatus throughout data taking.

3.2 Data samples

Data were taken under three principal conditions:

- i) At 94 GeV with $F = ST1$ (standard trigger): 900,000 triggers, yielding 233,192 $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ events.
- ii) At 63 GeV with the F counters used off line (ST2) (standard trigger): 500,000 triggers, 157,046 events.
- iii) At 94 GeV with $F = SHT$ (high-t trigger): 900,000 triggers, 207,890 events.

About 20% of the data were taken with various calibration triggers, in which one component of the standard trigger was relaxed. Small samples of data were also taken with the target empty in order to locate the ends of the target assembly.

Throughout the data taking, at different times of the day, NORD tapes were written with only a beam track trigger and with one or both magnets off. Tapes were also written with the three-prong trigger and both magnets off. These data were used to find the precise alignment of the spark chambers and to cross-calibrate the fields of the two magnets.

3.3 Off-line data processing

The positions of the magnets, spark chambers, and counter hodoscopes were determined relative to bench-marks on the floor of the experimental area, by optical surveying, at the beginning and end of each run. These measurements, together with the parameters of the various components of the spectrometer, provided an initial set of geometry constants for off-line analysis. In the first stage of off-line analysis, beam tracks and three-prong events were used to obtain more accurate numbers for the position and orientation of the spark-chamber planes. This was done by constructing straight-line tracks from the recorded sparks and then varying the position and orientation of the spark-chamber planes to minimize the root-mean-square deviation of the sparks from the tracks, using the program WIGGLE.

The data taken with a beam-track trigger and one or the other of the magnets off were then used to find the ratio of the bending powers of the two magnets, using the program BSPEC. (The absolute value of the bending power was determined most accurately at a later stage from the 3π mass found for $K^- \tau$ decays). The output from WIGGLE and BSPEC provided constants for the track reconstruction program TENPRO, which could handle events with up to ten charged secondaries.

TENPRO first searched for straight-line tracks in each group of spark chambers S1, S2, and S3-5. In each group each view (comprising all planes with wires in a given orientation) was taken in turn as the main view. The program then looped

over every pair of planes in this view, starting with the extreme planes and working inward. Each pair of sparks found was used to define a road and the sparks lying within this road were registered. If the number exceeded a specified number (four in S1), a track was declared and these sparks omitted from further main-view searches.

Having found up to 10 tracks in the main view, the program calculated the coordinates of all sparks in the other views projected on to a plane perpendicular to that of the main view, under the (false) assumption that all sparks in other views lie on a main-view track. The coordinates in this pseudoview of those sparks which do lie on the main-view track form a straight line: the rest are scattered. Thus a separate pseudoview was constructed for each main-view track and a track search carried out in each pseudoview.

Each main-view track projection was then paired with the corresponding pseudoview projection to define a road in three dimensions. A straight-line fit was performed to the sparks lying within the road, and any spark deviating in projection more than 1 mm from the fitted line was discarded. The product of this stage of the analysis was a set of straight-line parameters specifying up to 10 tracks in each group of chambers, not necessarily extending throughout the group.

The second stage of track reconstruction was to match tracks in adjacent groups of spark chambers through the intervening magnet. The angles in the horizontal plane of the two tracks on each side of a magnet were used to predict the z coordinate (down the spectrometer axis) at which the two extrapolated tracks would meet if they belonged to a single-particle trajectory. This was done with a simple algorithm established from the study of trajectories numerically integrated through the measured magnetic fields. The measured tracks were then extrapolated to this value of z and required to meet in the horizontal plane, within errors. The tracks were required to coincide in the vertical plane, after allowance for focusing effects. The matching was performed first through M1, then through M2, and finally a trajectory through the spectrometer was fitted to the associated track elements. The intersection of the beam track (defined by the

proportional wire chambers at the end of the beam line) with a secondary trajectory defined a vertex for that secondary. The initial data sample was defined by those events with three charged secondaries and a beam track meeting at a common vertex, with charge balance.

The program TENPRO, running in the Rutherford Laboratory IBM 360/195, processed three-prong trigger events in 200-300 ms per event. The output of TENPRO consisted of the parameters of the event (vertex coordinates and secondary-particle directions and momenta at the vertex) together with the best straight-line segments of each trajectory in each group of chambers (QLINES). The program also gave the individual wire efficiencies for the spark chambers for each NORD tape. The data tapes produced by TENPRO contained the counter bit patterns, ADC and TDC contents for each event, and were used to monitor, off line, spark-chamber and counter performance and quantities relevant to the physics. Examples from the raw data for the 94 GeV standard trigger sample are shown in fig. 7. The errors on fitted event parameters and other quantities are given in table 2.

Comparison of TENPRO output with the spark-chamber information recorded on the NORD tapes showed that the efficiency for finding a track segment in any given group of spark chambers depended on its distance R from the beam trajectory. The variation of track-finding efficiency as a function of R, with the mean values, are given in table 3. From these results, the average probability of losing a good event in reconstruction is calculated as $\sim 1\%$.

Study of a sample of individual events from a NORD tape resulted in the assignments given in table 4. The TENPRO loss agrees well with the figure calculated from individual track losses, and the observed losses due to secondary interactions and decays are in excellent agreement with calculated loss rates. Three quarters of all triggers were good three-prong events.

About two thirds of the good three-prong events had a large missing mass to the three-particle system, corresponding to one or more undetected fast neutrals. Those events surviving a broad cut on missing mass squared (-3 to $+5$ GeV^2), with

measured vertex within the central 46 cm of the hydrogen target, and with all three secondaries within the sensitive area of the spark chambers at least to the centre of S2, were written to a condensed data summary tape and passed to the next stage of the analysis. At this point each track was flagged with the Čerenkov information.

4. DATA ANALYSIS

4.1 Three-particle partial-wave analysis

The object of this experiment was to obtain the intensities and relative phases of all contributing 3π angular momentum states, as a function of three-pion mass and four-momentum transfer squared to the proton, t . All subsequent steps of the analysis were determined by this requirement, so a brief discussion of the principles is given at this stage.

Two methods of three-particle partial-wave analysis are in general use, one deriving from work at the University of Illinois [5-8] and the other from LBL-SLAC [9-11]. The basic principles (and defects) are common to both approaches.

The kinematics in production of a four-particle final state, such as

$$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$$

at given s , the square of the energy in the $\pi^- p$ centre of mass, are completely described by eight independent variables. These may be chosen so that three characterize production of the 3π system: the four-momentum transfer squared, t , the azimuthal angle of the produced system about the beam, ϕ , and the mass of the system, $M_{3\pi}$. The remaining five describe the decay. These may be taken as the effective mass squared of two of three possible two-particle combinations (Dalitz-plot variables) and three Euler angles giving the orientation of the three-particle plane in space with respect to a suitable set of axes. In this work the y axis was chosen normal to the production plane and the z axis along the direction of the incident π^- in the 3π centre of mass: the t -channel frame.

In the absence of information on the proton polarization, the production amplitude must be independent of ϕ , which is defined with respect to external

axes. The calculation of a model amplitude may proceed as follows. For a given proton amplitude (helicity flip or non-flip) let the 3π system be produced with mass $M_{3\pi}$, and with momentum transfer t , in a state with total angular momentum J and parity P , and in a specified magnetic substate M . The 3π system then decays into π_1^- and a $\pi_2^-\pi^+$ system. The angular distribution of π_1^- is determined by $J^P M$, the orbital angular momentum L between the π_1^- and the $\pi\pi$ system, and the angular momentum j of the $\pi\pi$ system. The associated distribution of j over the magnetic substates m of the $\pi\pi$ system is also specified. The $\pi\pi$ system then decays into two pions and the angular distribution of the pions in the dipion centre of mass is determined by jm . Each decay step introduces a polar and an azimuthal angle: the distribution in these four variables may be transformed into a distribution in four variables α, β, γ (a set of Euler angles) and $M_{\pi_1\pi}^{-+}$.

The behaviour of the amplitude as a function of $M_{\pi_2\pi}^{-+}$ was represented by a Breit-Wigner amplitude when the $\pi^+\pi^-$ is dominated by the ρ, f , or g mesons. When the $\pi\pi$ system is in an S-wave, the dipion amplitude was taken as $e^{i\delta} \sin \delta/q$, where δ is the (complex) S-wave $\pi\pi$ phase shift ('down' solution, see ref. [50]) and q the momentum in the $\pi\pi$ centre of mass. Centrifugal barrier terms in p , where p is the momentum of the odd pion in the 3π centre of mass, were also introduced, with characteristic radius 1 fermi (5 GeV^{-1}).

It is convenient to form linear combinations of the magnetic substates $\pm M$, so that the resulting amplitude is either even or odd under reflection in the production plane. Such states are characterized by M^η with $\eta = \pm 1$. The advantage is that states with different values of η do not interfere in the absence of proton-polarization information, and that at high energies states with $\eta = +1$ are produced by natural-parity exchange and states with $\eta = -1$ by unnatural-parity exchange. The angular momentum functions for given η are relatively real.

Finally the amplitude specified by $J^{P LM^\eta}$ is made Bose symmetric in the two negative pions.

An individual term in the over-all amplitude is thus given by

$$F_{\lambda_i \lambda_f}^{J^P LM \eta} (t, M_{3\pi}) \xi^{J^P LM \eta} (\alpha, \beta, \gamma) f^{J^P L} \quad (1)$$

and the over-all intensity is proportional to

$$\sum_{\lambda_f \lambda_i \eta} \left| \sum_{J^P LM} F_{\lambda_i \lambda_f}^{J^P LM \eta} \xi^{J^P LM \eta} f^{J^P L} \right|^2 \quad (2)$$

where F are a set of production amplitudes to be determined, with λ_i, λ_f labelling initial and final nucleon helicities, ξ are angular distribution factors, and f are functions of the Dalitz-plot variables. Any individual term in the intensity is of the form

$$\text{Re} \left[\left\{ F_{++}^{a*} F_{++}^b + F_{+-}^{a*} F_{+-}^b \right\} \xi^a \xi^b f^{a*} f^b \right], \quad (3)$$

where a, b represent sets of $J^P LM \eta$.

The set of coefficients $\{F_{++}^{a*} F_{++}^b + F_{+-}^{a*} F_{+-}^b\}$ constitutes an unnormalized density matrix with elements ρ_{ab} . It should be noted that because f^a and f^b are known complex functions, it is possible to extract both $\text{Re } \rho_{ab}$ and $\text{Im } \rho_{ab}$.

In the LBL-SLAC program the fitting variables are the amplitudes F, whereas in the Illinois approach the fitting variables are the real and imaginary parts of a normalized density matrix. The two approaches would be identical, were the density matrix constrained to have rank ≤ 2 , but in the absence of this constraint the Illinois program can handle, for example, an admixture of diffractive production with protons and N^* 's in the final state, without explicit introduction of new production amplitudes, and will be less sensitive to garbage. The Illinois program also has more room to absorb any incoherence due to averaging over 3π mass and t.

The Illinois program is described in detail elsewhere [12] and a discussion of general principles may be found in another work [13]. The output of either the Illinois or LBL-SLAC program consists of the intensities in the contributing partial waves, characterized by $J^P LM \eta$, and a set of relative phases. In this

paper the majority of results presented were obtained using the Illinois approach, and when the intensity of a wave is given, it refers to the total number of events at production in a given region of $M_{3\pi}, t$, multiplied by the appropriate diagonal element of the density matrix. When a phase is given, it is the phase of the appropriate off-diagonal element of the density matrix. The errors correspond to an increase in $\ln$ likelihood of $1/2$.

The amplitudes constructed using the general approach described in this section are well known not to satisfy the constraints of three-particle unitarity. However, studies which have been made of this problem show that while corrections for three-particle unitarity may affect both the intensity and the phase obtained for a given wave, the *variation* of both intensity and phase with three-particle mass should be little affected [14-17].

4.2 Method of analysis and treatment of losses

In an experiment in which no events were lost, or the losses were completely event independent, the theoretical amplitude would be fitted directly to the distribution of the data in the five decay variables. In this experiment there were losses which were event independent and losses which depended on the configuration of each event. The latter were dealt with by a Monte Carlo method in which events were generated according to three-particle phase space and fed through a computer model of the apparatus. At this stage all the criteria imposed either on or off line on the real events were imposed on the generated events. Those generated events which survived were weighted with a theoretical intensity, compared with the real events, and the fitting variables adjusted until the best representation of the data was achieved, using maximum likelihood. This resulted in not only the best set of partial-wave intensities and relative phases, but also in the best estimate of the number of events produced in any bin of kinematic variables. The latter number was used in calculating total and differential cross-sections, together with a numerical factor for those losses which were not dependent on configuration and so were not included in the Monte Carlo procedure. These external correction factors are listed in table 5 and include losses due to the G counters

(section 2.4) and losses due to secondary interactions and decays. The losses due to secondary interactions and decays were found in the following way. The probability of a track of given momentum and direction being lost because of secondary interaction (including elastic scattering) in the target or in the material of the spectrometer, or because of decay, was calculated. (The requirement was that a track could be located up to the middle of S2 and would point back to the vertex.) Events were then generated at various 3π masses with an isotropic 3π decay and weighted according to the probability of each track surviving without secondary interaction or decay. The probability of individual events surviving was found not to depend significantly on the kinematic configuration and so the correction was included among the external factors.

In the Monte Carlo program, events were generated at a given value of 3π mass, with a t distribution which was either exponential or the sum of two exponentials. The distribution in the decay variables was given by 3π phase space. These events were generated randomly with uniform distribution in ϕ , the azimuthal angle around the beam, and the position of the target vertex was assigned to give the distribution along the target arising from beam attenuation and secondary particle interaction in the target (although the normalization for these effects was done externally).

The secondaries for each event were then tracked through the model apparatus, and events were discarded if they did not satisfy the criteria imposed for real events. The model included the geometrical properties of the apparatus, the counter and chamber efficiencies as measured with data taken with calibration triggers, and a parametrization of multiple scattering in the apparatus. Events were discarded on purely geometrical criteria and on a statistical basis. In the former category, any event with a track not passing through the sensitive area of the spark chambers S2 was discarded, as was any event where two tracks went through the same element of P2/3 or a track went through a crack between two elements of P2/3. In the second category, events were discarded randomly according to the

efficiency of individual elements of P2/3 or if a track had too few segments to allow a fit including the vertex.

The probability of an event satisfying the F-counter criteria was applied as a weight.

Čerenkov tagging of the generated tracks was also done both geometrically and statistically. Tracks were marked as unidentifiable in a Čerenkov if they did not satisfy the geometric criteria imposed on real tracks. Secondly, tracks were marked as unidentifiable according to the probability of the geometry being spoiled by a secondary interaction or decay downstream of the middle of S2. Thirdly, the probability of an otherwise satisfactory track not firing the Čerenkov counter was taken from the observed threshold behaviour; tracks were discarded randomly to reproduce this.

Since the proportion of charged kaons among the three fast secondaries from pion-induced events is only $\sim 5\%$ (see fig. 6), the problem of event identification is not critical. The strongest criterion for event identification, using the Čerenkov information, is to require the beam track and all three secondaries to be identified as pions: only 20% of all events satisfied this criterion. The criterion finally adopted for accepting an event is that the event does not contain a secondary positively identified as a kaon or proton (or antiproton). This condition yielded a number of events at production consistent with the value obtained when demanding total identification. Some 90% of real events satisfied the less stringent criterion, which was satisfied by $\sim 95\%$ of Monte Carlo events: the real events contain some examples of $\pi^- p \rightarrow K^- \pi^- K^+ p$.

Finally the surviving events were assigned a further set of weights, each computed from a term $\xi^a \xi^b f^{a*} f^b$ in the theoretical intensity and a function only of the five decay variables. The coefficients of these terms were the fitting variables and the output of the Monte Carlo was thus the distribution of events expected experimentally for any given choice of amplitudes F or density matrix elements, the quantities which were varied in the fit.

Projections of the data in a single mass bin are compared in fig. 8 with the fitted intensity, generated with the Monte Carlo program.

4.3 Total and differential cross-sections

Each of the three sets of data (94 GeV and 63 GeV with standard trigger, 94 GeV with high- t trigger) was analysed with two modes of binning in $M_{3\pi}, t'$ ($t' = t - t_{\min}$). In order to study the variation of the three particle amplitudes with $M_{3\pi}$, each set of data was divided into two approximately equally populated t' bins ($0 \leq |t'| < 0.05$ and $0.05 \leq |t'| \leq 0.7$ GeV² for the standard trigger, $0.16 \leq |t'| < 0.3$ and $0.3 \leq |t'| \leq 0.7$ GeV² for the high- t trigger) and the data in each of the t' regions were analysed in 20 MeV $M_{3\pi}$ bins, matched to the resolution of the apparatus. In order to obtain the differential cross-section and the variation of the partial-wave amplitudes with t' , a coarser binning in mass was adopted (100 MeV) and the data analysed in smaller steps in t' , chosen so that each bin contained approximately the same number of events. The microbarn equivalents for each set of data are summarized in table 6.

The 3π mass spectra obtained from fitting in 20 MeV mass bins are shown in fig. 9 together with the raw 3π mass spectra. The over-all correction factor is ≤ 2 , smooth, and slowly varying as a function of 3π mass.

The data show a strong A_1 enhancement between 1 and 1.4 GeV, with a lesser A_3 enhancement in the region of 1.7 GeV. At low $|t'|$ there is no visible signal due to the A_2 meson, but as the value of $|t'|$ increases such a signal appears more and more clearly, although even in the highest $|t'|$ region it never contributes more than $\sim 50\%$ of the events at $M_{3\pi} \approx 1.3$ GeV.

The differential cross-section is shown as a function of 3π mass in fig. 10. There is a strong mass-slope correlation and a break in the slope at $|t'| \approx 0.4$ GeV², the effect being most marked at low masses. The differential cross-section is best fitted by the sum of two exponentials and the parameters are listed in table 7. Alternative forms fitted were an exponential with terms in both t' and t'^2 , and a sum of an exponential in t' and an exponential in t' multiplied by t' . The fits were acceptable, but inferior to the sum of two exponentials.

4.4 Partial-wave analysis of the data

The final wave sets used in the analysis are listed in table 8. The procedure adopted was to start with a minimal set and progressively add waves, usually of higher J, L, or M. Those waves which did not significantly affect the likelihood and had fitted intensity zero within errors were then discarded. The Illinois approach includes an amplitude FLAT, which is incoherent with all other amplitudes and proportional to 3π phase space. This wave is thus a garbage collector and although always included in the fit the intensity should be consistent with zero. It was found that this could be achieved by including in the partial-wave amplitudes, before fitting, appropriate t' and $M_{3\pi}$ dependence to allow explicitly for variation of the amplitudes over a given $t', M_{3\pi}$ bin. This was most important for the $2^+D1^+(\rho\pi)$ amplitude, since this amplitude must go to zero in the forward direction and is overwhelmingly dominated by the narrow (~ 100 MeV wide) A_2 meson. The intensities of the major waves 1^+S0^+ , 2^+D1^+ , 2^-S0^+ , together with relative phases, are shown in fig. 11 for the 94 GeV data taken with the standard trigger, $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$. For comparison, the results obtained using the LBL-SLAC approach are also shown. Agreement between the two approaches is, in general, extremely good: such quantitative discrepancies as do exist in the results for the principal waves would in no way affect any of the conclusions in this paper. The only over-all discrepancies which have been found are in minor waves, for $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$. The intensity in $1^+P0^+(\epsilon\pi)$ found with the LBL-SLAC approach is approximately twice that derived from the Illinois program. The total 1^+ intensity does not show this discrepancy, the excess 1^+P events being cancelled by a larger interference between 1^+S and 1^+P . The variation of 1^+S phase relative to 0^-P is also different. (Neither discrepancy appears in the region $0 \leq |t'| < 0.05 \text{ GeV}^2$.)

The t dependence of the principal $M = 0$ waves (94 GeV standard trigger) for three mass regions is given in table 9. The fits are to simple exponentials, and are excellent except for the 2^-P wave in the two lower mass regions, where the 2^-P intensity falls more rapidly than the fitted exponential at low t , and then flattens out. (See also section 5.8).

5. RESULTS OF PARTIAL-WAVE ANALYSIS: THE REGION 0.9-1.6 GeV

5.1 Introduction

The intensities and relative phases of the major $M = 0$ waves and the wave 2^+D1^+ are shown in figs. 12 and 13 for the summed 94 GeV and 63 GeV standard trigger data, $0.0 \leq |t'| \leq 0.7 \text{ GeV}^2$.

The $1^+S0^+(\rho\pi)$ wave dominates between 3π masses of 1.0 and 1.4 GeV, the enhancement having steep sides and a rounded top. [There is significant intensity in $1^+S1^+(\rho\pi)$: the A_1 enhancement is not produced with exact t-channel helicity conservation.] The $1^+P0^+(\epsilon\pi)$ wave is only $\sim 10\%$ of the dominant 1^+S component. The A_2 meson, in $2^+D1^+(\rho\pi)$, is picked out cleanly and has a clear Breit-Wigner shape with very little background. The phase of 2^+D1^+ increases through $\sim 90^\circ$ relative to 1^+S0^+ over the region of the A_2 resonance.

Results for the individual partial waves are presented in more detail and discussed in subsequent sections.

5.2 $J_{LM}^P = 2^+D1^+(\rho\pi)$: the A_2 meson

A 2^+ state decaying into three pions and formed with $M = 0$ has $\eta = -1$ and can only be produced by unnatural spin-parity exchange. The amplitude 2^+1^+ can be produced by natural exchange, and with a net helicity flip of 1 the intensity must vanish in the forward direction as $|t'|$: the fitted 2^+D amplitudes have just these features. There is no evidence for any contribution from $\eta = -1$ amplitudes and when fitting in 20 MeV bins 2^+D1^+ was the only 2^+ amplitude required. The variation of $2^+D1^+(\rho\pi)$ intensity with 3π mass is shown in fig. 14 together with the phase measured with respect to other waves. The intensity is the sum of the separate intensities at 94 and 63 GeV (standard trigger), in the region $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$, and the phases shown are the weighted averages.

The 2^+D1^+ intensity, in 20 MeV mass bins, has been fitted with the sum of a slowly varying exponential background term and a Breit-Wigner intensity. The form of the Breit-Wigner amplitude used is

$$BW(M) = \frac{M\Gamma}{M_0^2 - M^2 - iM\Gamma} ; \quad \Gamma = \Gamma_0 \frac{q}{q_0} \frac{M_0}{M} \frac{B(q^2 R^2)}{B(q_0^2 R^2)} \quad (4)$$

where $B(q^2 R^2)$ is a D-wave barrier factor [18] and the fit is shown in fig. 14a. The phase variation predicted by the Breit-Wigner fitted to the intensity alone is superimposed on the relative phase variations in figs. 14b and c.

The fits require a small amount of background under the A_2 resonance (represented here by a simple background incoherent with the resonance) and also require the centrifugal barrier factor in the expression for the width of the resonance, with a non-zero interaction radius R . The fits were not sensitive to the value of R , provided $100 > R^2 > 5 \text{ GeV}^{-2}$, and the value $R^2 = 10 \text{ GeV}^{-2}$ was used. The resonance parameters obtained for the A_2 are summarized in table 10. The total cross-section for A_2 meson production in each data sample was obtained from the fitted resonant component and is also given in table 10. In order to study the t dependence of A_2 production, fits were made in a single mass bin $1.26 \leq M_{3\pi} \leq 1.36 \text{ GeV}$ for a number of t' bins. (It should be remembered that the resonant variation of the $2^+ D1^+$ intensity was explicitly included in these fits.) The results were used to obtain the differential cross-section in $2^+ D1^+$ for each set of data fitted. The differential cross-sections were fitted to an expression of the form $t' e^{bt'}$ and values of the slope parameter b are given in table 10.

The $2^+ D1^+$ differential cross-section obtained from the 94 GeV data, using both the standard and high- t trigger samples, is shown in fig. 15. In this figure results from the data taken with the standard trigger are normalized to the total $2^+ D1^+$ cross-section in the high- t trigger data.

The $2^+ D1^+$ total cross-section in the region $1.26 \leq M_{3\pi} \leq 1.36 \text{ GeV}$ was obtained by integrating the fitted dependence over all t' . The two values thus obtained at 94 GeV, from the standard and high- t trigger samples, agree within errors despite the very different trigger conditions and do not differ significantly from the cross-section at 63 GeV. These results have been used to determine the dependence of the $2^+ D1^+$ cross-section on laboratory momentum. In fig. 16 the values

obtained from this experiment are shown together with results from a compilation given elsewhere [19]. The straight-line fit to the form $\sigma \propto p_{\text{lab}}^{-n}$, yields an exponent $n = 0.4 \pm 0.03$. This value is the same within errors as the exponent for the unnatural J^P states $1^+S(\rho\pi)$ in the mass region 1.0-1.2 GeV and $2^-S(f\pi)$ in the region 1.5-1.8 GeV (see sections 5.5 and 6.2 below).

The complete subdensity matrix for A_2 production is given in table 11, for the mass region 1.26-1.36 GeV, as a function of both the laboratory momentum and t' . In these fits to a mass region 100 MeV wide, contributions from 2^+D2^+ and $2^+\eta = -1$ amplitudes were allowed. At high $|t'|$ there is a significant interference term between 2^+D2^+ and 2^+D1^+ (although diagonal elements for $M = 2$ are consistent with zero). Otherwise there is no variation of the density matrix with t' .

The production phase of the A_2 resonance, measured with respect to the $1^+S(\rho\pi)$ amplitude in the mass region 1.26-1.36 GeV, is also listed in table 11. There is no significant variation of this phase as a function of t' .

The A_2 meson is produced by natural spin-parity exchange. The cross-section falls slowly with laboratory momentum ($\sigma \propto p_{\text{lab}}^{-0.4}$) and the production phase relative to $1^+S0^+(\rho\pi)$ does not vary with t' . All these features show that the A_2 meson is diffractively produced.

5.3 $J_{LM}^P \eta = 1^+S0^+(\rho\pi)$: the A_1 problem

The dominant wave in the low-mass region, out to $|t'| \approx 0.3 \text{ GeV}^2$, is $1^+S0^+(\rho\pi)$. An outstanding question in meson spectroscopy is whether or not there exists a resonance in this channel. In the naïve quark model, a P-wave $q\bar{q}$ system gives rise to two $I = 1$, $J^P = 1^+$ mesons. The positive G parity member of the lowest lying system, the $B(1235)$, has long been established. This state is not diffractively produced, because it has the wrong G parity, and decays to $\omega\pi$. The search for a 1^+ resonance in the diffractively produced 3π system is made difficult because diffractive dissociation of the incident pion into $\rho\pi$ through the Deck mechanism [20] populates predominantly the $1^+S0^+(\rho\pi)$ wave [21].

Previous studies of diffractive 3π production from a hydrogen target failed to show any substantial variation of the 1^+S phase measured relative to the waves present in the low-mass region [8,10,19,22-24]. This has been taken as evidence against the existence of a $1^+S(\rho\pi)$ resonance, the A_1 meson: the large $1^+S(\rho\pi)$ amplitude near threshold being attributed entirely to the Deck effect in ref. [8]. In contrast, a recent study of diffractive production from nuclei showed a substantial variation of the 1^+S phase with respect to 0^-P [25].

Several groups of authors have argued that the existence of an A_1 meson is not ruled out by the data from experiments with a hydrogen target, and that the data require both a 1^+S Deck amplitude and the existence of an A_1 meson [26-28]. Attempts to minimize the problem of background amplitudes by using charge and hypercharge exchange reactions [24,29,30] have suffered from low statistics and the complication of $I = 0$ final states. Attempts to find the A_1 meson produced by baryon exchange also suffer from small cross-section [31,32] and low statistics [32], although evidence has been claimed for an A_1 meson at a mass ~ 1.04 GeV.

The variation of 1^+S0^+ intensity with 3π mass obtained in this experiment is shown in fig. 17. In the t' regions $0 \leq |t'| < 0.05$ GeV² and $0.05 \leq |t'| \leq 0.7$ GeV² the data taken with the standard trigger at 94 GeV and 63 GeV have been combined, while the high- t trigger data at 94 GeV are shown separately. At low momentum transfer the intensity exhibits a broad peak around 1.1 GeV. As the mean value of $|t'|$ is increased, the peak moves higher in mass and the fall-off on the high-mass side steepens, until finally in the region $0.3 \leq |t'| \leq 0.7$ GeV², the intensity has taken on a resonance shape. These data are not consistent with production of a resonance alone, or with production of a Deck amplitude with no resonant component: such an amplitude peaks around 1.1 GeV and falls off far too slowly on the high-mass side (see section 5.4). The data on intensity suggest the presence of both Deck background and a resonance, the latter emerging more clearly at high $|t'|$.

The variation of the phase of the $1^+S0^+(\rho\pi)$ amplitude with respect to other waves is shown in fig. 18. These relative phases are independent of t' apart from

small over-all shifts ($\lesssim 10^\circ$) and so a weighted average has been taken of the results from all data sets. There is no indication of any change of phase with respect to $1^+P(\epsilon\pi)$ as the 3π mass increases through the A_1 enhancement, and the 1^+S phase increases by only $\sim 30^\circ$ with respect to $0^-S(\epsilon\pi)$. These results are consistent with those obtained from earlier experiments with a hydrogen target.

The phase of 1^+S measured with respect to $0^-P(\rho\pi)$ however increases through $\sim 70^\circ$ between a 3π mass of 0.9 and 1.4 GeV. This same feature was found in diffractive production of 3π from complex nuclei [25]. The phase with respect to the other $\rho\pi$ P-wave, $2^-P0^+(\rho\pi)$, increases through $\sim 50^\circ$ over the same mass region, and exhibits a sharp downturn above 1.4 GeV. This downturn can be attributed to the onset of the A_3 resonance at 1.7 GeV, which couples to the $2^-P(\rho\pi)$ channel (see section 6.2).

It is implausible that the two $\rho\pi$ P-waves have phases decreasing through $\gtrsim 50^\circ$ in a range of a few hundred MeV above threshold. The most plausible interpretation of these results is that the 0^-S , 1^+P , and 1^+S phases are rising and that the P-wave $\rho\pi$ phases remain approximately constant up to a mass ~ 1.4 GeV. This interpretation is confirmed by using the 2^+D1^+ amplitude as a reference [28]. With the high statistics obtained in this experiment, the relative phase of 2^+D1^+ measured with respect to the other waves could be obtained over the whole mass region 1.0-1.6 GeV. (Some results are shown in figs. 11 and 13.) In fig. 19 the phase of 1^+S0^+ relative to 2^+D1^+ is shown, taking a weighted average over all data sets. This relative phase initially rises through $\sim 40^\circ$, turns over and drops through zero at $M_{3\pi} \approx 1.3$ GeV, and finally levels out at about -40° . The maximum excursion of the 2^+D1^+ phase relative to 1^+S0^+ is thus only $\sim 90^\circ$ instead of $\sim 180^\circ$ expected from the A_2 resonance under the assumption of a constant 1^+S phase. Figure 19 also shows the result of subtracting from the 2^+D1^+ reference phase the phase of the A_2 Breit-Wigner amplitude, determined from the fits to the 2^+D1^+ intensity alone: the 1^+S phase measured relative to the A_2 production amplitude increases through $\sim 120^\circ$ between threshold and 1.5 GeV. The systematic change in shape of the 1^+S intensity as a function of t' and the evidence for an increase

in phase of $\sim 100^\circ$ over the A_1 enhancement suggest strongly the existence of a resonant $1^+S(\rho\pi)$ state, the A_1 meson. It is shown below that all these results are quantitatively explained in terms of an amplitude containing direct production of such a resonance and a coherent Deck background which is rescattered through the resonance.

5.4 Calculations of the Deck amplitude

In the process proposed by Deck [20], an incident pion dissociates into a ρ meson and a virtual pion. This virtual pion scatters diffractively from the target proton into a real state (fig. 20a). The amplitude for this process is thus

$$D(M_{\pi\pi}, t_\pi, t) = \frac{A_{\pi\pi}(M_{\pi\pi}, t_\pi) A_{\pi N}(s_{\pi N}, t)}{m_\pi^2 - t_\pi}, \quad (5)$$

where $A_{\pi\pi}$ is the $\ell=1$ $\pi\pi$ scattering amplitude through the ρ meson and $A_{\pi N}$ is the π^-p elastic scattering amplitude. It is assumed that the amplitude for scattering a virtual pion into a real state is the same as that for scattering of a real pion at the same values of the kinematic variables $s_{\pi N}, t$, and in this work $A_{\pi N}$ was taken as purely diffractive and represented by

$$A_{\pi N} = i s_{\pi N} \sigma_t e^{at}, \quad (6)$$

where σ_t is the total π^-p cross-section (taken as $25 \text{ mb} = 64 \text{ GeV}^{-2}$) and the slope parameter a was taken as 4.5 GeV^{-2} , from high-energy πp elastic scattering.

For $\pi\pi$ scattering through the ρ meson, the scattering amplitude was taken as a Breit-Wigner with a P-wave mass-dependent width, and the coupling to the exchanged pion and incident pion was taken off mass shell by replacing the mass squared of the exchanged pion with $(-t_\pi)$. Corresponding Deck amplitudes for $\pi\pi$ or $f\pi$ final states may be generated by setting $A_{\pi\pi}$ equal to the S- or D-wave amplitudes for $\pi^+\pi^- \rightarrow \pi^+\pi^-$.

In the limit $t \rightarrow 0$ and total energy in the centre of mass ($\sqrt{s}$) much greater than the target mass [21]

$$\frac{s_{\pi N}}{m_\pi^2 - t_\pi} \rightarrow \frac{s}{M_{3\pi}^2 - m_\pi^2}. \quad (7)$$

Both $s_{\pi N}$ and t_{π} depend on the orientation of the relative $\pi\rho$ momentum vector in the $\pi\rho$ centre of mass, but in such a way that this dependence cancels for $t \rightarrow 0$. Away from $t = 0$, the cancellation remains large, even though the relation (7) is no longer satisfied.

There are numerous possible modifications to these simple amplitudes. It has been popular in the past to Reggeize the exchanged pion [33,34], the most important feature being the introduction of a multiplicative factor $e^{i(\pi/2)\alpha(t_{\pi})}$ in the amplitude. There may be a form factor associated with the exchanged particle being off mass shell, introducing a multiplicative factor $e^{b(t_{\pi}-m_{\pi}^2)}$. A factor $e^{i(\pi/2)\alpha}$ shifts a growing proportion of the dominant S-waves into P and higher waves as the 3π mass increases, because of the angular dependence of t_{π} . Because the exponent is imaginary, there are large phase differences between the even and the odd waves [33], and the phase of all waves increases with 3π mass by $\sim 80^\circ \text{ GeV}^{-1}$ if α' is taken as 1 GeV^{-1} . If a form factor is also present, these effects on the phase are diluted.

Then there are other possible diagrams; in the $\pi\rho$ case virtual ρ exchange in addition to virtual pion exchange [35]. The importance of this contribution will depend on the strength of the form factors, since a virtual exchanged ρ is much further off mass shell than an exchanged pion. In the $\pi\epsilon$ case, no one knows how to represent the exchange of a broad $\pi\pi$ structure which is not even roughly approximated by an S-wave $\pi\pi$ Breit-Wigner amplitude. In the 0^-0^+ waves (both S-wave $\epsilon\pi$ and P-wave $\rho\pi$) there may be a contribution from a third diagram in which the incident pion is scattered into a virtual state (with mass $M_{3\pi}$) which then breaks up [36]. The contribution of this diagram too will depend on the form factors. Finally, there may be absorption of the bare amplitude calculated according to any of the above prescriptions [36,37]. The effect of all this is that while the qualitative behaviour of the major waves generated through the Deck mechanism is understood, the shape and relative proportions of the minor waves (P-waves and above) are very uncertain and the absolute cross-section is not well determined.

In this work, only the virtual pion exchange diagram has been included. The simple expression (5) has been modified only by the addition of a form factor $e^{b(t_{\pi}-m_{\pi}^2)}$ with $b = 2 \text{ GeV}^{-2}$. This value was selected because it was necessary in order to reproduce the t dependence of the data at $M_{3\pi} \sim 1 \text{ GeV}$ and also because it reproduced approximately the observed t -slope mass correlation (table 7). The calculated intensities of the S-wave and P-wave $\epsilon\pi$ and the S-wave $\rho\pi$ are shown in fig. 21, for $t' = 0$ and $t' = -0.15 \text{ GeV}^2$. The inability of the Deck mechanism to reproduce the observed rapid fall in 1^+S^0 with increasing mass is striking and this feature is independent of the detailed assumptions of the model. Furthermore, even if Reggeization of the Deck amplitude is permitted, the resulting phase increase in 1^+S is $\leq 30^\circ$ over the region where the observed increase is $\sim 100^\circ$.

5.5 Explanation of the behaviour of the 1^+S^0 amplitude

The Deck amplitudes exemplified by fig. 20a are not consistent with two-particle unitarity (treating the ρ as a stable particle) if there is any interaction between the π and ρ . If it is assumed that there is a resonance in the 1^+S $\rho\pi$ channel, then the Deck amplitude must be corrected for the effects of two-particle unitarity by adding to the bare Deck term a correction for rescattering through the resonance, fig. 20b. The rescattering process has been investigated by several groups and there is general agreement that the bare Deck amplitude D is to be replaced by a term

$$\sim D e^{i\delta} \cos \delta, \quad (8)$$

where δ is the S-wave $\rho\pi$ elastic phase shift [15,26,28,38,39]. In this analysis, the form taken for the Deck amplitude after rescattering was

$$D \left\{ e^{i\delta} \cos \delta + \frac{\alpha}{k} e^{i\delta} \sin \delta \right\}, \quad (9)$$

where k is $\rho\pi$ phase space. The value of α was 0.4 GeV , determined from the position of the pole in the simple Stodolsky approximation [21] to the Deck amplitude. This form is taken from ref. [39], and does not differ essentially from the result of the more elaborate treatment of ref. [28].

The model amplitude used to explain the 1^+S0^+ data therefore took the form

$$\frac{D[M_0 - m + (\alpha g^2/2)] + F_0 e^{at} e^{i\phi} (g^2/2)}{M_0 - m - i(g^2/2)k}, \quad (10)$$

where k is $\rho\pi$ phase space, calculated for a complex ρ mass to allow for the finite width of the ρ . The term $F_0 e^{at}$ represents direct production of the A_1 meson (fig. 20c), allowed to differ in phase from the Deck amplitude: a was taken as 3.5 GeV^{-1} .

This amplitude was fitted to the 1^+S0^+ intensity and the variation of phase relative to the A_2 production amplitude. The combined standard trigger data at 94 and 63 GeV in the two t' ranges and the 94 GeV high- t trigger data, again in two t' ranges, were fitted independently. The fitting variables were the mass M_0 (the position of the A_1 K-matrix pole), the coupling g^2 , the direct production variables F_0 and ϕ and a variable normalization A multiplying the intensity. In addition, the absolute value of the 1^+S0^+ phase relative to the A_2 production was allowed to float (ϕ_0) because only the variation is determined by the model amplitude. The results of these fits are summarized in table 12 and displayed in fig. 22. The fits are excellent, and the normalization factor A is close to unity except for the 94 GeV data in the region $0.3 \leq |t'| \leq 0.7 \text{ GeV}^2$. This means that except at high $|t'|$ the fitted Deck amplitude is in good agreement with the calculated bare Deck cross-section. It is entirely possible that, in the region $0.3 \leq |t'| \leq 0.7 \text{ GeV}^2$, the Deck amplitude is much smaller than given by the simple calculation: absorption of the Deck amplitude could achieve this.

The illustrated fits have an arbitrary production phase ϕ between the Deck amplitude and the direct production term. Although the fitted values for this quantity were $\sim 40^\circ$, acceptable fits were obtained when ϕ was constrained to be zero. In an alternative parametrization, essentially that of ref. [28], ϕ was constrained to be zero but δ was taken as the sum of a resonant phase shift and that of a repulsive scattering length term. The resulting fits required little direct production and more Deck intensity than the original fits. The results of all these fits are summarized in table 12.

The very high statistics data accumulated in this experiment yield compelling evidence for the existence of an A_1 meson. The behaviour of the $1^+S_0^+$ amplitude may be explained in detail by a model in which the Deck amplitude is rescattered through the A_1 meson, which may also be directly produced.

Uncertainties in the normalization of the Deck amplitude and in the correct form to be employed for the elastic phase shift δ make it impossible to determine in a model-independent way the magnitude and phase of the direct production term, but all fits in all t' regions require an A_1 resonance, regardless of the details of the model. The statistical errors on the mass of the A_1 are ≤ 10 MeV (except for the high- t trigger data at 94 GeV, $0.3 \leq |t'| \leq 0.7$ GeV², where the error is ~ 20 MeV) and on the width ~ 15 MeV. Systematic errors could be associated with the partial-wave analysis and with the details of the model. From the variation of the A_1 parameters obtained when fitting with different data sets and with variants of the model, the conclusion is that the A_1 meson has a mass of 1.28 ± 0.03 GeV and a width of 300 ± 50 MeV. These resonance parameters are in good agreement with the results of ref. [26], but differ substantially from the conclusions in refs. [15,28].

There is no evidence whatever in these data for a 1^+S ($\rho\pi$) resonance in the region 1.0-1.1 GeV. If it existed, it would leave an imprint on the rescattered Deck amplitude even if not directly produced. This could only be obliterated by a direct production amplitude so arranged that all trace of the resonance is cancelled. Fits were made with two resonances present and it proved entirely possible to smuggle in a second resonance at, say, 1.1 GeV with a width of ~ 100 MeV. The fitted intensity and phase were however indistinguishable from those for a single resonance.

The $1^+S_0^+$ cross-section, in the region 1.0-1.2 GeV, is shown in fig. 23 as a function of incident momentum. The three points from this experiment represent values obtained by integrating the fitted t dependence over all t for the 63 and 94 GeV standard trigger data, and the 94 GeV high- t trigger data. The other points are taken from the compilation given in ref. [19]. A fit with the form $\sigma \propto p_{lab}^{-n}$

yielded an exponent $n = 0.42 \pm 0.04$, similar to the values obtained both for A_2 production (section 5.2) and for the $2^-S(f\pi)$ intensity (section 6.2).

5.6 $J_{LM}^P = 1^+S1^+$

There is some production of 1^+S in the $M^\eta = 1^+$ substate. The variations of intensity and phase are consistent with results for $M^\eta = 0^+$ in the same t' regions, but the number of events is small and the data have not been fitted with the model amplitude. The subdensity matrix for 1^+S is given in table 13 as a function of t' .

5.7 The $J^P = 0^-$ amplitudes

The two 0^- waves, $0^-S(\epsilon\pi)$ and $0^-P(\rho\pi)$, are relatively minor contributors. The intensities and phases for these waves are shown in fig. 24, where the intensities are summed over all data sets and the phases are a weighted average. The reference phase in fig. 24 is the A_2 production phase.

The phase of 0^-S with respect to the A_2 production phase advances steadily through $\sim 60^\circ$ between 1.0 and 1.6 GeV. (Over this range the 1^+S phase increases through $\sim 100^\circ$.) The 0^-P phase is rather flat initially, rising from $\sim 70^\circ$ to $\sim 90^\circ$ between 1.0 and ~ 1.35 GeV, and then moving more rapidly to $\sim 150^\circ$ at 1.6 GeV.

This activity in the phases is suggestive of the presence of a 0^- resonance. A further pointer is the 0^-S intensity which peaks at ~ 1.2 GeV. The Deck model with an exponential form factor $e^{2(t_\pi - m_\pi^2)}$ gives a 0^-S intensity peaking at ~ 0.9 GeV (fig. 21). The Deck model will only give the maximum 0^-S intensity at a mass as high as 1.2 GeV if the form factor is constant.

There is no difficulty in fitting the 0^-S intensity and phase with a combination of rescattered Deck background and a directly produced resonance. Such a fit yields a resonance at ~ 1.4 GeV which is very broad, $\Gamma \sim 600$ MeV. However, it has not been found possible to obtain a good fit, which is also physically reasonable, in a coupled-channel analysis of 0^-S and 0^-P . The difficulty lies primarily in the value of the 0^-P phase, which is $\sim 70^\circ$ at 1 GeV, just above

threshold, where the 0^-S phase is $\sim 180^\circ$. If the absolute value of the 0^-P phase is included as a variable, an acceptable fit results ($\chi^2/\text{NDF} \sim 1.5$), but the 0^-P phase is shifted down by 50° .

The contributions of a resonance to the two channels would be relatively real, and without Reggeization of the exchanged pion the Deck amplitudes would be relatively real as well. The 2^-P amplitude is real up to about 1.5 GeV, with respect to both 0^-S and the A_2 production phase, but approximately imaginary with respect to 0^-P . The 2^-P , 0^-P , 2^+D1^+ and 1^+S0^+ waves are all $\rho\pi$ states and so an artificial shift of 0^-P alone through $\sim 50^\circ$ seems implausible.

An excellent fit to both intensity and phase in both channels has been obtained, but only through the expedient of adding constant terms to the resonance pole terms in the 2×2 K matrix used to describe the 0^- system. The values needed were very large (corresponding to scattering lengths $\sim 4 \text{ GeV}^{-1}$). A second implausible feature of this fit is that the fitted Deck intensity in 0^-P was ~ 3 times greater than that in 0^-S .

It must be concluded that while there is a strong indication of resonant activity in the 0^- amplitudes, the details are not understood. It may be relevant that a family of solutions in which 0^-S and 0^-P phases increased together, the amplitudes being relatively real, was found when using the LBL-SLAC approach with 94 GeV data in the region $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$. However, in the region $0.0 \leq |t'| \leq 0.05 \text{ GeV}^2$, the solution found had the same characteristics as that shown in fig. 24. An examination of likelihood as a function of the relative phase of 0^-S and 0^-P using the Illinois program did not uncover any rival solutions or ambiguities.

It should be noted that in the study of diffractive 3π production from complex nuclei, the preferred solution (Solution I) of ref. [25] shows the same behaviour of the 0^-P phase relative to the 0^-S amplitude as found here.

If there is a resonance in the 0^- system, it would be attributed to the first radial excitation of the pion. Similar evidence for a resonance in the analogous $K\pi\pi$ system has been presented in ref. [40].

5.8 $J_{LM}^P = 2^-P0^+$: an unexplained phenomenon

At low $|t'|$, the 2^-P amplitude rises smoothly from threshold to ~ 1.6 GeV. There is an indication of a peak at ~ 1.7 GeV, followed by a rapid fall to negligible intensity in the region of 1.9 GeV: these features may be attributed to the A_3 resonance.

At high $|t'|$ there is a striking and curious phenomenon in the 2^-P intensity. The intensity is shown in fig. 25 for the 94 GeV data taken with the high- t trigger. In the region $0.16 \leq |t'| < 0.3$ GeV² the 2^-P intensity is featureless, but in the region $0.3 \leq |t'| \leq 0.7$ GeV² the 2^-P intensity rises to a peak at about 1.2 GeV and then drops precipitously to less than one third of the peak intensity before resuming a rise. The total number of events in 2^-P in the low-mass region, for this selection on t' , exceeds the number in 1^+S . This structure in 2^-P intensity is not accompanied by any structure in the phase of 2^-P : for both selections on t' , the 2^-P phase, measured with respect to 1^+S , is slowly varying between 1.2 and 1.4 GeV. Mapping the likelihood as a function of 2^-P intensity did not reveal any alternative solutions.

This structure also appears in the 94 and 63 standard trigger data for $0.05 \leq |t'| \leq 0.7$ GeV², and can be seen in fig. 12. It is not a spurious effect caused by fitting in large bins of t' , because the effect remains when the t' dependence of the 2^-P amplitude is studied (using large bins in $M_{3\pi}$). The dependence of the 2^-P amplitude on t' is shown in fig. 26, together with that for 1^+S , for two regions of low 3π mass. At low 3π mass, the 2^-P intensity flattens at $t' \sim -0.4$ GeV²: at higher 3π masses the break in slope becomes less pronounced. It seems that the 2^-P amplitude dominates in the low-mass region for $|t'| > 0.5$ GeV² and is the most important source of the change in slope with t' observed in the total intensity (fig. 10).

The structure in t' suggests absorption of a 2^-P background amplitude, but the dominance of 2^-P at high $|t'|$ and the absence of any similar effect in other waves remain a mystery. A similar effect has been observed in the data of ref. [25].

6. RESULTS OF PARTIAL-WAVE ANALYSIS: $M_{3\pi} > 1.3$ GeV

6.1 Introduction

The dominant effect in the high-mass region, beyond the A_1 and A_2 mesons, is the A_3 enhancement at a mass ~ 1.7 GeV. The A_3 enhancement is dominated by $2^-S0^+(f\pi)$ and the question of whether there exists a resonance in this channel has not previously been resolved [7,19,22,24,41] because of problems of resolution and statistics. The analysis of the data taken in this experiment has shown that a 2^-A_3 meson dominates the diffractive production of 3π in the high-mass region and couples to four different $J^P = 2^-$ channels: all four 2^- partial waves are discussed in sections 6.2 and 6.3 below.

There have been reports of several other diffractively produced resonances: 1^+ at $M_{3\pi} \sim 1.7$ GeV [25], 2^+ at ~ 1.7 GeV [19] and 3^+ at ~ 2.2 GeV [24]. The data from this experiment on these states are discussed in sections 6.4-6.6.

All three data sets, for the full t' range, have been analysed independently, but because of the relatively low statistics in the high-mass region, the results have been combined by summing the intensities and taking weighted averages of the phases. The wave sets used in the analysis of the high-mass region are given in table 8. Figure 9e shows the 3π mass spectrum for the combined data.

6.2 The 2^-0^+ waves: the A_3 meson

The intensities of the four 2^-0^+ waves, S-wave $f\pi$, P-wave $\rho\pi$, D-wave $\epsilon\pi$ and D-wave $f\pi$ are shown in fig. 27. The $2^-S(f\pi)$ and $2^-D(\epsilon\pi)$ intensities have approximately resonant shapes, peaking at ~ 1.65 GeV. The $2^-P(\rho\pi)$ intensity also peaks at ~ 1.65 GeV, but there is clearly substantial background. The two-bin spike at 1.43 GeV in $2^-P(\rho\pi)$ is believed to be spurious. The reasons are, first that there is no evidence for such an effect in the total 3π mass spectrum (fig. 9e) and there is no narrow enhancement in the total 2^- intensity either, the enhancement in $2^-P(\rho\pi)$ being cancelled by negative interference with the other 2^- waves. Secondly, the 2^-P intensity is unusually volatile in this region, depending strongly on the assumed t dependence of the 1^+S0^+ amplitude.

The curves shown in figs. 27a to c are the results of simple fits to the intensities, assuming a resonance and an incoherent background. The resonance was represented by a relativistic Breit-Wigner with an S-wave mass-dependent width, and in each case the background was taken as an exponential in 3π mass, $e^{-AM_{3\pi}}$. The resonance parameters and the coefficient A were fitted independently in each channel. The results are summarized in table 14. The resonance parameters are dependent on the form chosen for the background, but it is clear that the peaks in $2^-P(\rho\pi)$ and $2^-D(\epsilon\pi)$ can be identified with the phenomenon producing the peak in $2^-S(f\pi)$. The appearance of these peaks in three separate channels, yielding within errors the same resonance parameters, is strong evidence for the existence of an A_3 meson. Table 14 lists the branching ratios into these three channels for the A_3 meson, assuming the spectra to be an incoherent sum of resonance and background. The $2^-D(f\pi)$ intensity peaks at ~ 1.85 GeV and cannot fit into this simple picture: it is discussed further below.

The resonant nature of the A_3 is also demonstrated by the variation of $2^-S0^+(f\pi)$ phase with respect to the $1^+S0^+(\rho\pi)$ and $0^-S0^+(\epsilon\pi)$ amplitudes. This is shown in fig. 28. The phase ($2^-S - 1^+S$) initially drops through $\sim 40^\circ$ between 1.3 and 1.45 GeV, a drop which may be attributed to the effect of the A_1 resonance in the reference wave. Thereafter the phase rises through $\sim 110^\circ$, flattening off at about 1.8 GeV, and perhaps falling thereafter. The phase ($2^-S - 0^-S$) shows similar behaviour. Previous experiments [7,19,22,24,41] have lacked the statistics to establish a convincing variation of 2^-S phase across the A_3 enhancement: in this experiment, the qualitative features of resonant-like peaks in several 2^- channels together with the associated variation of the 2^-S phase demonstrates conclusively the existence of the A_3 resonance.

The t' dependence of the 2^- intensities has been found from partial-wave analysis using narrow t' bins and a 3π mass bin $1.6 \leq M_{3\pi} \leq 1.7$ GeV. The differential cross-sections are well fitted with a simple exponential and the values of the slope parameters are given in table 14. The difference in values obtained for each channel must be due to different background behaviour. Fits with a resonance

and incoherent background have been made to the data analysed in 20 MeV mass bins for $0.0 \leq |t'| \leq 0.7$ (standard trigger data) and $0.16 \leq |t'| \leq 0.7 \text{ GeV}^2$ (high- t trigger). The results are consistent with a resonance component slope of $\sim 7.5 \text{ GeV}^{-2}$, with background steeper in the S-wave and flatter in the P-wave.

The integrated cross-section for $2^-S0^+(f\pi)$ in the range $1.5 \leq M_{3\pi} \leq 1.8 \text{ GeV}$ is shown as a function of laboratory momentum in fig. 29. The values obtained from this experiment together with those from previous experiments at lower energy have been fitted with the form $\sigma \propto p_{\text{lab}}^{-n}$, yielding an exponent $n = 0.43 \pm 0.06$, the same within errors as the values obtained for A_2 production and the 1^+S0^+ intensity between 1.0 and 1.2 GeV. The 2^-S subdensity matrix is given in table 15.

The phase of $2^-S(f\pi)$ measured relative to the other 2^- amplitudes may be found in fig. 30. The phase of $2^-S(f\pi)$ shows excursions only $\sim 30^\circ$ with respect to both $2^-P(\rho\pi)$ and $2^-D(\epsilon\pi)$: a large phase variation is not expected with the A_3 meson coupling to all three channels. The phase of $2^-S(f\pi)$ relative to $2^-D(f\pi)$ however exhibits a striking drop between 1.8 and 1.95 GeV. The 2^-S phase is roughly constant in this region: the conclusion must be that the phase of $2^-D(f\pi)$ increases through $\sim 130^\circ$ between 1.75 and 1.95 GeV. The $2^-D(f\pi)$ intensity (figs. 27 and 30) has a sharp peak in the middle of this range.

A simple qualitative explanation for these striking phenomena in the $2^-D(f\pi)$ amplitude is the existence of a second 2^- resonance at $\sim 1.85 \text{ GeV}$, coupling predominantly to $2^-D(f\pi)$. However, the proximity of the A_3 and the possibility of generating such an effect through interference with background must militate for caution in accepting this interpretation.

6.3 Fits to all four 2^- channels simultaneously

The simple-minded fits described in section 6.2 are inadequate in several respects. The fits were to intensity alone and the form chosen for the incoherent background was arbitrary, whereas the background is expected to be generated through the Deck mechanism and coherent with the resonance. The values obtained for the mass, width, and branching ratios of the A_3 can only be taken as indicative.

A single-channel fit to the $2^-S(f\pi)$ intensity and phase (measured with respect to 1^+S) was made using the model for the A_1 described in section 5.4 (but with the πf final state in place of the $\pi\rho$). The $2^-S(f\pi)$ intensity was very well described up to a mass ~ 1.95 GeV by this fit, which yielded a mass and width for the A_3 of ~ 1.7 GeV and ~ 300 MeV, respectively.

In order to use most effectively the data from all four channels in finding the A_3 parameters and determining the implications of the observed $2^-D(f\pi)$ peak in intensity and rapid phase change at ~ 1.85 GeV, a four-channel version of the model was constructed. This model had Deck background in all four channels, re-scattered through either one or two resonances which could also be directly produced. The amplitudes fitted are given by

$$\tilde{F} = (\tilde{1} - i \tilde{K}\rho)^{-1} \tilde{P} \quad (11)$$

where

$$\tilde{P} = (\tilde{1} + \alpha\tilde{K})\tilde{D} + \tilde{R} . \quad (12)$$

$\tilde{F}$ is a four-component column matrix giving the amplitude in each of the four channels, and $\tilde{K}$ is the 4×4 K matrix

$$K_{ij} = \frac{a_i a_j}{M_a - M} + \frac{b_i b_j}{M_b - M} . \quad (13)$$

The diagonal matrix ρ contains the phase space. $\tilde{P}$, $\tilde{D}$, and $\tilde{R}$ are four-component column matrices. $\tilde{D}$ contains the four bare Deck amplitudes, and the elements of $\tilde{R}$, the direct production terms, are given by

$$R_i = \frac{f_a a_i}{M_a - M} + \frac{f_b b_i}{M_b - M} . \quad (14)$$

With this formulation, the partial widths all have S-wave threshold factors. This is correct for $2^-S(f\pi)$, and the A_3 is well above $\rho\pi$ and $\epsilon\pi$ thresholds. The background amplitudes were taken as

$$D_i = D_{i0} e^{i\phi_i/(M_{3\pi}^2 - m_\pi^2)} \quad (15)$$

(the simple Stodolsky S-wave form). Introduction of D-wave threshold factors in the $2^-D(f\pi)$ partial widths and Deck term did not result in any significant differences in the fits.

The internal phase of the 2^- amplitudes was fixed by choosing f_a real. The production amplitude for the second resonance, f_b , was allotted a phase ϕ_b and the Deck amplitudes each had a phase ϕ_i .

These amplitudes were fitted to the four 2^- intensities and to the variation of relative phase of $2^-S(f\pi)$ with respect to $1^+S(\rho\pi)$, $2^-P(\rho\pi)$, $2^-D(\epsilon\pi)$, and $2^-S(f\pi)$. The origin of the $2^-S(f\pi)$ phase with respect to $1^+S(\rho\pi)$ was allowed to float. Much better fits were obtained when the origins of the phases of the other 2^- waves were also allowed to float, rather than being fixed by the model amplitude. This could allow for the absolute phases calculated being less reliable than the phase variations, or for any systematic shifts produced in the partial-wave analysis. It is to be regarded as a convenient parametrization of ignorance. The freeing of these internal 2^- phase origins did not change the nature of the fits or make significant changes to the extracted parameters. These offset phases, ideally zero, were 41° , -16° , and 26° , for $2^-D(f\pi)$, $2^-P(\rho\pi)$, and $2^-D(\epsilon\pi)$, respectively, in the fit shown in fig. 30.

The results of the best fit obtained, including two resonances, are shown in fig. 30, and the parameters are listed in table 16. The fit is not formally a good one ($\chi^2/\text{NDF} = 2197/339$), but serves to demonstrate that such a model can reproduce the qualitative features of the data. The mass and width of the A_3 indeed emerge as ~ 1.7 GeV and ~ 300 MeV, while the branching ratios of the A_3 are in quite good agreement with those extracted by assuming incoherent background in each channel (table 14). The second resonance has a fitted mass of 2.1 GeV rather than ~ 1.85 GeV, but the fits are not qualitatively different when the mass of the second resonance is within the range 2.1 ± 0.15 GeV. The data do not require a second resonance at ~ 1.85 GeV.

It is easy to see how a second resonance at 1.85 GeV, coupling mainly to $f\pi$, could produce the observed features of the $2^-D(f\pi)$ amplitude. The way in which two resonances, one at $\gtrsim 2$ GeV, produce these effects is also not difficult to understand. In a very simple model, the amplitudes of two resonances would simply be added, so that

$$F_i \sim \frac{f_{a_i}}{M_a - M - i(\Gamma_a/2)} + \frac{f_{b_i}}{M_b - M - i(\Gamma_b/2)} . \quad (16)$$

If the sign of f_{a_i} is the same as that of f_{b_i} (and the phase of f_b is approximately real) the amplitude has a minimum between the two resonances. If the signs are opposite the interference is constructive between the two resonances and destructive outside this range, giving a peak in the intensity between the two. The relative phase between two channels is given by the phase of the ratio

$$\frac{f_{a_j}[M_b - M - i(\Gamma_b/2)] + f_{b_j}[M_a - M - i(\Gamma_a/2)]}{f_{a_i}[M_b - M - i(\Gamma_b/2)] + f_{b_i}[M_a - M - i(\Gamma_a/2)]} , \quad (17)$$

so that if the relative sign of f_{a_j} , f_{b_j} is opposite to that for f_{a_i} , f_{b_i} , a rapid change of phase can result.

The best fit achieved with only one resonance had a χ^2 a factor of two greater than the two-resonance fit ($\chi^2/\text{NDF} = 4088/346$, to be compared with $2197/339$). The one-resonance fit indeed proved capable of producing a peak in $2^-D(f\pi)$ at ~ 1.85 GeV, together with a phase change relative to $2^-S(f\pi) \sim 100^\circ$, but the peak did not well match the data. The one-resonance fit totally failed to produce the second peak in $2^-D(\pi\pi)$.

It is worth noting that restricting the range of the fit to $M_{3\pi} < 2.1$ GeV, as in ref. [3], does not change significantly these conclusions.

The two-resonance fits show that all the qualitative features of the 2^- amplitudes can be reproduced with one resonance, the A_3 , at ~ 1.7 GeV with a width ~ 300 MeV, and a second, an A'_3 , at ~ 2 GeV with a width of ≥ 400 MeV; this solution is preferred to a solution in which the second resonance is placed at the peak of activity in the $2^-D(f\pi)$ amplitude, ~ 1.85 GeV. It seems likely that a second A'_3 resonance is manifesting itself in the data, but at a mass ~ 2.1 GeV rather than ~ 1.85 GeV. Indeed, in the simple quark model, a second 2^- resonance only 150 MeV above the A_3 would be surprising.

6.4 $J^P_{LM} = 1^+D0^+(\rho\pi)$ in the A_3 region

The $1^+S0^+(\rho\pi)$ intensity tails off beyond 1.4 GeV, with no indication of a secondary peak. The $1^+D0^+(\rho\pi)$ wave is small ($\sim 4\%$ of 1^+S intensity) until ~ 1.4 GeV,

but rises to a sharp peak at ~ 1.7 GeV. The intensity and phase, measured with respect to $1^+S_0^+(\rho\pi)$ are shown in fig. 31. The phase measured with respect to 1^+S passes upwards through $\sim 100^\circ$ between 1.4 and 1.8 GeV. The peak in intensity and variation of phase with respect to 1^+S have been previously observed in diffractive production of 3π from complex nuclei [25], and are suggestive of a resonance, an A_1' . This is however a minor wave and even in this experiment the statistics are too low to allow any firm conclusion to be drawn. In addition, there is no sign of phase variation with respect to $0^-S(\pi\pi)$. Thus there are indications of a resonance in $1^+D(\rho\pi)$ at ~ 1.7 GeV, but the evidence is meagre and conflicting.

6.5 $J^P_{M^{\eta}} = 2^+1^+$ in the A_3 region

A previous experiment [19] reported both a peak in $2^+P_1^+(f\pi)$ and an associated change of phase, in the region of 1.75 GeV. In fig. 32 are shown the intensities of $2^+D_1^+(\rho\pi)$ and $2^+P_1^+(f\pi)$ for masses greater than 1.3 GeV, together with the phases measured relative to 1^+S . The data from this experiment show a peak in 2^+P at about 1.75 GeV, and an increasing phase in this region. The 2^+D intensity has a minimum at ~ 1.75 GeV, followed by a broad secondary peak around 1.9 GeV. The data suggest a switch in the sign of the phase of the 2^+D amplitude, at around 1.7 GeV.

These results suggest the presence of a second 2^+ resonance, but the existence of such a state cannot be taken as established. The qualitative features of the data can be reproduced by a 2^+ two-channel K matrix containing two poles, but this model does not produce a glitch in the 2^+D phase as large as that suggested by the data. The solid curves shown in fig. 32 are for a second 2^+ resonance with a K-matrix pole at 1.69 GeV, and partial widths into D-wave $\rho\pi$ and P-wave $f\pi$ of 70 MeV and 165 MeV, respectively, the A_2 having a coupling to P-wave $f\pi$ consistent with zero.

6.6 $J^P_{M^{\eta}} = 3^+0^+$

No clear pattern can be perceived in the various 3^+ waves (fig. 33). These waves are found by the fit to be highly incoherent with the other waves in this region. The amplitude $3^+S(g\pi)$ turns on abruptly at threshold, $3^+P(f\pi)$ is present

throughout the region, while $3^+D(\rho\pi)$ has a peak at ~ 1.7 GeV with a small increase of phase with respect to $1^+S0^+(\rho\pi)$ between ~ 1.8 and 1.9 GeV. The phases of $3^+S(g\pi)$ and $3^+P(f\pi)$ are falling steadily with respect to 1^+S . A broad bump in $3^+S(g\pi)$ at a mass ~ 2.2 GeV has previously been reported [24]: there is no sign of such a structure in the data from this experiment.

7. CONCLUSIONS

7.1 Production mechanisms

The A_2 meson, extracted very cleanly from the data, is produced with no more than a few per cent background and with a differential cross-section $\propto t' e^{bt'}$, where $b = 7.3 \pm 0.1 \text{ GeV}^{-2}$. The 1^+S and 2^-S waves have contributions from both rescattered Deck and directly produced resonance amplitudes. In all three cases, the dependence of the cross-section on laboratory momentum can be represented by an expression of the form $\sigma \propto p_{\text{lab}}^{-n}$ and the three results for n are all consistent with a single value $n = 0.42 \pm 0.02$. All three waves are produced wholly by natural spin-parity exchange. The relative production phase of the A_2 meson and the A_1 enhancement is constant as a function of t . It may be concluded that all three states are diffractively produced, the diffraction process being characterized by $\sigma \propto p_{\text{lab}}^{-0.42}$. Diffractive production of the A_2 shows that the pomeron is not a simple 0^+ object.

7.2 Meson spectroscopy

The results of this experiment demonstrate conclusively the existence of the $A_1(1^+)$ and $A_3(2^-)$ mesons, diffractively produced both directly and through re-scattering of important background amplitudes. The mass of the A_1 is 1.28 ± 0.03 GeV and the width is 300 ± 50 MeV: there is no evidence in the data for an A_1 meson at lower mass. This result in no way conflicts with the meagre data on heavy-lepton decay $\tau \rightarrow 3\pi\nu$ [42-45], but is in disagreement with claims from baryon-exchange experiments [31,32]. In the first of these, the evidence is merely a shoulder on the low-mass side of a peak associated with the A_2 meson. In the second, a shoulder has been resolved into an asymmetric peak at ~ 1.04 GeV by picking out the $1^+S(\rho\pi)$ partial wave. The statistics are low and the data have

been selected so as to enhance any effect and, in addition, multi-Regge background may play a role similar to that played by the Deck process in diffractive production. Such data cannot be considered as evidence against the A_1 parameters found in this experiment.

A recent very high statistics study of the process

$$\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n \text{ at } 8.45 \text{ GeV}$$

has yielded good evidence for A_1 production by charge exchange [46]. The 1^+ signal peaks at ~ 1.15 GeV and is ~ 300 MeV wide. If this signal is fitted with a pure resonance formula, the resonance mass must be below 1.2 GeV. These results may be reconciled with the diffractive data by including a multi-Regge background in fitting the charge-exchange data, or by adding a background to the resonance in $\rho\pi$ scattering [47], a background which does not have to have the same diffractive production characteristics as the A_1 pole. Simultaneous fits to both the charge-exchange and diffractive data then yield an A_1 second sheet pole in the range 1.23-1.28 GeV, depending on the detailed assumptions.

A long-standing problem in meson spectroscopy has been resolved, and the $C = +1$ nonet made from P-wave $q\bar{q}$ pairs is now complete. The strange members Q_A have a mass ~ 1.34 GeV [11,48,49]. The isoscalar members are the D at 1.285 GeV [50,51] and the E at 1.425 GeV [52]. The D and E may be mixtures of octet and singlet $I = 0$ states and, given the mass of the A_1 , the Gell-Mann-Okubo formula may be used to calculate the mixing angle between the singlets, yielding a value $\sim 40^\circ$, close to ideal mixing. The D-E mixing is very sensitive to the value of the Q_A mass, reaching $\sim 50^\circ$ for $M_{Q_A} = 1.32$ GeV, as implied by the value of the Q mixing angle [48,49]. For a D-E mixing angle of 40° the measured width of the A_1 implies a width for $E \rightarrow K^* \bar{K} + \bar{K}^* K$ of ~ 80 MeV: for a mixing angle of 50° the value is ~ 60 MeV, not inconsistent with the results of a recent relatively high statistics experiment [52]. Recent discussions of the A_1 nonet, but assuming a low-mass A_1 , have been given in refs. [53,54].

The A_3 meson has a mass of 1.7 GeV and a width of 320 MeV, coupling to the channels $2^-P(\rho\pi)$, $2^-D(\epsilon\pi)$, and $2^-D(f\pi)$, in addition to the dominant channel $2^-S(f\pi)$.

The A_3 is approximately degenerate with the $J^{PG} = 3^{-+}$ g meson observed in $\pi\pi$ scattering and is to be assigned to the singlet D-wave $q\bar{q}$ configuration. No isoscalar candidates for this nonet are known, but the L enhancement in $K\pi\pi$ at ~ 1.8 GeV has $J^P = 2^-$ and there is now good evidence that at least one 2^- resonance exists in the L region [55].

The masses found for the A_1 and A_3 suggest that the effects of spin-spin and spin-orbit coupling in the P- and D-wave $q\bar{q}$ systems are quite small: $A_2(1.315)$, $A_1(1.28)$, $D(1.285)$, $B(1.235)$, and $g(1.69)$, $\omega(1.67)$, $A_3(1.70)$. This increases the difficulty of associating the 0^+ states $\delta(0.98)$, $S^*(0.98)$ with the triplet P-wave $q\bar{q}$ configuration, suggesting that the $\delta(1.3)$ and $\epsilon(1.3)$ are 0^+ $q\bar{q}$ states and that the 0^+ states around 1 GeV are to be associated with, for example, four-quark states [56].

There are indications in the data from this experiment for new states 0^- (somewhere in the range 1.2-1.6 GeV), 1^+ (D-wave $\rho\pi$ at ~ 1.7 GeV), 2^+ (~ 1.7 GeV), and 2^- (~ 2.1 GeV). If there are such states, they could all correspond to radial excitations in the quark model. The data are however not compelling in any of these instances: the effects are minor and it has taken the full power of 600,000 events in this experiment to elucidate the nature of the amplitudes in the dominant waves, $1^+S(\rho\pi)$ and $2^-S(f\pi)$. There is nonetheless a pattern to these possible states, illustrated in fig. 34.

This experiment has shown that diffractive production of three pions, far from being a featureless desert of Deck as mapped in ref. [8], is as rich in resonances as processes dominated by meson exchange.

REFERENCES

- [1] C. Daum et al., Phys. Lett. 89B (1980) 276.
- [2] C. Daum et al., Phys. Lett. 89B (1980) 281.
- [3] C. Daum et al., Phys. Lett. 89B (1980) 285.
- [4] G. Grayer et al., Nucl. Instrum. Methods 99 (1972) 579.
- [5] D.V. Brockway, University of Illinois report C00-1195-197 (1970).
- [6] G. Ascoli et al., Phys. Rev. Lett. 25 (1970) 962.
- [7] G. Ascoli et al., Phys. Rev. D 7 (1973) 669.
- [8] G. Ascoli et al., Phys. Rev. D 9 (1974) 1963.
- [9] D.J. Herndon et al., Phys. Rev. D 11 (1975) 3165, 3183.
- [10] M. Tabak et al., *in* Proc. 4th Int. Conf. on Experimental Meson Spectroscopy, Boston, 1974 (Ed. D. Garelick) (AIP, NY, 1974), p. 46.
- [11] G.W. Brandenburg et al., Phys. Rev. Lett. 36 (1976) 703.
- [12] J.D. Hansen et al., Nucl. Phys. B81 (1974) 403.
- [13] R.J. Cashmore, *in* Phenomenology of particles at high energies, Proc. 14th Scottish Universities Summer School, Middleton Hall, nr. Edinburgh, 1973 (Eds. R.L. Crawford and R. Jennings) (Academic Press, London, 1974).
- [14] R.L. Schult and H.W. Wyld, Phys. Rev. D 16 (1977) 62.
- [15] R. Aaron et al., Ann. Phys. (NY) 117 (1979) 56.
- [16] I.J.R. Aitchison and R. Golding, J. Phys. G4 (1978) 43.
- [17] K.R. Parker, Oxford Ph.D. Thesis (1979) unpublished.
- [18] J.D. Blatt and V.F. Weisskopf, Theoretical nuclear physics (Wiley, New York, 1952), p. 361.
- [19] Yu.M. Antipov et al., Nucl. Phys. B63 (1973) 153.
- [20] R.T. Deck, Phys. Rev. Lett. 13 (1964) 169.
- [21] L. Stodolsky, Phys. Rev. Lett. 18 (1967) 973.
- [22] G. Otter et al., Nucl. Phys. B80 (1974) 1.
- [23] G. Thompson et al., Phys. Rev. D 9 (1974) 560.
- [24] C. Baltay et al., Phys. Rev. Lett. 39 (1977) 591.
- [25] J. Pernegr et al., Nucl. Phys. B134 (1978) 436.

- [26] M.G. Bowler et al., Nucl. Phys. B97 (1975) 227.
- [27] R.S. Longacre and R. Aaron, Phys. Rev. Lett. 38 (1977) 1509.
- [28] J.-L. Basdevant and E.L. Berger, Phys. Rev. D 16 (1977) 657.
- [29] F. Wagner et al., Phys. Lett. 58B (1975) 201.
- [30] M. Cerrada et al., Nucl. Phys. B126 (1977) 241.
- [31] A. Ferrer et al., Nucl. Phys. B142 (1978) 77.
- [32] P. Gavillet et al., Phys. Lett. 69B (1977) 119.
- [33] G. Ascoli et al., Phys. Rev. D 8 (1973) 3894.
- [34] E.L. Berger, *in* Three particle phase shift analysis and meson resonance production (Eds. J.B. Dainton and A.J.G. Hey) (Daresbury Nuclear Physics Laboratory, Daresbury, nr. Warrington, 1975), Daresbury Study Weekend Series No. 8; DL-R 34.
- [35] E.L. Berger and J.T. Donahue, Phys. Rev. D 15 (1977) 790.
- [36] E.L. Berger and P. Pirilä, Phys. Rev. D 12 (1975) 3448.
- [37] A. Tsarev, Phys. Rev. D 11 (1975) 1864.
- [38] I.J.R. Aitchison and M.G. Bowler, J. Phys. G3 (1977) 1503.
- [39] M.G. Bowler, J. Phys. G5 (1979) 203.
- [40] G.W. Brandenburg et al., Phys. Rev. Lett. 36 (1976) 1239.
- [41] G. Thompson et al., Nucl. Phys. B69 (1974) 381.
- [42] G. Alexander et al., Phys. Lett. 73B (1978) 99.
- [43] J.A. Jaros et al., Phys. Rev. Lett. 40 (1978) 1120.
- [44] J.-L. Basdevant and E.L. Berger, Phys. Rev. Lett. 40 (1978) 994.
- [45] H. Goldberg and R. Aaron, Phys. Rev. Lett. 42 (1979) 339.
- [46] J.A. Dankowych et al., Paper submitted to 6th Int. Conf. on Experimental Meson Spectroscopy, Brookhaven, 26-27 April 1980.
- [47] R.S. Longacre, private communication.
- [48] R.K. Carnegie et al., Phys. Lett. 68B (1977) 287.
- [49] M.G. Bowler, J. Phys. G3 (1977) 775.
- [50] Particle Data Group, Phys. Lett. 75B (1978).
- [51] N.R. Stanton et al., Phys. Rev. Lett. 42 (1979) 346.

- [52] C. Dionisi et al., Nucl. Phys. B169 (1980) 1.
- [53] M. Mazzucato et al., Nucl. Phys. B156 (1979) 546.
- [54] A.C. Irving and H.R. Sepangi, Nucl. Phys. B139 (1978) 327.
- [55] This Collaboration, Diffractive production of strange mesons at 63 GeV/c,
to be submitted to Nucl. Phys. B.
- [56] R.L. Jaffe and F.E. Low, Phys. Rev. D 19 (1979) 2105.

Table 1

Spark chamber parameters

Spark-chamber bank	Wires: angle to vertical (degrees)	Number of planes	Sensitive area (horizontal $\times$ vertical) (m^2)	Distance from target (m)
S1 A	0	2	2×1	1.0
	+45	4		
	-45	4		
	90	2		
S1 B	0	4	2×1	1.6
	+45	2		
	-45	2		
	90	4		
S2	0	10	3.6×1	4.5
	+30	5		
	-30	5		
S3	0	8	3×1	8.0
	+30	4		
	-30	4		
S4	0	6	4.5×1	11.5
	-12	3		
	+12	3		
S5	0	4	4.5×2	24.5
	-12	2		
	+12	2		

Table 2

Resolution of the apparatus at 94 GeV: single tracks ^{a)}

	S1 and S2 only	Tracks present in S1, S2, S3-5
	$0.5 \lesssim p \lesssim 15 \text{ GeV}$	$5 \text{ GeV} \lesssim p$
$\Delta p/p$	$10^{-3} \sqrt{25 + 6P^2}$	5×10^{-3} (limited by multiple scattering)
Angular resolution (horizontal plane) (mrad)	$0.1 \sqrt{\frac{24}{p^2} + \frac{16}{p} + 9}$	$0.15 \sqrt{0.1 + \frac{90}{p^2}}$
Derived quantities		
$\sigma_{M_{2\pi}} = 9 \text{ MeV}$ (at mass of ρ meson) $\sigma_{M_{3\pi}} = 12 \text{ MeV}$ (averaged over range $1.0 \leq M_{3\pi} \leq 2 \text{ GeV}$) $\sigma_t = 0.005 \text{ GeV}^2$ $\sigma_{MM^2} = 0.85 \text{ GeV}^2$		

a) The errors on the derived quantities depend on the configuration of any given event and the values above are only representative.

Table 3

Efficiency E for finding track segments (QLINES) in TENPRO ^{a)}

Three-prong events		Three-prong events + extra beam track
S1	$E = \frac{R^2 + 92}{R^2 + 100}$ $\bar{E} = 0.97$	$E = \frac{R^2 + 35}{R^2 + 100}$ $\bar{E} = 0.81$
S2	$E = \frac{R^2 + 873}{R^2 + 900}$ $\bar{E} = 0.99$	$E = \frac{R^2 + 720}{R^2 + 900}$ $\bar{E} = 0.96$
S3-5	$\bar{E} = 0.99$	$\bar{E} = 0.98$

a) The quantity R is the mean distance of the track segment from the beam trajectory, in mm.

Table 4

Assignment of events from a detailed study of NORD tape

Assignment	Percentage of triggers
All triggers (21,567 events)	100
3 prongs, with charge balance and well-defined vertex (16,512 events)	76.6
3π events lost in TENPRO	1.2
Events lost because of beam track problems	1.9
Events lost because of a secondary scatter in the target	0.9 (calculated 0.9%)
Events lost because of secondary inter- actions in the apparatus	3.8 (calculated 3.9%)
Events lost because of secondary decay	1.6 (calculated 1.4%)
Acceptance loss in S2	0.3
Events containing K^0	0.7
$\pi^- e^+ e^-$ secondaries	1.6
2 prongs + e^-	1.5
$e^- \rightarrow e^- \gamma \rightarrow e^- e^+ e^-$	1.4
More than 3 prongs	5.2
0 or 1 prongs	2.4
Junk 3 prongs	0.6
Junk 2 prongs	0.3

Table 5
Event-independent corrections

Source	Effect
G counter veto	8% loss of events
Missing-mass squared cut (-3 to +5 GeV ²)	~ 1.5% (< 5%) gain of events due to background
Interaction, scattering or decay of secondaries resulting in failure to trigger	9% loss of events
Interaction, scattering or decay of secondaries resulting in failure to reconstruct	6% loss of events
Attenuation of beam in target	2.5% loss of beam track length
Interaction of beam between beam counters and target	1.3% loss of beam tracks
Electron contamination of beam	~ 1% gain in nominal beam tracks
Muon contamination of beam	~ 1% gain in nominal beam tracks
K ⁻ , $\bar{p}$ contamination of beam	~ 1.5% gain in nominal beam tracks

Table 6
Microbarn equivalents^{a)}

Data sets	Events/ μb
94 GeV standard trigger	860 $\pm$ 100
63 GeV standard trigger	500 $\pm$ 66
94 GeV high-t trigger	2400 $\pm$ 240

a) The table gives the number of events at production (that is, after the data has been corrected for all acceptance losses) corresponding to 1 μb cross-section.

For figures combining 63 and 94 GeV standard trigger data, the number of events corresponding to 1 μb is 1360 $\pm$ 120, and the weighted mean momentum is 81 GeV.

For figures combining 63 and 94 GeV standard trigger data and 94 GeV high-t trigger data, the cross-section for $0 \leq |t'| \leq 0.7 \text{ GeV}^2$ may be obtained approximately by dividing the number of events by 2100 events/ μb .

Table 7

Results of fits to the differential cross-section as a function of 3π mass for the 94 GeV standard trigger data using the expression

$$\frac{dN}{dt'} = N_0 \{ A P e^{At'} + B(1-P)^{Bt'} \}$$

Mass range (GeV)	A (GeV ⁻²)	B (GeV ⁻²)	N ₀	P
0.9-0.95	17.0 ± 0.3	2.0 ± 0.5	6447	0.93 ± 0.01
0.95-1.0	16.5 ± 0.2	2.7 ± 0.3	10393	0.92 ± 0.01
1.0-1.05	15.7 ± 0.3	3.2 ± 0.3	14247	0.92 ± 0.01
1.05-1.10	14.0 ± 0.2	2.5 ± 0.2	17879	0.94 ± 0.01
1.1-1.15	14.5 ± 0.5	4.7 ± 0.8	18916	0.86 ± 0.04
1.15-1.20	13.0 ± 0.3	4.2 ± 0.4	19822	0.86 ± 0.02
1.20-1.25	12.8 ± 0.2	3.9 ± 0.3	19456	0.82 ± 0.02
1.25-1.30	11.4 ± 0.4	3.8 ± 0.1	19482	0.74 ± 0.02
1.30-1.35	11.7 ± 0.7	4.4 ± 0.2	18247	0.55 ± 0.05
1.35-1.40	13.3 ± 0.3	5.0 ± 0.1	13656	0.47 ± 0.01
1.4-1.45	12.4 ± 0.6	4.0 ± 0.4	10603	0.63 ± 0.06
1.45-1.50	11.7 ± 0.3	4.1 ± 0.2	9147	0.64 ± 0.02
1.5-1.55	11.3 ± 0.3	4.3 ± 0.2	8762	0.64 ± 0.02
1.55-1.6	11.4 ± 0.6	4.2 ± 0.1	8944	0.63 ± 0.02
1.6-1.65	10.5 ± 0.4	4.6 ± 0.1	9379	0.64 ± 0.02
1.65-1.7	11.8 ± 1.0	4.9 ± 0.5	9557	0.53 ± 0.1
1.7-1.75	9.1 ± 0.7	3.1 ± 0.5	8631	0.8 ± 0.07
1.75-1.8	13.0 ± 2.0	4.9 ± 0.6	7788	0.43 ± 0.1
1.8-1.85	10.8 ± 0.6	5.1 ± 0.1	6498	0.44 ± 0.02
1.85-1.9	16.7 ± 1.1	5.6 ± 0.1	5427	0.24 ± 0.02

Table 8

Wave sets used in the partial-wave analysis a)

Mass range (GeV)	0.9-1.0	1.0-1.14	1.14-1.40	1.40-1.50	1.50-1.76	1.76-1.84	1.84-1.98	1.98-2.40	2.40-2.60
$0^- S0^+(\epsilon\pi)$									
$0^- P0^+(\rho\pi)$									
$1^+ S0^+(\rho\pi)$									
$1^+ P0^+(\epsilon\pi)$									
$1^+ S1^+(\rho\pi)$									
$1^+ P1^+(\epsilon\pi)$									
$2^- P0^+(\rho\pi)$									
$2^- D0^+(\epsilon\pi)$									
$2^+ D1^+(\rho\pi)$									
$1^+ D0^+(\rho\pi)$									
$1^+ D1^+(\rho\pi)$									
$2^- S0^+(\epsilon\pi)$									
$3^+ D0^+(\rho\pi)$									
$3^+ P0^+(\epsilon\pi)$									
$2^+ P1^+(\epsilon\pi)$									
$2^- D0^+(\epsilon\pi)$									
$2^- S1^+(\epsilon\pi)$									
$2^- P1^+(\epsilon\pi)$									
$2^- D1^+(\epsilon\pi)$									
$2^- D1^+(\rho\pi)$									
$4^- P0^+(\rho\pi)$									
$3^+ S0^+(g\pi)$									

a) Each wave is entered in the mass column in which it was first included, with a horizontal bar showing the range.

Table 9

Values of the parameter b in a fit to the t dependence
of the principal $M = 0$ partial-wave intensities using the form e^{bt}

Mass range (GeV)	1.08-1.14	1.26-1.36	1.6-1.7
0^-S	20.2 ± 1.0	15.5 ± 0.9	10.3 ± 2.5
0^-P	13.9 ± 1.6	15.5 ± 0.9	11.0 ± 3.0
1^+S	13.8 ± 0.5	9.8 ± 0.2	8.2 ± 0.8
1^+P	20.2 ± 1.5	9.7 ± 0.2	12.6 ± 3.0
2^-P	2.3 ± 0.3	6.9 ± 0.8	4.5 ± 3.0
2^-S			8.9 ± 0.3

Table 10

Resonance parameters for fits to A_2

	63 GeV $0.05 < t' < 0.7 \text{ GeV}^2$	94 GeV $0.05 < t' < 0.7 \text{ GeV}^2$	94 GeV $0.16 < t' < 0.7 \text{ GeV}^2$	All data combined
Mass (MeV)	1317 ± 3	1318 ± 3	1317 ± 3	1317 ± 2
Γ_0 (MeV) (resolution corrected, $\sigma = 10\text{-}12 \text{ MeV}$)	103 ± 12	105 ± 10	90 ± 10	96 ± 9
Background (%) (1.2-1.4 GeV region)	4.0 ± 0.6	6.3 ± 0.6	3.8 ± 0.4	4.6 ± 0.5
Exponential slope b in $t' \text{ ebt}'$ fit (GeV^{-2})	7.6 ± 0.2	7.5 ± 0.2	7.1 ± 0.1	7.3 ± 0.1
Background-subtracted total A_2 ($2^{+}D1^{+}$) cross- section (μb)	22.0 ± 3.4	18.8 ± 2.6	22.9 ± 2.5	-

Table 11
A₂ production characteristics, $1.26 < M_{3\pi} < 1.36$ GeV.

P _{lab} (GeV)	Standard trigger		High-t trigger						
	63 GeV	94 GeV	94 GeV						
t' limits (GeV ²)	0.0-0.7	0.0-0.7	0.16-0.18	0.18-0.21	0.21-0.25	0.25-0.30	0.30-0.38	0.38-0.47	0.47-1.00
Intensity of 2 ⁺ D (events)	3402	5256	1351	1904	2292	2299	2606	2301	3841
Production phase with respect to 1 ⁺ S0 ⁺ (deg)	-5 ± 11	-5 ± 9	-3 ± 6	-8 ± 4	-5 ± 4	-6 ± 4	-11 ± 4	-12 ± 5	-1 ± 7
$\eta = +1$									
ρ_{11}	0.90 ± 0.03	0.91 ± 0.03	0.91 ± 0.06	0.89 ± 0.04	0.92 ± 0.04	0.92 ± 0.04	0.88 ± 0.04	0.92 ± 0.03	0.90 ± 0.03
ρ_{22}	0.05 ± 0.03	0.03 ± 0.02	0.03 ± 0.06	0.03 ± 0.05	0.03 ± 0.04	0.03 ± 0.04	0.04 ± 0.03	0.04 ± 0.03	0.06 ± 0.02
Re (ρ_{21})	0.12 ± 0.02	0.10 ± 0.02	0.11 ± 0.03	0.09 ± 0.02	0.11 ± 0.02	0.11 ± 0.02	0.09 ± 0.02	0.12 ± 0.02	0.19 ± 0.02
Im (ρ_{21})	-0.08 ± 0.08	-0.04 ± 0.07	-0.07 ± 0.19	-0.01 ± 0.14	0.00 ± 0.12	-0.05 ± 0.10	-0.02 ± 0.11	-0.05 ± 0.10	-0.03 ± 0.06
$\eta = -1$									
Trace of ρ	0.05 ± 0.04	0.06 ± 0.03	0.06 ± 0.08	0.08 ± 0.06	0.05 ± 0.05	0.05 ± 0.05	0.07 ± 0.05	0.03 ± 0.04	0.04 ± 0.03

Table 12

Fits to the 1^+S amplitude

Data sample	94 GeV + 63 GeV standard trigger		94 GeV high-t trigger	
$ t' $ limits (GeV^2)	0-0.05	0.05-0.7	0.16-0.3	0.3-0.7
<i>Basic model (ref. [39])</i>				
M_0 (GeV)	1.29	1.28	1.26	1.23
g^2 (GeV)	1.1	1.0	0.95	1.0
ϕ (degrees)	-37	-47	-54	-106 ^{a)}
F_0	0.18	0.21	0.25	0.65
ϕ_0 (degrees)	13	25	25	82 ^{b)}
A (normalization)	1.04	0.89	1.0	0.14 ^{a)}
$F_0\sqrt{A}$	0.18	0.20	0.25	0.25
Resonance peak (GeV)	1.31	1.30	1.28	1.25 ^{c)}
Width (MeV)	320	310	290	300 ^{c)}
χ^2/NDF	54/59	105/59	87/59	60/59
<i>Model including scattering length + resonance</i>				
M_0 (GeV)	1.28	1.28	1.26	1.25
g^2 (GeV)	1.4	1.3	1.6	1.3
b (scattering length) (GeV^{-1})	-1.6	-1.7	-3.3	-3.4
F_0	-0.01	0.02	-0.03	0.01
ϕ_0 (degrees)	22	25	41	39
A (normalization)	1.8	1.9	4.3	3.2
χ^2/NDF	51/59	119/59	61/59	56/59

- a) In the region $0.3 \leq |t'| \leq 0.7$ the angle ϕ is not well determined if the amount of Deck may be arbitrarily small. If ϕ is allowed any value, the normalization A could be as large as 1.0, with $\phi = -115^\circ$. A = 1, $\phi = -50^\circ$ however gives $\chi^2 = 114$. These data in the highest t' region will not tolerate a significant interference term between rescattered Deck and direct production.
- b) ϕ_0 is determined largely by the value of ϕ .
- c) The quoted values for resonance peak position and width were taken from the direct production intensity alone.

Table 13

Subdensity matrix for 1^+S obtained in the interval $1.26 \leq M_{3\pi} \leq 1.36$ GeV

$ \tau' \text{ (GeV}^2\text{)}$	0.16-0.18	0.18-0.21	0.21-0.25	0.25-0.30	0.30-0.38	0.38-0.47	0.47-1.0
ρ_{00}	0.89 ± 0.03	0.87 ± 0.03	0.89 ± 0.03	0.88 ± 0.03	0.88 ± 0.03	0.90 ± 0.05	0.93 ± 0.08
ρ_{11}	0.11 ± 0.03	0.13 ± 0.03	0.11 ± 0.03	0.12 ± 0.03	0.12 ± 0.03	0.10 ± 0.05	0.07 ± 0.08
$\text{Re } \rho_{10}$	-0.26 ± 0.04	-0.28 ± 0.04	-0.26 ± 0.03	-0.25 ± 0.04	-0.24 ± 0.04	-0.20 ± 0.05	$+0.03 \pm 0.05$
$\text{Im } \rho_{10}$	-0.01 ± 0.17	$+0.01 \pm 0.14$	$+0.08 \pm 0.16$	$+0.08 \pm 0.15$	$+0.08 \pm 0.17$	$+0.11 \pm 0.26$	$+0.10 \pm 0.4$

Table 14

A₃ parameters for simple fits to intensity alone

	$2^{-}S(f\pi)$ intensity		All decay channels			
	Standard trigger		High-t trigger	Summed data		
	63 GeV	94 GeV	94 GeV	$2^{-}S(f\pi)$	$2^{-}P(\rho\pi)$	$2^{-}D(\epsilon\pi)$
Mass (MeV)	1669 ± 3	1670 ± 4	1671 ± 3	1671 ± 2	1643 ± 2	1655 ± 5
Fitted width (MeV)	182 ± 15	219 ± 10	221 ± 8	207 ± 10	230 ± 20	228 ± 60
Incoherent background model (1500-1800 MeV) (%)	21 ± 3	21 ± 2	10 ± 1	24 ± 1	28 ± 1	18 ± 3
Integrated cross-section (including background) (μb)	34.6 ± 5.4	29.3 ± 4.1	31.5 ± 3.9			
Branching ratio, $\pi^{-}\pi^{-}\pi^{+}$ final state (%)				57 ± 3	32 ± 6	11 ± 5
$d\sigma/dt'$ slope parameter for exponential fit				8.5 ± 0.3	4.1 ± 0.3	10.2 ± 0.8
				8.3 ± 0.3	5.2 ± 0.3	10.7 ± 1.1

Table 15

Subdensity matrix for 2^-S obtained in the interval $1.60 < M_{3\pi} < 1.70$ GeV
for the 94 GeV standard trigger data

$ t' $ (GeV ²)	0.0-0.02	0.02-0.05	0.05-0.08	0.08-0.16	0.16-0.18	0.18-0.21	0.21-0.25	0.25-0.30	0.30-0.47	0.47-1.0
$\eta = +1\rho_{00}$	0.92 ± 0.10	0.92 ± 0.07	0.87 ± 0.09	0.85 ± 0.06	0.85 ± 0.10	0.84 ± 0.08	0.77 ± 0.06	0.81 ± 0.06	0.74 ± 0.05	0.52 ± 0.06
ρ_{11}	0.03 ± 0.04	0.05 ± 0.04	0.07 ± 0.04	0.10 ± 0.03	0.13 ± 0.04	0.11 ± 0.04	0.14 ± 0.03	0.14 ± 0.04	0.21 ± 0.03	0.41 ± 0.05
ρ_{22}	0.03 ± 0.04	0.01 ± 0.03	0.04 ± 0.04	0.03 ± 0.03	0.02 ± 0.04	0.03 ± 0.03	0.03 ± 0.03	0.02 ± 0.04	0.03 ± 0.02	0.04 ± 0.03
Re ρ_{10}	0.07 ± 0.04	0.14 ± 0.03	0.21 ± 0.03	0.23 ± 0.03	0.26 ± 0.04	0.27 ± 0.03	0.29 ± 0.03	0.28 ± 0.04	0.32 ± 0.02	0.32 ± 0.04
Im ρ_{10}	-0.03 ± 0.09	-0.03 ± 0.07	0.00 ± 0.07	-0.01 ± 0.05	-0.04 ± 0.07	-0.03 ± 0.06	-0.07 ± 0.05	-0.07 ± 0.05	-0.08 ± 0.04	0.00 ± 0.05
Re ρ_{20}	-0.03 ± 0.03	0.01 ± 0.02	-0.04 ± 0.03	-0.02 ± 0.02	-0.04 ± 0.03	-0.04 ± 0.02	-0.07 ± 0.02	-0.02 ± 0.03	-0.04 ± 0.02	-0.07 ± 0.07
Im ρ_{20}	0.02 ± 0.07	0.01 ± 0.05	0.02 ± 0.05	0.01 ± 0.04	0.02 ± 0.06	-0.01 ± 0.05	0.01 ± 0.03	0.01 ± 0.04	-0.04 ± 0.02	-0.06 ± 0.04
$\eta = -1$										
Trace of ρ	0.02 ± 0.06	0.02 ± 0.04	0.02 ± 0.05	0.02 ± 0.04	0.02 ± 0.06	0.02 ± 0.05	0.02 ± 0.04	0.02 ± 0.05	0.03 ± 0.03	0.03 ± 0.04

Table 16

Resonance parameters from two resonance fits
to 2 S waves (GeV) a)

	A_3	A'_3
Mass	1.71 ± 0.02	2.10 ± 0.15
$\Gamma(\text{S-wave } f\pi)$	0.154 ± 0.03	0.168 ± 0.04
$\Gamma(\text{D-wave } f\pi)$	0.035 ± 0.015	0.065 ± 0.035
$\Gamma(\text{P-wave } \rho\pi)$	0.092 ± 0.007	0.123 ± 0.03
$\Gamma(\text{D-wave } \varepsilon\pi)$	0.031 ± 0.015	0.295 ± 0.03

- a) The error on the mass of the higher resonance is very uncertain, the fits not changing qualitatively in the range shown. The errors on other quantities were obtained from the spread in values when refitting with various masses for the upper resonance, and can be taken as indicative only.

Figure captions

- Fig. 1 : A plan view of the spectrometer. The 50 cm long liquid-hydrogen target (T) is shown surrounded by a box of lead-scintillator sandwich counters (F). Sandwich counters G and H are also shown. The two packs of wire chambers S1 are separated from the chambers S2 by the first bending magnet (M1). Following the second bending magnet (M2) are the two Čerenkov counters (C1, C2) and three packs of spark chambers (S3, S4, S5). The double hodoscope of scintillation counters P2/3 measured multiplicity after the first magnet, and a measurement of multiplicity after the target could be obtained from the double proportional wire chamber P1. The spectrometer was approximately 30 m long.
- Fig. 2 : An exploded view of the F counters which surrounded the target (not shown). Each of the outer counters consisted of six layers of lead-scintillator, feeding a single photomultiplier. Except for the counters on the upstream face of the box, each outer counter was covered by an inner single layer of scintillator, feeding a separate photomultiplier.
- Fig. 3 : The fraction of good three-prong events at 94 GeV surviving the imposition of the F counter condition ST1 (see text) is shown as a function of vertex position within the hydrogen target (z), the four-momentum transfer squared to the proton (t) and the missing mass recoiling against the 3π system. The distribution in missing mass squared is from a sample of approximately 10,000 events with missing mass squared between -3 and 5 GeV^2 . The loss is constant and about 0.2 between -3 GeV^2 and 1 GeV^2 , and then increases with missing mass squared because of rejection of events with N^* states recoiling against the 3π system. The distributions in z and t are taken from a subsample of approximately 5000 events with missing mass squared below the proton peak, almost uncontaminated with $3\pi N^*$ events. The projections of the correction algorithm are shown.

- Fig. 4 : The dimensions of the double hodoscope P2/3. The seven central strips, 2 cm wide, were split in the horizontal plane to increase the spatial resolution in this region of greatest particle density.
- Fig. 5 : The results of calibration studies of the Čerenkov counters, carried out with secondaries from the reactions $\pi^-p \rightarrow \pi^-\pi^+n$, $\pi^-p \rightarrow K^-K^+n$ at 63 GeV. The ordinate is calibrated with the detection efficiency E , and the scale is logarithmic. For $E < 50\%$ $\log E$ is plotted and for $E > 50\%$ the quantity $2 \log 50 - \log (100 - E)$ is plotted.
- a) Detection efficiency of C1 for pions and for kaons in the threshold region. (The calibration for kaons is restricted to the region around 27 GeV, because one particle was required to be positively identified and the nature of the other then inferred from conservation of strangeness.) The solid curve shows the expected efficiency, with eight photoelectrons produced as $\beta \rightarrow 1$. The discrepancy between the expected and observed efficiency is caused by local inefficiencies at the mirror edges.
- b) Detection efficiency of C2 for pions. The solid curve shows the expected efficiency with six photoelectrons produced as $\beta \rightarrow 1$.
- Fig. 6 : The response of the Čerenkov counters C1 and C2 as a function of secondary particle momentum, for events accepted prior to particle identification. The figure shows the fraction of secondary particles giving rise to a signal in the appropriate cell. The nominal π and K thresholds are indicated.
- Fig. 7 : Raw data from two NORD tapes (94 GeV, standard trigger) processed through the program TENPRO. The data sample consists of 15,050 events with vertex in the hydrogen target and missing mass squared between -3 and $+5 \text{ GeV}^2$.
- a) Vertex distribution along the beam. The ends of the target, determined from a target empty run, are shown cross-hatched and the arrows indicate the cuts applied to the vertex position in order to accept only events occurring in hydrogen.

- Fig. 7 : b) The distribution of missing mass squared.
(cont.)
c) The 3π mass distribution, showing peaks in the A_1/A_2 and A_3 regions.
d) The $\pi^+\pi^-$ mass distribution, each event yielding two combinations, showing a weak f signal in addition to the prominent ρ meson.
e) The distribution in four-momentum transfer squared, t .

Fig. 8 : Comparison of data with projections of intensity from the fitted amplitudes for $1.64 \leq M_{3\pi} \leq 1.66$ GeV, $0.0 \leq |t'| \leq 0.7$ GeV². (This mass bin was chosen for display, because the $\pi^+\pi^-$ spectrum has a strong f signal in addition to the ρ .) In this figure the data points are shown with statistical errors and the projections of the intensity obtained from the fitted amplitudes are shown as solid points. The number of events is plotted as a function of $\pi^+\pi^-$ mass, $\pi^-\pi^-$ mass, the Euler angles θ , ϕ , γ of the π^+ in the t -channel frame, and π^+p mass. The latter distribution shows that at this laboratory momentum (94 GeV) the 3π system is uncontaminated by events in which Δ^{++} is formed.

Fig. 9 : The 3π invariant mass distribution for a number of selections. The crosses are the raw data, and the points give the distribution after acceptance corrections made in the course of partial-wave analysis. Data in the mass range $0.7 \leq M_{3\pi} \leq 1.6$ GeV for four selections:
a) summed 63 and 94 GeV data (standard trigger), $0.0 \leq |t'| < 0.05$ GeV²;
b) summed 63 and 94 GeV data (standard trigger), $0.05 \leq |t'| \leq 0.7$ GeV²;
c) 94 GeV (high- t trigger), $0.16 \leq |t'| < 0.3$ GeV²;
d) 94 GeV (high- t trigger), $0.3 \leq |t'| \leq 0.7$ GeV²;
e) Data in the mass range $1.3 \leq M_{3\pi} \leq 2.3$ GeV, summed 63 GeV data (standard trigger) and 94 GeV data (standard trigger and high- t trigger).

Fig. 10 : Dependence of production of 3π events at 94 GeV (standard trigger) as a function of the variable t' .

- a) The number of events in each t' bin is plotted for five bins of 3π mass, corrected only for the effects of the F counters. The curves are the results of fitting a sum of two exponentials to the data.
- b) The slope parameters A and B (see table 7) resulting from fits with a sum of two exponentials in t' .

Fig. 11 : The results of the two independent partial-wave analyses for the major waves 1^+S0^+ , 2^+D1^+ , 2^-S0^+ . Results obtained from the Illinois program are shown as points with error bars, results obtained using the SLAC-LBL approach as rectangles.

- a) 1^+S0^+ intensity, $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$;
- b) 2^+D1^+ intensity, $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$;
- c) The phase of 2^+D1^+ measured relative to the 1^+S0^+ amplitude;
- d) 2^-S0^+ intensity, $0.0 \leq |t'| \leq 0.7 \text{ GeV}^2$;
- e) 2^-S0^+ phase measured relative to the 1^+S0^+ amplitude (94 GeV standard trigger data).

Fig. 12 : An over-all view of the relative importance, in the A_1 - A_2 region, of the waves 1^+S0^+ , 1^+P0^+ , 0^-S0^+ , 0^-P0^+ , 2^+D1^+ and 2^-P0^+ is given in this figure, where the intensities obtained from the 63 and 94 GeV standard trigger data, $0.0 \leq |t'| \leq 0.7 \text{ GeV}^2$, are shown. The same scale is used for all intensities. All other waves are very minor.

Fig. 13 : The relative phases of the six waves for which the intensities are displayed in fig. 12. All phases are measured with respect to 1^+S0^+ . It will be noted that 1^+P0^+ and 0^-S0^+ show little variation of phase with respect to 1^+S0^+ , while 0^-P0^+ and 2^+P0^+ (both $\rho\pi$ waves) have phases decreasing through $\approx 50^\circ$. The phase of 2^+D1^+ has the qualitative behaviour expected from the A_2 resonance, but a maximum excursion relative to $1^+S \approx 90^\circ$ rather than $\approx 180^\circ$.

Fig. 14 : a) Variation of 2^+D1^+ intensity with 3π mass for the summed 63 and 94 GeV data taken with the standard trigger, $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$. The curve is the result of a fit with the sum of a D-wave Breit-Wigner intensity and an exponential background intensity. The background under the A_2 resonance is only a few per cent.

b,c) The phase of the 2^+D1^+ amplitude measured with respect to the other five waves important in the mass region $1.0 \leq M_{3\pi} \leq 1.6 \text{ GeV}$. The curves are the phase of the fitted D-wave Breit-Wigner amplitude superimposed on the data. No fit was made to the data on the 2^+D1^+ phase: these curves result from the fit to the mass distribution alone.

Fig. 15 : The differential cross-section for A_2 production. The figure shows the number of 2^+D1^+ events in the range $1.26 \leq M_{3\pi} \leq 1.36 \text{ GeV}$, as a function of t' . The standard trigger data (from both the 63 and 94 GeV samples) has been normalized to the total cross-section obtained from the high- t trigger (94 GeV) data. The curve is the result of a fit with the form

$$\frac{dN}{dt'} \propto t' e^{bt'}$$

Fig. 16 : The cross-section for A_2 production as a function of the laboratory momentum of the incident pion. The points represent the 2^+D1^+ cross-section integrated over all t' and the range $1.2 \leq M_{3\pi} \leq 1.4 \text{ GeV}$. The three independent measurements from this experiment are plotted with the results of the compilation in ref. [19]. The result of a fit with the form $\sigma \propto p_{\text{lab}}^{-n}$ is shown.

- Fig. 17 : Variation of $1^+S_0^+$ intensity with 3π mass, for four selections on t' .
- a) Summed 63 and 94 GeV standard trigger data, $0.0 \leq |t'| \leq 0.05 \text{ GeV}^2$;
 - b) Summed 63 and 94 GeV standard trigger data, $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$;
 - c) 94 GeV high- t trigger data, $0.16 \leq |t'| \leq 0.3 \text{ GeV}^2$;
 - d) 94 GeV high- t trigger data, $0.3 \leq |t'| \leq 0.7 \text{ GeV}^2$.
- Fig. 18 : The relative phase of the $1^+S_0^+$ amplitude, measured with respect to:
- a) $0^-S_0^+$, b) $1^+P_0^+$, c) $0^-P_0^+$, d) $2^-P_0^+$. The results from all three data samples have been averaged. The 1^+S amplitude has a substantial phase variation relative to 0^-P and 2^-P .
- Fig. 19 : The relative phase of the $1^+S_0^+$ amplitude, measured with respect to $2^+D_1^+$. The results from all three data sets have been averaged. The broken crosses show the phase relative to the $2^+D_1^+$ production amplitude with a variation of over 100° between 1.0 and 1.5 GeV.
- Fig. 20 : Three terms in the amplitude for diffractive $\rho\pi$ production:
- a) One-pion exchange Deck mechanism;
 - b) Deck mechanism with rescattering through $\rho\pi$ final state interactions;
 - c) Direct diffractive A_1 meson production.
- Fig. 21 : Differential cross-sections for the waves $1^+S_0^+$, $0^-S_0^+$, $1^+P_0^+$, calculated with the one-pion exchange Deck model described in the text.
- Fig. 22 : Results of fitting $1^+S_0^+$ intensity and phase, measured with respect to the A_2 production amplitude. The solid curves are the result of fitting with a rescattered Deck amplitude plus a direct resonance production term, as described in the text. The broken curves show the contribution to the intensity of the fitted pure Deck term alone.
- a) Standard trigger data (63 and 94 GeV), $0.0 \leq |t'| \leq 0.05 \text{ GeV}^2$;
 - b) Standard trigger data (63 and 94 GeV), $0.05 \leq |t'| \leq 0.7 \text{ GeV}^2$;
 - c) High- t trigger data (94 GeV), $0.16 \leq |t'| \leq 0.3 \text{ GeV}^2$;
 - d) High- t trigger data (94 GeV), $0.3 \leq |t'| \leq 0.7 \text{ GeV}^2$.

- Fig. 23 : The cross-section for 1^+S0^+ production in the range $1.0 \leq M_{3\pi} \leq 1.2$ GeV, as a function of the laboratory momentum of the incident pion. The three independent measurements from this experiment are plotted with the results of the compilation in ref. [19]. The result of a fit with the form $\sigma \propto p_{lab}^{-n}$ is shown.
- Fig. 24 : Properties of the 0^-S0^+ and 0^-P0^+ waves. The intensities are summed over all three data sets. The phases, measured with respect to the A_2 production phase, are weighted averages. a) 0^-S intensity, b) 0^-S phase, c) 0^-P intensity, d) 0^-P phase. The substantial phase variations suggest the existence of a 0^- resonance, but no acceptable quantitative explanation of these data has been found.
- Fig. 25 : 2^-P0^+ intensity in the mass range $0.9 \leq M_{3\pi} \leq 1.6$ GeV, 94 GeV high- t trigger data.
a) $0.16 \leq |t'| < 0.3$ GeV²;
b) $0.3 \leq |t'| \leq 0.7$ GeV².
The peak at $M_{3\pi} \approx 1.2$ GeV, appearing only at very high t' , is unexplained.
- Fig. 26 : The variation of 2^-P intensity with t' compared with that of 1^+S for two bins in $M_{3\pi}$ (94 GeV high- t trigger data).
a) 1^+S , $1.16 \leq M_{3\pi} \leq 1.26$ GeV;
b) 1^+S , $1.26 \leq M_{3\pi} \leq 1.36$ GeV;
c) 2^-P , $1.16 \leq M_{3\pi} \leq 1.26$ GeV;
d) 2^-P , $1.26 \leq M_{3\pi} \leq 1.36$ GeV.
- Fig. 27 : Intensities of the four fitted 2^-0^+ partial waves as a function of 3π mass (all data samples combined):
a) $2^-S0^+(f\pi)$;
b) $2^-P0^+(\rho\pi)$;
c) $2^-D0^+(\epsilon\pi)$;
d) $2^-D0^+(f\pi)$.
The curves shown in (a) to (c) are the results of fits with Breit-Wigner intensities and incoherent background.

- Fig. 28 : The phase of the dominant $2^-S0^+(f\pi)$ amplitude measured relative to:
a) $1^+S(\rho\pi)$;
b) $0^-S(\epsilon\pi)$.

In both cases there is a large phase variation indicative of a resonance in 2^-S .

- Fig. 29 : The cross-section for 2^-S0^+ production in the range $1.5 \leq M_{3\pi} \leq 1.8$ GeV, as a function of the laboratory momentum of the incident pion. The three independent measurements from this experiment are plotted with the results of the compilation in ref. [19]. The result of a fit with the form $\sigma \propto p_{lab}^{-n}$ is shown.

- Fig. 30 : The figure shows the intensities of the four 2^-0^+ waves in the range $1.3 \leq M_{3\pi} \leq 2.6$ GeV, together with the phase of 2^-S0^+ measured relative to 1^+S0^+ and the other 2^-0^+ amplitudes. The curves are the result of fitting with the model containing two resonances and coherent backgrounds, described in the text.

- Fig. 31 : The intensity of $1^+D0^+(\rho\pi)$ and phase measured with respect to $1^+S0^+(\rho\pi)$, for all data combined.

- Fig. 32 : The behaviour of the 2^+1^+ amplitudes in the high-mass region is shown, using all data samples combined.

- a) $2^+D1^+(\rho\pi)$ intensity;
- b) $2^+P1^+(f\pi)$ intensity;
- c) $2^+D1^+(\rho\pi)$ phase, measured with respect to 1^+S ;
- d) $2^+P1^+(f\pi)$ phase, measured with respect to 1^+S .

The curves were obtained with a model containing two resonances coupling to only these two channels.

- Fig. 33 : The intensity and phase variation of the 3^+0^+ waves. The phases are measured with respect to 1^+S0^+ . There is no evidence for a 3^+ resonance at ~ 2.2 GeV and no clear pattern can be perceived. The data from all three samples have been combined in this figure.

Fig. 34 : Meson spin plotted against mass squared for long established states, states established by this experiment, and states suggested by this experiment. The horizontal error bars denote the width of the corresponding states.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

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Fig. 7

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Fig. 26

Fig. 27

Fig. 28

Fig. 29

Fig. 30

Fig. 31

Fig. 32

Fig. 33

- Well established states
- States established in this experiment
- States suggested by these data

Fig. 34