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GRAPHICAL MASS FACTORIZATION

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ABSTRACT

We point to the close analogy between (multiplicative) BPHZ renormalization and mass factorization. Adaptation of the forest formula to mass singular graphs allows an alternative proof of mass factorization. A diagrammatic method is developed to carry out diagram-by-diagram mass factorization with the mass singularities being subtracted by counter terms which built up the operator matrix element. The reasoning is exposed for deep inelastic (DI) scattering and for the Drell-Yan (DY) process.

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Several ways to prove mass factorization have been suggested¹⁾, most of them following a reasoning similar to the Dyson-Salam²⁾ proof of (multiplicative) renormalization. In this line of thought the authors of Ref. 3) recently pointed to (and expanded on) the analogy between renormalization and mass factorization. In this paper we develop the mass factorization analogue to BPHZ renormalization⁴⁾. First the analogy is exposed, then the necessary terminology is introduced and finally the forest formula for mass singular graphs is formulated in order to prove mass factorization. The reasoning is exemplified in the DI/DY processes. Mass factorization is carried out for each diagram permitting a pictographical presentation. Our method permits elegant determination of:

- i) any cut-graph (Feynman diagram with unitarity cut) in terms of the Callan-Symanzik (CS) expansion coefficients,
- ii) the Wilson coefficient function in DI scattering,
- iii) its analogue in the DY process, and
- iv) the form of the DY correction term.

We limit ourselves initially to an AF field theory without spin and colour (as ϕ_6^3) and generalize later on.

We develop the analogy between BPHZ renormalization and mass factorization by considering a 1PI truncated unrenormalized Green's function $\Gamma(p, M, g(M))$ with external momentum p , regulator mass M and bare coupling constant $g(M)$. Multiplicative renormalization, changing M to the renormalization point μ ,

$$\Gamma_R(p, \mu, g(\mu)) = \bar{Z}^{-1}(M, \mu, g(\mu)) \Gamma(p, M, g(M)) \quad (1)$$

involves two steps: coupling constant renormalization and wave function renormalization. Coupling constant renormalization substitutes

$$g^2(M) = g^2(\mu) \left[1 + g^2(\mu) b_1 \ln \frac{M^2}{\mu^2} + \dots \right] \quad (2)$$

into the right-hand side of Eq. (1) leaving

$$\Gamma_R(p, \mu, g(\mu)) = \bar{Z}^{-1}(M, \mu, g(\mu)) \tilde{\Gamma}(p, M, \mu, g(\mu)) \quad (3)$$

Now we deal exclusively with multiplicative Green's function renormalization where all remaining M dependence is absorbed in the wave function renormalization constant Z^{-1} . This step removes all products of singular times finite terms as well as all remaining UV singularities. Following the BPHZ scheme, this procedure allows a graph-by-graph diagrammatic presentation which is well known. Comparing Eq. (3) with the mass factorization in DI scattering

$$\mathcal{C}(q, \mu, g(\mu)) = \bar{A}^{-1}(p, \mu, g(\mu)) \cdot \sigma(p, q, \mu, g(\mu)) \quad (4)$$

we observe the resemblance between multiplicative renormalization of the Green's function $\tilde{\Gamma}$ in Eq. (3) and mass factorization of the DI cross-section σ in Eq. (4). All quantities in Eq. (4) are moments and any further reference to cross-sections, operator matrix elements, etc., shall always be understood as their moments. The role of the UV singularity M is taken over by the mass singularity p^2 , Z^{-1} is replaced by the inverse of the operator matrix element A^{-1} and the renormalized Green's function Γ_R is replaced by the Wilson coefficient $\mathcal{C}$. All diagrams contributing to A^{-1} serve as counter terms for the DI cross-section σ , A^{-1} absorbs all mass singularities connected with the initial state and its form is unchanged if different processes are considered (universality of mass singularities). Note, mass factorization proceeds in one step only; there is no mass analogue to coupling constant renormalization.

Analysis of the cut-graphs on their M singular characteristics reveals in ϕ_6^3 the simple rule: a graph is only mass singular if there is a pair of equal propagators, and every additional pair introduces new mass singularities which appear via higher powers in $\ln p^2$ (ld). In analogy to the terminology used in BPHZ renormalization, we classify all cut-graphs according to their mass factorization characteristics. We distinguish:

- a) mass singular graphs contributing new counter terms which have
 - a₁) primitive divergences only
 - a₂) no internal self-energy and strong vertex corrections
 - a₃) internal self-energies and strong vertices
- b) mass finite graphs
- c) mass singular graphs contributing no new counter terms, meaning that one of the subgraphs belongs to class b).

The a₁ graphs contain only one pair of equal propagators with $\overset{p^2 \rightarrow 0}{\sim} \ln p^2$. The a₂ graphs with n loops (n rungs) go as $\overset{p^2 \rightarrow 0}{\sim} (\ln p^2)^n$. From dimensional arguments these cut-graphs reveal $(\ln q^2/p^2)^k$ dependence which results in the

"overlapping divergences"⁵⁾ $\ln^i (q^2/p^2) \cdot \ln^j (\mu^2/p^2)$. The insertion of self-energies and strong vertices in a_1, a_2 graphs does not introduce additional overlapping divergences. The a_3 graphs behave as $\overset{p^2 \rightarrow 0}{\sim} (\ln p^2)^{n+i}$ where n refers to the a_1, a_2 graphs (see earlier) and the set of self-energy/vertex insertions contributing $\overset{p^2 \rightarrow 0}{\sim} (\ln p^2)^i$.

Following closely the work of Zimmermann on BPHZ renormalization^{4b)} we present in the following the (analogous) notions and arguments for a proof of mass factorization. We limit ourselves here to DI scattering and to the class a_1 graphs; the extensions are indicated later. Let us consider graphs of class a_1 . $\sigma_\Gamma(p^2, q^2)$ represents the cross-section of a specific graph Γ of order $(g^2)^n$. Its mass subtraction term follows from a Taylor expansion around $q^2 = \mu^2$, in formal notation $(-t^\Gamma)\sigma_\Gamma$, which is identified with A_Γ^{-1} [see Eq. (4)]. Since all mass divergences are logarithmic, only the lowest order term in the Taylor expansion is needed. The operator matrix element A_Γ follows from graph Γ by a contraction of two electromagnetic vertices to one point. It is also of order $(g^2)^n$ and primitively divergent, which allows the relation $A_\Gamma^{-1} = -A_\Gamma$. As a result, the mass factorized Γ cross-section reads

$$\mathcal{C}_\Gamma = (1 - t^\Gamma) \sigma_\Gamma \quad (5)$$

We consider graphs of classes a_2, a_3 . First some definitions. Imagine a graph Γ being broken into a set of mass singular subgraphs $\{\gamma_1 \dots \gamma_c\}$ (Fig. 1a). They are disjoint (meaning $\gamma_i \cap \gamma_j = \emptyset, i \neq j, \gamma_i \neq \Gamma$), connected and 1PI. In analogy to Ref. 4b) we call them mass factorization parts. A reduced diagram $F \equiv \Gamma / \{\gamma_1 \dots \gamma_c\}$ is defined by contracting each γ_i to a point. The classes a_2, a_3 give rise to overlapping divergences (in the sense of 't Hooft-Veltman⁵⁾) which are removed by Bogoliubov's R operation. The reduced cross-section $\bar{\sigma}_\Gamma$ has all overlapping divergences removed; it is recursively defined by

$$\bar{\sigma}_\Gamma = \sigma_\Gamma + \sum_F \sigma_F \overset{c}{\pi} (-t^{\gamma_c}) \bar{\sigma}_{\gamma_c} \quad (R \text{ operation}) \quad (6)$$

(see Fig. 1b). The sum extends over all sets $\{\gamma_1 \dots \gamma_c\}$ of mass factorization parts of Γ . The final over-all subtraction is carried out via $(-t^\Gamma)\bar{\sigma}_\Gamma = A_\Gamma^{-1}$ with $(-t^\Gamma)\sigma_\Gamma = -A_\Gamma$ (Fig. 1c). Note that the inverse of the operator matrix element leads to combinations of lower order diagrams. $\bar{\sigma}_\Gamma$ is substituted into

$$\mathcal{C}_\Gamma = (1-t^\Gamma) \bar{\sigma}_\Gamma \quad (7)$$

(see Fig. 1d) leading finally to the Wilson coefficient which is cast in the forest formula

$$\mathcal{C}_\Gamma = \sum_u \pi_{\lambda \in u} (-t^\gamma) \sigma_\Gamma \quad (8)$$

The sum extends over all possible Γ forests of U . U is a set of non-overlapping mass factorization parts of Γ which includes Γ and ϕ . $(-t^\lambda) \sigma_\Gamma$ removes the λ subgraph from Γ and replaces it by the counter term $A_\lambda^{-1} \equiv (-t^\lambda) \bar{\sigma}_\lambda$. Following Ref. 4b) we define the BPHZ overlapping divergences. Two mass factorization parts are said to overlap if none of the three following relations holds true: $\gamma_1 \subset \gamma_2$, $\gamma_2 \subset \gamma_1$, $\gamma_1 \cap \gamma_2 = \emptyset$. The absence of any BPHZ overlapping divergences (as for instance in DI scattering) allows reformulation of Eq. (8) as

$$\mathcal{C}_\Gamma = \pi_\gamma (1-t^\gamma) \sigma_\Gamma \quad (9)$$

with γ representing all mass factorization parts of (and including) Γ . For every cut-graph Γ , Eq. (9) therefore allows graphical specification of its corresponding contribution to the Wilson coefficient.

The proof of mass factorization makes use of the subgraph ordering: $\gamma_1 \subset \gamma_2 \subset \dots \subset \gamma_n \equiv \Gamma$ where $\gamma_i = O((g^2)^i)$. From Eqs (6), (7) and (9) we derive

$$\sigma_\Gamma = \mathcal{C}_\Gamma - \sum_{\gamma_i} \sigma_{\Gamma/\gamma_i} (-t^{\gamma_i}) \bar{\sigma}_{\gamma_i} - (-t^\Gamma) \bar{\sigma}_\Gamma \quad (10)$$

The first term on the right-hand side is mass finite, the second term contains all overlapping divergences $\ln^i(q^2/p^2) \ln^j(\mu^2/p^2)$ and the last one behaves as $\ln^i(\mu^2/p^2)$. We go back to Eq. (4) where the mass singularities of the DI cross-section are expected to be cancelled by the mass singular function A^{-1} . The cross-section form and the ansatz for A^{-1} are

$$\sigma = \sum_{n=0}^{\infty} (g^2)^n \sum_{\Gamma} \sigma_{\Gamma}^{(n)}, \quad A^{-1} = \sum_{m=0}^{\infty} (g^2)^m \sum_{\Gamma} \{ A_{\Gamma}^{(m)} \}^{-1} \quad (11)$$

The $(g^2)^n$ term of $A^{-1} \cdot \sigma$ has the form

$$\sum_{\Gamma} \left[\sigma_{\Gamma}^{(n)} + \sum_{i=1}^n \{ A_{\Gamma}^{(i)} \}^{-1} \cdot \sigma_{\Gamma}^{(n-i)} \right] \quad (12)$$

where $\sigma_{\Gamma}^{(n)}$ is replaced by Eq. (10). Identifying subsequently

$$A_{\Gamma}^{(i)-1} \equiv (-t^{\gamma_i}) \bar{\sigma}_{\gamma_i}, \quad \sigma_{\Gamma/\gamma_i} \equiv \sigma_{\Gamma}^{(n-i)} \quad (13)$$

Eq. (7) is immediately seen to be mass finite.

We sketch the analogous reasoning for the DY process. Its parton cross-section in mass factorized form reads

$$W(p_1, p_2, Q) = \sigma(p_1, Q) \cdot \sigma(p_2, Q) \cdot \Delta(1, \bar{g}) \quad (14)$$

where σ stands for the DI parton cross-section, and the DY correction term $\Delta(1, \bar{g})$ is expanded in $\bar{g}^2$ (and for later purposes in g^2) giving

$$\Delta = [1 + \bar{g}^2 \Delta_1 + \bar{g}^4 \Delta_2 + \dots] = [1 + g^2 \Delta_1 + g^4 (\Delta_2 + b_1 \Delta_1 \ln \frac{Q^2}{\mu^2}) + \dots] \quad (15)$$

Mass factorization of the DY cross-section implies that the DY analogue of the Wilson coefficient function

$$C^{DY}(Q, p, g) = [A(p_1, p, g) \cdot A(p_2, p, g)]^{-1} \cdot W(p_1, p_2, Q) \quad (16)$$

is mass finite, and the universality of mass factorization identifies the $A(p, \mu, g)$ [in Eq. (16)] with the DI operator matrix elements.

We point to the new aspects of this process. $W(p_1, p_2, Q)$ reveals mass singularities from both incoming lines which can be traced back to the 2PR graphs^{1d)}. As in DI scattering each DY cut graph Γ is broken into disjoint mass factorization parts $\gamma_\tau \subset \Gamma$ (Fig. 2a). Let Γ_i ($i = 1, 2$) be the maximal mass factorization part containing the mass singularities of one incoming state p_i only. It reveals the topology of the DI cut-graphs and therefore the same mass singularities are found. This is the deeper reason why the operator matrix elements can be used as counter terms for the DY graphs. Contrary to DI scattering, the DY graphs give rise to BPHZ overlapping divergences; Γ_1 and Γ_2 in particular are overlapping.

Mass factorization proceeds in an analogous way as in DI scattering (Fig. 2b,c). The DY coefficient function is derived from Eqs (6), (7) and (8). Some manipulations as in DI scattering lead us to the DY generalized forest formula

$$\mathcal{C}_\Gamma^{\text{DY}} = \left[\prod_{\delta \in \Gamma_1} \pi(1-t^\delta) \right] \left[\prod_{\lambda \in \Gamma_2} \pi(1-t^\lambda) \right] \sigma_\Gamma^{\text{DY}} \quad (17)$$

(see Fig. 2d). The product extends over all mass factorization parts of Γ which belong either to Γ_1 or Γ_2 ; in this example γ_2 therefore does not appear.

The proof of mass factorization follows the DI line of reasoning with the DY cross-section defined as in Eq. (11) and the ansatz

$$[A(p_1)A(p_2)]^{-1} = \sum_{m=0}^{\infty} (g^2)^m \sum_{k=0}^m \sum_{\gamma, \lambda} \{ A_\gamma^{(k)} A_\lambda^{(m-k)} \}^{-1} \quad (18)$$

The $(g^2)^n$ term of $(A_1 A_2)^{-1} \sigma$ reads

$$\sum_\Gamma \left[\sigma_\Gamma^{(n)} + \sum_{m=1}^n \sum_{k=0}^m \sum_{\gamma, \lambda} \{ A_\gamma^{(k)} A_\lambda^{(m-k)} \}^{-1} \cdot \sigma_\Gamma^{(n-m)} \right] \quad (19)$$

$\gamma \subset \Gamma_1, \lambda \subset \Gamma_2$

Using Eqs (6), (7) and (17) we replace the DY cross-section by

$$\sigma_\Gamma^{(n)} = \mathcal{C}_\Gamma^{(n)} - \sum_{\gamma_m \subset \Gamma} \sigma_{\Gamma/\gamma_m}(-t^{\gamma_m}) \bar{\sigma}_{\gamma_m} - (-t^\Gamma) \bar{\sigma}_\Gamma^{(n)} \quad (20)$$

Upon identification $\sigma_{\Gamma/\gamma_m}^{(n)} \equiv \sigma_{\Gamma}^{(n-m)}$ and

$$(-t^{\Gamma}) \bar{\sigma}_{\Gamma}^{(n)} \equiv \left\{ A_{\Gamma_1}^{(k)} A_{\Gamma_2}^{(n-k)} \right\}^{-1} \Big|_{\Gamma_1, \Gamma_2 \subset \Gamma}, \quad (-t^{\gamma_m}) \bar{\sigma}_{\gamma_m}^{(n)} \equiv \left\{ A_{\Gamma_1}^{(k)} A_{\Gamma_2}^{(n-k)} \right\}^{-1} \Big|_{\Gamma_1, \Gamma_2 \subset \gamma_m} \quad (21)$$

Eq. (19) is found to be mass finite. Note that $(-t^{\Gamma}) \sigma_{\Gamma}^{(n)} = -A_{\Gamma_1}^{(k)} A_{\Gamma_2}^{(n-k)} \Big|_{\Gamma_1, \Gamma_2 \subset \Gamma}$.

The above exposed subtraction technique can also be used to determine the DY correction term since

$$\Delta(1, \bar{g}) = \left[\sigma(p_1, q) \cdot \sigma(p_2, q) \right]^T W(p_1, p_2, q) \quad (22)$$

Comparing with Eq. (16) we realize that the DI cross-section moments serve as counter terms for the DY process. The diagrammatic rules developed for that case apply here as well once the operator graphs are replaced by the DI cut graphs. Seeking explicit evaluation of the expansion coefficients Δ_i [see Eq. (2.2)] we first carry out BPHZ subtraction in the usual way and subsequently set $Q^2 = \mu^2$. The second step removes from Eq. (15) the term $b_1 \Delta_1 \ln(Q^2/\mu^2)$ and reduces $\bar{g}(Q^2) \rightarrow g$. In Fig. 3 we give examples and illustrate our reasoning on graphs.

We discuss the changes if operator mixing (e.g., in QCD) is admitted. The parton cross-section and the Wilson coefficient function will be vectors and the operator matrix element will be a matrix as

$$\vec{C}^{(n)} = \begin{pmatrix} C_q^{(n)} \\ C_g^{(n)} \end{pmatrix}, \quad \mathbb{A}^{(n)} = \begin{pmatrix} A_{qq}^{(n)} & A_{qg}^{(n)} \\ A_{gq}^{(n)} & A_{gg}^{(n)} \end{pmatrix} \quad (23)$$

In DI scattering they are related by

$$\vec{\sigma}^{(n)} = \mathbb{A}^{(n)} \cdot \vec{C}^{(n)} \quad (24)$$

Aiming at mass factorization in the above sketched way the matrix elements of the inverse matrix $\mathbb{M}^{-1}$ serve as counter terms for $\vec{\sigma}^{(n)}$. All the earlier arguments apply in an analogous though slightly more complicated way.

Whilst generalizing these ideas to gauge theories we come across the IR singularities. Their complete cancellation is necessary if mass factorization is to be carried out. Considering the integrated parton cross-sections, all final state associated M and IR singularities disappear as a consequence of the Kinoshita-Lee-Nauenberg theorem⁶⁾ (or unitarity). In QED, cancellation of the initial state IR singularities is guaranteed by the Bloch-Nordsieck theorem⁷⁾. Limiting the initial state to one coloured particle, this theorem extends to QCD⁸⁾. However, it no longer applies if two coloured particles are admitted, since the order g^4 IR singularities no longer cancel⁹⁾. Fortunately, as shown by the authors of Ref. 9), they manifest themselves in the higher twist contributions which, in the Bj limit are suppressed.

We comment on the gauge choice. Although our reasoning was presented for non-gauge theories it can immediately be carried over to QCD (or QED) provided the axial gauge is chosen. The mass singularities manifest themselves here in the same way as in ϕ_6^3 field theory.

Choosing a different gauge (e.g., Feynman gauge), the above simple rule no longer applies. Mass singularities appear in graphs which in the axial gauge are M finite. They are removed by the corresponding operator matrix elements which are calculated in the same gauge.

We point to the value of the above insight and first list the points of principle:

- a) We start from an equivalence between BPHZ renormalization and mass factorization [see Eqs (3) and (4)].
- b) Mass factorization is proven in two steps: the ('t Hooft-Veltman) overlapping divergences are removed by the R operation and the over-all subtraction is carried out by the forest formula.
- c) The graph-by-graph subtraction procedure is exposed by illustrative examples.

Apart from its (solid) proof by an alternative method, mass factorization can be presented pictographically as shown in Figs 1d, 2d. Our method permits elegant determination of: (i) all log coefficients of an order $(g^2)^n$ cut graph once the lower order graphs are fully known and the order $(g^2)^n$ operator matrix elements are specified up to the logarithmic terms, (ii) the DY coefficients function $\Delta(1, \bar{g})$ giving information on its $\ln^i(1-\tau)$ expansion, and (iii) the determination of the DI Wilson coefficient and its DY analogue in a transparent way. Point (i) is limited to the logarithmic singularities, in particular

the mass singularities which appear exclusively in the 2PR graphs^{1d)}. Therefore, all 2PI graphs have to be evaluated by a direct computation of the cut graphs. Points (i) and (ii) lead us to wonder as to the validity of our reasoning for the higher twist terms and other fundamental features of mass factorization.

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FIGURE CAPTIONS

- Fig. 1 : Mass factorization in DI scattering: (a) cut-graph, (b) R operation, (c) over-all counter terms, (d) Wilson coefficients.
- Fig. 2 : Mass factorization in the DY process: (a) cut-graph, (b) R operation, (c) over-all counter terms, (d) DY Wilson coefficient.
- Fig. 3 : Graphical evaluation of the DY correction term.

A diagram showing a gluon jet (represented by a curly line) that splits into a quark-antiquark pair (represented by straight lines). The jet is shown as a series of nested dashed squares, with the splitting occurring at the center. A vertical line is drawn through the center of the diagram.

$\bar{b}_L^{(3)} =$

$$(-t^T) \overline{G}_T^{(3)} = -$$

$$\mathcal{O}_T^{(3)} =$$

Fig. 1

$$\Delta^{DY} = \text{[Diagram 1]} - \text{[Diagram 2]} \left[\text{[Diagram 3]} \text{[Diagram 4]} \right] - \text{[Diagram 5]}$$

(a)

$$\Delta^{DY} = \text{[Diagram 1]} - \text{[Diagram 2]}$$

(b)

Fig. 3