Ref.TH.3064-CERN

INFRA-RED BEHAVIOUR OF YANG-MILLS THEORY AT HIGH TEMPERATURES

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A B S T R A C T

We employ Monte Carlo methods to probe the infra-red structure of $SU(2)$ Yang-Mills theory at high temperatures. In particular, we study the free energy of the gauge invariant non-Abelian magnetic flux. The Monte Carlo data support the conjecture that the magnetic flux is screened. The inverse magnetic screening length provides a non-perturbative infra-red cut-off for the quantum statistics of the $SU(2)$ gauge theory and is estimated to be about $0.24 g^2 T$.

Ref.TH.3064-CERN

13 April 1981

In recent years a great deal of attention has focused on understanding the behaviour of Yang-Mills theories at non-zero temperatures. It has been argued that these theories undergo a phase transition at some critical temperature T_c ¹⁾⁻³⁾. Both spontaneously broken and confining non-Abelian gauge theories are supposed to undergo such a phase transition, and the properties of the high temperature phase ($T > T_c$) are expected to be similar in the two cases.

Monte Carlo studies ⁴⁾ of the pure $SU(2)$ gauge system at non-zero temperature support the conjecture by Polyakov ²⁾ and Susskind ³⁾ that the theory undergoes a transition from a confined (glueball) phase for $T < T_c$ to an unconfined (gluon) phase for $T > T_c$. For $SU(2)$, T_c is estimated to be between 200 and 230 MeV.

Perturbative attempts at understanding the nature of the phase above T_c have so far been hampered by the presence of virulent infra-red divergences that arise from the static magnetic sector of the theory ⁵⁾. More precisely, the leading infra-red divergences in the quantum statistics of Yang-Mills theory for $T > T_c$ are those of a three-dimensional Yang-Mills theory. It has been conjectured ⁵⁾ that for $T > T_c$, the theory develops a non-perturbative infra-red cut-off $m(T)$ (the "magnetic" mass) of order $g^2(T)T$, where $g(T)$ is the effective gauge coupling at temperature T . This would imply the screening of static non-Abelian magnetic fields beyond $m^{-1}(T)$ ^{*)}.

Motivated by these considerations we investigate in this note the large distance behaviour of Yang-Mills theory for $T > T_c$. Our discussion employs the following ingredients : a) the gauge invariant non-Abelian magnetic flux introduced by 't Hooft ⁷⁾ ; b) Wilson's $SU(2)$ lattice gauge theory ⁸⁾; and c) Monte Carlo methods. The results that we obtain are consistent with the conjecture that at temperatures $T > T_c$, the non-Abelian gauge theory possesses a non-zero magnetic mass $m(T)$. For $SU(2)$, we estimate $m(T)$ to be approximately $0.24 g^2(T)T$. The mechanism that causes the magnetic screening is not well understood. The screening is probably due to the presence of a plasma of magnetic excitations ^{**)} [see Gross et al., Ref. 5)].

^{*)} It is generally accepted ^{5),6)} that static non-Abelian electric fields are Debye screened beyond $m_e^{-1}(T)$, with $m_e(T) \approx \sqrt{\frac{2}{3}} g(T)T$ for $SU(2)$

^{**)} Non-Abelian magnetic fields of a confining gauge symmetry are also screened below T_c . This is probably due to some magnetic condensate in the vacuum.

We consider Wilson's lattice gauge theory with gauge group $SU(2)$ on a four-dimensional hypercubic lattice. The lattice spacing is denoted by a and the number of lattice sites is $N_s \times N_s \times N_s \times N_t$. Here N_s denotes the number of sites along any spatial axis and N_t the number of sites along the Euclidean time axis. The partition function is given by

$$Z = \int \prod_{\{ij\}} dU_{ij} \exp[-b_0 S(U)] , \quad (1)$$

where the link variable U_{ij} , associated with each nearest neighbour pair of sites i and j , is an element of $SU(2)$. The $SU(2)$ invariant action S is a sum over all elementary plaquettes P in the lattice,

$$S(U) = \sum_P \left[1 - \frac{1}{2} \text{Tr}(U_P) \right] , \quad (2)$$

where

$$U_P = U_{ij} U_{jk} U_{kl} U_{li} . \quad (3)$$

The indices i, j, k and l label the sites of the plaquette P . In Eq. (1), $b_0 = 4/g_0^2$, where g_0 denotes the bare coupling on the lattice. In the continuum limit ($a \rightarrow 0$), we have the one-loop renormalization group relation

$$\Lambda_L a \simeq \exp\left(-\frac{3\pi^2}{11} b_0\right) , \quad (4)$$

where $\Lambda_L \simeq 2 \text{ MeV}$ is the lattice scale parameter. It follows from Ref. 9) that (4) is well satisfied for $b_0 \geq 2.17$.

Throughout the paper we shall employ periodic boundary conditions on the $N_s^3 \times N_t$ lattice. The periodicity along the spatial axes is imposed to eliminate surface effects, whereas periodicity along the Euclidean time direction introduces a non-zero temperature $T \equiv \beta^{-1} = (N_t a)^{-1}$, when $N_t \ll N_s$. For $N_t = N_s$ the system is supposed to be effectively at zero temperature.

It has been shown in Ref. 4) that the $SU(2)$ gauge system undergoes a transition from a confined phase to an unconfined phase at a critical temperature $T_c \approx 230 \text{ MeV}$ ^{*}). The quantity that is normally employed to study this phase transition is the thermal Wilson loop :

$$L = \text{Tr}(U_{i_1 i_2} U_{i_2 i_3} \cdots U_{i_{N_t} i_1}) , \quad (5)$$

where $i_1, i_2, \dots, i_{N_t}$ denotes a sequence of successive lattice sites along the positive Euclidean time axis. The expectation value of L determines the phase of the system. It is related to the free energy F of an isolated quark in a thermal bath at temperature T by

$$\langle L \rangle = \exp(-\beta F) . \quad (6)$$

The action given by (2) is invariant under the global symmetry transformation $U \rightarrow -U$ on each link that starts from a selected spacelike hyperplane and points towards the positive time direction. Under such a transformation $L \rightarrow -L$. In the infinite volume limit ($N_s a \rightarrow \infty$) and $T \leq T_c$, $\langle L \rangle = 0$ which implies quark confinement. For $T > T_c$, $\langle L \rangle \neq 0$ which means the above global symmetry is broken and quarks are liberated. In practice, however, the space volume is always finite so that, in the latter case, there exists a non-zero probability for L to flip sign. Due to this tunnelling phenomenon, the expectation value of L may become zero and quark confinement will persist, even though the temperature is bigger than T_c .

We have studied this phenomenon on $4^3 \times 2$ and $4^3 \times 3$ lattices. For the Monte Carlo calculations we employ the non-Abelian 120 element icosahedral subgroup of $SU(2)$ which provides a very good approximation to the full $SU(2)$ group for $b_0 \lesssim 5$ ¹⁰⁾. Some typical results are presented in Figs. 1 and 2. For the $4^3 \times 2$ lattice, the critical temperature T_c corresponds to $b_0 \approx 1.9$. We find that L does not flip sign for $b_0 \geq 2.2$ (Fig. 1) (the number of lattice sweeps is of order 10^4). Therefore, the $4^3 \times 2$ system, when probed with the number of sweeps of order 10^4 , is in the unconfined phase for $b_0 \geq 2.2$. For the $4^3 \times 3$ lattice, T_c corresponds to $b_0 \approx 2.17$. However, we now find an appreciable flipping in the sign of L , at least till $b_0 \approx 4$ (Fig. 2) (the number of sweeps is again of order 10^4). The expectation value of L essentially averages to zero. Thus,

^{*}) This estimate follows if one uses the one-loop renormalization group result (4). For the two-loop case, $T_c \approx 210 \text{ MeV}$. We shall employ the one-loop result throughout.

the $4^3 \times 3$ system, when probed with sweeps of order 10^4 , is in the confined phase in the interesting range of values of b_0 . Finite volume effects in this case prevent the phase transition from the confined to the unconfined phase.

[Note that for a given temperature $T = (N_t a)^{-1}$, the space volume $(N_s a)^3$ of the $4^3 \times 2$ lattice is a factor $(1.5)^3$ larger than the corresponding space volume of the $4^3 \times 3$ lattice.]

In order to probe the large distance behaviour of the SU(2) magnetic fields in the unconfined (gluon plasma) phase, we consider the quantity ^{7),11),12)}

$$\mu(b_0, N_s, N_t) = -\frac{1}{2} \ln\left(\frac{Z_t}{Z}\right), \quad (7)$$

where Z is defined in (1) with periodic boundary conditions (p.b.c.) and Z_t is also given by (1) with p.b.c., except that S is now replaced by the twisted action

$$S_t = \sum_P \left[1 - \frac{1}{2} \epsilon_P \text{Tr}(u_P) \right]. \quad (8)$$

Here $\epsilon_P = -1$ for all plaquettes P that protrude from the $x_1 = x_2 = 0$ plane with links in the positive x_1 and x_2 directions. Otherwise $\epsilon_P = +1$. The quantity T_t represents the free energy, at temperature $T = (N_t a)^{-1}$, of one unit of SU(2) magnetic flux in a box of space size $(N_s a)^3$. The magnetic flux is along the x_3 direction.

The quantity μ cannot, however, be reliably computed by Monte Carlo methods. We compute instead its derivative

$$\frac{\partial \mu(b_0, N_s, N_t)}{\partial b_0} = \frac{1}{2} \left[\langle S_t \rangle_t - \langle S \rangle \right], \quad (9)$$

where $\langle \dots \rangle_t$ denotes the expectation value in the ensemble defined by (1), with S replaced by S_t .

The Monte Carlo data for the quantity $\partial \mu / \partial b_0$ on a $4^3 \times 2$ lattice are shown in Fig. 3. The data have been obtained with the number of lattice sweeps per point ranging between 8,000 and 48,000. A large number of sweeps is necessary since the difference between the two quantities in (9) happens to be rather small.

We now wish to argue that the Monte Carlo data in Fig. 3 are consistent with the conjecture ⁵⁾ that, at high temperatures, the non-Abelian magnetic flux is screened for distances much bigger than the screening length $m^{-1}(T) = (\kappa g^2(T)T)^{-1}$ [$m(T)$ is the infra-red cut-off or the so-called magnetic mass], where $g(T)$ is the effective gauge coupling. We estimate κ to be about 0.24.

For $T > T_c$ and $N_s a \gtrsim m^{-1}(T)$ we expect, if screening exists, that μ falls off exponentially with the cross-section of the space box perpendicular to the magnetic flux direction. Thus, in this situation,

$$\mu \simeq c e^{-m^2(T) (N_s a)^2} = c e^{-16 \kappa^2 \left(\frac{N_s}{N_t}\right)^2 b^{-2}(T)}, \quad (10)$$

where ¹³⁾

$$b(T) \equiv \frac{4}{g^2(T)} \simeq \frac{11}{3\pi^2} \ln\left(\frac{4T}{\Lambda}\right). \quad (11)$$

Here Λ is the continuum renormalization scale parameter which is related to Λ_L by $\Lambda \simeq 57.5 \Lambda_L$ ¹⁴⁾. From Eqs. (11) and (4),

$$b(T) \simeq b_0 - \frac{11}{3\pi^2} \ln\left(\frac{57.5 N_t}{4}\right), \quad (12)$$

which is expected to hold for $T > T_c$ and $b_0 \gtrsim 2.17$. Differentiating (10) with respect to b_0 one obtains

$$\frac{\partial \mu}{\partial b_0} \simeq 32 c \kappa^2 \left(\frac{N_s}{N_t}\right)^2 b^{-3}(T) \exp\left[-16 \kappa^2 \left(\frac{N_s}{N_t}\right)^2 b^{-2}(T)\right]. \quad (13)$$

Within its region of validity, this formula fits well the Monte Carlo data of Fig. 3 for $c \simeq 1.53$ and $\kappa \simeq 0.24$. Thus, the conjecture that static non-Abelian magnetic fields are screened for $T > T_c$ is fully consistent with the Monte Carlo data.

Note that the spatial dimension of the box, $N_s a$, is equal to the magnetic screening length $(\kappa g^2 T)^{-1}$ for

$$b_0 \simeq 4 \kappa \left(\frac{N_s}{N_t}\right) + \frac{11}{3\pi^2} \ln\left(\frac{57.5 N_t}{4}\right) \simeq 3.2. \quad (14)$$

Formula (13) is not expected to hold for $b_0 \gg 3.2$. In this case, $N_s a \ll (\kappa g^2 T)^{-1}$ and screening effects cannot be seen. One finds ^{11),15)}, instead, that $(\partial\mu/\partial b_0) \approx \alpha_1 b_0^{-2} + \dots$. For the case under discussion the perturbative behaviour starts at $b_0 \sim 5$.

In Fig. 4 we present Monte Carlo data on $\partial\mu/\partial b_0$ for the $4^3 \times 3$ lattice. The data, which have been taken with the number of lattice sweeps per point ranging between 8,000 and 48,000, provides further evidence that confinement persists in this case even though the temperature exceeds T_c . Indeed, the data can be reasonably well fitted by employing the considerations of Ref. 11) that are relevant in the confining phase. The considerations that led us to Eq. (13) do not apply to the $4^3 \times 3$ lattice.

To summarize, we have presented evidence in support of the conjecture put forward by several authors that non-Abelian magnetic fields are screened in the high temperature unconfined phase ^{*)}. The inverse screening length provides an infra-red cut-off in the quantum statistics of Yang-Mills theory.

ACKNOWLEDGEMENTS

We wish to thank C. Lang and C. Rebbi for useful discussions and for their computer programme. We also thank J. Ellis and W. Nahm for comments.

^{*)} This is also expected to hold for spontaneously broken gauge theories for $T > T_c$.

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FIGURE CAPTIONS

Figure 1 Typical results for the thermal Wilson loop L on a $4^3 \times 2$ lattice for $b_0 = 2.3$ (Fig. 1a), and $b_0 = 2.8$ (Fig. 1b). The points shown are averages over 100 consecutive sweeps.

Figure 2 Typical results for the thermal Wilson loop L on a $4^3 \times 3$ lattice for $b_0 = 2.4$ (Fig. 2a), and $b_0 = 3$ (Fig. 2b). The points shown are averages over 100 (Fig. 2a) and 300 (Fig. 2b) consecutive sweeps.

Figure 3 Plot of $\partial\mu/\partial b_0$ versus b_0 for $4^3 \times 2$ lattice. The number of lattice sweeps per point ranges between 8,000 and 48,000. The curve corresponds to Eq. (13) with $c \approx 1.53$ and $\kappa \approx 0.24$.

Figure 4 Plot of $\partial\mu/\partial b_0$ versus b_0 for $4^3 \times 3$ lattice. The number of lattice sweeps per point ranges between 8,000 and 24,000.

Figure 1a

Figure 1b

Figure 2a

Figure 2b

Figure 3

Figure 4