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DYNAMICAL GAUGE BOSONS FROM FUNDAMENTAL FERMIONS

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ABSTRACT

In a class of four dimensional, non-linear fermionic Lagrangians, a classical gauge symmetry induces dynamical gauge fields at the quantum level. The construction requires flavour (or families) as well as the 'unification' of all gauge couplings at the common Landau pole mass.

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1. Fermions are an essential constituent in nature. It is natural to ask if they could be the only fundamental constituents. For instance, could gauge fields be dynamically generated from fermions? It is the purpose of this note to pose this question and show explicitly how this appealing suggestion might be realized.

There already exist attempts to convert a global symmetry of a classical Lagrangian into a local symmetry of the quantum theory, with the generation of the corresponding vector bosons¹⁾. The success of this approach, which needs to circumvent a no-go theorem²⁾, is an open question³⁾. Alternatively, it has been possible to generate massless composite vectors starting from a classical Lagrangian with local invariance, but no explicit gauge field. We refer here to the two dimensional CP^{n-1} models⁴⁾ and their non-Abelian generalizations⁵⁾. Following this second line of approach, we discuss in this note a set of four dimensional Lagrangian models which produce dynamical massless vectors as bound states of constituent fermions. These vectors act as gauge fields of an effective standard gauge Lagrangian, equivalent to the original one at small energies relative to an ultra-violet cut-off Λ .

The CP^{n-1} models, being two dimensional, are renormalizable in spite of their high degree of non-linearity. In four dimensions we will be able to give an unambiguous meaning to our Lagrangian only by introducing the cut-off Λ , which cannot be removed, at least with our present understanding of the theory. Indeed, the absence of any dimensional parameter might ultimately result in the theory being well behaved in the ultra-violet, as has been suggested in Ref. 6) for a similar case with scalar fields. In any case, the point is that in order to establish the presence of a dynamical gauge boson, renormalizability in a conventional sense, is not crucial if a "decoupling"⁷⁾ occurs between the massless particles and an uncontrollable spectrum at Λ .

In this note, we shall successively discuss a QED like model, a non-Abelian model, an Abelian Goldstone model and finally an Abelian Higgs-like model. A characteristic of the broken chiral $U(1)$ case is a relation between the mass of the gauge boson, that of the constituent fermion and their $U(1)$ charges. In the non-Abelian case, the actual construction requires two general conditions: i) flavour (or families), since the theory ultimately has to be infra-red free; ii) for a non-simple group, a "unification" of all gauge couplings has to occur at the common Landau pole mass of the generated effective theory.

2. Let us consider the formal Lagrangian

$$\mathcal{L} = i \bar{\psi} \not{\partial} \psi - i \bar{\psi} \gamma_{\mu} \psi \frac{\bar{\psi} \overleftrightarrow{\partial}_{\mu} \psi}{\bar{\psi} \psi} \quad (1)$$

where ψ is a Dirac field. $\mathcal{L}$ is invariant under a local $U(1)$ gauge transformation, $\psi(x) \rightarrow e^{i\alpha(x)} \psi(x)$, which cannot be eliminated in a Lorentz covariant manner^{*)}. It is also scale invariant and possesses no free, dimensionless parameter.

We are unable to handle (1) in its present form. Instead, we take as our starting point the theory defined by the generating functional^{**)}

$$Z[j] = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu \mathcal{D}\lambda_\mu \mathcal{D}\lambda \exp \left\{ - \int d^4x \left[\bar{\psi} (\not{\partial} + iA) \psi + i\lambda (\bar{\psi}\psi - v) + i\lambda_\mu (vA_\mu - i\bar{\psi}\not{\partial}_\mu\psi) + \bar{j} \cdot \psi + \bar{\psi} \cdot j \right] \right\} \quad (2)$$

After elimination of the auxiliary A_μ , λ_μ and λ fields, the Lagrangian in (2) reduces to that of (1), supplemented with a constraint $\bar{\psi}\psi = v$.

Instead of a constraint^{***)}, the vacuum expectation value (v.e.v.) for $\bar{\psi}\psi$ could have been enforced through a potential without modifying the conclusions of this paper. The theory, as defined by (2), requires an ultra-violet regularizing scale, Λ , to be regarded as an independent parameter. The cut-off procedure must respect gauge invariance; we use Pauli-Villars regulators.

Integrating over fermions in (2) yields

$$Z[j] = \int \mathcal{D}A_\mu \mathcal{D}\lambda_\mu \mathcal{D}\lambda \exp \left\{ - \Gamma - \int d^4x \bar{j} [\not{\partial} + \lambda_\mu \not{\partial}_\mu + i\lambda] j \right\} \quad (3)$$

where

$$\Gamma = \int d^4x \left\{ i\lambda_\mu A_\mu - i\lambda v \right\} - \text{tr} \ln [\not{\partial} + \lambda_\mu \not{\partial}_\mu + i\lambda]$$

*) In the scalar CP^{n-1} model, on covariant elimination of a redundant degree of freedom, the local invariance of the Lagrangian disappears.

**) A Lagrangian of this type, but without the auxiliary field λ , λ_μ (hence with a vanishing current as a constraint) was advocated in Ref. 8) for the generation of gauge fields.

***) We are aware of conceptual difficulties that may be implied by the substitution of a Fermi-bilinear with a c number

We now do a saddle-point approximation in the remaining fields and find a Lorentz invariant solution

$$A_\mu = \lambda_\mu = 0, \quad i\lambda = m \quad (4)$$

at the minimum. m is determined from

$$V = -\text{tr} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{i\cancel{p} + m} = -4 \int \frac{d^4 p}{(2\pi)^4} \left[\frac{m}{p^2 + m^2} - \sum_i c_i \frac{\Lambda_i}{p^2 + \Lambda_i^2} \right] \quad (5)$$

$$\sim -\frac{1}{4\pi^2} m \Lambda^2 \quad \text{for } m \ll \Lambda_i \sim \Lambda$$

and is the generated fermion mass^{*)}.

Let us now consider the quadratic terms in λ , A_μ and λ_μ around the saddle-point solution of (4). These can be simply understood in terms of Feynman diagrams. The results are as follows. As a consequence of gauge invariance, there is no A_μ - λ mixing. In momentum space

$$\frac{\delta^2 \Gamma}{\delta \lambda \delta \lambda} = O(\Lambda^2) \quad (6)$$

for all momenta $q \ll \Lambda$. Thus, for such momenta, $\lambda(x)$ is a non-propagating field. The vector sector (A_μ, λ_μ) is more interesting and complicated. The quadratic term in A_μ is just the standard one-loop vacuum polarization diagram of QED (with $e=1$).

$$\frac{\delta^2 \Gamma}{\delta A_\mu \delta A_\nu} = (q_\mu q_\nu - q^2 g_{\mu\nu}) I(q^2, m^2, \Lambda^2) \quad (7)$$

*) In (5), chirality plays a crucial role, requiring m to be non-zero and proportional to v . A similar role is played by the infra-red divergence in the two dimensional CP^{n-1} models, which forces the Lagrange multiplier, analogous to λ , to acquire a non-vanishing v.e.v.^{4),9)} and yields restoration of the local symmetry. This would not be the case in more than two dimensions¹⁰⁾. More precisely, applying our ideas and, of course, our approximation scheme, to the bosonic model would not lead to a quantum mechanical realization of the gauge symmetry of the classical Lagrangian. No symmetry restoration takes place once it has been broken.

$$I(q^2, m^2, \Lambda^2) \xrightarrow{\Lambda^2 \gg q^2, m^2} \frac{1}{12\pi^2} \ln\left(\frac{\Lambda^2}{q^2 + 4m^2}\right) \quad (7) \text{ cont.}$$

However, for λ_μ we obtain

$$\frac{\delta^2 \Gamma}{\delta \lambda_\mu \delta \lambda_\nu} = g_{\mu\nu} O(\Lambda^4) + (q_\mu q_\nu - q^2 g_{\mu\nu}) O(\Lambda^2) \quad (8)$$

showing, upon rescaling $\tilde{\lambda}_\mu = \lambda_\mu \Lambda$, that it is a non-propagating field for $q \ll \Lambda$. Finally, the crucial $A_\mu, \tilde{\lambda}_\nu$ mixing, which could make A_μ massive, takes the harmless form

$$\frac{\delta^2 \Gamma}{\delta A_\mu \delta \tilde{\lambda}_\nu} = O\left(\frac{m}{\Lambda}\right) (q_\mu q_\nu - q^2 g_{\mu\nu}) \quad (9)$$

because of the cancellation, obtained after the use of (5), between the tree contribution, $-\frac{v}{\Lambda} g_{\mu\nu}$, and the one loop diagram. From Eqs. (7) - (9), the $A_\mu \tilde{\lambda}_\nu$ mixing is completely negligible and A_μ describes a rigorously massless photon in interaction with fermions of mass m .

We should proceed further with a systematic analysis of the exact generating functional (2) by considering higher derivatives in the various fields. From the results obtained for the quadratic part, and inspection of some relevant cases in higher order (i.e., considering the insertions of "heavy" fields λ, λ_μ) we are led to conclude that the exact action, with cut-off Λ , is equivalent to the standard QED Lagrangian at small momenta, q , with respect to Λ , with power law deviations in q/Λ . The only quantities for which the insertions of heavy fields give renormalization of $O(1)$ are the fermion mass and the two-point function of Eq. (7). This amounts to a decoupling between the "heavy" fields and the light ones, A_μ and ψ . These are protected from acquiring a mass $\approx \Lambda$ by gauge and chiral invariance respectively^{*)}. More precisely, by rescaling the A_μ field so that its kinetic field is the standard $-1/4 F_{\mu\nu} F^{\mu\nu}$, the renormalized charge in the effective QED Lagrangian at the one loop fermion level is

$$e_R \Big|_{q=0} = \left[\frac{1}{12\pi^2} \ln\left(\frac{\Lambda^2}{m_R^2}\right) + O(1) \right]^{-1} \quad (10)$$

*) In the action (2), chiral invariance is broken only by the presence of the parameter v .

where $O(1)$ includes the effect due to the virtual heavy fields. Analogously, for the fermion mass, one has $m_R = O(m)$. Equation (10) illustrates the identification of the cut-off Λ with the approximate position of the Landau pole for the effective QED Lagrangian.

At this point we note that, in the limit $v \rightarrow 0$, $m \rightarrow 0$ and we are left with massless QED. Except for having to define an off-shell renormalized charge, we see no sign of trouble in taking this limit, even though v was initially introduced to define the theory. The physics of this case is apparently the same as that obtained by our method from the Lagrangian

$$\mathcal{L} = i \bar{\Psi} \not{\partial} \Psi + i \lambda_\mu \bar{\Psi} \not{D}_\mu \Psi \quad (11)$$

which is formally equivalent to that of Eq. (1)

We can generalize our Lagrangian to include N_f flavours. By rescaling the fields we obtain the same fermion mass as before and a one loop renormalized charge

$$e_R^2 = \left[\frac{N_f}{12\pi^2} \ln \left(\frac{\Lambda^2}{m_R^2} \right) + O(1) \right]^{-1} \quad (12)$$

in agreement with the expected modification of the β function. The λ, λ_μ couplings to fermions are $O(\sqrt{1/N_f})$. Diagrams with λ, λ_μ insertions now have an extra $1/N_f$ suppression factor with respect to the previous discussion. Thus, even though we have not seen any problem for the $N_f = 1$ case, the $1/N_f$ expansion is a further justification of the saddle-point method we have used and of the subsequent λ, λ_μ decoupling we obtained.

Finally, we point out that the classical, gauge invariant Lagrangian (1) possesses no Noether current, since the standard construction leads to a trivial identity ($0=0$), as in the CP^{n-1} models. This is a consequence of only having an auxiliary gauge field to start with. The quantum mechanical generation of a kinetic term for A_μ leads to a non-trivial conserved current, which coincides with that of QED at low energies.

3. A generalization of our construction to the non-Abelian case is possible. A Lagrangian with local $U(n)$ invariance is

$$\mathcal{L} = i \bar{\Psi}_\alpha^i \not{\partial} \Psi_\alpha^i + \bar{\Psi}_\alpha^i \not{W}_{\alpha\beta} \Psi_\beta^i \quad (13)$$

where

$$W_\mu = \frac{i}{2} M_\mu M^{-1} + h.c.,$$

$$M_{\alpha\beta} = \psi_\beta^{iT} C^{-1} \psi_\alpha^i, \quad (M_\mu)_{\alpha\beta} = -\psi_\beta^{iT} C^{-1} i\partial_\mu \psi_\alpha^i$$

with the (left-handed) Weyl spinors, ψ_α^i , having a flavour index $i = 1, \dots, f$ and a gauged index $\alpha = 1, \dots, n$. C is the charge conjugation matrix. Indeed, one can check that (13) is invariant under

$$\psi_\alpha^i(x) \longrightarrow U_{\alpha\beta}(x) \psi_\beta^i(x)$$

with

$$U(x) U^\dagger(x) = U^\dagger(x) U(x) = \mathbb{1}$$

The case in which only a subgroup, G , of $U(n)$ is gauged can be obtained from (13) by the replacement

$$W_\mu = \sum_{a \in U(n)} \lambda^a \text{Tr}(W_\mu \lambda^a) \longrightarrow \sum_{b \in G} \lambda^b \text{Tr}(W_\mu \lambda^b)$$

where λ^a are the (properly normalized) generators of $U(n)$ in the fundamental representation and λ^b are the subset of the generators of $U(n)$ spanning the subgroup G .

In analogy to the Abelian case, a definite meaning can be given to (13) by going to the non-Abelian generalization of (11). For the general non-Abelian case, it is important that we are able to use this form. If we were to use the non-Abelian generalization of (2), the analogue of v would automatically break the general gauge group down to a vector like subgroup. Hence, we would be unable to describe $SU(2)_L$, at least using the standard fermions as constituents. However, we could use this construction for $SU(3)_C \times U(1)_{e.m.}$.

Assuming that the cut-off procedure^{*)} can be extended to the non-Abelian case, we are able to establish i) the dynamical generation of the massless gauge bosons; ii) interactions between the gauge bosons and their interactions with the massless fermions ψ_α^i , in a way indistinguishable from the standard gauge theory, at low energies. We thus obtain an effective running coupling constant of $O(1)$ at Λ . Only if the theory is infra-red free will the coupling constant decrease with decreasing q^2 , and thus define a meaningful theory at low energies ($q^2 \ll \Lambda$). This requires the number of flavours f to be large enough so that the fermion contribution to the β function dominates over that of the gluons. Furthermore, for non-simple groups, the different couplings must be unified at Λ , which plays the role of a universal Landau pole for the effective gauge couplings. Therefore, the couplings are related at low energies by the corresponding β coefficients. The picture stemming from our construction is thus very similar to the one proposed in Ref. 11) on the basis of different motivations.

4. A realistic model necessitates the treatment of broken gauge theories. As a first example, we consider a broken Abelian case. We will comment on the intrinsic difficulties of the analogous non-Abelian problem.

Consider the Lagrangian

$$\mathcal{L} = i \bar{\psi} \not{\partial} \psi + i \bar{\chi} \not{\partial} \chi - \left(\frac{e_1 \bar{\psi} \gamma_\mu \psi + e_2 \bar{\chi} \gamma_\mu \chi}{e_1 + e_2} \right) \times \quad (14)$$

$$\times \left(\frac{\bar{\psi} i \partial_\mu \chi}{\bar{\psi} \chi} + \frac{\bar{\chi} i \partial_\mu \psi}{\bar{\chi} \psi} \right) + \lambda (\bar{\psi} \chi \bar{\chi} \psi - v^2)$$

where ψ and χ are two Weyl spinors, one left, one right. Equation (14) has a local $U(1)$ symmetry $\psi \rightarrow e^{ie_1 \alpha(x)} \psi$, $\chi \rightarrow e^{ie_2 \alpha(x)} \chi$ and a global $U(1)$ invariance $\psi \rightarrow e^{ic} \psi$, $\chi \rightarrow e^{-ic} \chi$. When $\bar{\psi} \chi$ and $\bar{\chi} \psi$ acquire v.e.v., two possible breaking patterns arise:-

$$U(1)_{\text{local}} \times U(1)_{\text{global}} \supset U(1)_{\text{local}} \quad \text{if } e_1 = e_2$$

$$\supset U(1)'_{\text{global}} \quad \text{if } e_1 \neq e_2$$

where $U(1)'_{\text{global}}$ is a combination of the original $U(1)_{\text{local}}$ and $U(1)_{\text{global}}$. We treat the two cases simultaneously with the treatment described previously.

^{*)} There seems to be no problem for a vector like theory, e.g., $SU(3)$. However, a problem will arise for the physical group $SU(3) \times SU(2)_L \times U(1)_C$ due to the lack of a gauge invariant regularization procedure for theories with fermions in complex representations. It is possible that our construction requires a doubling of the usual fermions, as also advocated for conventional gauge theories¹²⁾.

To solve (14) we must make it quadratic in ψ and χ . Consequently, we introduce two complex scalar fields in the combinations

$$\eta (\bar{\psi}\chi - \phi^*) + \eta^* (\bar{\chi}\psi - \phi)$$

After integration over fermions and minimization we fix an arbitrary phase so that $\phi = \phi^* = v$ at the minimum. We then obtain the quadratic terms.

For $e_1 = e_2$, the gauge invariance is unbroken and the quadratic term for A_μ is as in (7). However, we now have a matrix for the scalar fields (η, ϕ, λ) . This has a determinant proportional to q^2 and, on diagonalization, yields an eigenvector $(\eta - \eta^* + \phi - \phi^* \equiv \pi')$ with eigenvalue proportional to q^2 , whilst all other eigenvectors have eigenvalues of $O(\Lambda^2)$. Besides the fermions, we are left with a massless 'pion' and a massless 'photon' in the light sector.

The identification of the 'pion' as the Goldstone boson of chiral symmetry breaking can be made manifest if we identify ψ and χ as the left and right components of a Dirac field. The broken $U(1)_{\text{global}}$ is then just chiral symmetry. If a small, explicit chiral breaking term, of the form $\epsilon(\bar{\psi}\chi + \bar{\chi}\psi)$, is added to (14) then the pion acquires a mass

$$m_\pi^2 = -\frac{2\epsilon v}{F_\pi^2} + O(\epsilon^2) \quad (15)$$

$$F_\pi^2 = m^2 \frac{1}{8\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right) = \frac{3}{2} \frac{m^2}{e_R^2} \quad (16)$$

in agreement with current algebra.

For $e_1 \neq e_2$, the quadratic term for A_μ of (7) is modified to

$$\begin{aligned} & \frac{(e_1^2 + e_2^2)}{12\pi^2} (q_\mu q_\nu - q^2 g_{\mu\nu}) \ln\left(\frac{\Lambda^2}{q^2 + 4m^2}\right) \\ & + \frac{1}{4\pi^2} g_{\mu\nu} m^2 (e_1 - e_2)^2 \ln\left(\frac{\Lambda^2}{q^2 + 4m^2}\right) \end{aligned} \quad (17)$$

There is also a coupling between the pion and A_μ of the form $(e_1 - e_2) i \partial_\mu A_\mu \pi$. On diagonalization of the matrix of quadratic terms, a massive vector emerges. The combination $(e_1 - e_2) \pi - i \partial_\mu A_\mu$ does not appear, whilst other scalars get a mass of $O(\Lambda)$ as before. This is just the Higgs phenomenon. The pion has given rise to the longitudinal part of the vector. The mass of the gauge boson is

$$m_W^2 = 3 m^2 \frac{(e_1 - e_2)^2}{(e_1^2 + e_2^2)} \left[1 + O\left(\frac{1}{e_1} \left(\frac{\Lambda^2}{m^2}\right)\right) \right] \quad (18)$$

Comparing with (10) and (16) we see the standard proportionality, $m_W \sim e F_\pi$ emerging. In this example, we do not find a light Higgs scalar. This could have been expected from the absence of any symmetry protecting the prospective Higgs scalar from acquiring a mass of $O(\Lambda)$, i.e., naturality. However, for this Abelian example, a light Higgs is not required for renormalizability.

When extending our scheme to the broken non-Abelian case, we will inherit the conventional difficulties in generating (naturally) a light Higgs particle. Of course, we could introduce fundamental Higgs scalars or, alternatively, combine our scheme with that of technicolour. We are currently exploring these possibilities in order to discuss realistic models in the context of our Landau pole unification.

To summarize, we have shown the appealing possibility that gauge bosons can be composites of elementary fermions. As a consequence, an unconventional unification picture is obtained, in which couplings eventually grow in the ultra-violet towards a value of order unity at a common scale Λ . This theory, which is equivalent to a standard gauge theory for $E \ll \Lambda$, leads us to expect the appearance of further families¹¹⁾.

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