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HIERARCHICAL FERMION MASSES IN SU(5)

R. Barbieri and D.V. Nanopoulos

CERN--Geneva

D. Wyler

Physikalisches Institut
der Universität Bonn

ABSTRACT

Radiative masses are generated for the first and the second fermion families by exploiting the idea that their chirality is a symmetry of the "low energy" $SU(3) \times SU(2) \times U(1)$ approximation of $SU(5)$, broken only by including the effects of superheavy particles. With only 5's of Higgs coupled to fermions and getting a non-zero vacuum expectation value, we unavoidably get

- i) $m_s = m_\mu/3$ at the grand unification scale;
- ii) the charm quark needing a direct mass as the third family;
- iii) neutrino masses of size $\lesssim (\alpha/\pi) M_W^2/M$.

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1. In this note we come back to the problem of the fermion masses in an $SU(5)$ grand unified model. As is well known, the so-called "minimal" version of it - only one $\underline{5}$ of Higgs with Yukawa couplings to all fermion families - leads to a successful prediction: $m_\tau/m_b = 1$ at the grand unification mass $M \approx 10^{14}$ GeV, and to massless neutrinos. In contrast, one gets, for the lighter families, $m_\mu/m_s = m_e/m_d$ which is not compatible with the current algebra relation $m_d/m_s = m_\pi^2/2m_K^2$, and also $m_\mu/m_s = 1$ at M which leads, after renormalization, to a value of m_s about $3 \div 5$ times larger than the presumably observed one¹⁾.

It is tempting to observe that these diseases might simply reflect the very fact that the relative smallness of the lighter families (hierarchy) is in itself not understood within the minimal model. The purpose of this note is to reconsider these problems by focusing precisely on the hierarchical structure of the fermion masses. We will, in fact, show that such a basic feature of the fermion spectrum can be "naturally" produced in $SU(5)$, exploiting the idea²⁾ that the smallness of some fermion masses is due to an exact global symmetry of the "low energy" $SU(3) \times SU(2) \times U(1)$ approximation of $SU(5)$, broken only by including the effects of superheavy particles. More precisely, we demand that the third family (t, b, τ) gets a direct mass through a $\underline{5}$ of Higgs ($\underline{5}_H$) and that the Yukawa Lagrangian possesses a global symmetry such that the fermions of the second family (c, s, μ) have only a radiatively generated one-loop mass and those of the first family (u, d, e) only a two-loop mass, with loops involving superheavy particles.

Insisting on the mentioned successes of the minimal $SU(5)$ model, we shall not allow any $\underline{45}$'s and/or $\underline{15}$'s of Higgs to couple to the fermions since their vacuum expectation values (v.e.v) might respectively undo the relation $m_b = m_\tau$ and give direct masses to neutrinos.

Irrespective then of our specific Lagrangian, we shall show that the one-loop generated mass for the second family leads necessarily to $m_\mu = 3m_s$ at the unification mass, whereas the two-loop generated mass for the first family gives rise to a ratio m_d/m_e non-simply related to an $SU(5)$ Clebsch-Gordan coefficient. Furthermore, the only possible one-loop mass for the charge $Q = 2/3$ sector (a candidate for the charm quark mass) being necessarily off-diagonal in family space, leads to an eigenvalue of the same size as that of an effective two-loop mass. As such, we are forced to also give the charm quark a direct mass, something which of course may not be unrelated to its actual value.

Finally, again as a general consequence of our basic philosophy and in contrast with the minimal $SU(5)$ model, we are bound to have non-vanishing neutrino masses of size $\approx \alpha/\pi M_W^2/M$.

2. The idea that naturally small fermion masses are generated through loops with virtual superheavy particles is an appealing one, especially since it may lead to masses of order $\alpha/\pi M_W$ or $(\alpha/\pi)^2 M_W$ (one-loop and two-loop, respectively) independent of the value of the exchanged large masses $\approx M^2$. In unified models based on a group G where standard (f) and superheavy (F) fermions sit together in an irreducible representation of G , the necessary breaking of chirality for the standard fermions may be provided by gauge couplings of the type $\bar{f}_Y A_\mu F$ where A_μ is a superheavy boson. These kinds of couplings are, however, absent in the $SU(5)$ model with standard fermions assigned to the representation $\underline{5} + \underline{10}$; possibly relevant in this case are Yukawa couplings $f^T C^{-1} f \phi$ to superheavy scalars ϕ . The general way of guaranteeing the "calculability" (the finiteness) of a radiatively generated fermion mass is to demand that the Yukawa Lagrangian possesses some global symmetry, broken only by soft terms (quadratic/cubic interactions) in the Higgs potential. The global symmetry prevents a direct mass at the tree level, whereas the soft breaking makes it occur in higher orders through finite contributions.

Within this context, the general one-loop diagram for a radiative mass is given in Fig. 1 with tadpoles denoting Higgs vacuum expectation values. The tadpole connected with the fermion line is necessarily a $\underline{5}$ of Higgs. The other Higgs lines attached to the fermion can be a $\underline{5}_H$ (coupled to $\underline{5} \times \underline{10}$ and $\underline{10} \times \underline{10}$), a $\underline{10}_H$ (coupled to $\underline{5} \times \underline{5}$) and/or a $\underline{50}_H$ (coupled to $\underline{10} \times \underline{10}$). Of these Higgs fields only the $\underline{5}_H$ can get a v.e.v.*) consistent with unbroken $SU(3)_C \times U(1)_{em}$, since the others have no neutral, colourless component. The tadpoles connected with the internal superheavy Higgs line, which are necessarily there to provide the breaking of the global symmetry, must all be singlets under $SU(3) \times SU(2) \times U(1)$ in order not to give a totally negligible mass term $\lesssim \alpha/\pi M_W^2/M$ (see later).

Without making any restriction on the Higgs content of the model, other than excluding $\underline{45}_H$ and $\underline{15}_H$ coupled to fermions for the above-mentioned reasons, it follows that there exists only one specific diagram of the type in Fig. 1. The diagram is given in Fig. 2. It necessitates the introduction of a $\underline{50}$ and a $\underline{75}$ of Higgs. In this diagram all tadpoles connected with the internal Higgs line must form altogether a $\underline{75}$. Its uniqueness follows by the explicit construction, asking which products of two Higgs fields possibly coupled to fermions contain, in their $SU(5)$ reduction, a self-conjugate representation, the only one with singlet components under $SU(3) \times SU(2) \times U(1)$.

*) We actually make the natural assumption that all neutral colourless Higgs components get non-zero v.e.v.

Interestingly enough, the diagram of Fig. 2 corresponds to an effective $\underline{45}_H$ coupled to $\underline{\bar{5}} \times \underline{10}$ or to $\underline{10} \times \underline{10}$. The $\underline{45}$ is, in fact, the only representation which is contained in $\underline{75} \times \underline{5}$ and couples to the fermion mass terms ($\underline{\bar{5}} \times \underline{10}$ and $\underline{10} \times \underline{10}$). Now, since the coupling $\underline{45}_H \times \underline{10} \times \underline{10}$ is antisymmetric in the two $\underline{10}$'s, the diagram of Fig. 2 cannot give a diagonal mass term. Therefore, a sizeable contribution to m_c can only come from a direct coupling to a $\underline{5}_H$, as for the fermions of the heavy family. On the other hand, it is known that the coupling $\underline{45}_H \times \underline{\bar{5}} \times \underline{10}$ gives, at the grand unification mass M , a ratio of 3 between the lepton and the $Q = -1/3$ quark masses³⁾. For the second family, we are thus led to $m_\mu = 3m_s$ at the unification mass scale and to $m_s \approx (3/5)m_\mu$ at low energy, which is in spectacular agreement with the usually quoted value¹⁾ of $m_s \approx 100 - 150$ MeV.

To estimate the over-all size of the diagram, we associate a unique mass scale to the Higgs system, most likely (but not necessarily) the unification mass M and one dimensionless parameter h perhaps not too different from the gauge coupling itself. More specifically, we think of a situation where, after the breaking, all Higgs fields will have masses of order M except for one "light" Glashow-Weinberg-Salam doublet^{*)}, Yukawa couplings of order h ^{**)} and trilinear couplings of order hM . Then we have, for the diagram of Fig. 2

$$\delta m^{(1)} \approx \frac{h^2}{4\pi^2} m \quad (1)$$

where m is a direct mass, typical of the third family.

3. We now ask if a "natural" two-loop mass can be given to the first family (e,d,u). The answer is yes, provided one breaks the chirality of the corresponding $\underline{\bar{5}}_1 + \underline{10}_1$ SU(5) representation (the subscript 1 stands for the first family) only through Yukawa couplings to Higgs fields $\underline{10}_H$ and $\underline{50}_H$ which cannot get any v.e.v. Typical terms are $\underline{\bar{5}}_1 \underline{\bar{5}}_3 \underline{10}_H$ and $\underline{10}_1 \underline{10}_3 \underline{50}_H$. The corresponding minimal two-loop diagrams are given in Fig. 3, where it is understood that any number of superlarge tadpoles (e.g., $\underline{24}_H$) can stick out of any internal Higgs line and go into the vacuum without extra-suppressing the diagram. Both diagrams give finite masses of order

*) This minimizes the unnaturalness of the gauge hierarchy problem.

**) Of course, $h \approx e(m/M_W)$ where m is an average value of the masses for the third generation. Based on this we need not impose the artificial condition $h \ll e$.

$$\delta m^{(2)} \simeq \left(\frac{R^2}{4\pi^2} \right)^2 m. \quad (2)$$

In particular, the diagram 3a gives masses to d and e in a ratio which is, however, not simply related to an $SU(5)$ Clebsch-Gordan coefficient, (as in the case of Fig. 2), because for different large tadpole insertions the $SU(5)$ transformation properties are different^{*)}. Actually, by allowing any superlarge tadpole, [Higgs with $SU(3) \times SU(2) \times U(1)$ singlet components], other two-loop diagrams can be constructed. They are always finite and no one-loop mass term can be found for the first generation.

4. The features of the generation of radiative masses that we have discussed so far are general. The fact that they can be "naturally" implemented in a specific model is shown by the following Yukawa Lagrangian

$$\begin{aligned} \mathcal{L}_Y = & a \bar{5}_3 10_3 \bar{5}_H + b 10_3 10_3 5_H + c \bar{5}_2 10_3 \bar{5}'_H + d 10_2 10_2 5'_H \\ & + e 10_3 10_2 \bar{5}_H + f 10_3 10_1 \bar{5}'_H + g 5_1 5_3 \bar{10}_H + h.c. \end{aligned} \quad (3)$$

$\mathcal{L}_Y$, together with the standard gauge Lagrangian, is readily seen to possess a global $U(1)^4$ which forbids any other coupling of the fermions to the same Higgs fields. Through the Higgs self-interactions, the neutral components of $5_H, 5'_H$ will get a v.e.v., giving direct masses to t, b, τ and c . The global $U(1)^4$ is broken by cubic terms of the Higgs potential, e.g.,

$$\begin{aligned} \mathcal{L}_H^{(3)} = & h_1 10_H 5_H 5'_H + (h_2 5_H + h_3 5'_H + h_4 \bar{5}_H + h_5 \bar{5}'_H) 10_H 10_H \\ & + (h_6 5_H \bar{5}_H + h_7 5_H \bar{5}'_H + h_8 5'_H \bar{5}_H + h_9 5'_H \bar{5}'_H) 75_H + h.c. \end{aligned} \quad (4)$$

^{*)} We have calculated a few diagrams which, in fact, show that the mass term is indeed a mixture of a $\underline{5}$ and a $\underline{45}$, as it is required to reproduce the correct ratio m_d/m_e .

where a $\underline{75}_H$ has also been introduced. $\mathcal{L}_Y$, along with $\mathcal{L}_H^{(3)}$ and the remainder of the Higgs potential (respecting the $U(1)^4$ symmetry), is easily seen to give radiative masses as indicated^{*)}.

As in the minimal $SU(5)$ model, the global symmetry of $\mathcal{L}_Y$ leads, even after spontaneous symmetry breaking, to the conservation of B-L, baryon minus lepton number. Here, however, the necessary breaking of the global symmetries by $\mathcal{L}_H^{(3)}$, in order to produce radiative masses, also spoils the conservation of B-L itself. "Calculable" neutrino masses are therefore radiatively generated. At one-loop, the only contributing diagram is given in Fig. 4 with mass terms of typical order

$$\delta m_\nu^{(1)} \simeq \frac{h^2}{4\pi^2} \left(\frac{h}{e}\right)^2 \frac{M_W^2}{M} \quad (5)$$

For $h \simeq e$ and $M \simeq 10^{14}$ GeV, $\delta m_\nu^{(1)} \simeq 10^{-3}$ eV. The possible relevance of these masses in astrophysics has already been emphasized⁴⁾.

The particular Lagrangian ($\mathcal{L}_Y$) gives, in leading order, only two neutrino masses as well as a definite mixing pattern. We cannot, however, really say how much these special features have a general character or if they rather depend on the Lagrangian chosen. The postulates stated at the beginning strongly restrict its form, but not completely. To understand its physical naturalness, the incorporation of $SU(5)$ in a more complete physical theory (with gravity in it?) seems required.

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*) The uniqueness of the one-loop mass generation mechanism presented here and contained in this Lagrangian is tied to the natural assumption that all colourless neutral Higgs components get a non-zero v.e.v. By relaxing it, and having say a $\underline{5}_H$ without a v.e.v., we might construct a model in which the mass generation is stepwise, that is in which the mass of a family comes as a one-loop effect of the mass of the preceding family. This mechanism would exhibit an interesting feature: while m_u is proportional to m_s , m_d is proportional to m_c , thus explaining the puzzling inversion of the fermion spectrum $m_c > m_s$, $m_u < m_d$.

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FIGURE CAPTIONS

- Fig. 1 : A general one-loop mass term. The solid lines are fermions and the dashed lines are Higgs fields.
- Fig. 2 : Actual one-loop mass diagram.
- Fig. 3 : Possible two-loop mass diagrams.
- Fig. 4 : Neutrino mass diagram.

Fig. 1

Fig. 2

Fig. 3

Fig. 4