CABIBBO ANGLES WITHOUT SCALARS

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ABSTRACT

A simple model of dynamical symmetry breaking is presented in which the mass matrices of the up and down quarks are related to a (ETC) vector boson mass-matrix in the 10-100 TeV range. Realistic quark masses and Cabibbo angles can be obtained. The problem of flavour-changing neutral interactions in theories of dynamical symmetry breaking is discussed in the context of this model.
Over the past couple of years, a programme has been developed to replace the
elementary Higgs scalar of the Weinberg-Salam model by a composite one. Mecha-
nisms have been found to break $SU_L(2) \times U_Y(1) + U^c(1) \times U'_{\Delta}(1)$ in the desired way $^1$ pre-
serving $M_W = M_Z \cos \theta_W$ and giving masses to the quarks and leptons $^2$. We will
call "extended technicolour" (ETC) the resulting type of theory. Although several
ETC models have been presented in the literature, we know of none which generate
Cabibbo angles in a simple way. We feel that this lacuna has made it difficult to
discuss certain questions explicitly such as the problem of neutral flavour chan-
ging interactions $^3$ and the problem of introducing CP violation (à la Kobayashi-
Maskawa $^4$) into ETC-type theories. It is our purpose here to fill this lacuna.

THE MODEL

Let us restrict ourselves to models in which ETC commutes with the fa-
miliar quantum numbers :

$$SU_L(2) \times U_Y(1) \times SU(3) \times G^{ETC}$$

$$(2, \frac{1}{6}, 3, m)_L$$

$$(1, \frac{2}{3}, 3, m')_R$$

$$(1, -\frac{1}{3}, 3, m'')_R$$

(1)

The numbers in parentheses give the transformation properties of the fermions under
the gauge group. To have colour anomaly-free we require $2n = n' + n''$. The ano-
malies involving the hypercharge $Y$ are to be cancelled by the introduction of
leptons, those involving ETC by the introduction of additional colour singlet
fermions (as in the example below). To have ETC instantons conserve baryon number $^5$,
we require $2t = t' + t''$ where $t$, $t'$ and $t''$ are the traces of the squares of
the representation matrices for $n$, $n'$ and $n''$. Finally, we want $M_W = M_Z \cos \theta_W$
and $m_u \neq m_d$ $^6$. A simple solution to these conditions is :

$$SU_L(2) \times U_Y(1) \times SU(3) \times SU^{ETC}(N+3)$$

$$[2, \frac{1}{6}, 3, N+3]_L = \{ U_{Li} \} \begin{bmatrix} t_L & c_L & u_L \\ \end{bmatrix}$$

$$[1, \frac{2}{3}, 3, N+3]_R = \{ U_{Ri} \} \begin{bmatrix} t_R & c_R & u_R \\ \end{bmatrix}$$

$$[1, -\frac{1}{3}, 3, (N+3)^*]_R = \{ D_{Ri} \} \begin{bmatrix} b_R & s_R & d_R \\ \end{bmatrix}$$

(2)
where \( i = 1, 2, \ldots, N \) and where \( SU^{ETC}(N+3) \) is spontaneously broken down to \( O^{TC}(N) \) by V.E.V.'s of order a few TeV to a few hundred TeV. The TC gauge coupling constant becomes strong at a scale of order 1 TeV (provided \( N \geq 5 \)) and the technifermion condensates \(^7\) :

\[
\langle \bar{U} U \rangle_c = \langle \bar{D} D \rangle_c = \mu_{TC}^3
\]

(3)

break \( SU_L(2) \times SU_Y(1) \times SU^C(3) \rightarrow U^6.M.(1) \times SU^C(3) \) in the usual way.

The adjoint representation decomposes as follows :

\[
\left( \frac{(N+3)^2 - 1}{2} \right)_{ETC} = \left( \frac{1}{2} N(N-1) \right) + \left( \frac{1}{2} N(N+1) - 1 \right) + 6(N) + 9(1) \]

(4)

On the right-hand side of Eq. (4), the first term represents the massless TC gluons, the second term represents massive vector bosons which couple only to techniquarks, the third term represent massive vector bosons \( (W^a_i) \) and \( W^i_a = W^a_{i+} \), \( a = 1, 2, 3 \) is the generation index; that couple techniquarks to ordinary quarks, and finally the fourth term represents nine vector bosons which couple ordinary quarks to each other.

The ordinary quarks acquire mass through the four-fermion interactions mediated by the \( W^a_i \). The \( (mass)^2 \) matrix of the latter has the general form :

\[
W^a_i \mu^a \mu^b W^i_b + \frac{1}{2} \left( W^a_i \mu^a \mu^b W^b_i + h.c. \right)
\]

(5)

where \( \mu^a = u \) and \( \mu^\dagger = u' \). The first term in (5) is \( SU(n) \) symmetric whereas the second is only \( O^{TC}(n) \) symmetric. In terms of the Hermitian fields :

\[
W^1_{aj} = \frac{1}{\sqrt{2}} \left( W^a_j + W^d_i \right)
\]

\[
W^2_{aj} = \frac{1}{i\sqrt{2}} \left( W^a_j - W^d_i \right)
\]

(6)
the \((\text{mass})^2\) matrix is 6x6 dimensional, real and symmetric:

\[
\frac{1}{2} \begin{pmatrix}
W_{1j}^1 & W_{1j}^2 \\
W_{2j}^1 & W_{2j}^2
\end{pmatrix}
\begin{pmatrix}
(\mu_1 + \mu'_1)_{ad} & (\mu_2 - \mu'_2)_{ac} \\
-(\mu_2 + \mu'_2)_{bd} & (\mu_1 - \mu'_1)_{bc}
\end{pmatrix}
\begin{pmatrix}
W_{dj}^1 \\
W_{cj}^2
\end{pmatrix}
\] (7)

where \(u = \mu_1 + i\mu_2\), \(u' = \mu_1' + i\mu_2'\) (\(\mu_1\), \(\mu_1'\) and \(\mu_2\) are symmetric; \(\mu_2\) is antisymmetric).

The four-fermion interactions mediated by the \(W^a_1\) are:

\[
\frac{g^2}{2} \left[ J_{i\mu}^a \lambda^b \gamma^\mu J_{i\mu}^{i\mu} + \frac{i}{2} \left( J_{i\mu}^a \lambda^{b\mu} J_{i\mu}^{b\mu} + \text{c.c.} \right) \right]
\] (8)

where:

\[
\begin{align*}
J_{i\mu}^a &= \bar{u}_a Y^a \gamma_\mu U^c + \bar{d}_a Y^a \gamma_\mu D^c - \bar{D}_R^i \gamma_\mu D_R^a \\
\lambda &= \lambda_1 + i\lambda_2 = \lambda^\dagger \\
\lambda' &= \lambda_1' + i\lambda_2' = \lambda'^\dagger
\end{align*}
\] (9)

and

\[
\begin{pmatrix}
\lambda_1 + \lambda'_1 & \lambda_2 - \lambda'_2 \\
-\lambda_2 - \lambda'_2 & \lambda_1 - \lambda'_1
\end{pmatrix} = M^{-2}
\] (10)

is the inverse of the \((\text{mass})^2\) matrix given in (7). The ordinary quark masses can readily be calculated by Fierz transforming (8). One finds:

\[
\begin{align*}
m_d &= \frac{g^2}{2} (\mu^a_{\tau c})^3 \lambda' = m_d \dagger \\
m_u &= \frac{g^2}{2} (\mu^a_{\tau c})^3 \lambda^\dagger = m_u \dagger
\end{align*}
\] (11)
In general, $\lambda'$ and $\lambda^T$ are not simultaneously diagonalizable and a non-trivial Cabibbo-Kobayashi-Maskawa 4) mixing matrix results.

$\mu_u$ and $\mu_d$ must satisfy certain constraints following from the positive-definiteness of $M^2$. In the CP conserving limit where $\lambda_2 = \lambda'_2 = 0$, we have:

$$m_u = O_u \Delta_u O_u^T \quad m_d = O_d \Delta_d O_d^T$$

$$\Delta_u = \text{diag} \left( \epsilon_u m_u, \epsilon_e m_e, \epsilon_\tau m_\tau \right)$$

$$\Delta_d = \text{diag} \left( \epsilon_d m_d, \epsilon_s m_s, \epsilon_b m_b \right)$$

where each $\epsilon_q$ can independently be chosen to be $\pm 1$ and the matrices $O_u$ and $O_d$ are orthogonal. The matrix of weak mixing angles is:

$$O_L = O_u^T O_d$$

In the case of two generations, the positive-definiteness of $M^2$ implies:

$$\text{Tr} \left( m_u \pm m_d \right) > 0, \quad \text{Det} \left( m_u \pm m_d \right) > 0$$

These constraints appear to be satisfied in the real world, e.g. (all masses in GeV):

$$m_e = 1.5 \quad m_s = 1.15 \quad m_d = 0.1$$

$$m_u = 0.007 \quad -\sin^2 \theta_c = 0.05$$

provided that $\epsilon_u = \epsilon_c = +1, \quad \epsilon_s \epsilon_d = -1$.

A particular choice of (elementary or composite) Higgs representation content to break ETC + TC will of course imply additional constraints. Consider for example, the model:
\[ \text{SU}_L(2) \times U_Y(1) \times \text{SU}_E^0(3) \times \text{SU}_A(8) \times O_T^{T'} \]

\[
\begin{align*}
(2, \frac{1}{6}, 3, 8, 1)_L \\
(1, \frac{2}{3}, 3, 8, 1)_R \\
(1, -\frac{1}{3}, 3, \bar{8}, 1)_R \\
(1, 0, 1, \bar{8}, 6)_L \\
(1, 0, 1, 1, 6)_L
\end{align*}
\]

which is anomaly-free with respect to colour, ETC and T'C. The T'C gauge coupling becomes strong at \( \Lambda^{T'C} = 100 \text{ TeV} \). The fermions that carry T'-colour form condensates that transform as \( \frac{1}{3}, \bar{8} \) and \( \frac{5}{3} \)-sym = \( \frac{36}{36} \) of \( \text{SU}^{ETC}(8) \). Let us represent the non-singlet condensates \( \frac{8}{} \) and \( \frac{36}{} \) by the V.E.V.'s of scalar fields \( \phi^i \) and \( \phi^{ij} = \phi^{ji} \). \( \text{SU}^{ETC}(8) \) breaks down to \( O_T^{T'}(5) \) if:

\[
\langle \phi^i \rangle_0 = (x \ y \ z \ o \ o \ o \ o \ o)
\]

\[
\langle \phi^{ij} \rangle_0 = e^{i\delta}
\begin{pmatrix}
    a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & b & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & c & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & d & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & e & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & f & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & g & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{g}
\end{pmatrix}
\]

Note that \( O(5) \) invariant \( \langle \phi^i \rangle_0 \) and \( \langle \phi^{ij} \rangle_0 \) can always be rotated into the form (17) with \( a, b, c \) and \( v \) real, and with \( x, y, z \) complex.

Equation (17) implies (for two generations in which case we forget \( z \) and \( c \)):
Comparing (18), (10) and (11) shows that Cabibbo-Kobayashi-Maskawa \(^4\) angles are naturally present in the model. Moreover, CP violation is introduced in a novel way which contrasts with Ref. 8). The \(\theta\) parameters of the various non-Abelian gauge groups in Eq. (16) can all be set equal to zero before symmetry-breaking by chiral rotations on the fermion multiplets. The Lagrangian of the model is thus CP conserving. However CP is violated spontaneously if the ETC breaking condensates have certain relative phases, e.g., \(\text{Im}(\mathbf{x}^\dagger_y) \neq 0\). The ETC vector boson mass matrix, and consequently the up and down quark mass matrices, are then CP violating. It is natural in this model that \(\arg \det n_u = 0\) since \(n_u^\dagger = n_u\) and since it is natural that the TC condensates be CP conserving \(^6\). Unfortunately \(\arg \det n_d\) is in general different from zero once CP violation has been introduced. This implies that the \(\theta\) parameter of QCD is not in fact zero for this model; however, it would be zero in any model which had \(n_d = n_d^\dagger, n_u = n_u^\dagger\).

Perhaps our model is (11) half-way there.

When restricted to two generations, the model does not appear quite able to reproduce the "real world" parameters of Eq. (15). A best fit produced:

\[
\begin{align*}
\alpha &= 10.4 \text{ TeV}, \quad b = -1.6 \text{ TeV}, \quad N = 4.3 \text{ TeV} \\
\lambda &= 38 \text{ TeV}, \quad \text{and} \quad \gamma &= 14 \text{ TeV}
\end{align*}
\]

(19)

which yields:

\[
\begin{align*}
m_c &= 1.5 \text{ GeV}, \quad m_s = 117 \text{ MeV}, \quad m_u = 28 \text{ MeV}, \\
m_d &= 7.5 \text{ MeV}, \quad \text{and} \quad \theta_c = 13^\circ
\end{align*}
\]

(20)
It appears that in the two-generation case at least, the ETC representation content of the scalar fields whose V.E.V.'s break $SU^2_{TC}(7) \rightarrow U^2_{TC}(5)$ needs to contain more than just the $7$ and $28$ of SU(7). In principle, it is possible even in the model of Eq. (16) that such higher dimensional representations may be present at some level since we cannot exclude condensates of $4,6,\ldots$ $T'$ coloured fermions. We did not attempt to fit the data on three-generation parameters.

**FLAVOUR CHANGING NEUTRAL CURRENTS**

ETC theories risk conflict with the experimental constraints on the $K_1-K_2$ and $D_1-D_2$ mass differences in at least two different ways: flavour-changing neutral (FCN) processes can be mediated both by light pseudo-Goldstone bosons (PGB) and by the heavy ETC vector bosons which are $T_1$ singlets – see Eq. (14) – and gauge the generation group. Let us discuss these successively within the context of our model.

When completed by the introduction of leptons and technileptons, our model has the light colour-singlet pseudoscalars $P^0, P^3, P^\pm$. The $P^\pm$ acquire mass from the electroweak interactions of order 8–14 GeV. None of them acquire mass from the ETC interactions. Quark-lepton unification is necessary to give masses to the $P^0$ and $P^3$. Such unification can be done in our model, for example in the manner of Ref. 5. The masses of $P^{0,3}$ must then be smaller than 2 or 3 GeV as a result of the experimental upper limit on $K_L^0 \rightarrow \mu e$ decay 5,12).

Because our model is monophasic, the order $0(g^2 m^2_{\nu}/M_{\nu})$ couplings of $P^{0,3}$ are flavour-diagonal 13). However, even in monophasic models, one must expect small $0(g^2 m^2_{
u}/M_{\nu}^2)$ flavour changing couplings of the $P^{0,3}$ to quarks 14). These can be explicitly calculated in our model. We find:

$$\frac{F_P}{4 f^3 \langle FF \rangle_0} \left\{ (P^0 + P^3) \bar{u} (m_u)^2 i Y_{\ell} u + (P^0 - P^3) \frac{1}{2} \bar{d} ([m_d, m_u] + i Y_{\ell} \{m_d, m_u\}) d \right\}$$

(21)
where \( F_p = 250 \text{ GeV} \), \( <FF> = \frac{3}{15N} \approx (290 \text{ GeV})^3 \sqrt{3/N} \), \( m_u = m_{u_1} + m_{u_2} = m_u^+ \) and \( m_d \) has been diagonalized. The couplings (19) do not violate the experimental constraints on the \( K_1^\pm - K_2^\pm \) mass difference if \( m_{p,0,3} \geq 500 \text{ MeV} \), but they do suggest \(^{14}\) that the \( p_{0,3} \) be seen in the decays of the bottom quark: \( b \to s + p_{0,3} \).

Let us now turn to the flavour-changing neutral processes mediated by the heavy ETC vector bosons which are TC singlets and gauge the generation group. In our model, they are due to the four-fermion interaction:

\[
\sum_{a,b=0}^{3} \frac{9}{2} J_{\mu}^{a} \left( \frac{1}{\sqrt{m^2}} \right)^a b J_{b\mu} \tag{22}
\]

where \( \mathcal{M} \) is the \( 4 \times 4 \) (in the case of two generations) real symmetric mass matrix of those vector bosons, and:

\[
J_{\mu}^{a} = \bar{u} \gamma_{\mu} \frac{1}{2} \tau^{a} u + \bar{d} \gamma_{\mu} \frac{1}{2} \tau^{a} d - \bar{d} \gamma_{\mu} \frac{1}{2} \tau^{a_T} d
\tag{23}
\]

for \( a = 0, 1, 2, 3 \); \( \gamma^0 = \sqrt{N/N+2} \mathbf{1} \). It is clear that (22) will give too large \( K_1^\pm - K_2^\pm \) and \( D_1^\pm - D_2^\pm \) mass differences unless \( \mathcal{M} \) is large enough.

In the model of Eq. (16), \( \mathcal{M} \) can be readily calculated from the V.E.V.'s (17)-(19), the resulting four-fermion interaction (22) worked out and its \(|\Delta S| = 2\), \(|\Delta C| = 2\) pieces extracted. We omit the details but give the resulting orders of magnitude. For \(|\Delta S| = 2\) interactions,

\[
\begin{align*}
\frac{L}{|\Delta S| = 2} &= O\left( \frac{1}{\chi^2} \right) \left( \bar{\mathcal{D}} \gamma_\alpha d \right)^2 \\
&\quad + O\left( \frac{1}{\chi^2}, \frac{\theta_c^2}{\alpha^2} \right) \left( \bar{\mathcal{D}} \gamma_\alpha \gamma_5 d \right)^2 + \text{l.c.}
\end{align*}
\tag{24a}
\]
and putting in the numerical values (19) of the V.E.V.'s we get:

\[
\mathcal{L}^{0}_{|\Delta S|=2} = \left[ O(10^{-9}) (\bar{\tau}_Y \gamma_\mu d)^2 + O(10^{-9}) (\bar{\tau}_Y \gamma_\mu s d)^2 \right] \text{GeV}^{-2} + h.c. \tag{24b}
\]

Comparing this numerical estimate with the coefficient \(O(10^{-12})\ \text{GeV}^{-2}\) of the \(|\Delta S|=2\) interaction responsible for \(K^0 - \bar{K}^0\) mixing, we find a discrepancy by about three orders of magnitude. The estimate (24) agrees roughly with the guess of Ref. 3). A more interesting situation arises for \(|\Delta C|=2\) transitions:

\[
\mathcal{L}^{0}_{|\Delta C|=2} = O\left( \frac{a_1^2}{x^2} \right) (\bar{u} Y d c)^2 + h.c. \tag{25a}
\]

and putting in the numerical values (19) of the V.E.V.'s we get

\[
\mathcal{L}^{0}_{|\Delta C|=2} = O(10^{-11}) \ \text{GeV}^{-2} (\bar{u} Y d c)^2 + h.c. \tag{25b}
\]

which is on the borderline of phenomenological acceptability. The estimate (25) is considerably smaller than the guess of Ref. 3), due to identifiable extra suppression factors.

While the \(|\Delta C|=2\) interaction is characterized a priori by the exchange of bosons of \((\text{mass})^2 = x^2\) [corresponding to the breakdown of SU(7) to SU(6) in the two-generation version of our model] it can be rotated away unless the symmetry between the \(c\) quark and the charge \(\frac{2}{3}\) techniquarks is broken [SU(6) \(\rightarrow\) SU(5) in our model] entailing the extra suppression factor of \(y^2/x^2\) in (25a). But a vector representation \(\phi^1\) cannot break SU(7) to SU(5) by itself: there must be another representation \(\phi^2\) in our case) with respect to which the orientation of \(\phi^1\) is fixed. This is done by the V.E.V. a which entails an extra suppression factor of \(a^2/x^2\) in (25a). One naturally asks why there is no similar suppression of the \(|\Delta S|=2\) interaction. This seems to be connected with the fact \(11\) that the charge \(-\frac{1}{3}\) quark mass matrix is symmetric while that for the charge \(\frac{2}{3}\) quarks is hermitian, implying that orthogonal subgroups of SU(7) play an essential rôle in discussing \(|\Delta S|=2\) interactions. It is clear that orthogonal generators will contain pieces that both "raise" and "lower" the generation index, whereas unitary generators may be chosen as pure "raising" or "lowering" operators. Unitary groups therefore offer the possibility of constructing a "schizon" model in which the \(\Delta G = \pm 1\) bosons are forbidden to mix by some extra symmetry. The construction of such models was pursued \(15\) before 1974 as an alternative to the GIM \(16\) mechanism. We cannot \(3\).
implement GIM in the standard ETC scenario - can we make a schizoid ETC model?
Our model succeeds in making Cabibbo half-schizophrenic, but not totally. Perhaps
a model with Hermitian mass matrices in both the charge $\frac{2}{3}$ and charge $-\frac{1}{3}$ sectors
could be completely schizoid, as well as solving the problem of the QCD $\theta$ parameter as mentioned earlier.

We have looked for other ways to avoid discrepancies with the $K^0-\bar{K}^0$ and
$D^0-\bar{D}^0$ phenomenology, but without much success. If the dimensionality of
the V.E.V. representations breaking ETC $\rightarrow$ TC could be made arbitrarily high,
then $\lambda_0$ could be made arbitrarily large compared to the mass-matrix $M$ of the
vector bosons that give masses to the quarks \cite{17}. However, it goes against the
grain of a dynamical scheme to produce condensates of very high ETC representa-
tion content. A more plausible mechanism might involve the fact that $M$ is
singular when $\Delta_\pi = \det(m_\pi^2 - m_\pi) = 0$. It so happens that both $\Delta_\pi$ and $\Delta_-$(\textit{\textsuperscript{r}})
are relatively small when computed in terms of the two-generation parameters \cite{15}. It is possible that in the case of three generations $\Delta_\pi$ and $\Delta_-$ might be even
closer to vanishing. In that case even relatively large V.E.V.'s can still yield
large quark masses, and one might hope to avoid the conflict \cite{3} between the sizes
of the quark masses and the rates of flavour-changing neutral processes.

We hope that, despite its failure to solve the problems of flavour-
changing neutral interactions and CP violation, our model may play a constructive
role in future model-building.
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17) One of us (P.S.) thanks Larry Abbott for useful conversations on this point.