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THE ELUSIVE AXION

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A B S T R A C T

In view of the uncertainties attached to the interpretation of existing axion searches we consider the possibility of probing the direct axion-quark coupling in K and B decays.

Experimental evidence against the existence of the axion has been accumulating since the particle was first discussed by Weinberg¹⁾ and Wilczek²⁾, as a necessary corollary of the Peccei-Quinn³⁾ solution of the strong CP violation problem.

We shall briefly review the present situation, and argue that, even if the presence of a light axion now appears unlikely (see however ref. 4), considerable uncertainties persist in the interpretation of each individual experiment. We shall argue that clean tests can only come from processes where the axion directly couples to (heavy) quarks. We study more precisely the B and K decays into axions, with the conclusion that the heavy quarks contributions might be observable in the first case, depending on the mass of the charged physical scalar involved; the second process is unclear due to the possible cancellations between the various contributions.

We first list the various ways in which the axion is expected to interact with ordinary matter. All have been considered in experimental tests

- (i) direct interaction with quarks, proportional to their "current" mass
- (ii) mixing with pseudoscalar mesons ($\pi, \eta, \dots$)
- (iii) direct interaction with leptons
- (iv) electromagnetic interactions (2γ decays).

The simplest model involving an axion consists of a Weinberg-Salam Lagrangian including (at least) two Higgs doublets¹⁾. In order to avoid unwanted strangeness-changing neutral currents, all quarks bearing the same charge must receive their mass from the same scalar field. Since the lagrangian needs further to be invariant under two independent phase transformations of the right-handed quarks, the Yukawa couplings take the unique structure:

$$\begin{aligned} \mathcal{L}_{\text{Yuk.}} = & \frac{\Gamma_{ij}^u}{v_1} \bar{u}_{R_i} (\phi_1^0, -\phi_1^+) \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L + \\ & + \frac{\Gamma_{ij}^d}{v_2} \bar{d}_{R_i} (\phi_2^-, \phi_2^{0*}) \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L \end{aligned} \quad (1)$$

In the above expression the coefficients Γ may be reexpressed in terms of the quark masses and mixings. The only free parameter, as long as we stick to the above conditions, is the ratio v_1/v_2 of the vacuum expectation values of the ϕ_1 and ϕ_2 fields.

Rewriting the interaction in terms of mass eigenstates, and of the physical and unphysical scalar fields, we have: [we omit the two heavy neutral (flavor diagonal) scalars]

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} = & -\frac{ig}{2M_W} h \left(\frac{v_2}{v_1} \bar{u}_i m_{u_i} \gamma_5 u_i + \frac{v_1}{v_2} \bar{d}_i m_{d_i} \gamma_5 d_i \right) - \\
& -\frac{g}{2} H \left(\bar{u}_i m_{u_i} \gamma_5 u_i - \bar{d}_i m_{d_i} \gamma_5 d_i \right) - \\
& -\frac{g}{\sqrt{2} M_W} h^+ \left(\bar{u}_i \left[m_{u_i} (KM)_{ij} \frac{v_2}{v_1} \left(\frac{1-\gamma_5}{2} \right) + (KM)_{ij} m_{d_j} \frac{v_1}{v_2} \left(\frac{1+\gamma_5}{2} \right) \right] d_j \right) + \text{h.c.} \\
& -\frac{g}{\sqrt{2} M_W} H^+ \left(\bar{u}_i \left[m_{u_i} (KM)_{ij} \left(\frac{1-\gamma_5}{2} \right) - (KM)_{ij} m_{d_j} \left(\frac{1+\gamma_5}{2} \right) \right] d_j \right) + \text{h.c.}
\end{aligned} \tag{2}$$

where h is the axion field, $h^\pm$ a physical charged scalar, and H represents the would-be Goldstone bosons, which accompany the gauge field. (KM) stands for the Kobayashi-Maskawa mixing matrix; v_2/v_1 is usually denoted x . We have omitted the two heavy neutral scalars.

If no strong interaction were present, the axion field would stay massless. In the presence of strong interactions, it mixes with the other pseudoscalar states: if one considers only the π^0 and η mixing for instance, the mixing angles and mass of the axion are: (N is the number of quark flavors considered)

$$\begin{aligned}
m_{\text{axion}} & \approx 23 \text{ keV} \cdot \frac{N}{2} \cdot \left(x + \frac{1}{x} \right) \\
\xi_\pi & = \xi \cdot B_\pi \\
B_\pi & = \left(\frac{3m_d - m_u}{m_d + m_u} \cdot x - \frac{3m_u - m_d}{m_u + m_d} \cdot 1/x \right) \\
\xi_\eta & = \xi \left(\sqrt{3} \cdot x + \frac{1}{\sqrt{3}} \cdot 1/x \right) \\
\xi & = 1.9 \cdot 10^{-4} .
\end{aligned} \tag{3}$$

We insist once again on the fact that all the couplings above are quite unique, unless more quarks are added, or the requirement of the absence of tree level flavor changing neutral couplings is lifted⁵⁾.

So much cannot be said of the lepton couplings. Since flavor conservation is here automatic, the only requirement is that a right-handed lepton couples to at most one of the scalar doublets ϕ_1, ϕ_2 . In a two-doublet model, the coupling of a charged lepton to the axion may thus occur either with a coefficient ($-x$) or ($1/x$). More possibilities exist if extra scalars are allowed.

Finally we turn to the 2γ decay of the axion. This coupling is mediated by the sum of triangle graphs involving all fermions coupled to the axion. In the analysis of Bardeen and Tye, for 6 quarks with standard couplings

$$\tau(h \rightarrow 2\gamma) = \tau(\pi^0 \rightarrow 2\gamma) \cdot \left(\frac{m_\pi}{m_h}\right)^5 \cdot \frac{g \cdot z \cdot (1+z)^{-2}}{[3 + z(1+z)^{-1} - N_1]^2} \quad (4)$$

where N_1 is the number of charged leptons with coupling $(-x)$ to the axion, and $z = m_u/m_d$.

This lifetime is quite sensitive to the (arbitrary) lepton couplings (in particular, if $N_1 = 1$!).

Having stated those couplings, we now turn to the experimental tests. We first evoke as an example the $K^+ \rightarrow \pi^+ h$ process. Since the process $(K^+ \rightarrow \pi^+ h)$ would have contaminated the search for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, this provides an upper limit on the branching ratio

$$\text{B.R.}(K \rightarrow \pi h) = \frac{\Gamma(K^+ \rightarrow \pi^+ h)}{\Gamma(K^+ \rightarrow \text{all})} < 1.4 \cdot 10^{-6} \quad (5)$$

Goldman and Hoffman⁶⁾ have argued that this bound could be pushed down to $2.7 \cdot 10^{-7}$ (90% C.L.) due to the different kinematics of the 2-body decay. When it comes to compare this value to theoretical estimates the situation however gets much more intricate.

Ref. 6 quotes a lower bound, due to π mixing only of $B > 2 \cdot 10^{-8}$. Weinberg¹⁾ argues that the η contribution could be much bigger, but it is difficult to estimate. Ref. 7 gives two different estimates: $B \geq 7.8 \cdot 10^{-6}$, based on η mixing only, and $B \approx (5.7x - 3.4/x)^2 \cdot 10^{-6}$ from a full $SU(4)$ treatment with coherent contributions from $\pi^0, \eta, \eta', \dots$. Note that this last estimate may be suppressed for some values of x , while the estimation based on the η contribution alone cannot.

We will come back in the following to this process, and show that further uncertainties are in fact added by extra contributions due to a direct coupling of the axion to heavy quarks.

One of the most stringent tests on axions seems to come from production in reactor experiments, and subsequent detection through 2 photon decay. The multiple uncertainties of this experiment (coming essentially from the excited nuclei axion production) have been discussed in some detail in ref. 8.

The difficulties encountered above in evaluating the effective hadronic interaction of axions with light quarks is met again in the interpretation of beam

dump experiments, where at least the production or detection phase is hadronic. Purely hadronic experiments provide us with an upper limit on $\sigma_{\text{prod.}} \cdot \sigma_{\text{int.}}$. Typical values are $\sigma_{\text{prod.}} \cdot \sigma_{\text{int.}} < 10^{-67} \sim 10^{-68} \text{ cm}^4$ ⁹⁾. The simplest interpretation of these values consists in a direct comparison with similar pion cross-sections, allowing for a scaling with ξ_π : the data then suggest a coupling $B_\pi^2 < 0.12 \sim 0.04$.

This last step is however not reliable since interference with other production/interaction mechanisms (via $\eta, \dots$ mixing) should be taken into account.

Ellis and Gaillard ¹⁰⁾ have suggested considering the incoherent production of $u\bar{u}$ and $d\bar{d}$ pairs, and were led to replace B_π^2 by $(B_\pi^2 + 1/3 B_\eta^2)$ in the treatment of $\sigma(pp \rightarrow h + X)$, or by $(B_\pi^2 + 2/3 B_\eta^2)$ in $\sigma(h + p \rightarrow X)$. Even if in this precise approximation the interference happens to increase the cross-sections (B_η does not have a zero), it must clearly be kept in mind that a totally reliable estimation of these processes has certainly not been obtained.

From a phenomenological point of view, purely leptonic experiments appear extremely attractive, due to their cleaner interpretation ¹¹⁾. The results of purely leptonic experiments ^{8), 12)} however suffer, as expected, from lower statistics. Assuming a coupling between leptons and axions $\propto 1/x$, the SLAC beam dump experiment expects $5.5/x^4$ leptonic pairs, but sees none. The low energy electron experiments ¹²⁾ based on electron-bremsstrahlung production and e^+e^- or $\gamma\gamma$ decay exclude the values $.074 \leq x \leq 1.2$ and $9.2 \leq x \leq 70$.

Another purely leptonic experiment has been recently suggested ¹³⁾, which consists in looking for a rare decay mode of positronium $\rightarrow$ axion $+$ γ , with an expected branching ratio of $3 \cdot 10^{-8}$ for $x \approx 1$.

We should however stress that while the interpretation of purely leptonic experiments is quite unambiguous, the leptonic couplings to the axion are not really constrained, even in the simplest model, as we have already noted.

The inclusion of leptonic couplings may also affect considerably the 2γ decay of the axion, as already noted in eq. (4). Furthermore the possible presence of massive neutrinos, which would also couple to the axion with either $(-x)$ or $1/x$ opens new channels for the decay. The width for $h \rightarrow \nu\bar{\nu}$ with mass m_ν and coupling $(-x)$ is found to be

$$\Gamma(h \rightarrow \nu\bar{\nu}) = \frac{G_F \cdot m_\nu^2 \cdot m_h \cdot \sqrt{2}}{8\pi} x^2. \quad (6)$$

Although this would not affect usual experiments, it could be of some importance when cosmological bounds are considered.

The cleanest way to test the axion hypothesis (and probably the only possibility to rule it out completely) would be to probe the direct coupling with (heavy) quarks current masses. This possibility has been envisaged in ψ decays¹⁾, with $BR(\psi \rightarrow h + \gamma) \approx 6 \cdot 10^{-4}$.

Another possibility to test the direct axion-quark coupling has recently been suggested by M.B. Wise¹⁴⁾, in a slightly different framework (the model he considers there has all the quarks coupled to one Higgs boson, and therefore does not qualify for solving the strong CP violation problem). He pointed out that the graph fig. 1-1 would bring an important contribution to the $K \rightarrow \pi h$ (and $B \rightarrow \pi h$) process, leading to a branching ratio of order $[x^2 \cdot 10^{-4}]$. This should however be at best regarded (as discussed below) as an order of magnitude estimation, since the graph 1.1 is not by itself gauge invariant. We have calculated explicitly the sum of graphs appearing in fig. 1, in the framework of the Weinberg-Salam-Peccei-Quinn model in Landau gauge (the relevant couplings are given by eq. (2); the physical scalar - unphysical scalar - axion vertex being simply proportional to the square of the physical Higgs mass).

Since in all cases of interest ($s \rightarrow d$ via c, t , or $b \rightarrow s$ via t) the mass² of external quarks is negligible with respect to the internal ones, we have put them equal to zero in the calculation of the integrals, keeping only the leading contributions. We get, for the contribution of quark q_3 to the $q_1 \rightarrow q_2 + h$ amplitude

$$A_{q_1, q_2}^{q_3} = -\frac{1}{16\pi^2} \cdot \frac{g^3}{8M_W^3} \cdot m_3^2 \cdot \bar{u}_2(p-q) \cdot \not{q} \cdot \frac{1-\gamma_5}{2} \cdot u_1(p) \quad (7)$$

$$\times [\text{mixing angles}] \times [x \cdot A_1 + x^3 \cdot A_2]$$

where q is the axion 4-momentum.

The mixing angles are:

$$\begin{aligned} [\text{mixing angles}] &= [c_1 c_2 c_3 - s_2 s_3] [s_1 c_2] & (s \rightarrow d \text{ via } c) \\ &[c_1 s_2 c_3 + c_2 s_3] [s_1 s_2] & (s \rightarrow d \text{ via } t) \\ &[c_1 s_2 s_3 - c_2 c_3] [c_1 s_2 c_3 + c_2 s_3] & (b \rightarrow s \text{ via } t) \end{aligned} \quad (8)$$

and the functions A_1 and A_2 (tabulated in fig. 2, for $m_{q_3} = 1.2, 20, 30$ GeV, as a function of the physical charged scalar mass M_H) respectively read:

$$\begin{aligned}
A_1[m] = & \frac{-3 M_W^4}{(M_W^2 - m^2)(M_W^2 - M_H^2)} \left\{ 1 + \frac{M_H^2 - m^2}{M_W^2 - m^2} \right\} \ln \frac{m^2}{M_W^2} \\
& + \frac{M_H^2}{(M_H^2 - m^2)} \left\{ \frac{6 M_W^2}{M_W^2 - M_H^2} + \frac{M_H^2}{M_H^2 - m^2} \right\} \ln \frac{m^2}{M_W^2} \\
& + 2 + \frac{3 M_W^2}{M_W^2 - m^2} + \frac{M_H^2}{M_H^2 - m^2}
\end{aligned} \tag{9}$$

$$A_2[m] = -2 \left[1 + \frac{M_H^2}{M_H^2 - m^2} \cdot \ln \frac{m^2}{M_H^2} \right] \frac{m^2}{M_H^2 - m^2} \tag{10}$$

Clearly the first observation is that this complete calculation has brought a new parameter into our analysis, namely the mass M_H of the physical charged scalar boson (note that the neutral physical scalars play no rôle, since they are by construction flavor-diagonal). As for K decays, we also remark that the importance of the t quark contribution should in general be expected to dominate the transition, since this contribution is multiplied by $s_1(s_2^2 + s_2s_3) \cdot m_t^2$, to be compared to $s_1 m_c^2$, i.e., with $s_3 \approx 0$, $s_2 \approx s_1$, $m_t = 30$ GeV, a factor 16 bigger.

Following ref. 14, we obtain the branching ratio of $K \rightarrow \pi h$ by introducing the semi-leptonic $K_{\ell 3}$ decay constant f_+ :

$$BR(K^+ \rightarrow \pi h) = \kappa \cdot \left\{ [xA_1(m_c) + x^3 A_2(m_c)] + \frac{m_t^2}{m_c^2} (s_2^2 + s_2s_3) [xA_1(m_t) + x^3 A_2(m_t)] \right\}^2 \tag{11}$$

$$\kappa = \frac{s_1^2 G^3}{256 \cdot 16 \cdot \pi^5 \cdot \sqrt{2}} \cdot m_K^3 m_c^4 f_+^2(0) \cdot \frac{1.24 \cdot 10^{-8}}{6.6 \cdot 10^{-25}} = 0.78 \cdot 10^{-6} \tag{12}$$

(the masses being expressed in GeV.)

It appears quite clearly from fig. 1 that for most values of M_H both the contributions of c and t would separately prove bigger than the experimental bound. Unless m_H is in the interval (12-20 GeV) the c contribution cannot be made to vanish, even by fitting x , since for $M_H < 12$ GeV, A_1 and A_2 are of the same sign, while for $M_H > 20$ GeV, A_2 is too small. For those values of

M_H however, the t contribution is big and only a conspiracy between s_2 and s_3 might suppress it. A possibility of compensation of the amplitudes (including the previously estimated contributions from $\pi, \eta \dots$ mixing) however exists for m_H around 50-120 GeV, since the two "hard" terms could be of same importance in that area.

The situation stays thus unconclusive (and the axion just appears a little more unlikely to exist).

A cleaner test could be found by considering B decays, since the c contribution is negligible in front of the t -quark one. We obtain for the branching ratio:

$$\text{B.R.}(B \rightarrow Xh) \approx \frac{G^3 m_b^3 m_t^4}{256 \cdot 16 \cdot \pi^5 \cdot \sqrt{2}} \frac{[xA_1(m_t) + x^3 A_2(m_t)]^2}{\frac{N \cdot G^2}{192 \pi^3} m_b^5} \quad (13)$$

where N is the number of open channels for $(b \rightarrow c + X)$. This branching ratio could be sizeable, for instance, for $(m_t = 30 \text{ GeV}, M_H = 15 \text{ GeV}, x = 1)$ we get $\text{BR}(B \rightarrow K h) \sim 2\%$.

We thus conclude that, depending on the value of the physical charged Higgs mass, the axion might stay an elusive particle for quite a while.

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When this work was completed we received a preprint by L.J. Hall and M.B. Wise, where the same calculation was performed (Harvard Preprint HUTP 81/A010), with a slightly different technique. Those authors also argue that strong corrections would reduce the c contribution by a factor of 3. This however does not affect our conclusions.

REFERENCES

- 1) S. Weinberg, Phys. Rev. Letters 40 (1978) 223.
- 2) F. Wilczek, Phys. Rev. Letters 40 (1978) 279.
- 3) R.D. Peccei, H.R. Quinn, Phys. Rev. D16 (1977) 1791.
- 4) H. Faissner, Search for axion decays, report to the International Neutrino Conference at Erice (1980), Aachen preprint PITHA 81/03.
- 5) W. Bardeen and S.-H.H. Tye, Phys. Letters 74B (1978) 229.
- 6) T. Goldman and C.M. Hoffman, Phys. Rev. Letters 40 (1978) 279.
- 7) J. Kandaswamy, P. Salomonson, J. Schechter, Phys. Letters 74B (1978) 377.
- 8) T.W. Donnelly, S.J. Freedman, R.S. Lytel, R.D. Peccei and M. Schwartz, Phys. Rev. D18 (1978) 1607.
- 9) P. Coteus et al., Phys. Rev. Letters 42 (1979) 1438;
A. Soukas et al., Phys. Rev. Letters 44 (1980) 564;
P. Alibrant et al., Phys. Letters 74B (1978) 134;
P.C. Bossetti et al., Phys. Letters 74B (1978) 143;
T. Hansl et al., Phys. Letters 79B (1978) 139.
- 10) J. Ellis and M.K. Gaillard, Phys. Letters 74B (1978) 374.
- 11) W. Bardeen, S.-H.H. Tye and J.A.M. Vermaseren, Phys. Letters 76B (1978) 580.
- 12) D.J. Bechis et al., Phys. Rev. Letters 42 (1979) 1511.
- 13) G. Carboni, A method to measure the axion mass, CERN preprint EP/81/20.
- 14) M.B. Wise, Radiatively induced flavor changing neutral Higgs boson couplings, Harvard preprint HUTP 80/A086.

FIGURE CAPTIONS

Fig. 1 W^+ propagator

- unphysical charged scalar (ghost) (massless in Landau gauge)
- { physical massive charged scalar (internal lines)
axion (external lines)
- fermions. 1, 2, 3 are quark labels, 3 being the (heavy) internal quark.

Fig. 2 A_1 and A_2 , as defined in formulas (9),(10), in fonction of the physical charged Higgs mass.

1.1

1.2

1.3

1.10

1.11

1.4

1.5

1.6

1.12

1.13

1.7

1.8

1.9

FIG.1

FIG. 2