A REFINED ESTIMATE OF THE FINE STRUCTURE CONSTANT

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ABSTRACT

The fine structure constant $\alpha$ is arbitrary for quantum electrodynamics (QED) considered in isolation. It has been argued previously that if QED is to be incorporated into a grand unified theory of all elementary particle interactions then $\alpha$ should be less than $1/25$. In this paper we show that a grand unifiable fine structure constant must in fact lie between about $1/120$ and $1/170$.

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Quantum electrodynamics (QED) is an immensely powerful theory whose validity extends over many decades of distance scales from $10^{-15}$ cm to light years. But it is an unsatisfactory theory in many ways, not the least of which is the arbitrariness of the electromagnetic coupling strength, the fine structure constant $\alpha$, not to mention the incomprehensible commensurability of the electric charges of different particles. Nowadays when one seeks to extend QED to distances smaller than $10^{-15}$ cm it is commonplace to combine it first with the weak interactions and subsequently with the strong interactions into a grand unified theory (GUT) of all elementary particle interactions. It has been emphasized that the incorporation of QED and the other fundamental interactions into a GUT is only possible if these interactions and the particles experiencing them satisfy some very non-trivial constraints. As far as QED is concerned, it is well-known that its incorporation into a simple unifying group imposes charge quantization. Some progress has also been made in understanding the magnitude of $\alpha$ in the context of grand unification. Stability of baryons in GUTs requires the grand unification energy scale $m_X$ to be $\geq 10^{14}$ GeV. Couplings in renormalizable field theories such as QED evolve logarithmically with energy, and in order for the effective $\alpha$ to be less than unity at the grand unification scale — necessary if the GUT is to make sense — it has been shown that one must have $\alpha < 1/25$. In this paper we point out that within the same standard set of grand unifying assumptions as used previously, one is able to make a much more refined estimate of the fine structure constant: $1/120 < \alpha < 1/170$. We do not use any specific GUT, but just the general observations that light fermions can be grouped into "families" or "generations" of 15 helicity states, that the strong interactions become strong on a scale of order 1 GeV, and that if a grand unification energy scale exists then it must lie in the range between $10^{14}$ GeV (for baryons to be stable enough) and $10^{19}$ GeV (the energy at which gravity becomes of unit strength). We know of no other theoretical framework for particle interactions which is able to provide such convincing and restrictive constraints on $\alpha$, and we find it hard to believe that the experimental value of $\alpha = 1/137$ is a coincidence. Surely some variety of GUT must be true?

We will briefly recall some basic features of the GUTs before going on to derive our upper and lower bounds on $\alpha$: more complete descriptions can be found in Refs. 3, 4 and 6. The strong interactions are described by a non-Abelian SU(3) gauge group, while the weak and electromagnetic interactions are described by a theory based on the group SU(2)$\times$U(1). The ratio of the gauge coupling constants $g_2$ and $g_1$ in this theory is arbitrary, and is parametrized by $\sin^2\theta_W$ and the fine structure constant $\alpha = \alpha_2 \sin^2\theta_W$, where $\alpha_2 \equiv g_2^2/4\pi$. The strong, weak and electromagnetic interactions act on fundamental fermions, the
quarks and leptons, which seem to be arranged \(^2\) in sets of 15 helicity states called "families" or "generations". GUTs combine the different SU(3), SU(2) and U(1) gauge interactions into a simple group \(G\), and assign the fermions to 15 dimensional representations of \(G\). It has been emphasized \(^3\) that it is by no means trivial that the observed fermions fall into representations of "low energy" symmetries which can be grand unified. Since they are kind enough to do so, it seems churlish to spurn the opportunity provided for constructing a GUT.

Grand unification introduces new gauge interactions between the fermions which violate baryon and lepton number conservation. They are mediated by heavy gauge bosons of masses \(m_X\) heavier than \(10^{14}\) GeV if protons and bound nucleons are not to have lifetimes shorter than the present experimental lower limit \(^7\) of \(10^{30}\) years. In their present formulations, GUTs do not include gravitation \(^8\).

Since quantum gravity effects (one-graviton exchange, etc.) become important at energies comparable with the Planck mass of \(1.2 \times 10^{19}\) GeV, it follows that the grand unification mass scale must be less than \(10^{19}\) GeV if this GUT philosophy is to make any sense.

The grand unification mass scale can be estimated \(^9\) using the renormalization group \(^10\). The idea is that while the SU(3), SU(2) and U(1) coupling constants are equal above this scale, below it they have different logarithmic dependences on the effective energy scale at which they are measured. This is how \(^9\) the SU(3) interactions can become strong at energies of order 1 GeV, while the SU(2)\(\times\)U(1) weak interactions can be weak at low energies. For the GUT philosophy to be viable, the "low energy" weak interactions must be neither too strong - in which case grand unification would occur at too low a mass scale and baryons would decay too fast - nor too weak - in which case grand unification would not occur before the Planck mass and it would have been nonsense not to have included gravity in the theory. We remarked before that the fine structure constant \(\alpha = \alpha_2 \sin^2 \theta_W\), and it is well-known \(^2\),\(^9\) that \(\sin^2 \theta_W\) is determined by the GUT philosophy to be \(3/8\) in the symmetry limit above the grand unification scale. (The variation of \(\sin^2 \theta_W\) below this scale has been computed \(^9\),\(^11\),\(^12\), and the value of 0.21 to 0.22 found at low energies lies within one standard deviation of the present experimental value \(^13\).) Knowing the symmetry value of \(\sin^2 \theta_W\) we are able to translate the "grand unifiability" restriction on the weakness of the weak interactions into upper and lower bounds on \(\alpha\) itself. In the leading order approximation, the renormalization group equations at energy scales \(Q\) between 100 GeV (\(m_W^+\), \(m_Z\)) and the grand unification scale \(m_X\) tell us that

\[
\frac{1}{\alpha(Q)} = \frac{1}{\pi} \ln(m_X/Q) + \frac{8}{3} \frac{1}{\alpha_3(Q)}
\]
if we just include GUT representations of fermions and gauge bosons. The renormalization group also tells us that if $\alpha_3(Q)$ is of order 1 on a typical hadronic energy scale then it should be of order 0.1 to 0.2 when $Q = 100 \text{ GeV}^{14}$. There is also a change in the effective $\alpha(Q)$ between its value at the Thompson limit $Q = 0$, which is where we define and measure the fine structure constant $\alpha$, and its value at $Q = 100 \text{ GeV}$:

$$\frac{1}{\alpha} \approx \frac{1}{\alpha(100 \text{ GeV})} + q$$

(2)

If we use these low energy renormalizations of $\alpha_3(Q)$ and $\alpha(Q)$ and require $10^{14} \text{ GeV} < m_X < 10^{19} \text{ GeV}$ in Eq. (1) we find $1/119 < \alpha < 1/174$ which we have rounded off to the values quoted earlier. Including a possible light Higgs boson into the leading order renormalization group equations changes these bounds by about one percent.

It must come as no surprise to a theoretist working on grand unification that $\alpha$ is so tightly constrained. She or he has known for some time that

$$\frac{m_X}{\Lambda_{\alpha_0}} = \exp \left\{ \frac{O(1)}{\alpha} + O(1) \ln \alpha + O(1) \alpha^0 + \cdots \right\}$$

(3)

and has put in the known value of $\alpha$ to determine $m_X$ in minimal GUTs within a factor of 10 or less $^{15}$. However, to our knowledge no-one has inverted the procedure and pointed out how tightly constrained a grand unifiable $\alpha$ must be. Indeed the previous $^4$ upper bound on $\alpha$ from GUTs was much weaker than ours here. The most precise estimates $^{15}$ of $m_X$ have gone beyond our leading order formula (1) to include all the $\ln \alpha$ and $\alpha^0$ terms of Eq. (3). In view of our modest intention to convey a qualitative message, we will not repeat all those calculations here. However, we would like to point out that they should enable one to make an even more refined estimate of $\alpha$ if baryon decay is ever observed. The reason is that the baryon lifetime $^{15}$ is very sensitive to the value of $m_X$:

$$\tau_{\text{baryon}} \approx 8 \times 10^{30^{11} \text{ years}} \times \left( \frac{m_X}{6 \times 10^{19} \text{ GeV}} \right)^4$$

(4)

Hence a determination of $\tau_{\text{baryon}}$ should enable us to fix $m_X$ within a factor of 1.5 or so. This 10-fold improvement in bounding $m_X$ would in turn enable us to estimate $\alpha^{-1}$ an order of magnitude more precisely, to ±3 say. The fact that
\( \alpha \) lies closer to the upper bound we have established gives us hope that \( m_x \) is closer to \( 10^{14} \) GeV than to \( 10^{19} \) GeV, and more detailed estimates in popular GUTs strongly suggest \(^{15}\) that the baryon lifetime is sufficiently short to be detectable in the forthcoming generation of experiments. If this turns out to be the case, and \( \sin^2 \theta_W \) and \( \alpha \) still lie within the range of GUTs, we will find it difficult to contain our enthusiasm. To an even greater extent than was realized before, things must be as they are.
REFERENCES

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Earlier attempts at unification of all the particle interactions adopted a
different philosophy from that pursued here :


5) For alternative philosophies of unification, see : J.C. Pati and A. Salam -
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6) J. Ellis - Lectures presented at the 21st Scottish Universities' Summer


8) For foolhardy attempts to do so, see :
J. Ellis, M.K. Gaillard, L. Maiani and B. Zumino - "Unification of the Funda-
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14) In agreement with recent direct measurements in $e^+e^-$ annihilation :

15) See, for example :