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QCD CORRECTIONS TO DIRECT LEPTON PRODUCTION
IN HADRONIC COLLISIONS

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ABSTRACT

We present detailed calculations for the QCD α_s corrections to the single lepton spectrum in the process hadron + hadron $\rightarrow W^\pm, Z^0$ or $\gamma \rightarrow \ell^\pm$ + anything. The correction terms are analyzed using two different factorization prescriptions and numerical results are shown for two different parton distribution parametrizations.

We find rather large α_s corrections, which significantly modify the shape of the lowest order Drell-Yan contribution. Their magnitude is remarkably insensitive to the different conventions we have studied.

1. INTRODUCTION

Quantum chromodynamics (QCD) in its perturbative form has been used to explain an imposing variety of hadronic phenomena at high energies. Calculations of deep inelastic scattering (DIS) to first non-leading order in α_s have been completed¹⁾, and the total e^+e^- annihilation cross-section is known to order α_s^2 ²⁾. In hadronic collisions the lepton pair production (LPP) process is known completely to order α_s ³⁾, and partially to order α_s^2 ⁴⁾. Similarly, some non-leading effects have been calculated for large P_T direct photon production⁵⁾ (α_s^2 diagrams), and for large P_T hadron production⁶⁾ (α_s^3 diagrams). When QCD results and experiments are confronted, the perturbative calculations can reproduce the data rather well. In LPP particularly, the factor $K \approx 2 - 2.5$, which has been found experimentally to multiply the lowest order Drell-Yan prediction⁷⁾, is consistent with the $O(\alpha_s)$ QCD calculations. However, these successes should not overshadow the fact that the convergence of the QCD perturbative series is not yet well established⁸⁾, as is exemplified by the size of the first order corrections in LPP. Although the precise magnitude of the corrections depends on the renormalization scheme used, and on the definition of the parton distributions beyond the leading approximation⁹⁾, some general statements can be made about their origin. Large corrections in the next-to-leading order calculations arise from diagrams with infra-red or soft singularities. For example, in LPP the quark-antiquark QCD diagrams give a much bigger correction than the quark-gluon QCD diagrams, even for proton-proton collisions. Conversely, the $O(\alpha_s^2)$ corrections from the $qq \rightarrow qq\gamma$ subprocess in direct photon production are small compared to the leading order result.

In this paper we study the $O(\alpha_s)$ QCD corrections to direct lepton production (DLP), where the lowest order (parton model) contribution is given by the Drell-Yan quark-antiquark annihilation mechanism¹⁰⁾. This process is interesting in the forthcoming pp and $\bar{p}p$ colliders, where the weak vector bosons $W^\pm$ of the standard $SU(2) \times U(1)$ model are expected to be discovered through their leptonic decay modes $W^\pm \rightarrow \ell^\pm \nu$ ^{11,12)}. The diagrams we study are the same as in massive lepton pair production, but the observed final state is different. In LPP, one integrates over the momenta of the decay leptons keeping their invariant mass fixed, whereas in DLP the momentum of one of the leptons is fixed and the other lepton (neutrino for $W^\pm$) momentum is integrated over.

We use the method of dimensional regularization, where the infinities from the calculation of the QCD diagrams become poles in $\epsilon = (4-n)/2$. The main motivation for this approach is its relative simplicity compared to mass regularization schemes, and especially the fact that the complete calculation of DIS beyond

leading order has been done using this scheme. According to the factorization theorem of QCD¹³⁾, the pole parts of the parton level cross-section are then absorbed into the parton distributions defined in DIS. This procedure, together with a discussion of axial currents in dimensional regularization, will be the subject of Section 2, where we also present the formulae for the matrix elements we study. In Section 3 we give successively the γ , Z^0 , and $W^\pm$ contributions to the single lepton spectrum, and the size of the QCD correction terms is discussed and compared with the lowest order prediction¹⁴⁾. Section 4 deals with the question of the vector boson transverse momentum, and reviews different ways to estimate its effect on the single lepton spectrum. Two appendices contain respectively kinematical and other notations, and the derivation of some n dimensional phase space integrals which can be used for any process with $2 \rightarrow 3$ particle kinematics.

2. DESCRIPTION OF THE HIGHER ORDER CALCULATIONS

We aim at calculating the spectrum of a charged lepton produced via the Feynman diagrams of Fig. 1. This involves integration over the unobserved momenta in the diagrams of Figs. 1b, 1c, and 1d, which leads to mass and infra-red singularities. Going to n dimensional momentum space, the divergences appear as poles in $\epsilon = (4-n)/2$. Although the rules for the Dirac γ matrix manipulations in n dimensions are well known for amplitudes involving vector currents¹⁵⁾, there are ambiguities in the case of axial currents. We choose here γ_5 to anticommute with all γ^μ 's in n dimensions:

$$\begin{aligned} \{\gamma_5, \gamma^\mu\} &= 0 & \mu = 1, \dots, n \\ \gamma_5^2 &= 1 \end{aligned} \quad (1)$$

which has been shown to be consistent with the Ward identities¹⁶⁾. We are now left with ambiguities in the evaluation of traces involving an odd number of γ_5 matrices. It has been shown in Ref. 16) that it must be so, and that it is a manifestation in n dimensions of the axial anomaly. However, since we are calculating physical cross-sections, our final results should be unambiguous. More precisely, the following traces are not well defined to order ϵ :

$$\text{Tr} [\gamma^\alpha \gamma^\beta \gamma^\delta \gamma^\epsilon \gamma_5] = (1 + c_1 \epsilon) 4i \epsilon^{\alpha\beta\delta\epsilon} \quad (2)$$

$$\text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\delta \gamma^\epsilon \gamma_\mu \gamma^\mu \gamma_5] = (n + c_2 \epsilon) \text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\delta \gamma^\epsilon \gamma_5] \quad (3)$$

where in the latter trace, the result depends on whether one uses $\gamma_\mu \gamma^\mu = n \cdot I$ or anticommutes γ_μ with γ_5 to perform the sum over repeated indices. The term $1 + c_1 \epsilon$ factorizes in front of the axial part of the matrix element, and the term $c_2 \epsilon$ gives rise to constant terms proportional to c_2 in the final result. We dispose of the ambiguities by noting that in a calculation of DIS (with axial current) the same $1 + c_1 \epsilon$ term factorizes in front of the axial part and the same terms proportional to c_2 appear. Thus, in a comparison of the two processes, the ambiguities cancel out, and we can choose $c_1 = c_2 = 0$ to be used consistently for all processes.

We turn now to the evaluation of the relevant matrix elements. Throughout the paper we work in the Feynman gauge, and the strong interaction coupling constant is denoted by $\alpha_s = g^2/4\pi$ (see Appendix A for other notations). The Drell-Yan matrix element, including the virtual corrections at order α_s (Figs. 1a and 1b), is proportional to

$$\begin{aligned} T_{DY} \Big|_{A_q, B_q}^{A_l, B_l} &= \text{Tr}[k_2 \gamma_\mu k_1 \gamma_\nu (A_l + B_l \gamma_5)] \\ &\quad \otimes \text{Tr}[p_2 \gamma^\mu p_1 \gamma^\nu (A_q + B_q \gamma_5)] \cdot \left(1 + \frac{\alpha_s}{2\pi} \langle \frac{4}{3} \rangle \cdot V\right) \\ &= 8 \cdot [A_l A_q (\hat{k}_1^2 + \hat{k}_2^2 - \epsilon \hat{s}^2) + B_l B_q (\hat{k}_1^2 - \hat{k}_2^2)] \cdot \left(1 + \frac{\alpha_s}{2\pi} \langle \frac{4}{3} \rangle \cdot V\right) \end{aligned} \quad (4)$$

where the virtual contribution V is given in Eq. (B26) in Appendix B. The matrix element due to the QCD annihilation diagrams of Fig. 1c is proportional to

$$\begin{aligned} T_A \Big|_{A_q, B_q}^{A_l, B_l} &= -g^2 \cdot \text{Tr}[k_2 \gamma_\mu k_1 \gamma_\nu (A_l + B_l \gamma_5)] \\ &\quad \otimes \text{Tr}\left[p_2 \left(\gamma^\sigma \frac{1}{p_1 - k} \gamma^\mu + \gamma^\mu \frac{1}{k - p_2} \gamma^\sigma\right) p_1 \left(\gamma^\nu \frac{1}{p_1 - k} \gamma^\nu + \gamma^\nu \frac{1}{k - p_2} \gamma^\nu\right) (A_q + B_q \gamma_5)\right] \\ &= -g^2 \cdot 16 A_l A_q \cdot k^2 \left\{ 1 + \frac{\hat{s} - 2\hat{k}_1 - k^2}{\hat{k}} - \frac{\hat{k}_1^2 + \hat{k}_2^2 + \hat{s}(\hat{k}_1 + \hat{k}_2 + k^2)}{\hat{k}\hat{u}} \right. \\ &\quad \left. - \epsilon \left[1 + 2 \frac{\hat{s} - \hat{k}_1 - \hat{k}_2 - k^2}{\hat{k}} - \frac{(\hat{k}_1 + \hat{k}_2 + \hat{s})^2 - \hat{s}^2 + 2\hat{s}k^2}{\hat{k}\hat{u}} \right] + [\hat{u} \leftrightarrow \hat{k}, \hat{k}_1 \leftrightarrow \hat{k}_2] \right\} \end{aligned} \quad (5)$$

$$-g^2 16 B_l B_q \left\{ \frac{k^2(\hat{s} + 2\hat{k}_2 + k^2)}{\hat{s}} + \frac{k^2(\hat{k}_1 - \hat{k}_2)}{\hat{k}\hat{u}} + \varepsilon \frac{\hat{s}\hat{k}_1 - k^2(2\hat{k}_1 + \hat{k}_2) - k^4}{\hat{s}} - [\hat{u} \leftrightarrow \hat{k}, \hat{k}_1 \leftrightarrow \hat{k}_2] \right\} \quad (5) \text{ cont.}$$

where we have dropped ε terms, which do not contribute to the final result.

The matrix element for the QCD Compton diagrams of Fig. 1d can be obtained by crossing symmetry from the annihilation result (here p_2 is the gluon momentum)

$$\begin{aligned} T_c |_{A_l, B_l}^{A_q, B_q} &= -g^2 16 A_l A_q \cdot k^2 \left\{ \frac{\hat{k} - 2(\hat{k}_1 + k^2)}{\hat{s}} + \frac{\hat{s} + 2(\hat{k}_1 + \hat{k}_2)}{\hat{k}} \right. \\ &+ \frac{2}{\hat{s}\hat{k}} [(\hat{k}_1 + \hat{k}_2 + k^2)^2 + \hat{k}_1^2 - \hat{k}_2 k^2] - 2\varepsilon \left[\frac{\hat{s} + \hat{k}_2 + k^2}{\hat{k}} + \frac{\hat{k}_2^2 + k^4}{\hat{s}\hat{k}} \right] \Big\} \\ &- g^2 16 B_l B_q \left\{ k^2 \left[\frac{2(\hat{k}_1 + k^2) - \hat{k}}{\hat{s}} + \frac{\hat{s} + 2(\hat{k}_1 + \hat{k}_2)}{\hat{k}} \right] \right. \\ &- \left. \frac{2k^2(2\hat{k}_1 + \hat{k}_2 + k^2)}{\hat{s}\hat{k}} \right] + \varepsilon \frac{\hat{s}\hat{k}_1 - k^2\hat{k}_2}{\hat{k}} \Big\} \end{aligned} \quad (6)$$

where we have again neglected non-contributing ε terms in the axial part. The spin- and colour-averaged square matrix elements can now be written in the form ($i = DY, A, C$)

$$|\overline{\mathcal{M}}_i|^2 (q\bar{q} \text{ or } qg \rightarrow W^+ \rightarrow l^+ X) = \frac{e^4}{16 \sin^4 \theta_w} \frac{C_i}{(k^2 - M_W^2)^2 + (\Gamma_W M_W)^2} \cdot T_i \Big|_{\frac{1}{2}, -\frac{1}{2}}^{\frac{1}{2}, \frac{1}{2}} \quad (7)$$

$$\begin{aligned} |\overline{\mathcal{M}}_i|^2 (q\bar{q} \text{ or } qg \rightarrow \gamma, Z^0 \rightarrow l^+ X) &= \frac{e^4}{4} \cdot C_i \cdot \\ &\left\{ \frac{Q_q^2}{k^4} \cdot T_i \Big|_{1,0}^{1,0} + \frac{1}{\sin^4 \theta_w (\cos^4 \theta_w)} \frac{1}{(k^2 - M_Z^2)^2 + (\Gamma_Z M_Z)^2} \cdot T_i \Big|_{A_l, B_l}^{A_q, B_q} \right. \\ &- 2Q_q \frac{1}{\sin^2 \theta_w \cos^2 \theta_w} \cdot \frac{k^2 - M_Z^2}{(k^2 - M_Z^2)^2 + (\Gamma_Z M_Z)^2} \cdot T_i \Big|_{a_q, b_q}^{a_l, b_l} \Big\} \end{aligned} \quad (8)$$

where the colour factors C_i are given by

$$C_{DY} = \frac{1}{3} \quad , \quad C_A = \frac{4}{9} \quad , \quad C_C = \frac{1}{6} \quad (9)$$

The formulae are written down for the case where p_1 is the quark momentum and p_2 the antiquark (DY and A cases) or the gluon (C case) momentum. The change quark $\leftrightarrow$ antiquark in the initial state or $\ell^+ \leftrightarrow \ell^-$ in the final state amounts to a change of sign of the axial part.

The rules of integration given in Appendix B can now be used to derive the direct lepton cross-sections. Before quoting the final results, some general features of the calculation will be reviewed.

In the annihilation process, the $1/\hat{t}\hat{u}$ terms in Eq. (5) lead to an $1/\epsilon^2$ divergence, which is compensated by a similar term coming from the vertex corrections. After this cancellation, the annihilation and the Compton processes have the same structure. Independent of the $SU(2) \times U(1)$ gauge bosons involved, they have diverging terms of the form

$$-\frac{1}{\epsilon} \left(\frac{4\pi e^{-\gamma_E} \mu^2}{k_{1T}^2} \right)^\epsilon \frac{\alpha_s}{2\pi} \int_0^1 dz P_{qp}(z) \cdot k_0 \frac{d\hat{\sigma}^{DY}}{d^3k_1} \left(x_1, \frac{x_2}{z} \right) \quad (10)$$

arising from the vanishing of $\hat{t} = -2p_2 \cdot p_3$ (similar terms with $x_1 \leftrightarrow x_2$ come from the vanishing of $\hat{u}$). $P_{qp}(z)$'s are the Altarelli-Parisi parton decay distributions, and $z = x_2/(1-x_1)$ (defined in Appendix B) is the fraction of momentum of parton p carried by the quark or antiquark in the parallel configuration ($\hat{t}=0$). Following the well known procedure, one absorbs such terms as in Eq. (10) into the Born cross-section by a redefinition of the parton distributions

$$f_q(x, \mu^2) \rightarrow f_q(x, \mu^2) - \frac{1}{\epsilon} \left(\frac{4\pi e^{-\gamma_E} \mu^2}{k_{1T}^2} \right)^\epsilon \frac{\alpha_s}{2\pi} \sum_p \int_x^1 \frac{dz}{z} P_{qp}(z) f_p\left(\frac{x}{z}\right) \quad (11)$$

In the $\overline{MS}$ renormalization scheme, terms proportional to $-(1/\epsilon) \cdot (4\pi e^{-\gamma_E})^\epsilon$ are dropped, and choosing $\mu^2 = k_{1T}^2$, the left-hand side of Eq. (11) becomes the non-scaling parton distribution, the scale of which is set by the transverse momentum of the observed lepton.

The parton distributions defined above are process-independent and satisfy QCD evolution equations (beyond the leading approximation), which depend only on the anomalous dimensions. This definition will be denoted by BKKU¹⁷⁾. In another, more phenomenological approach¹⁸⁾ (denoted by AEM), the quark distributions are defined to $O(\alpha_s)$ in deep inelastic scattering, in the same way as to leading order, namely by $F_2(x, Q^2) = x \sum_q Q_q^2 q(x, Q^2)$. Now α_s corrections for DIS are absorbed into

the quark distributions, and the relation between the two definitions is given by

$$f_q^{AEM}(x, Q^2) = f_q^{BKKU}(x, Q^2) + \frac{\alpha_s}{2\pi} \sum_{i=q, \bar{q}} C_i \int_x^1 \frac{dz}{z} F_i(z) \cdot f_i\left(\frac{x}{z}, Q^2\right) \quad (12)$$

with

$$\begin{aligned} F_q(z) &= [z^2 + (1-z)^2] \cdot \ln \frac{1-z}{z} + 6z(1-z) \\ F_q(z) &= a \delta(1-z) + b(z) \\ a &= -\left(\frac{9}{2} + \pi^2/3\right) \\ b(z) &= (1+z^2) \cdot \left[\frac{\ln(1-z)}{1-z} \Big|_+ - \frac{\ln z}{1-z} \right] - \frac{3}{2} \frac{1}{1-z} \Big|_+ + 3 + 2z \end{aligned} \quad (13)$$

and the colour factors are $C_q = 4/3$ and $C_g = 1/2$. When using the AEM prescription for other processes, care has to be taken to subtract the terms proportional to $F_1(z)$ from the final result. In the following, we will present results for both prescriptions in common formulae with the definition $F_g = F_q = 0$ for the BKKU case.

In the actual numerical calculation and in comparison with experiments, it is a good approximation to use the same parton distribution parametrizations for both cases. In direct lepton production at collider energies, the range at k_{1T} covered by the detectors probes mainly the small x region of parton distributions, and it has been shown from a fit to present deep inelastic scattering data¹⁹⁾ that the low n moments ($n \leq 4$) of parton distributions in the two cases coincide within error bars.

In the virtual γ production case, there exists another divergent term, which is associated with the pole in the photon propagator when the final state leptons are collinear. This term is of the form

$$-\frac{1}{\epsilon} \left(\frac{4\pi e^2 \mu^2}{k_{1T}^2} \right)^\epsilon \frac{\kappa}{2\pi} \int \frac{d\xi}{\xi} D_{\ell\gamma}(\xi) \cdot k_{10} \frac{d\hat{\sigma}}{d^3k_1} qP \rightarrow \gamma X \quad (14)$$

where $\xi = x_1 + x_2$ is the fraction of momentum of γ carried by the observed lepton and $D_{\ell\gamma}(\xi)$ is the corresponding decay function. This divergence related to the $\gamma \rightarrow \ell^+ \ell^-$ vertex appears because we have assumed the leptons to be massless. To take the mass effects into account, we have adopted the following procedure. Since the $\hat{t} = 0$ (or $\hat{u} = 0$) divergence does not overlap with $k^2 = 0$ divergences,

we can separate the latter ones and perform the phase space integrations with massive leptons in four dimensions ($\epsilon = 0$) in this sector. We find exactly the same result as for the $m_\ell = 0$ case in n dimensions, except that $-1/\epsilon$ is replaced by $\ln(k_{1T}^2 \cdot \xi^2/m_\ell^2)$. In this way, we can correctly take into account the potentially large correction terms, which appear when the mass setting the strength of the scaling violations in the structure functions is different from the maximum kinematically allowed for the diverging propagator. We have neglected terms of order m_ℓ^2/k_{1T}^2 and also finite contributions coming from terms like m_ℓ^2/k^4 , which arise from the calculation of the matrix elements with non-zero lepton masses. Past experience has taught us that they are numerically irrelevant.

Having thus disposed of all diverging terms appearing in the evaluation of diagrams in Fig. 1, we can now write down the $O(\alpha_s)$ QCD corrections to the single lepton spectrum. This will be done in the next section, where we analyze in turn the γ and then the $W^\pm$ and Z^0 contributions.

3. ANALYSIS OF THE QCD CORRECTION TERMS

3.1 Analytical formulae

We first give our results from our calculation of the Compton and annihilation processes for direct lepton production via the virtual photon. The Compton result is, at the parton level:

$$k_{10} \frac{d\hat{\sigma}}{d^3k_1} (q\bar{q} \rightarrow \gamma^* q \rightarrow \ell^\pm X) = \langle \frac{1}{6} \rangle 2\alpha^2 Q_q^2 \frac{\alpha_s(k_{1T}^2)}{2\pi} \frac{1}{s^2} \cdot \frac{1}{x_2} \cdot \left\{ \left[(1-x_1)^2 + x_1^2 \right] \cdot \left[P_{qg}(z) \cdot \ln \frac{1-z}{x_1 z} - F_g(z) \right] - P_{qg}(z) + \left[\frac{x_2^2 + \xi^2}{\xi^2} \cdot \ln \frac{1-\xi}{x_1 x_2} - 2 \frac{x_2^2 + \xi^2 - x_2 \xi}{\xi^2} \right] \cdot D_{\ell\gamma}(\xi) - 2 \left[1 - 2\xi + 2\xi^2 + 2x_2^2 \right] \cdot \ln \frac{\xi}{z} - 2x_1 z^2 + 4(1-x_1+x_1^2) - \frac{x_2^2 - x_1 x_2 (1-\xi)}{\xi^2} \right\} \quad (15)$$

with $P_{qg}(z) = (1-z)^2 + z^2$, $D_{\ell\gamma}(\xi) = (1-\xi)^2 + \xi^2$, and $F_g(z)$ is given in Eq. (13) for the AEM case ($F_g = 0$ for the BKKU case). We also give here the formula for the direct photon conversion cross-section, the derivation of which was discussed at the end of the previous section:

$$k_{10} \frac{d\hat{\sigma}}{d^3k_1} (q\bar{q} \rightarrow \gamma q ; \gamma \rightarrow \ell^\pm X) = 2\alpha\alpha_s(k_{1T}^2) Q_q^2 \langle \frac{1}{6} \rangle \frac{1}{s^2} \cdot \frac{1}{x_2} \cdot \left(\frac{x_2^2 + \xi^2}{\xi^2} \cdot \frac{\alpha}{2\pi} D_{\ell\gamma}(\xi) \cdot \ln \frac{k_{1T}^2 \xi^2}{m_\ell^2} \right) \quad (16)$$

We write down the total $q\bar{q}$ parton cross-section (i.e., the sum of the Drell-Yan, virtual and annihilation contributions) in the form

$$\begin{aligned}
 & k_{10} \frac{d\hat{\sigma}}{d^3k_1} (q\bar{q} \rightarrow \gamma^* X \rightarrow \ell^\pm X) \\
 &= k_{10} \frac{d\hat{\sigma}^{DY}}{d^3k_1} \cdot \left\{ 1 + \left\langle \frac{4}{3} \right\rangle \frac{\alpha_s(k_{1T}^2)}{2\pi} \cdot \left[\pi^2 - 8 - 2a - \ln^2 \frac{x_1}{1-x_1} - 3 \ln x_1 / (1-x_1) \right] \right\} \\
 &+ \left\langle \frac{4}{9} \right\rangle 2\alpha^2 Q_q^2 \cdot \frac{\alpha_s(k_{1T}^2)}{2\pi} \cdot \frac{1}{3^2} \left\{ \frac{1}{x_2} \left[\left[(1-x_1)^2 + x_1^2 \right] \cdot \left[P_{q\bar{q}}(z) \cdot \left(\ln \frac{1-z}{x_1 z} + z \ln \frac{1-z}{1-x_1} \right) \right. \right. \right. \\
 &\quad \left. \left. \left. - b(z) \right] + 1-z + z(1-z+2x_1 z) \ln \frac{1-z}{1-x_1} + 2 \left[(1-x_1)^2 + x_1^2 + 2x_2^2 \right] \ln z \right] \right. \\
 &\quad \left. + \frac{1}{x_1} \left[x_1 \leftrightarrow x_2 \right] \right. \\
 &\quad \left. - \frac{2}{\xi} \left[\frac{x_1^2 + x_2^2}{x_1 x_2 \xi} \cdot D_{\ell\gamma}(\xi) + 2 \right] \cdot \ln \xi + \frac{x_1^2 + x_2^2}{x_1 x_2 \xi} \cdot D_{\ell\gamma}(\xi) \cdot \ln \frac{1-\xi}{x_1 x_2 \xi} \right\} \quad (17)
 \end{aligned}$$

where $P_{q\bar{q}}(z) = (1+z^2)/(1-z)|_+$, and a and $b(z)$ are given in Eq. (13) for the AFM case (again they vanish for the BKKU case). The Drell-Yan cross-section is

$$k_{10} \frac{d\hat{\sigma}^{DY}}{d^3k_1} = \left\langle \frac{1}{3} \right\rangle 2\alpha^2 Q_q^2 \frac{1}{3^2} \delta(1-x_1-x_2) \cdot [x_1^2 + x_2^2] \quad (18)$$

and the photon conversion contribution is now

$$\begin{aligned}
 & k_{10} \frac{d\hat{\sigma}}{d^3k_1} (q\bar{q} \rightarrow g\gamma; \gamma \rightarrow \ell^\pm X) = 2\alpha\alpha_s(k_{1T}^2) Q_q^2 \left\langle \frac{4}{9} \right\rangle \frac{1}{3^2} \cdot \\
 & \quad \otimes \frac{x_1^2 + x_2^2}{x_1 x_2 \xi} \cdot \frac{\alpha}{2\pi} D_{\ell\gamma}(\xi) \cdot \ln \frac{k_{1T}^2 \xi^2}{m_\ell^2} \quad (19)
 \end{aligned}$$

We will now write down the corresponding formulae for the massive vector boson case. As the interference term between γ and Z^0 is numerically negligible at the Drell-Yan level, we do not consider its QCD corrections here at all. The $W^\pm$ and Z^0 production results can be written in common formulae with the differences hidden into the vector and axial couplings $A_{\ell q}^A$ and $B_{\ell q}^B$, and into the normalization factors $N = \sin^4\theta_w$ for $W^\pm$ and $N = \sin^4\theta_w \cos^4\theta_w/4$ for Z^0 . For the Compton process we get

$$\begin{aligned}
 k_{10} \frac{d\hat{\sigma}}{d^3k_1} (q\bar{q} \rightarrow q W^\pm \text{ or } Z^0 \rightarrow l^\pm X) &= \langle \frac{1}{6} \rangle \cdot \frac{\alpha^2}{N} \cdot \frac{\alpha_s(k_\pi^2)}{2\pi} \\
 &\left\{ \frac{A_L A_q [x_1^2 + (1-x_1)^2] + B_L B_q [(1-x_1)^2 - x_1^2]}{2} \cdot \frac{z^2}{x_2} \right. \\
 &\cdot \left[BW(z\hat{s}) \cdot [P_{qq}(z) \cdot \left(\ln \frac{1-z}{x_1 z} - \ln r - \frac{z\hat{s}-M^2}{\Gamma M} \cdot \theta \right) + 1 - F_q(z)] \right. \\
 &\quad \left. + P_{qg}(z) \cdot \frac{\theta}{z\hat{s}\Gamma M} \right] + \frac{A_L A_q - B_L B_q}{2} \cdot \frac{1}{z^2} \\
 &\cdot \left[J(x_1, x_2) - \frac{2z}{x_2} \left[(x_1 + z - 2x_1 z - \frac{M^2}{z}) \cdot \frac{M\theta}{\Gamma} + (x_1 + z - 2x_1 z - 2\frac{M^2}{z}) \ln r \right] \right] \Bigg\}
 \end{aligned} \tag{20}$$

where

$$BW(k^2) = \frac{1}{(k^2 - M^2)^2 + (\Gamma M)^2}$$

and

$$\begin{aligned}
 J(x_1, x_2) &= \frac{x_2(1-2x_1-2x_2)}{(x_1+x_2)^2} \cdot \left(\ln r_0 + \frac{M\theta_0}{\Gamma} \right) - 2 \frac{x_2 - x_1(1-x_1-x_2)}{(1-x_1)^2} \\
 &+ \frac{x_1 x_2}{x_1 + x_2} \left(2 + \frac{x_1(1-x_1) - x_2(1+x_1)}{(x_1+x_2)^2} \right) \cdot \left[\frac{\xi\hat{s}}{k_{1r}^2} + \frac{2M^2}{k_\pi^2} \cdot \ln r_0 + \frac{M^3\theta_0}{k_{1r}^2\Gamma} \right]
 \end{aligned}$$

The functions r_0 , θ_0 , r and θ are defined in Appendix B, Eqs. (B21) and (B25), and the couplings are given in Appendix A. Calculation of the $q\bar{q}$ cross-section to $O(\alpha_s)$ gives

$$\begin{aligned}
 k_{10} \frac{d\hat{\sigma}}{d^3k_1} (q\bar{q} \rightarrow q W^\pm \text{ or } Z^0 \rightarrow l^\pm X) \\
 = k_{10} \frac{d\hat{\sigma}}{d^3k_1} \frac{dy}{dy} \cdot \left\{ 1 + \langle \frac{4}{3} \rangle \frac{\alpha_s(k_\pi^2)}{2\pi} \cdot [\pi^2 - 8 - 2a - \ln^2 \frac{x_1}{1-x_1} - 3 \ln x_1(1-x_1)] \right\} \\
 + \langle \frac{4}{9} \rangle \frac{\alpha^2}{N} \frac{\alpha_s(k_\pi^2)}{2\pi} \cdot \left\{ \frac{A_L A_q [x_1^2 + (1-x_1)^2] + B_L B_q [(1-x_1)^2 - x_1^2]}{2} \cdot \right. \\
 \cdot \frac{z^2}{x_2} \cdot \left[BW(z\hat{s}) \cdot [P_{qq}(z) \cdot \left(\ln \frac{1-z}{x_1 z} - \ln r - \frac{z\hat{s}-M^2}{\Gamma M} \cdot \theta \right) - b(z) + 1 - z] \right. \\
 + BW(\hat{s}) \cdot \frac{1}{z} P_{qg}(z) \cdot \left(\ln \frac{1-z}{1-x_1} + \ln r + \frac{\hat{s}-M^2}{\Gamma M} \cdot \theta \right) \Bigg] + \frac{A_L A_q - B_L B_q}{2} \cdot \frac{z}{x_2} \\
 \cdot \left[BW(\hat{s}) \cdot (1-z+2x_1 z) \cdot \left(\ln \frac{1-z}{1-x_1} + \ln r + \frac{\hat{s}-M^2}{\Gamma M} \cdot \theta \right) - \frac{1}{z} \ln r - (1-z+2x_1 z + \frac{M^2}{z}) \frac{\theta}{\hat{s}\Gamma M} \right] \\
 \left. + [x_1 \leftrightarrow x_2, B_q \rightarrow -B_q] - \frac{A_L A_q}{2} \cdot \frac{1}{z^2} \cdot \frac{2}{x_1+x_2} \cdot \left(\ln r_0 + \frac{M\theta_0}{\Gamma} \right) \right\}
 \end{aligned} \tag{21}$$

The Drell-Yan cross-section is given by

$$k_{10} \frac{d\hat{\sigma}^{DY}}{d^3k_1} = \langle \frac{1}{3} \rangle \frac{\alpha^2}{N} \cdot 8W(\xi) \cdot \delta(1-x_1-x_2) \cdot \frac{A_1 A_2 [(1-x_1)^2 + x_1^2] + B_1 B_2 [(1-x_1)^2 - x_1^2]}{2} \quad (22)$$

In formulae (20) - (22), the sign of the axial part corresponds to the case $q(p_1) + \bar{q}(p_2) \rightarrow W^+ \rightarrow \mu^+ X$. The definition of the distributions $1/(1-z)|_+$ and $\ln(1-z)/(1-z)|_+$ is well known, however we must modify the results in Eq. (17) and (21) because of the non-zero lower limits of integration over z [see Ref. 6] for a detailed discussion].

Before going to the numerical evaluation of the cross-sections, we point out some regularities in our final results, Eqs. (15) - (22). As in LPP, $\ln(1-z)$ terms appear which are large at the boundary of phase space. Technically they arise from the phase space factor [see Eq. (B8)], which is proportional to $(1-z)^{-\epsilon}$ in DLP and in DIS, but to $(1-z)^{-2\epsilon}$ in LPP. However, a new feature in DLP is that the annihilation diagram has an extra $(1-z)^{-\epsilon}$ factor [Eq. (B13)], and the $\ln(1-z)$ terms have different weights for the annihilation and Compton contributions. The $\ln(1-z)$ terms are presented for both γ and W in a similar fashion, so that taking the limit $M \rightarrow 0$ in Eqs. (20) - (21), we retain from the W case the $\ln(1-z)$ terms of the γ case. The $\ln r$ and $\ln r_0$ terms in $W^\pm$ and Z^0 production arise from the presence of a second effective scale M^2 besides the lepton transverse momentum k_{1T}^2 . The limit $M \rightarrow 0$ gives divergent terms (as $r \sim M^2$ and $r_0 \sim 1/M^2$), and their coefficients produce in this limit the direct photon conversion cross-sections of Eqs. (16) and (19) for the $q\bar{q}$ and qg cases respectively. This serves as a check of our formulae, as the computations are much more complicated for $W^\pm$ and Z^0 production than for γ production.

3.2 Numerical evaluations

The single muon spectrum from virtual photon decays is presented in Fig. 2 for the ISR energies. It has been calculated using two different non-scaling parton distribution parametrizations, which both obey the leading logarithmic QCD evolution equation numerically: the Owens-Reya parametrization²⁰⁾ (denoted by OR) and the Beier-Engels-Petersson parametrization²¹⁾ (BEP). The difference between the OR and BEP curves in Fig. 2 is explained by the larger gluon contribution at high k_{1T} values for the BEP case. The curves including the QCD corrections of Eqs. (15) - (19) are presented only for the BKKU factorization description. However, in Fig. 3 we have shown the relative contributions (normalized by the DY cross-section) of the different QCD correction terms for both BKKU and AEM cases (now at the $\bar{p}p$ collider energies). The factorizable α_s contribution is denoted by F [with $DY+F = DY(1+\alpha_s \dots)$ on the first line of Eq. (17)], A is the rest

of the QCD $q\bar{q}$ corrections of Eq. (17) and it includes also the photon conversion term of Eq. (19), and finally C is the Compton contribution of Eq. (15), including the photon conversion term of Eq. (16). The total $O(\alpha_s)$ correction is $\alpha_s = F + A + C$, and hence the single lepton cross-section to order α_s is $DY + \alpha_s$. The ratios R^C and R^{α_s} agree totally for the BKKU and AEM cases; however, the total $q\bar{q}$ correction is divided quite differently between the contributions A and F in the two cases: for AEM, F is large with $R^F \approx 0.8$ and A is negligible, whereas for BKKU, F and A are comparable, with $R^F \approx 0.3$ and $R^A \approx 0.5$. An interesting note is that the fast-decreasing C contribution is given almost totally by the photon conversion cross-section of Eq. (16), which is thus the main mechanism for direct lepton production at small k_{1T} .

Next we turn to the QCD corrections for $W^\pm$ and Z^0 production. In Fig. 4 we have again separated the QCD contributions F , A and C , where F is the factorizable α_s term on the first line of Eq. (21), A is the rest of Eq. (21) and C is given by Eq. (20). In our previous letter [Ref. 14], we presented a similar figure with sea-quarks not included. We can now note that for $\bar{p}p$ collisions, sea amounts to a factor 2 at small k_{1T} values and can be neglected at $k_{1T} > M/2$. Thus the valence-sea collision cannot be neglected in general, although the sea-sea terms are vanishingly small. We first see that the Compton terms are not important and only at large k_{1T} ($> M/2$) do they become comparable with the other contributions. For the BKKU case, the F contribution is small, with $R^F \approx 0.1$. The A contribution is at $k_{1T} < M/2$, a 50% correction to the lowest order Drell-Yan result, but it becomes relatively much more important at $k_{1T} > M/2$, where $R^A \approx 3$ at $k_{1T} = 50$ GeV/c. The large QCD correction in this region is not a sign of a breakdown at the perturbation expansion, but is due to the kinematical vanishing of the DY cross-section. In the zero width approximation, there is actually a cut-off for DY at $k_{1T} = M/2$, so that the shape of the k_{1T} spectrum above this value reflects the W (or Z) width. This is because in the lowest order calculation the annihilating quarks have no transverse momentum, and the vector boson is produced with $k_T = 0$.

The typical form of the annihilation (A) and Compton (C) QCD corrections with a pronounced negative peak at $k_{1T} = M/2$ deserves a closer analysis. We note first that the QCD corrections' being negative is quite acceptable as they are really factorization prescription-dependent, and choosing different references scale in parton distributions $f(x, Q^2)$ amounts to changes in these α_s correction terms. Only the total cross-section to order α_s ($DY + \alpha_s$ corrections) should remain always positive. The Compton cross-section of Eq. (20) is simpler to analyze,

as it has fewer terms than the annihilation cross-section. The main contributions come from terms proportional to

$$P_{qg}(z) \cdot \left[\left(\frac{1}{z\hat{s}M} - \frac{z\hat{s}-M^2}{M} \cdot BW(z\hat{s}) \right) \cdot \theta - \ln r \right] \quad (23)$$

where the θ term provides a positive, large and rather flat contribution. The $\ln r$ term is negative, rising fast and becoming larger than the θ term at $k_{1T} \simeq 36$ GeV/c, but then dropping very fast at $k_{1T} > M/2$, so that at large k_{1T} the Compton term is again given by the θ term alone.

The annihilation cross-section of Eq. (21) (DY and F terms not included) we write symbolically as

$$\sigma^A = \sigma_0^A + \sigma_1^A \frac{1}{1-z|_+} + \sigma_2^A \frac{\ln(1-z)}{1-z|_+} \quad (24)$$

and note first that σ_0^A is not important for the understanding of the shape of the A curve in Fig. 4. The σ_2^A term is (for the BKKU case) important only around $k_{1T} = M/2$, where it has a positive peak. The σ_1^A term is the main contribution to σ^A , and, writing it in pieces by $\sigma_1^A = A_0 + A_1 \cdot \ln r + A_2 \cdot \theta$, the importance of the different terms is similar to the Compton case: A_0 does not contribute significantly, $A_2 \cdot \theta$ is positive and large, and $A_1 \cdot \ln r$ has a negative peak at $k_{1T} = M/2$, the magnitude of which is larger than the magnitude of the positive peak from $\sigma_2^A \cdot \ln(1-z)/(1-z)|_+$. This completes the analysis of the important of different contributions to the annihilation and Compton cross-sections.

We have presented the final $O(\alpha_s)$ cross-sections for $\bar{p}p \rightarrow W^\pm \rightarrow \mu^\pm X$ and for $\bar{p}p \rightarrow Z^0 + \gamma \rightarrow \mu^\pm X$ at the $\bar{p}p$ collider in Figs. 5 and 6, together with the DY predictions. The results are essentially the same for the BKKU and the AEM factorization prescriptions. For the two parton distribution parametrizations (OR and BEP) we also get results which agree within less than 10% over the k_{1T} range considered.

Comparison of Figs. 5 and 6 shows that the tail from the $W^\pm$ decays above $k_{1T} > M_W/2$ makes the observation of Z^0 in the single lepton spectrum very hard; fortunately it can be seen more easily in the lepton pair invariant mass distribution²²⁾.

4. THE QUESTION OF k_T OF THE VECTOR BOSON

The purpose of this section is to compare our work with other approaches, especially with regard to the transverse momentum k_T of the vector boson. First let us note that in LPP experiments, events with large k_T ($k_T \geq 4$ GeV/c) have been observed²³⁾. The QCD annihilation and Compton processes provide a natural explanation for these large values of lepton pair transverse momentum, as well as for the observed increase of $\langle k_T \rangle$ with increasing M ²⁴⁾. Two approaches have been used to implement the non-zero k_T effects into the single lepton cross-section^{*)}. In the first one²⁵⁾, the k_T distribution from the annihilation and Compton processes (without the decay leptons) is convoluted with a probability function for W to give the observed lepton. A large contribution is obtained at $k_{1T} > M/2$, but this approach suffers from the divergence of the k_T distribution at small k_T , which must be regularized phenomenologically. The second approach^{14), 26)} is the one used in this paper, namely to calculate the QCD diagrams including the decay leptons, as in Fig. 1. Now the divergence of small k_T can be handled in a way which is consistent with the factorization theorem of QCD, and we also include, unlike in previous works, the virtual diagrams to order α_s .

It has been suggested²⁷⁾ that one should take into account leading logarithmic effects to all orders at α_s by including the Sudakov form factor:
 $S = \exp \left[-2\alpha_s/3\pi \cdot \ln^2(k^2/k_T^2) \right]$. This was shown to reduce rather drastically the average $\langle k_T \rangle$ of the vector boson from its value $\langle k_T \rangle = 10$ GeV/c (for W^+), based on a straightforward evaluation of the Compton and annihilation diagrams^{**)}. We would like to argue against such a modification in the second approach. The form factor arises when, besides the total energy, two large mass scales (here k^2 and k_T^2) are present. Fixing k_T^2 prevents the cancellation of the infra-red singularity from the virtual diagrams [now proportional to $\delta(k_T^2)$], and the real diagrams give the famous $4/3 \cdot \alpha_s/2\pi \cdot \ln^2(k^2/k_T^2)$ terms, which exponentiate in the calculation done to all orders of perturbation theory. In our case, we integrate over both the virtuality (k^2) and the transverse momentum (k_T) of the vector boson, which leads to the exact cancellation of the infra-red singularity as shown by the vanishing of the $1/\epsilon^2$ terms. Traces of this infra-red problem survive in the form of the $\ln(1-z)/(1-z)|_+$ terms, which become large only at the edge of

*) In early works [Ref. 11)], the DY process was assumed to give the longitudinal motion at W^+ correctly, and a distribution for its transverse momentum was put in by hand. As a consequence, the peak of the lepton spectrum was smeared and a tail at $k_{1T} > M/2$ appeared.

**) When single lepton production is viewed as a sequential process involving the production of the weak boson (on mass shell) at fixed k_T , such an approach may be justified.

phase space. Although in our numerical evaluations ($\bar{p}p$ collider energies) they are not so important, they remain a theoretical problem and might be summable to all orders of perturbation theory²⁸⁾.

Finally we note that one might also test the QCD processes at fixed k_T ²⁹⁾, as experimentally one can observe k_T indirectly by measuring the transverse imbalance due to the escaping lepton (neutrino). In fact, this method will be used in the CERN $\bar{p}p$ collider experiments³⁰⁾ to separate the $W^\pm$ signal from the backgrounds.

5. CONCLUSIONS

We have performed a complete (including non-leading terms) first order QCD calculation of the single lepton inclusive distribution from γ , $W^\pm$ and Z^0 decays in hadronic collisions. The QCD corrections are found to be important over the studied range of lepton transverse momentum. For γ production, the effect of the first order diagrams is to multiply the Drell-Yan prediction by a factor 2. For $W^\pm$ and Z^0 production, the cross-section is increased by 50% for the lepton transverse momentum below $M/2$. Above this value, the corrections are much bigger, and they should be considered as coming from a separate production mechanism. As a result, the Jacobian peak at $M/2$ is smeared and displaced towards a lower value. We find a remarkable stability of our predictions against the different factorization descriptions, and also against the different parton distribution parametrizations we have used.

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APPENDIX A: KINEMATICS AND COUPLINGS

The Feynman diagrams we discuss in this paper are shown in Fig. 1 (the vector boson is γ , $W^\pm$ or Z^0). At the parton level we define the following invariants for the Drell-Yan process of Fig. 1a ($p_1 + p_2 = k_1 + k_2 \equiv k$):

$$\hat{s} = 2p_1 \cdot p_2, \quad \hat{t}_1 = -2p_1 \cdot k_1, \quad \hat{t}_2 = -2p_2 \cdot k_1 \quad (A1)$$

and for the QCD processes of Figs. 1c and 1d ($p_1 + p_2 = p_3 + k_1 + k_2 \equiv p_3 + k$, Fig. 7), we introduce in addition

$$\hat{t} = -2p_2 \cdot p_3, \quad \hat{u} = -2p_1 \cdot p_3, \quad k^2 = 2k_1 \cdot k_2 = \hat{s} + \hat{t} + \hat{u}$$

It is useful to define the scaling variables

$$x_1 = \frac{k_{1T}^2}{2p_1 \cdot k_1}, \quad x_2 = \frac{k_{1T}^2}{2p_2 \cdot k_1} \quad (A2)$$

in terms of which the above invariants become

$$\hat{s} = \frac{k_{1T}^2}{x_1 x_2}, \quad \hat{t}_1 = -\frac{k_{1T}^2}{x_1}, \quad \hat{t}_2 = -\frac{k_{1T}^2}{x_2} \quad (A3)$$

Furthermore, the boundary condition $(p_1 + p_2 - k_1)^2 \geq 0$ implies $x_1 + x_2 \leq 1$.

Similarly, at the hadron level, we define for the process $P_1 + P_2 = k_1(\ell^+) + P_X$ the variables

$$X_1 = \frac{k_{1T}^2}{2P_1 \cdot k_1}, \quad X_2 = \frac{k_{1T}^2}{2P_2 \cdot k_1} \quad (A4)$$

with the constraint $X_1 + X_2 \leq 1$ from $P_X^2 \geq 0$. These variables can be expressed in terms of the centre of mass system muon variables, rapidity y and $x_T = 2k_{1T}/\sqrt{s}$, as

$$X_1 = \frac{x_T}{2} e^y, \quad X_2 = \frac{x_T}{2} e^{-y} \quad (A5)$$

The parton model assumption $p_i = \xi_i P_i$ ($i = 1, 2$) gives the relation

$$X_i = \xi_i x_i \quad (i = 1, 2) \quad (A6)$$

The single muon cross-section is now given as the following convolution over the parton cross-sections (denoted by hat $\hat{}$) and the parton distributions

$$k_{10} \frac{d\hat{\sigma}}{d^3k_1}(X_1, X_2, k_{1\pi}^2) = \sum_{j,k} \int_{X_1}^{1-X_2} \frac{dx_1}{x_1} \int_{X_2}^{1-x_1} \frac{dx_2}{x_2} \cdot$$

$$\bullet \left[\frac{X_1}{x_1} f_j^1\left(\frac{X_1}{x_1}\right) \right] \cdot \left[\frac{X_2}{x_2} f_k^2\left(\frac{X_2}{x_2}\right) \right] \cdot k_{10} \frac{d\hat{\sigma}}{d^3k_1}(x_1, x_2, k_{1\pi}^2) \quad (A7)$$

The couplings of the weak vector bosons to fermions are defined by the interaction term $g_B \bar{f} \gamma_\mu (a + b\gamma_5) f$, where for $W^\pm$

$$g_W = \frac{e}{\sqrt{2} \sin \theta_W} \quad , \quad a_\ell = a_q = \frac{1}{2} \quad , \quad b_\ell = b_q = -\frac{1}{2} \quad (A8)$$

and for Z^0

$$g_Z = \frac{e}{\sin \theta_W \cos \theta_W} \quad a_\nu = \frac{1}{4} \quad , \quad b_\nu = -\frac{1}{4} \quad , \quad a_\ell = -\frac{1}{4} + \sin^2 \theta_W \quad , \quad b_\ell = \frac{1}{4}$$

$$a_{u,c} = \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W \quad , \quad b_{u,c} = -\frac{1}{4} \quad (A9)$$

$$a_{d,s} = -\frac{1}{4} + \frac{1}{3} \sin^2 \theta_W \quad , \quad b_{d,s} = \frac{1}{4}$$

When evaluating the squared matrix elements the following combinations appear

$$A_f = a_f^2 + b_f^2 \quad , \quad B_f = 2a_f b_f \quad (f = \ell, q) \quad (A10)$$

APPENDIX B: PHASE SPACE INTEGRALS

We are interested in the one-particle inclusive cross-section for $2 \rightarrow 3$ processes, which in n dimensions is related to the matrix element M by

$$k_{10} \frac{d\hat{\sigma}}{d^3k_1} = \frac{1}{4s} \frac{1}{(2\pi)^3} \cdot I(PS) \otimes \overline{|M|^2} \quad (B1)$$

with

$$I(PS) \otimes F = \mu^{4-n} \int \frac{d^n p_3}{(2\pi)^{n-1}} \delta^+(p_3^2) \int \frac{d^n k_2}{(2\pi)^{n-1}} \delta^+(k_2^2)$$

$$\otimes (2\pi)^n \delta^n(p_1 + p_2 - p_3 - k_1 - k_2) \cdot F \quad (B2)$$

$$= \mu^{4-n} (2\pi)^{2-n} \int \frac{d^{n-1} p_3}{2p_{30}} \delta((p_1 + p_2 - k_1 - p_3)^2) \cdot F$$

and where the kinematics are defined in Appendix A and in Fig. 7. μ is a mass scale introduced to give correct dimensions to the cross-section. We can carry out the integrations of Eq. (B2), for example, in the over-all centre of mass system, where we choose the z axis along $\vec{k}_1$, and $\vec{p}_1$ and $\vec{p}_2$ to lie on the

xz plane. Defining angular variables θ_i ($i = 1, \dots, n-1$) in $n-1$ dimensional space, we can write down the momenta [with the notation $P = (P_0, P_z, P_x, P_y, \dots)$]

$$\begin{aligned} k_1 &= k_{10} (1, 1, 0, 0, \dots) \\ p_1 &= \frac{\sqrt{s}}{2} (1, \cos\alpha, \sin\alpha, 0, \dots) \\ p_2 &= \frac{\sqrt{s}}{2} (1, -\cos\alpha, -\sin\alpha, 0, \dots) \\ p_3 &= p_{30} (1, \cos\theta_1, \sin\theta_1 \cos\theta_2, \sin\theta_1 \sin\theta_2 \cos\theta_3, \dots) \end{aligned} \quad (B3)$$

Integration over irrelevant angles gives

$$I(PS) = \mu^{4-n} \frac{(2\pi)^{2-n} \pi^{(n-3)/2}}{2 \Gamma((n-3)/2)} \cdot \int_0^{\frac{\sqrt{s}}{2} - k_{10}} dp_{30} p_{30}^{n-3} \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 \cdot \delta(k^2) \quad (B4)$$

Changing variables to

$$\begin{aligned} \chi &= \left(\frac{\sqrt{s}}{2} - p_{30} \right) / k_{10} \\ \eta &= (1 - \cos\theta_2) / 2 \end{aligned} \quad (B5)$$

and doing the θ_1 integration with the help of the δ function, one arrives at

$$I(PS) = \frac{1}{8\pi^2} \left(\frac{\pi \mu^2}{-\hat{k}_1 - \hat{k}_2} \right)^\epsilon \frac{\sqrt{\pi}}{\Gamma(\frac{1}{2} - \epsilon)} \cdot \int_0^1 d\chi [\chi(1-\chi)]^{-\epsilon} \int_0^1 d\eta [\eta(1-\eta)]^{-\frac{1}{2} - \epsilon} \quad (B6)$$

The only invariants depending on χ and η are (results for $\hat{u}$ can be obtained by changing $p_1 \leftrightarrow p_2$ in the final results for $\hat{t}$):

$$\begin{aligned} k^2 &= -(\hat{k}_1 + \hat{k}_2) \chi \\ \hat{k} &= -p_{30} \sqrt{s} \cdot [1 + \cos\alpha \cos\theta_1 + \sin\alpha \sin\theta_1 (1 - 2\eta)] \end{aligned} \quad (B7)$$

The phase space integrals we have to calculate are of the form $I(PS) \otimes f(k^2) \cdot \hat{t}^q$ ($q = -1, 0, 1$). For $q = 0, 1$ the integrations are straightforward and the results can be expressed in terms of Euler's beta-functions, which can easily be expanded in powers of ϵ . Thus we concentrate on the case $q = -1$, which gives rise to a mass singularity when p_2 and p_3 are collinear ($\hat{t} = 0$). The η integration yields a hypergeometric function, and after a quadratic transformation of its argument and a change of integration variable, we get

$$\begin{aligned} I(PS) \otimes \frac{f(k^2)}{\hat{k}} &= -\frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{3 + \hat{k}_1 + \hat{k}_2} \right)^\epsilon \cdot \frac{1}{3 + \hat{k}_2} \\ &\otimes \left\{ R^{-\epsilon} \int_0^1 dx x^{-1-2\epsilon} (1-x)^{-\epsilon} (1+Rx)^\epsilon f(k^2 = z\hat{s}(1+Rx)) \cdot F(-\epsilon, -2\epsilon; 1-\epsilon; \frac{1-x}{1+Rx}) \right\} \end{aligned} \quad (B8)$$

$$+ R^\epsilon \int_0^1 dx x^{-1-2\epsilon} (1-x)^{-\epsilon} \left(1 + \frac{x}{R}\right)^\epsilon f(k^2 = z\hat{s}(1-x)) \cdot F(-\epsilon, -2\epsilon; 1-\epsilon; \frac{1-x}{1+\frac{x}{R}}) \Big\} \quad (B8)$$

cont.

where $R = (1-x_1-x_2) \cdot x_1/x_2 = x_1(1-z)/Z$ and $z = x_2/(1-x_1)$. The hypergeometric function F has the following power expansion in ϵ :

$$F(-\epsilon, -2\epsilon; 1-\epsilon; w) = 1 + 2\epsilon^2 Li_2(w) \quad (B9)$$

Equations (B8) and (B9) are the basic formulae from which all integrals we need can be derived.

Virtual photon production

For virtual photon production we need integrals of the form $I(PS) \otimes (k^2)^P / \hat{t}$ ($p = -1, 0, 1$). The singular behaviour of the integrands in Eq. (B8) at $x = 0$ (corresponding to $\hat{t} = 0$) or at $x = 1$ (corresponding to $k^2 = 0$) leads at most to $1/\epsilon$ divergences. Therefore we can safely take $F = 1$ in Eq. (B8), and after integration and expansion in ϵ , we get:

$$\begin{aligned} I(PS) \otimes \frac{1}{k^2 \hat{t}} &= -\frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{x_1^2 x_2}{k_{1T}^4} \left[-\frac{2}{\epsilon} - 2 \ln \frac{x_1(x_1+x_2)}{1-z} \right] \\ I(PS) \otimes \frac{1}{\hat{t}} &= -\frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{x_1 z}{k_{1T}^2} \left[-\frac{1}{\epsilon} - \ln \frac{x_1 z}{1-z} \right] \\ I(PS) \otimes \frac{k^2}{\hat{t}} &= -\frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{z}{1-x_1} \left[-\frac{1}{\epsilon} - 1 + x_1(1-z)/z - \ln \frac{x_1 z}{1-z} \right] \end{aligned} \quad (B10)$$

In the QCD annihilation diagrams, we encounter integrals of the form $I(PS) \otimes (k^2)^P / \hat{t}\hat{u}$. To evaluate them, we use the decomposition

$$\frac{1}{\hat{t}\hat{u}} = \frac{1}{k^2 - \hat{s}} \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right) \quad (B11)$$

which shows the overlapping infra-red singularity ($k^2 = \hat{s}$) and mass singularity ($\hat{t} = 0$ or $\hat{u} = 0$) generating $1/\epsilon^2$ divergences. Neglecting for the time being the ϵ^2 term in Eq. (B9), we can perform the integration in Eq. (B8)

$$\begin{aligned} I(PS) \otimes \frac{1}{(k^2 - \hat{s}) \hat{t}} &= -\frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{x_1^2 x_2^2}{1-x_1} \cdot \frac{1}{k_{1T}^4} \\ &\cdot \frac{\beta(1-\epsilon, -\epsilon)}{(1-z)^{1+2\epsilon}} \cdot \left\{ x_1^{2\epsilon} F_2(-2\epsilon, -\epsilon, -2\epsilon; 1-3\epsilon; -\frac{x_2}{x_1}; z) \right\} \end{aligned} \quad (B12)$$

$$+ (x_1 + x_2)^{2\epsilon} \cdot F_2(-2\epsilon, -\epsilon, 1; 1-3\epsilon; \frac{x_1(1-z)}{x_1+x_2}; \frac{x_1}{x_1+x_2}) \} \quad (B12) \text{ cont.}$$

where F_2 is the hypergeometric function of two variables³¹⁾. The infra-red divergent point is at $z = 1$, as can be seen from Eq. (B7), where $k^2/\hat{s} = (x_1 + x_2) \cdot \chi$ can reach the value one only if $x_1 + x_2 = 1$. The most singular part of Eq. (B12) is given by $B(1-\epsilon, -\epsilon)/(1-z)^{1+2\epsilon} \sim 1/2\epsilon^2 \delta(1-z)$. This remark allows us to simplify the expansion in powers of ϵ of the expressions in brackets in Eq. (B12), as the $O(\epsilon^2)$ terms are needed only at $z = 1$, for which the F_2 functions reduce to ordinary hypergeometric functions. To complete the evaluation of $I(\text{PS}) \otimes 1/k^2 - \hat{s}/\hat{t}$, we note that the hypergeometric function in Eq. (B8) needs to be known to $O(\epsilon^2)$ only at $x = 0$, where it reduces to $1 + 2\epsilon^2 \text{Li}_2(1) = 1 + 2\epsilon^2 \pi^2/6$. The final result for the integral $I(\text{PS}) \otimes 1/\hat{u}\hat{t}$ is obtained after symmetrization in x_1 and x_2 according to Eq. (B11), and it reads

$$I(\text{PS}) \otimes \frac{1}{\hat{u}\hat{t}} = \frac{1}{\epsilon} \frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{1}{k_{1T}^4} \cdot \left\{ \frac{x_1^2 x_2^2}{1-x_1} \cdot \frac{1}{(1-z)^{1+2\epsilon}} \cdot \left[1 + \epsilon \ln x_1 x_2 + \epsilon^2 \left[\ln^2 x_1 + \ln^2(1-x_1) + \text{Li}_2\left(\frac{1-x_1}{x_1}\right) + 3\text{Li}_2\left(\frac{x_1}{1-x_1}\right) + 2\text{Li}_2(1) \right] \right] \right\} + (x_1 \leftrightarrow x_2) \} \quad (B13)$$

where we still have to make the replacement

$$\frac{F(z)}{(1-z)^{1+2\epsilon}} = -\frac{1}{2\epsilon} F(1) \delta(1-z) + \frac{F(z)}{1-z} - 2\epsilon F(z) \frac{\ln(1-z)}{1-z} + \quad (B14)$$

The most general integral of the form $I(\text{PS}) \otimes (k^2)^p/\hat{t}\hat{u}$ can be obtained from Eqs. (B10) and (B13) with the help of the recursion relation

$$\frac{(k^2)^p}{\hat{t}\hat{u}} = (k^2)^{p-1} \cdot \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right) + \hat{s} \frac{(k^2)^{p-1}}{\hat{t}\hat{u}} \quad (B15)$$

This completes the evaluation of all integrals needed for the case of inclusive lepton production via virtual photon decay. More generally, Eqs. (B10), (B13) and (B15) can be used for the computation of the single particle spectrum from any $2 \rightarrow 3$ subprocesses involving only massless quanta.

Massive vector boson production

For the case of massive vector boson production, the corresponding formulae are somewhat more complicated. Let M and Γ be the mass and the width of the weak vector boson. We encounter terms of the form $BW(k^2) (k^2)^P / \hat{t}$ in the matrix element, where

$$BW(k^2) = \frac{1}{(k^2 - M^2)^2 + (\Gamma M)^2} = \frac{1}{\Gamma M} \mathcal{I}_M \frac{1}{k^2 - M^2 - i\Gamma M} \quad (B16)$$

Using again Eq. (B9) and expanding to $O(\epsilon)$, we get

$$\begin{aligned} I(PS) \otimes BW(k^2) \cdot \frac{(k^2)^P}{\hat{t}} &= -\frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi M^2}{k_{ir}^2} \right)^\epsilon \cdot \frac{x_1 \bar{z}}{k_{ir}^2} \\ &\cdot (\hat{z}\hat{s})^P \cdot BW(\hat{z}\hat{s}) \cdot \left\{ -\frac{1}{\epsilon} - \frac{1}{\Gamma M} \mathcal{I}_M G^P \right\} \end{aligned} \quad (B17)$$

where

$$\begin{aligned} G^P &= (\hat{z}\hat{s} - M^2 + i\Gamma M) \cdot \left\{ \left(\frac{z_b}{z_b - 1} \right)^P \cdot (\ln z_a + \ln z_b) \right. \\ &\quad \left. + \sum_{k=2}^P \frac{P!}{k! (P-k)!} \frac{1}{(z_b - 1)^k} \sum_{j=1}^{k-1} \frac{(1-z_a)^j + (1-z_b)^j}{j} \right\} \end{aligned} \quad (B18)$$

z_a and z_b are defined by

$$z_a = 1 + \frac{\hat{s}(\xi - \bar{z})}{\hat{z}\hat{s} - M^2 - i\Gamma M}, \quad z_b = 1 - \frac{\hat{z}\hat{s}}{\hat{z}\hat{s} - M^2 - i\Gamma M} \quad (B19)$$

where $\xi = x_1 + x_2$. Using the formula

$$\begin{aligned} \ln(x + iy) &= \ln \sqrt{x^2 + y^2} + i \arctan \frac{y}{x} \\ \arctan \frac{y}{x} &= \overline{\arctan} \frac{y}{x} + \pi \theta(-x) \cdot [\theta(y) - \theta(-y)] \end{aligned} \quad (B20)$$

the final result will depend on

$$\begin{aligned} r &= |z_a| |z_b| = \frac{M^2 \sqrt{(\xi\hat{s} - M^2)^2 + (\Gamma M)^2}}{(\hat{z}\hat{s} - M^2)^2 + (\Gamma M)^2} \\ \theta &= \theta_a + \theta_b = \arctan \frac{\Gamma}{M} + \arctan \frac{\Gamma M}{M^2 - \xi\hat{s}} - 2 \arctan \frac{\Gamma M}{M^2 - \hat{z}\hat{s}} \end{aligned} \quad (B21)$$

As an example we quote

$$I(PS) \otimes \frac{BW(k^2)}{\hat{s}} = -\frac{1}{8\pi} \cdot \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{x_1 z}{k_{1T}^2} \cdot BW(\hat{s}) \cdot \left\{ -\frac{1}{\epsilon} - \ln r - \frac{\hat{s} - M^2}{M^2} \cdot \Theta - \ln \frac{x_1 z}{1-z} \right\} \quad (B22)$$

Terms like $I(PS) \otimes BW(k^2)^{p\hat{u}}$ again give overlapping mass and infra-red singularities. These terms can be brought into the following form:

$$BW(k^2) \cdot \frac{(k^2)^p}{\hat{s}\hat{u}} = BW(\hat{s}) \frac{(k^2)^p}{\hat{s}\hat{u}} - \frac{1}{M^2} \mathcal{J}_M \left[\frac{1}{\hat{s} - M^2 - i\Gamma M} \cdot \frac{(k^2)^p}{k^2 - M^2 - i\Gamma M} \cdot \left(\frac{1}{\hat{s}} + \frac{1}{\hat{u}} \right) \right] \quad (B23)$$

where now the first term contains all of the $1/\epsilon^2$ divergences and is known from the virtual photon case, Eq. (B13). The second term can be obtained from Eq. (B22). Finally, to conclude this enumeration of integrals, we mention the non-singular terms of the type $I(PS) \otimes BW(k^2) \cdot (k^2)^p$, an example of which is

$$I(PS) \otimes BW(k^2) \cdot k^2 = \frac{1}{8\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{k_{1T}^2} \right)^\epsilon \frac{x_1 z}{k_{1T}^2} \left(\ln r_0 + \frac{M}{\Gamma} \Theta_0 \right) \quad (B24)$$

where

$$r_0 = \frac{\sqrt{(\hat{s} - M^2)^2 + (\Gamma M)^2}}{M^2}, \quad \Theta_0 = \arctan \frac{\hat{s} - M^2}{\Gamma M} + \arctan \frac{M}{\Gamma} \quad (B25)$$

Virtual diagrams

We just quote the result from the calculation of the diagram in Fig. 1b [see Altarelli et al., Ref. 3)]. Continuation from spacelike to timelike momenta gives the well known large π^2 factor

$$V = \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{\hat{s}} \right)^\epsilon \cdot \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 \right] \quad (B26)$$

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FIGURE CAPTIONS

Fig. 1 : The Feynman diagrams for the hadronic production of vector boson ($V = \gamma, Z^0, W^\pm$);

- a) the Drell-Yan process $q\bar{q} \rightarrow \ell\bar{\ell}$;
- b) QCD virtual correction for $q\bar{q} \rightarrow \ell\bar{\ell}$;
- c) the QCD annihilation process $q\bar{q} \rightarrow g + \ell\bar{\ell}$;
- d) the QCD Compton process $qg \rightarrow q + \ell\bar{\ell}$.

Fig. 2 : The direct lepton spectrum $k_{10}d\sigma/d^3k_1$ in proton-proton collisions at $\sqrt{s} = 63$ GeV and $y = 0$. The Drell-Yan contribution (DY) and the cross-section to order α_s (HO) are shown for two different non-scaling parton distribution parametrizations: the Owens-Reya (dashed line) and the BEP parametrization (full line).

Fig. 3 : The ratios $R^i = k_{10}d\sigma^i/d^3k_1 / k_{10}d\sigma^{DY}/d^3k_1$ ($i = F, A, C$ or α_s as explained in text) for $\bar{p}p \rightarrow \gamma^* X \rightarrow \ell^+ X$ at $\sqrt{s} = 540$ GeV and $y = 0$. Case I refers to the AEM definition of the parton distributions and case II to the BKKU definition. In both cases the Owens-Reya parton distributions were used.

Fig. 4 : Different QCD α_s contributions to the cross-section $k_{10}d\sigma/d^3k_1$ ($\bar{p}p \rightarrow W^+ X \rightarrow \mu^+ X$) at $\sqrt{s} = 540$ GeV and $y = 0$. The curves $k_{10}d\sigma^i/d^3k_1$ ($i = F, A, C, DY$) were calculated for the BKKU factorization description and with the BEP parametrization.

Fig. 5 : The $W^+ \rightarrow \mu^+ X$ spectrum in the $\bar{p}p$ collider. The HO curve is the cross-section to order α_s , and DY is the lowest order Drell-Yan contribution.

Fig. 6 : The $Z^0 + \gamma \rightarrow \mu^+ X$ spectrum in the $\bar{p}p$ collider. The HO and DY curves have the same meaning as in Fig. 5.

Fig. 7 : Kinematics for the $2 \rightarrow 3$ process.

Fig. 1

Fig. 2

Fig. 3

$k_{1T} \text{ [GeV/c]}$

Fig. 4

Fig. 5

Fig. 6

Fig. 7