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GAUGE FIELDS, EXTENDED OBJECTS AND STATES OF MATTER

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A B S T R A C T

We discuss the interaction of dual extended objects with the Yang-Mills potential $\mathcal{A}_\mu$ and antisymmetric potentials $\mathcal{A}_{\mu\nu}$ and $\mathcal{A}_{\mu\nu\lambda}$. The following is shown.

- a) In the field of $\mathcal{A}_\mu$, strings, shells and bags superconduct.
- b) A shell (like a string) is a source of vorticity in the fluid described by $\mathcal{A}_{\mu\nu}$. It is also opaque to the fluid. The fluid tends not to occupy the interior of a bag.
- c) The scalar field χ described by $\mathcal{A}_{\mu\nu\lambda}$ also tends not to occupy the interior of a bag (Aurilia et al. have shown before that a shell behaves like a bubble in the field of χ , creating a jump discontinuity in an otherwise constant χ).
- d) In all these cases, it is shown that a dual object at rest does not dissipate energy, although it may carry a current which is a source of a gauge field. (For $\mathcal{A}_\mu$, this is the phenomenon of superconductivity.)

It is pointed out that a dual object in a gauge field tends to behave like a dense collection, or a condensate, of dual objects of lower dimension (which can couple to the gauge field).

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1. - INTRODUCTION

In this paper, we are going to argue that the interaction of gauge fields to point particles and extended objects furnishes idealized descriptions of different phases (or states) of matter. Examples of such states are : a) the normal and superconducting phases of matter, b) the irrotational and turbulent regions in a fluid, and c) a fluid and cavities (regions of inhomogenities) in that fluid.

The gauge fields we discuss are the Yang-Mills potentials $\mathcal{A}_\mu$ and the antisymmetric potential $\mathcal{A}_{\mu\nu}$ and $\mathcal{A}_{\mu\nu\lambda}$ ^{1),*)}. We are interested in their coupling to point particles, and to dual strings, shells and bags.

The coupling of $\mathcal{A}_\mu$ to point particles has been discussed before⁴⁾ as also the coupling to extended objects^{5),6)}. In the latter case, it has been shown that the extended objects exhibit features which resemble Abelian and non-Abelian (electric) superconductivity. The derivation of this result [due to 5)] will be recalled and extended in the text. In particular, it will be shown that a dual object at rest does not dissipate energy, although it may carry a current which is a source of any of the gauge fields $\mathcal{A}_\mu$, $\mathcal{A}_{\mu\nu}$ or $\mathcal{A}_{\mu\nu\lambda}$. A model which exhibits magnetic superconductivity has also been discussed elsewhere, and its utility for describing the MIT bag model and quark confinement has been emphasized⁷⁾.

It has been pointed out especially by Lund and Regge⁸⁾ that in the absence of sources, the potential $\mathcal{A}_{\mu\nu}$ describes a four-dimensional generalization of an irrotational, incompressible fluid. It does not couple to a point particle. When it couples to a string, the string acts as a source of vorticity⁸⁾. Here we study the interaction of $\mathcal{A}_{\mu\nu}$ to shells and bags as well. These extended objects are also sources of vorticity. Further, in either case, the "fluid" current^{**) $\mathcal{M}_\mu = \frac{1}{3!} \epsilon_{\mu\nu\lambda\rho} \mathcal{F}^{\nu\lambda\rho}$} [associated with the field strength $\mathcal{F}^{\nu\lambda\rho}$] is constrained to flow tangentially to the boundary of the object. The latter is thus

*) The non-Abelian generalization of the antisymmetric fields has been studied by Freedman and Townsend²⁾. In a paper under preparation, we will investigate the interactions of these non-Abelian fields to extended objects and discuss their relevance to QCD. Kalb has also recently studied³⁾ a non-Abelian generalization of antisymmetric tensor fields. His construction, however, involves the use of singular gauge transformations.

**) We will use a space-like metric and the convention $\epsilon_{0123} = -\epsilon^{0123} = 1$.

opaque to the fluid. In addition, the fluid cannot in general exist in the interior of the bag. Therefore, we can interpret this interior as a region where irrotationality is destroyed (or as a region of turbulence ⁹⁾).

The potential $\mathcal{A}_{\mu\nu\lambda}$ describes a scalar field χ which is obliged to be a constant in a source free region. It cannot couple to a particle or a string. Its interaction to a shell has been studied by Aurilia et al. ¹⁰⁾. They show that χ undergoes a jump when the shell surface is crossed. They interpret χ in terms of pressure and argue that the differential pressure inside and outside the shell can account for the pressure term in the MIT bag model ¹¹⁾. In this paper, we will extend their work to the interaction of $\mathcal{A}_{\mu\nu\lambda}$ to bags ^{*}). We can then find a dynamics where χ is identically zero inside the bag (while it need not vanish inside a shell). We may note here that :

- a) if χ is interpreted in terms of a pressure in a fluid, a closed shell can be interpreted as enclosing a cavity in a fluid, the pressure inside and outside the cavity being different ;
- b) if χ is interpreted in terms of fluid density, a closed shell can be interpreted as enclosing, for instance, a liquid bubble in a gas ;
- c) the bag can be interpreted as a "vacuum" domain in a fluid.

The lowest dimensional object to which $\mathcal{A}_\mu$ couples is a point particle. When $\mathcal{A}_\mu$ couples to an extended object, the extended object superconducts. Thus the latter describes a condensed phase of point particles. Similarly, the lowest dimensional object to which $\mathcal{A}_{\mu\nu}$ couples is a string (a vortex source). When it couples to a higher dimensional object, say a shell, then the latter appears to describe a dense tangle of vortices ⁹⁾. Thus it seems in general that if the lowest dimensional object to which a gauge field couples is of dimension n , then objects of dimension greater than n in this gauge field can be regarded as a dense distribution of n dimensional objects. The fact that (as in superconductivity), static extended objects in a gauge field do not lose energy (even if there is a flow of current) further supports this point of view.

The present paper is organized as follows. In Section 2, we derive an identity fulfilled by the Lagrangian of a dual system. This identity is not new ^{14),5)}. It does not depend on the details of the Lagrangian and is a

^{*}) A somewhat related construction is given by Aurilia in Ref. 12) who suggests that gauge fields of the fourth kind could be relevant for confinement. This question has been discussed by Lüscher in the context of quantum field theories with instantons 13).

consequence of the reparametrization invariance of the corresponding action. It leads to constraints involving field strengths and currents.

Section 3 recalls and refines earlier results on the interaction of $\mathcal{A}_\mu$ to extended objects⁵⁾. The identity of Section 2 is shown to imply that an extended object at rest does not dissipate energy, even though it carries a current. Thus charged strings, shells and bags superconduct. This result is easily adapted to any extended object in any gauge field. We also generalize this result and show that if there is a constant (non-zero) four vector ξ which is everywhere tangent to the extended object (considered as a submanifold in the Minkowski space-time), then it does not dissipate the component of four momentum in the direction of ξ .

Section 4 discusses the potential $\mathcal{A}_{\mu\nu}$ and its interaction with extended objects. It is shown that they are capable of forming vortices and cavities in the fluid described by $\mathcal{A}_{\mu\nu}$.

Section 5 describes the interaction of $\mathcal{A}_{\mu\nu\lambda}$ to extended objects. A dual bag which can expel the field strength of $\mathcal{A}_{\mu\nu\lambda}$ is discussed. Its possible relevance to the MIT bag model is indicated.

Section 6 finally develops Lagrangian descriptions for the equations of motion of the previous sections. It is pointed out that the conventional dual Lagrangian gives a degenerate bag (of space-time dimension three and not four). Its interpretation as a system of points, strings and shells is suggested.

2. - REPARAMETRIZATION INVARIANCE

An extended object of dimension n in the Minkowski space-time M^4 is parametrized by n variables

$$\sigma \equiv (\sigma^0, \sigma^1, \dots, \sigma^{n-1}) \quad (2.1)$$

Its co-ordinates in M^4 are thus

$$Z(\sigma) \equiv (Z^\mu(\sigma)) \quad (2.2)$$

The dynamics of the dual extended object is given by the Nambu-Goto action ¹⁵⁾

$$\mathcal{S}_0 = \int d^n \sigma \mathcal{L}_0 \quad (2.3)$$

$$\mathcal{L}_0 = -c \sqrt{-\det M} \quad (2.4)$$

$$M_{ab} = \partial_a \bar{z}^\mu \partial_b z_\mu \quad (2.5)$$

$$\partial_a \equiv \frac{\partial}{\partial \sigma^a} \quad (2.6)$$

where c is a constant.

This action is invariant under the infinitesimal reparametrization transformation

$$\delta z^\mu = \epsilon^b \partial_b \bar{z}^\mu \quad (2.7)$$

Since

$$\begin{aligned} \delta \mathcal{L}_0 &= \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} \partial_a \delta \bar{z}^\mu = \\ &= \partial_a \left(\frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} \delta \bar{z}^\mu \right) - \delta \bar{z}^\mu \partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} \end{aligned} \quad (2.8)$$

the identity

$$\partial_b \bar{z}^\mu \partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} \equiv 0 \quad (2.9)$$

follows from the reparametrization invariance ^{*)}.

^{*)} We note that if $\mathcal{L}_0$ is not specifically of the form (2.4), but depends also on z^μ say, reparametrization invariance requires instead the identity

$$\partial_b \bar{z}^\mu \left(\partial_a \mathcal{L}_0 / \partial (\partial_a \bar{z}^\mu) - \partial \mathcal{L}_0 / \partial z^\mu \right) \equiv 0$$

3. - SUPERCONDUCTIVITY

In this section, we shall recall and extend the discussion in Ref. 5). For simplicity, we discuss mostly Abelian gauge fields.

3.1. - Strings as sources

In analogy with the electrodynamics of point particles, we postulate the following equations for the Abelian gauge dynamics of strings ($n = 2$) :

$$\partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} = - F_{\mu\nu}(z) \partial_a \bar{z}^\nu J^a(\sigma) \quad (3.1)$$

$$\partial_\mu F^{\mu\nu}(x) = \int d\sigma \delta^4(x - z(\sigma)) J^a(\sigma) \partial_a \bar{z}^\nu(\sigma) \quad (3.2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (3.3)$$

The "internal" vector $J^a(\sigma)$ is to be interpreted as the current density on the extended object. In terms of I^a in Ref. 5), it is eI^a where e is the $U(1)$ charge.

The identity (2.9) gives

$$F_{ab}(z) J^b(\sigma) = 0 \quad (3.4)$$

$$F_{ab}(z) \equiv F_{\mu\nu}(z) \partial_a \bar{z}^\mu \partial_b \bar{z}^\nu \quad (3.5)$$

The identity

$$\partial_\nu \partial_\mu F^{\mu\nu} \equiv 0 \quad (3.6)$$

gives the current conservation

$$\partial_a J^a(\sigma) = 0 \quad (3.7)$$

[and boundary conditions on the current if the object has a boundary, cf. Ref. 5) or Eq. (6.17)].

To have a qualitative understanding of the physical content of the equation (3.4), consider a static string and let $x^0 = z^0 = \sigma^0$. Assume also that the current and the tangential component of the electric field are proportional : $J_1 = \sigma F_{0v} \partial_1 z^v$ [The metric $g_{ab} \equiv \partial_a z^\mu \partial_b z_\mu$ is diagonal in this case. Conductivity has been denoted by σ]. If there is a current on the string [$J_1 \neq 0$] (3.4) says that $F_{0v} \partial_a z^v = 0$. Thus the conductivity is infinite.

We now show that the following are consequences of (3.1) and (3.4) ^{*)}. (The results are valid for any extended object as we shall see in Section 3.2)

- a) a static string does not dissipate energy even though it may carry a current (for such a string, in an arbitrary inertial frame, the corresponding conserved quantity is just its invariant mass) ;
- b) more generally, let translations in the direction of a constant four vector ξ map the string S onto itself :

$$z(\sigma) \in S \implies z(\sigma) + \lambda \xi \in S \quad (3.8)$$

Here λ can depend on z . Then the component of the four momentum in the direction of ξ is not dissipated by S .

Note the following : i) for $\xi = (0,0,0,1)$, b) implies a) ; ii) a string (or any extended object) which fulfils the condition in b) is necessarily unbounded in the direction of ξ . If ξ is space-like, say $\xi = (1,0,0,0)$, then it is infinite in the x^1 direction and its constant x^1 slices are all identical. (Further, the x^1 component of momentum is not dissipated). If ξ is a null vector, it is moving by rigid translations with the velocity of light (parallel to ξ).

The proofs of a) and b) are based on the observation that the energy momentum tensor

^{*)} The argument which follows is due to V.P. Nair.

$$T_{\mu\nu}^E(x) = \int d\sigma^n \delta^4(x - z(\sigma)) \partial_a z_\mu \frac{\partial \mathcal{L}_0}{\partial (\partial_a z^\nu)} \quad (3.9)$$

of the string ($n = 2$) fulfils

$$\partial^\mu T_{\mu\nu}^E(x) = - \int d\sigma^n \delta^4(x - z(\sigma)) F_{\nu\mu}(z) J^a \partial_a z^\mu \quad (3.10)$$

Now if there exists $\alpha^a(\sigma)$ such that

$$\alpha^a(\sigma) \partial_a z^\mu(\sigma) = \xi^\mu \quad (3.11)$$

is a constant four vector, then

$$\partial^\mu (T_{\mu\nu}^E \xi^\nu) = 0$$

by (3.4), and the momentum in the direction of ξ is not dissipated by S . It is therefore enough to show that (3.11) is equivalent to (3.8). Since any multiple of $\alpha^a \partial_a$ is a vector field on S , the associated finite transformations map S to S . The latter are of the form (3.8). The result is thus proved.

The results a) and b) are also valid for non-Abelian $\mathcal{A}_\mu$ and for $\mathcal{A}_{\mu\nu}$ and $\mathcal{A}_{\mu\nu\lambda}$ potentials. The proofs are essentially the same. For the Lorentz-Maxwell equations and for the analogue of (3.4) and (3.7) for these potentials, see Refs. 5), 1) and also Sections 4 and 5 of this paper.

3.2. - Shells and bags

The equations for shells and bags in an Abelian $\mathcal{A}_\mu$ are (3.1) and (3.2) with $n = 3$ and 4 , respectively. Thus (3.4) and (3.7) follow with a and b running over n values. It is also easy to see as before that :

- a) a static shell or bag does not dissipate energy ; and
- b) if translations in the direction of a constant four vector ξ map a shell or a bag onto itself, then neither dissipates the component of its four momentum in the direction of ξ .

The results (3.4) and (3.7) as well as a) and b) above generalize with ease to non-Abelian $\mathcal{A}_\mu$ and to $\mathcal{A}_{\mu\nu}$ and $\mathcal{A}_{\mu\nu\lambda}$ potentials [the equations defining these systems are discussed in Refs. 5), 1), 10) and in Sections 4 and 5 of this paper].

We now make a few scattered remarks regarding (3.4) which, as noted, is valid for any n .

For a (four-dimensional) bag B , the Jacobian $\partial(z)/\partial(\sigma)$ is not zero. Thus for B , we can also write (3.4) in the form

$$F_{\mu\nu}(z) \partial_\rho F^{\rho\nu}(z) = 0 \quad (3.13)$$

by using (3.2). Then non-Abelian version of this equation is

$$\text{Tr}(F_{\mu\nu}(z) \mathcal{D}_\rho F^{\rho\nu}(z)) = 0 \quad (3.14)$$

where the field strengths are Lie algebra valued and $\mathcal{D}_\rho$ denotes covariant differentiation.

By a suitable variational principle (see Section 6) it is possible to strengthen the equation (3.4) into

$$F_{ab}(z) = 0 \quad (3.15)$$

for any extended object. We shall assume this stronger condition below.

For a shell, (3.15) means that the electric field tangential to the shell, and the magnetic field perpendicular to the shell (considered as a two-dimensional object at a given time) are zero. For a bag, (3.15) implies also the vanishing of electric and magnetic fields in its interior. Thus we get the usual gross features of superconductivity. Note, however, that one gets zero penetration depth in the Meissner effect ^{*)}.

^{*)} Elsewhere we will discuss systems with non-zero penetration depth as well as systems with a characteristic coherence length (as in superconductivity).

4. - IRROTATIONAL FLUIDS

4.1. - No sources

The gauge potential

$$A_{\mu\nu} = -A_{\nu\mu} \quad (4.1)$$

has the associated field strength

$$F_{\mu\nu\lambda} = \partial_\mu A_{\nu\lambda} + \partial_\nu A_{\lambda\mu} + \partial_\lambda A_{\mu\nu} \quad (4.2)$$

It fulfils the ("Bianchi") identity

$$\epsilon^{\mu\nu\lambda\rho} \partial_\mu F_{\nu\lambda\rho} = 0 \quad (4.3)$$

and the field equation

$$\partial_\mu F^{\mu\nu\lambda} = 0 \quad (4.4)$$

when there are no sources.

It is known ^{1),8)} that (4.3) and (4.4) describe a massless scalar field. In fact ^{*)}, due to (4.4), we can write

$$F^{\mu\nu\lambda} = \epsilon^{\mu\nu\lambda\rho} \partial_\rho \varphi \quad (4.5)$$

Then (4.3) becomes

$$\partial_\rho \partial^\rho \varphi = 0 \quad (4.6)$$

The field φ is thus a phonon-like field. [The velocity of propagation in (4.6) need not necessarily be interpreted as the velocity of light.]

Let

$$M_\mu = \frac{1}{3!} \epsilon_{\mu\nu\lambda\rho} F^{\nu\lambda\rho} \quad (4.7)$$

^{*)} This argument was pointed out to one of the authors (A.P.B.) by S. Hojman.

Then (4.3) and (4.4) give

$$\partial_\mu \mathcal{M}^\mu = 0 \quad (4.8)$$

$$\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu = 0 \quad (4.9)$$

If we retain only the spatial components $\mathcal{M}_i$ and ∂_i in these equations, and identify $\mathcal{M}$ as constant times velocity, these equations describe an incompressible irrotational fluid⁸⁾. Equations (4.8) and (4.9) are thus the four-dimensional generalizations of the equations for an incompressible irrotational fluid. We have no satisfactory interpretation of these equations (in the four-dimensional form) in terms of any familiar macroscopic fluid^{*)}. In this connection, we observe the following. Equation (4.8) resembles a continuity equation and suggests the identifications : a) $\mathcal{M}^\mu / (-\mathcal{M}_0 \mathcal{M}^0)^{\frac{1}{2}} = (\text{normalized}) \text{ four velocity}$; b) $(-\mathcal{M}_0 \mathcal{M}^0)^{\frac{1}{2}} = N$, the number of particles (making up the fluid) per unit volume in a frame where the volume element is at rest. Such identifications would imply that pressure and proper energy density are proportional to N^2 since the energy momentum tensor (obtained from the Lagrangian in Section 6) is proportional to

$$\mathcal{M}_\mu \mathcal{M}_\nu - \frac{1}{2} \eta_{\mu\nu} \mathcal{M}_\rho \mathcal{M}^\rho \quad (4.10)$$

[Cf. Ref. 17)]. Further, with these identifications, (4.9) does not admit a simple interpretation. [The equality of pressure and proper energy density, implied by (4.10), is expected since phonon-like modes describe all the excitations.]

The following may also be noted. The integral

$$M \equiv \int_{\mathbb{R}^3} d^3x \mathcal{M}_0 \quad (4.11)$$

$$= \frac{1}{3!} \int_{\mathbb{R}^3} d^3x \epsilon_{ijk} F^{ijk} \quad (4.12)$$

*) Sugamoto has pointed out¹⁶⁾ that if one couples Higgs scalars to the relativistic hydrodynamics of Kalb, Ramond and Nambu, one obtains the same partition function as in the "dual" transformation of the Abelian Higgs model.

can be written as an integral over the "boundary" ∂R^3 of R^3 :

$$M = \frac{1}{2} \int_{\partial R^3} \mathcal{H}_{ij} dx^i \wedge dx^j \quad (4.13)$$

It is therefore superselected. This superselection rule resembles the mass superselection rule in Galilei invariant theories¹⁸⁾. There are also infinitesimal symmetry transformations which resemble Galilei velocity transformations in the theory. Consider

$$\delta \mathcal{H}_{\mu\nu} = \frac{1}{3} \epsilon_{\mu\nu\rho\sigma} x^\rho n^\sigma, n^\sigma \text{ constant} \quad (4.14)$$

under which

$$\delta \mathcal{F}_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\sigma} n^\sigma \quad (4.15)$$

$$\delta \mathcal{M}_\mu = n_\mu \quad (4.16)$$

This is a symmetry of the source free equations and the corresponding action^{*)} (Section 6). Since

$$\delta M = n_0 \int_{R^3} d^3x \quad (4.17)$$

we should require $n_0 = 0$ for the finiteness of δM . With this restriction, such transformations resemble Galilei boosts on velocities [see Eq. (4.16)]. The corresponding finite transformations shift the free action by infinite constants and therefore cannot be implemented.

4.2. - Strings as sources

According to Kalb and Ramond¹⁾, the interaction of a dual string and $\mathcal{A}_{\mu\nu}$ is described by the equations

*) Such transformations can be generalized to $\mathcal{A}_\mu$ and $\mathcal{A}_{\mu\nu\lambda}$:

$$\delta \mathcal{A}_\mu = \epsilon_{\mu\nu\lambda\rho} x^\nu n^\lambda, \delta \mathcal{A}_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\rho} x^\rho n$$

where $n^{\lambda\rho}$ and n are constants.

$$\partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} = -\frac{1}{2} F_{\mu\nu\lambda}(\bar{z}) \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda J^{ab} \quad (4.18)$$

$$\partial_\mu F^{\mu\nu\lambda}(x) = \int d\sigma \delta^4(x - z(\sigma)) J^{ab} \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda \quad (4.19)$$

where the "current" J^{ab} is of the form

$$J^{ab}(\sigma) = \epsilon^{ab} \omega(\sigma) \quad (4.20)$$

$$\epsilon^{10} = -\epsilon^{01} = 1 \quad (4.21)$$

These equations can be postulated in analogy with the Lorentz-Maxwell equations. (The correct factors on the right-hand sides of these equations follow from the Lagrangian in Section 6.)

The identity (2.9) gives nothing since

$$F_{\mu\nu\lambda} \partial_c \bar{z}^\mu \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda \equiv 0 \quad (4.22)$$

The analogue $\partial_\mu \partial_\nu F^{\mu\nu\lambda} \equiv 0$ of the identity (3.6) shows that

$$\partial_a \omega = 0 \quad (4.23)$$

that is ω is a constant. (Kalb and Ramond assume from the beginning that ω is a constant. For later purposes, we have shown here that it is a consequence of the field equation.)

We consider only strings with no boundary (closed or infinitely long strings). Thus the identity $\partial_\mu \partial_\nu F^{\mu\nu\lambda} \equiv 0$ does not lead to boundary conditions on ω .

Following Lund and Regge⁸⁾, we now observe that (4.19) is equivalent to

$$\partial_\mu \mathcal{M}_\nu(x) - \partial_\nu \mathcal{M}_\mu(x) = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} \omega \int d\vec{\sigma} \delta^4(x - z(\tau)) \epsilon^{ab} \partial_a z^\lambda \partial_b z^\rho \quad (4.24)$$

Thus the string is a vortex line. The circulation around it is

$$\begin{aligned} \oint_C \mathcal{M}_\mu dx^\mu &= \frac{1}{2} \int_\Delta (\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu) dx^\mu \wedge dx^\nu \\ &= \frac{1}{4} \epsilon_{\mu\nu\lambda\rho} \omega \int_\Delta dx^\mu \wedge dx^\nu \int \delta^4(x - z(\tau)) dz^\lambda \wedge dz^\rho \\ &= -6\omega \end{aligned} \quad (4.25)$$

where Δ is any surface with boundary C and transverse to the two-dimensional surface of the string.

4.3. - Shells as sources

We consider only closed shells.

In analogy with (4.18) and (4.19), we postulate the equations

$$\partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a z^\mu)} = -\frac{1}{2} F_{\mu\nu\lambda}(z) \partial_a z^\nu \partial_b z^\lambda J^{ab} \quad (4.26)$$

$$\partial_\mu F^{\mu\nu\lambda}(x) = \int d\vec{\sigma} \delta^4(x - z(\tau)) J^{ab} \partial_a z^\nu \partial_b z^\lambda \quad (4.27)$$

where now a and b run over 0, 1 and 2.

In terms of the current $\mathcal{M}_\mu$, (4.27) reads

$$\partial_\mu \mathcal{M}_\nu(x) - \partial_\nu \mathcal{M}_\mu(x) = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} \int d^3\sigma \delta^4(x - z(\tau)) J^{ab} \partial_a z^\lambda \partial_b z^\rho$$

so that the current J^{ab} is a source of vorticity.

The identity (2.9) gives

$$F_{abc}(z) J^{bc} = 0 \quad (4.29)$$

while the analogue of (3.6) gives current conservation :

$$\partial_a J^{ab} = 0 \quad (4.30)$$

Equation (4.29) is a generalization of the superconductivity equation (3.4). If any one component of J^{ab} is not zero, it gives

$$F_{abc} = \epsilon_{\mu\nu\lambda\rho} \mathcal{M}^\mu (\partial_a z^\nu \partial_b z^\lambda \partial_c z^\rho) = 0 \quad (4.31)$$

Thus the current $\mathcal{M}_\mu$ flows tangentially to the surface of the shell. A shell $\mathcal{M}$ with a non-zero vortex current J^{ab} is opaque to the fluid.

Note that the results of Section 3 on the non-dissipation of energy or momentum can be adapted to the present system using (4.26) and (4.29).

Let

$$J^{ab} = \epsilon^{abc} \omega_c \quad (4.32)$$

Then current conservation gives

$$\partial_a \omega_b - \partial_b \omega_a = 0 \quad (4.33)$$

Let us compute the circulation

$$\omega = \oint_C \mathcal{M}_\mu dx^\mu \quad (4.34)$$

If S is the shell regarded as a submanifold of Minkowski space M^4 , then C is a closed curve in the complement $M^4 \setminus S$ of S . Since $\mathcal{M}$ is irrotational in $M^4 \setminus S$, ω is zero by Stoke's theorem, if C can be shrunk to a point without crossing S . Suppose now that C is not homotopic to a point. Then (4.28) gives

$$\omega = \frac{1}{4} \epsilon_{\mu\nu\lambda\rho} \int_{\Delta} dx^\mu \wedge dx^\nu \wedge \int_S \delta^4(x-z(\tau)) dz^\lambda \wedge dz^\rho \wedge \omega_a d\tau^a = -6 \int_{\Delta \cap S} \omega_a d\tau^a \quad (4.35)$$

where Δ is any two surface in M^4 with boundary C and transverse to S .

If S is simply connected, then by (4.33) there is a potential ξ defined globally on S such that

$$\omega_a = \partial_a \xi \quad (4.36)$$

Thus

$$\omega = 0 \quad (4.37)$$

If S is not simply connected, ξ need not be globally defined and ω need not vanish.

4.4. - Bags as sources

We assume in this section that the bag B is a four-dimensional manifold (possibly with boundary) in Minkowski space M^4 . That is, we exclude the case where B is degenerate (cf. Section 6).

The analogues of (4.18) and (4.19) now are *)

$$\partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} = -\frac{1}{2} F_{\mu\nu\lambda}(z) \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda J^{ab} \quad (4.38)$$

$$\partial_\mu F^{\mu\nu\lambda}(x) = \int d^4\sigma \delta^4(x - z(\sigma)) J^{ab} \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda \quad (4.39)$$

The left-hand side of (4.38) is identically zero (cf. Section 6). Thus the energy-momentum tensor inside the bag is zero and

$$F_{\mu\nu\lambda}(z) \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda J^{ab} = 0, z \in B \quad (4.40)$$

For x inside the bag B , (4.39) becomes

$$\partial_\sigma F^{\sigma\nu\lambda}(z) = \frac{\partial(\sigma)}{\partial(z)} J^{ab} \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda \quad (4.41)$$

It follows that

$$F_{\mu\nu\lambda}(z) \partial_\sigma F^{\sigma\nu\lambda}(z) = 0 \quad (4.42)$$

that is

$$\mathcal{M}^\mu(z) \cdot (\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu)(z) = 0 \quad (4.43)$$

This equation is similar to (3.13)

The identity $\partial_\mu \partial_\nu F^{\mu\nu\lambda} \equiv 0$ gives current conservation

$$\partial_a J^{ab} = 0 \quad (4.44)$$

inside B and the condition

*) For boundary conditions on ∂B , see Section 6 from (6.17) onwards.

$$n_a J^{ab} \partial_b z^\lambda = 0 \quad (4.45)$$

on the boundary ∂B where n_a is normal to ∂B . That is, if $\sigma^a(\tau^0, \tau^1, \tau^2)$ describes ∂B , n_a is defined by

$$n_a \frac{\partial \sigma^a}{\partial \tau^k} = 0 \quad (4.46)$$

We now analyze (4.43). There are two cases to be considered.

- a) $\mathcal{M}_\mu$ is null (or zero), $\mathcal{M}_\mu(z)\mathcal{M}^\mu(z) = 0$. In this case, (4.43) allows solutions with $(\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu)(z) \neq 0$ and $\partial_\mu \mathcal{M}^\mu(z) = 0$. For example, if a and b are two constant orthogonal vectors and a is null, we can set

$$\mathcal{M}_\mu(z) = (b \cdot z) a_\mu \quad (4.47)$$

- b) $\mathcal{M}_\mu(z)\mathcal{M}^\mu(z) \neq 0$. In this case, if the differential form $\mathcal{M} = \mathcal{M}_\mu dz^\mu$ admits an integrating factor (for $z \in B$) ^{*)}, then we can prove that $(\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu)(z)$ [and hence J^{ab} by (4.39)] are zero [note that in the example of case a), $\mathcal{M}$ admits an integrating factor: it fulfils (4.49) below]. In such a case, it would be as though the bag did not exist for $\mathcal{M}_\mu$.

In Section 6, we will show how to arrange for the validity of the equation

$$\mathcal{M}_\mu(z) = 0 \quad (4.48)$$

from a variational principle.

If $\mathcal{M}$ admits an integrating factor, then the vectors orthogonal to $\mathcal{M}^\mu$ are tangent to a three-dimensional surface.

The condition for $\mathcal{M}$ to admit an integrating factor in B is $\mathcal{M} \wedge d\mathcal{M} = 0$ in B , that is,

*) W. Nahm has constructed an example which shows that without this condition, the result is false. Its construction helped us to locate an error in an earlier version of the proof.

$$\epsilon^{\mu\nu\lambda\rho} \mathcal{M}_\nu(z) (\partial_\lambda \mathcal{M}_\rho - \partial_\rho \mathcal{M}_\lambda)(z) = 0 \quad (4.49)$$

Now by (4.43),

$$(\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu)(z) = \epsilon_{\mu\nu\lambda\rho} \mathcal{M}^\lambda(z) C^\rho(z) \quad (4.50)$$

for some $C(z)$. From these two equations, we find

$$(\mathcal{M}_\rho C^\rho)(z) \mathcal{M}^\mu(z) - (\mathcal{M}_\rho \mathcal{M}^\rho)(z) C^\mu(z) = 0 \quad (4.51)$$

Thus in case b) (where $\mathcal{M}_\rho \mathcal{M}^\rho \neq 0$ in B), we solve for $C(z)$ from (4.51) and find from (4.50) that

$$(\partial_\mu \mathcal{M}_\nu - \partial_\nu \mathcal{M}_\mu)(z) = 0 \quad (4.52)$$

With the condition (4.48), the current $\mathcal{M}_\mu$ is zero in and on B . If we put a surface current J^{ab} on the shell ∂B as in Section 4.3, the current $\mathcal{M}_\mu$ can flow tangentially to ∂B , but cannot penetrate into B .

5. - THE PRESSURE FIELD

5.1. - No sources

The gauge potential is now a totally antisymmetric tensor $\mathcal{A}_{\mu\nu\lambda}$. The field strength is

$$\begin{aligned} F_{\mu\nu\lambda\rho} &= \partial_\mu \mathcal{A}_{\nu\lambda\rho} - \partial_\nu \mathcal{A}_{\lambda\rho\mu} + \partial_\lambda \mathcal{A}_{\rho\mu\nu} - \partial_\rho \mathcal{A}_{\mu\nu\lambda} \\ &= -\epsilon_{\mu\nu\lambda\rho} \chi \end{aligned} \quad (5.1)$$

The analogue of the Bianchi identity is trivial while the field equation

$$\partial_\mu F^{\mu\nu\lambda\rho} = 0 \quad (5.2)$$

gives

$$\partial_\mu \chi = 0 \quad (5.3)$$

Thus χ is a constant in source free regions.

5.2. - Shells as sources

The potential $\mathcal{A}_{\mu\nu\lambda}$ does not couple to particles or strings. Its interaction with closed shells has been discussed by Aurilia et al.¹⁰⁾. We briefly outline some of their results.

The equations which generalize (4.26) and (4.27) are

$$\partial_a \frac{\partial \mathcal{L}_0}{\partial (\partial_a \bar{z}^\mu)} = -\frac{1}{3!} F_{\mu\nu\lambda\rho}(z) \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda \partial_c \bar{z}^\rho J^{abc} \quad (5.4)$$

$$\partial_\mu F^{\mu\nu\lambda\rho}(x) = \int d^3\sigma \delta^4(x-z(\sigma)) J^{abc} \partial_a \bar{z}^\nu \partial_b \bar{z}^\lambda \partial_c \bar{z}^\rho \quad (5.5)$$

where the current has the form

$$J^{abc} = \epsilon^{abc} \omega \quad (5.6)$$

The identity (2.9) gives nothing [due to (5.1)] while the analogue of (3.6) gives current conservation :

$$\partial_a J^{abc} = 0 \quad (5.7)$$

Thus

$$\partial_a \omega = 0 \quad (5.8)$$

or ω is a constant.

We can write (5.5) in the form

$$\partial_\mu \chi = \frac{1}{3!} \epsilon_{\mu\nu\lambda\rho} \omega \int_S \delta^4(x-z(\sigma)) d\bar{z}^\nu d\bar{z}^\lambda d\bar{z}^\rho \quad (5.9)$$

where S is the shell. Thus χ is a constant χ_0 outside S and another constant χ_I inside S where

$$\chi_I - \chi_0 = -4\omega \quad (5.10)$$

5.3. - Bags as sources

Since the left-hand side of (4.38) now vanishes, the equations describing the dynamics become

$$0 = F_{\mu\nu\lambda\rho}(z) \partial_a z^\nu \partial_b z^\lambda \partial_c z^\rho J^{abc}, z \in B \quad (5.11)$$

$$\partial_\mu F^{\mu\nu\lambda\rho}(x) = \int d\tau \delta^4(x - z(\tau)) J^{abc} \partial_a z^\nu \partial_b z^\lambda \partial_c z^\rho \quad (5.12)$$

where B is the bag. Thus

$$\partial_\mu \chi(x) = 0, \quad x \in M^4 \setminus B \quad (5.13)$$

and

$$\partial_\mu F^{\mu\nu\lambda\rho}(z) = \frac{\partial(\tau)}{\partial(z)} J^{abc} \partial_a z^\nu \partial_b z^\lambda \partial_c z^\rho \quad (5.14)$$

where B has been assumed to be four-dimensional. Multiplying (5.14) by $F_{\alpha\nu\lambda\rho}(z)$ and using (5.11), we find

$$\chi(z) \partial_\mu \chi(z) = 0, \quad z \in B \quad (5.15)$$

By a variational principle, we will strengthen this equation in Section 6 to

$$\chi(z) = 0, \quad z \in B \quad (5.16)$$

If we define the dual bag B as the complement $M^4 \setminus B_{MIT}$ of the MIT bag B_{MIT} (where B_{MIT} is compact) and χ as the pressure from the vacuum [compare the discussion in Ref. 7)], we see from (5.13) and (5.16) that the pressure is zero outside B_{MIT} and can be a constant inside B_{MIT} *).

6. - THE LAGRANGIAN FORMALISM

The free Lagrangian densities $\mathcal{L}_0$ for extended objects were given in Section 2. The free Lagrangian densities for the $\mathcal{A}_{\mu\nu}$ and $\mathcal{A}_{\mu\nu\lambda}$ fields are¹⁾

$$\mathcal{L}_F = -\frac{1}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} \quad (6.1)$$

and

$$\mathcal{L}_F = -\frac{1}{48} F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} \quad (6.2)$$

(suitable natural units are understood depending on the physical interpretation of the fields). We shall now discuss the interaction Lagrangians.

The potential $\mathcal{A}_{\mu_1 \dots \mu_m}$ interacts only with extended objects of dimension $n \geq m$. With this in mind, the interaction Lagrangian density can be written as

$$\mathcal{L}_I = -\frac{1}{m!} \varepsilon^{a_1 \dots a_n} \left(\mathcal{H}_{\mu_1 \dots \mu_m}^{(\mathbb{Z})} \partial_{a_1} \mathbb{Z}^{\mu_1} \dots \partial_{a_m} \mathbb{Z}^{\mu_m} \right. \\ \left. - \partial_{[a_1} \wedge a_2 \dots a_m] \right) \omega_{a_{m+1} a_{m+2} \dots a_n} \quad (6.3)$$

*) If we identify $\mathcal{A}_{\mu\nu\lambda}$ with $\varepsilon_{\mu\nu\lambda\rho} k^\rho$ where k^ρ is the current for instanton density 13), then the instanton density is non-zero only in the MIT bag.

Here $\Lambda_{a_2 \dots a_m}(\sigma)$ and $\omega_{a_{m+1} a_{m+2} \dots a_n}(\sigma)$ are totally antisymmetric in their indices. (For $m = 1$, $\partial_{[a_1} \Lambda \dots] \equiv \partial_{a_1} \Lambda$, where Λ is a scalar function.) Further, as we shall see, we must vary ω freely to get conditions like $F_{ab \dots} = 0$ on the extended object ^{*)}. To find weaker conditions like (4.29), we can set 5)

$$\omega_{a_{m+1} a_{m+2} \dots a_n} = \bar{\omega}_{b_{m+1} \dots b_n} \frac{\partial \varphi^{b_{m+1}}(\sigma)}{\partial \sigma^{a_{m+1}}} \dots \frac{\partial \varphi^{b_n}(\sigma)}{\partial \sigma^{a_n}} \quad (6.4)$$

where $\bar{\omega} \dots$ are constants, and vary the functions $\varphi^{bj}(\sigma)$. [If m and n are such that $\omega \dots$ is a scalar, interpret (6.4) as $\omega = \bar{\omega} = \text{constant}$.]

The action associated with $\mathcal{L}_I$ is

$$\mathcal{S}_I = \int d\sigma^n \mathcal{L}_I \quad (6.5)$$

This action is : a) reparametrization invariant if $\omega \dots$ and $\Lambda \dots$ are transformed as covariant tensors and φ^{bj} and z^μ are transformed as scalars ;

b) invariant under the gauge transformations

$$\mathcal{A}_{\mu_1 \mu_2 \dots \mu_m}^{(x)} \longrightarrow \mathcal{A}_{\mu_1 \mu_2 \dots \mu_m}^{(x)} + \partial_{[\mu_1} \mathcal{B}_{\mu_2 \dots \mu_m]}^{(x)} \quad (6.6)$$

$$\Lambda_{a_2 a_3 \dots a_m} \longrightarrow \Lambda_{a_2 a_3 \dots a_m} + B_{a_2 a_3 \dots a_m} \quad (6.7)$$

$$B_{a_2 a_3 \dots a_m} \equiv \partial_{a_2} z^{\mu_2} \partial_{a_3} z^{\mu_3} \dots \partial_{a_m} z^{\mu_m} \mathcal{B}_{\mu_2 \mu_3 \dots \mu_m}(z(\sigma))$$

where $\mathcal{B} \dots$ is totally antisymmetric.

^{*)} In the interaction Lagrangian (6.2) of Ref. 5) for the non-Abelian field $\mathcal{A}_\mu$, the full variation of $K \dots$ leads to $D_{a_0} g = 0$. This has the integrability condition $F_{ab}(z) = 0$ on the extended object. This variational principle to derive the non-Abelian version of (3.15) was pointed out to us by Trahern.

For a string in the potential $\mathcal{A}_{\mu\nu}$,

$$\mathcal{L}_I = -\frac{1}{2} \epsilon^{ab} (\mathcal{A}_{\mu\nu}(z) \partial_a z^\mu \partial_b z^\nu - \frac{1}{2} (\partial_a \wedge_b - \partial_b \wedge_a)) \omega \quad (6.8)$$

For $\omega = \text{constant}$ [Eq. (6.4)], this differs from the Kalb-Ramond Lagrangian density ¹⁾ only by total derivatives. On the other hand, if ω is varied fully, we find, for any n ,

$$\mathcal{A}_{\mu\nu}(z) \partial_a z^\mu \partial_b z^\nu = \frac{1}{2} (\partial_a \wedge_b - \partial_b \wedge_a) \quad (6.9)$$

Therefore,

$$F_{abc}(z) \equiv \mathcal{F}_{\mu\nu\rho}(z) \partial_a z^\mu \partial_b z^\nu \partial_c z^\rho = 0 \quad (6.10)$$

For a string, which is two dimensional, this is an identity.

For general m and n , the results of the variations are as follows.

A) The variation of Λ gives

$$\epsilon^{a_1 a_2 \dots a_n} \partial_{a_1} \omega_{a_{m+1} a_{m+2} \dots a_n} = 0 \quad (6.11)$$

so

$$\omega_{a_{m+1} a_{m+2} \dots a_n} d\tau^{a_{m+1}} \wedge d\tau^{a_{m+2}} \wedge \dots \wedge d\tau^{a_n} \quad (6.12)$$

is a closed form. We can interpret (6.11) as a current conservation :

$$\partial_{a_1} \mathcal{J}^{a_1 a_2 \dots a_m} = 0 \quad (6.13)$$

$$\mathcal{J}^{a_1 a_2 \dots a_m} = \epsilon^{a_1 a_2 \dots a_n} \omega_{a_{m+1} \dots a_n} \quad (6.14)$$

Note that (6.4) identically fulfils (6.11).

B) The general variation of ω gives an equation of the type (6.9). Thus

$$F_{a_1 a_2 \dots a_{m+1}}(z) \equiv F_{\mu_1 \mu_2 \dots \mu_{m+1}}(z) \partial_{a_1} z^{\mu_1} \partial_{a_2} z^{\mu_2} \dots \partial_{a_{m+1}} z^{\mu_{m+1}} = 0 \quad (6.15)$$

For (6.4), the variation of φ^{bj} gives instead

$$F_{a_1 a_2 \dots a_{m+1}}(z) J^{a_1 a_2 \dots a_m}(\nabla) = 0 \quad (6.16)$$

as required. [To derive this equation, multiply the result of the variation of φ^{bj} by $\partial_p \varphi^{bj}$. The resultant equation involves only F and ω . Now substitute for ω in terms of J from (6.14).]

- C) Variation of z leads to the generalized Lorentz force equations when using (6.13) while the variation of $\mathcal{A}$ leads to the generalized Maxwell equations.
- D) In the preceding sections, we have assumed that strings and shells are closed. Hence, for strings and shells, no boundary terms arise in the variation of S_I . For bags, it seems physically essential to allow for boundaries [see, e.g., Ref. 7]. We can allow for such boundaries also for any extended object during the variations. For simplicity, let us assume that ω is varied freely. Then no boundary terms arise from its variation. The variation of Λ in S_I shows that

$$n_{a_1} J^{a_1 \dots a_m} = 0 \quad (6.17)$$

on any such boundary, where n is any normal to the boundary. [It is defined as in (6.46). See also Eq. (4.3) in Ref. 5]. The boundary terms from varying z in S_I thus vanish. Hence the boundary conditions are (6.17) and the usual dual model boundary conditions from S_0 . The latter are treated further below. See also Ref. 15).

We shall now show that the left-hand side of (4.38) vanishes for a bag.

We have

$$\mathcal{L}_0 d^m \nabla = -c dz^0 \wedge dz^1 \wedge dz^2 \wedge dz^3 \quad (6.18)$$

since $\mathcal{L}_0$ is proportional to the Jacobian $\partial(z)/\partial(\sigma)$. Thus

$$\int_B \mathcal{L}_0 d\sigma = \frac{1}{4!} c \int_{\partial B} \epsilon_{\alpha\beta\gamma\sigma} z^\alpha dz^\beta \wedge dz^\gamma \wedge dz^\sigma \quad (6.19)$$

where ∂B is the boundary of B . It follows that

$$\partial_a \frac{\partial \mathcal{L}_0}{\partial(\partial_a z^\mu)} \equiv 0 \quad (6.20)$$

inside B .

It is interesting that the right-hand side in (6.19) is not the action for the dual shell even though it is an integral over a three-dimensional surface. It is more akin to the action of Rasetti and Regge⁸⁾ for the string. Its variation gives

$$\epsilon_{\alpha\beta\gamma\sigma} dz^\beta \wedge dz^\gamma \wedge dz^\sigma = 0, \quad z \in \partial B \quad (6.21)$$

This implies that ∂B is at most two-dimensional and hence that B is at most three-dimensional. If this conclusion is awkward, the bag Lagrangian can be modified by the addition of the dual shell Lagrangian (or the interaction Lagrangian with a gauge field) for its surface. Our previous considerations are not affected by such surface terms (they are, however, affected if the "bag" is only three-dimensional).

The case where B is three-dimensional seems capable of describing some interesting situations. The time slices of B are then in general at most two-dimensional. They will consist of one or several connected components, that is of shells and lower dimensional objects. The number of such objects need not be conserved in time. Such a model can perhaps be adapted to describe the formation and interaction of objects like vortices and bubbles in an ambient medium. [Note also the possibility that B exists only for an instant $z^0 = \bar{z}^0$ of time. Such a possibility is allowed by (6.21).]

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