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SECOND ORDER CORRECTIONS TO THE DRELL-YAN PROCESS
AT LARGE TRANSVERSE MOMENTUM

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A B S T R A C T

The gluon radiative corrections to parton processes producing lepton pairs at large transverse momentum are calculated. The corrections are large and positive and bring theory into better agreement with the data.

In this letter we present the results of a calculation of the gluon radiative corrections to parton processes which lead to the production of lepton pairs at large transverse momentum. In particular we consider the processes which contribute to the difference of the cross-sections for beams of particles B and antiparticles $\bar{B}$ scattering off a target T . Examples of these "non-singlet cross-sections" are the differences $\pi^- - \pi^+$ or $\bar{p} - p$. These combinations are experimentally attractive because they eliminate possible backgrounds from leptons resulting from hadronic production processes. Theoretically, these combinations remove the parton subprocesses involving initial gluons which would be subject to large uncertainties because of the ill-determination of the gluon distributions. As a side benefit it significantly reduces the number of Feynman diagrams which contribute.

Nevertheless the number of remaining diagrams is sufficiently large that we considered very carefully what we would learn over and above the $O(\alpha_s)$ calculation ¹⁾ before we undertook this project. Our motivation was twofold; the prediction of a large positive correction ²⁾ in the total Drell-Yan cross-section and its subsequent experimental measurement ³⁾ leaves an open question. Does the transverse momentum distribution have large corrections and if so are those corrections uniform throughout the range of all transverse momentum? There are experimental indications that the answer to this question is affirmative ⁴⁾. Our second motivation was more theoretically oriented. A large amount of theoretical effort ⁵⁾ has been devoted to the consideration of the transverse momentum distribution in a regime in which $\Lambda \ll q_T \ll Q$ (Q is the invariant mass of the muon pair) and an algorithm has been proposed ⁶⁾ which describes the transverse momentum distribution to all orders in $\ln Q/q_T$. An explicit second-order calculation is an important test of such a scheme and an important step in its implementation.

The factorization theorem ⁷⁾ implies that the Drell-Yan scattering cross-section differential in the rapidity y of the muon pair may be written in the form

$$\frac{d\sigma}{dQ^2 dq_T^2 dy} = \sum_{i,j} \int dx_1 dx_2 \left[\sigma_{ij}(\alpha_s(M^2), p_1, p_2) \right] \bigg|_{\substack{p_1 = x_1 P_1 \\ p_2 = x_2 P_2}} f_i(x_1, M^2) f_j(x_2, M^2) \quad (1)$$

where M^2 is a normalization scale of order of the large invariants and σ_{ij} is the parton cross-section with the incoming parton momenta set equal to rescaled values of the incoming hadron momenta. The sum on i and j runs over all parton types which contribute at a given order in α_s . For massless on-shell partons the perturbative expansion of σ_{ij} in powers of α_s is,

$$\sigma_{ij} \equiv \frac{s d\tilde{\sigma}_{ij}}{dQ^2 dt du} = \left[\frac{s d\sigma_{ij}^{(1)}}{dQ^2 dt} \delta(s+t+u-Q^2) + \frac{s d\tilde{\sigma}_{ij}^{(2)}}{dQ^2 dt du} \theta(s+t+u-Q^2) + \dots \right] \quad (2)$$

The superscripts in parentheses indicate the order in $\alpha_s(M^2)$ and $s = (p_1+p_2)^2$, $t = (p_1-q)^2$, $u = (p_2-q)^2$.

For the special case of the non-singlet combination which we calculate Eq. (1) reduces to

$$\frac{d\sigma^{B-\bar{B}}}{dQ^2 dq_T^2 dy} = \sum_{ff'} \int dx_1 dx_2 \Delta_{ff'} \bigg|_{\substack{p_1=x_1 p_1 \\ p_2=x_2 p_2}} \cdot \left(\bar{q}_f^B(x_1, M^2) - q_f^B(x_1, M^2) \right) \left(q_{f'}^T(x_2, M^2) - \bar{q}_{f'}^T(x_2, M^2) \right) \quad (3)$$

where in the derivation of the above equation we have used the equality of the distribution of quarks in particle B and antiquarks in antiparticle $\bar{B}$. The sum of f and f' runs over all flavours. In this paper we report on a calculation of the quantity $\Delta_{ff'} \equiv (\sigma_{q_f q_{f'}}, -\sigma_{q_f q_{f'}})$ and the numerical values which we obtain for the differential cross-section for the particular case of pion beams.

In lowest order in α_s only annihilating quarks of the same flavour contribute as shown in the diagrams of Fig. 1. The result is

$$\frac{s d\sigma_{\bar{q}q}^{(1)}}{dQ^2 dt} = \frac{2\alpha_s^2 e_q^2}{9Q^2} \frac{4}{3} \frac{\alpha_s}{s} \left(\frac{u}{t} + \frac{t}{u} + \frac{2sQ^2}{ut} \right) \quad (4)$$

In $O(\alpha_s^2)$ the diagrams which contribute to the difference $\Delta_{ff'}$ are shown in Figs. 2, 3, 4. Fortunately for the quantity $\Delta_{ff'}$ there are extensive cancellations in these diagrams. In particular $\Delta_{ff'}$ is zero unless $f = f'$. For $f = f'$, as well as the virtual and real gluon radiation corrections to the lowest order process Figs. 2, 3a, we have the processes

$$q + \bar{q} \rightarrow q + \bar{q} + \gamma^* , \quad q + q \rightarrow q + q + \gamma^* \quad (5)$$

shown in Fig. 3b. Here again extensive cancellations occur when we form the quantity $\Delta_{ff'}$ but some direct and interference terms remain. The analytic results for these diagrams calculated using the programme SCHOONSCHIP will be presented elsewhere ⁸⁾.

The final object of our calculation is the second term in Eq. (2) from which all the ultra-violet divergences have been removed and the mass-singularities have been factored. The divergences present in the intermediate stages of the cancellation were controlled by dimensional regularization. Renormalization is performed in the $\overline{MS}$ scheme. The explicit form of the finite cross-section $d\tilde{\sigma}^{(2)}$ depends on the size of the finite pieces which were factored along with the mass singularities. In our calculation this ambiguity is removed by defining the quark densities beyond the leading order in terms of the deep inelastic scattering function F_2 ^{*)}.

In Fig. 4 we have plotted the numerical results for the non-singlet pion cross-section as a function of q_T . Our parton densities for the proton are derived from the CDHS deep inelastic scattering data ⁹⁾ for F_2

$$u - \bar{u} = \frac{2.27}{\sqrt{x}} (1-x)^\beta, \quad d - \bar{d} = \frac{1.29}{\sqrt{x}} (1-x)^{\beta+1}, \quad \beta = 2.8$$

and for the π^- distribution we take

$$\bar{u} - u = d - \bar{d} = \frac{.75}{\sqrt{x}} (1-x)$$

in agreement with the valence value sum rules. We have included an intrinsic parton transverse momentum k_T using the procedure of Altarelli et al., Ref. 1) with a Gaussian distribution for k_T and $\langle k_T^2 \rangle = 1 \text{ GeV}^2$. The normalization of the differential cross-section is sensitive to the magnitude and method of inclusion of the intrinsic transverse momentum but for values of $q_T \gtrsim 2\langle k_T \rangle$ the slope of the distribution is determined by perturbation theory alone. However, no parametrization of the intrinsic transverse momentum can make the first order formula alone [Eq. (4)] fit both the experimental transverse momentum distribution and its average value. The second order corrections appear to remove this discrepancy. Although strictly speaking our formulas are only applicable for the non-singlet cross-sections for beams of incident particles containing valence antiquarks, the contribution of the gluons is small especially at larger values of τ . For example, in this region our formula could be applied directly to $\pi^- N$ cross-sections. In Fig. 5 we plot the quantity :

$$K(q_T) = \left(\frac{1}{q_T} \frac{d\sigma}{dQ d q_T} \right) [O(\alpha_s) + O(\alpha_s^2)] / \left(\frac{1}{q_T} \frac{d\sigma}{dQ d q_T} \right) [O(\alpha_s)]$$

*) For a detailed description see especially Ref. 2) last paper. The alternative procedure (i.e., to define quark densities in terms of Drell-Yan total cross-section) will be considered in Ref. 8).

to show that the correction does not simply multiply the lowest order result by a constant factor, but it varies as q_T changes.

In conclusion the correction to the non-singlet cross-section for the production of lepton pairs at large transverse momentum is large and positive. The situation thus seems very similar to the case of the Drell-Yan total cross-section. As before reliable predictions will require the isolation of the large terms and their resummation. But already at a phenomenological level the second-order term seems to be needed to find agreement with the data.

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FIGURE CAPTIONS

- Fig. 1 Lowest order diagrams for the process $q + \bar{q} \rightarrow \gamma^* + G$.
- Fig. 2 Virtual corrections to $q + \bar{q} \rightarrow \gamma^* + G$.
- Fig. 3 (a) Diagrams for the process $q + \bar{q} \rightarrow \gamma^* + G + G$.
 (b) Diagrams for the process $q + \bar{q} \rightarrow \gamma^* + q + \bar{q}$. The lower row of diagrams also represents $q + q \rightarrow \gamma^* + q + q$.
- Fig. 4 $(1/q_T)(d\sigma^{\pi^-\pi^+}/dQ dq_T)$ vs. q_T . The lower line is the $O(\alpha_s)$ contribution and the upper line is the total including $O(\alpha_s^2)$ corrections. The cross-section is per nucleon for scattering off a target which is 60% neutrons. The scale of the running coupling constant is Q^2 and $\Lambda^2 = 0.16 \text{ GeV}^2$. The dependence of the result on the choice of the parameters and on scaling violations which were not included will be described in Ref. 8). The average value of the transverse momentum is $\langle q_T \rangle = 0.7 \text{ GeV}$ in lowest order and $\langle q_T \rangle = 1.0 \text{ GeV}$ at order α_s^2 .
- Fig. 5 We show $K(q_T)$ (see text) vs. q_T for $q_T \gtrsim 2.5 \text{ GeV}$. All the parameters have been chosen as for Fig. 4. The full line refers to the results obtained without including an intrinsic transverse momentum ; the dashed line contains the effects of an intrinsic transverse momentum as explained in the text.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5