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HIERARCHICAL FERMION MASSES IN E_6

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A B S T R A C T

A recent proposal for getting hierarchical fermion masses in GUTs is realized by using E_6 as a prototype model. The existence of superheavy fermions is crucial. An interesting consequence is a possible explanation of $m_c > m_s \sim m_\mu$.

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1. The idea that the standard model is only the infra-red limit of a more fundamental physical theory at distances M^{-1} much shorter than M_W^{-1} , leads to the following possibility ¹⁾ : the smallness of some masses and/or some mixing angles may be attributed to some exact global symmetry of the standard model broken only by couplings of the "light" particles (with masses $m \lesssim M_W$) to heavy degrees of freedom of the theory (with masses $\approx M$). Such a more basic theory will have to provide not only the necessary extra couplings, but also give an explanation of the global symmetries occurring in the light sector. As is indeed well known, all such symmetries, except for B , L_e , L_μ and L_τ , would look rather accidental if seen from the point of view of the pure $SU(3) \otimes SU(2) \otimes U(1)$ low energy theory. On the other hand, since most of these possible global symmetries are associated with the family replicas, the programme has a chance of being pursued only after achieving a non-trivial description of such a replication.

A particular mechanism to produce naturally small masses, making use of superheavy fermions, has been suggested in Ref. 1). Here, a realization of it is given in the context of a grand unified E_6 model ^{2),3)}, where superheavy fermions naturally occur ³⁾. On the basis of what we have just said, this model is not complete in so far as it does not provide any clue to the appearance of the mentioned global symmetries, some of which are, in fact, assumed at the beginning. It presents, however, two results : i) giving a driving mass term $O(M_W)$ to the third family (t, b, τ) , the second (c, s, μ) and first families (u, d, e) will get masses $O(\alpha/\pi M_W)$ and $O((\alpha/\pi)^2 M_W)$ respectively, as a result of radiative corrections ; ii) some plausible reasons for the hierarchy $m_c > m_\mu \sim m_s$ appear naturally in our scheme.

2. - The fermions of each family are assigned to the fundamental $\underline{27}$ representation of E_6 . Its decomposition under $SU(5)$:

$$\underline{27} = \underline{10} + \overline{\underline{5}} + \underline{1} + \underline{1} + \overline{\underline{5}} + \underline{5} \quad (1)$$

reveals that the $\underline{27}$ contains the usual 15-plet of the known fermions (f) plus a self-conjugate 12-plet of fermions (F) getting a superheavy mass according to the "chirality" argument known as "survival hypothesis" ^{*)}.

^{*)} Low mass fermions are only those that cannot receive $SU(3)_C \otimes SU(2)_W \otimes U(1)$ invariant masses.

The classification of the F's according to their colour, weak isospin and electric charge is given by :

$$\begin{aligned}
 \underline{5} & : N(1, 1/2, 0) , E(1, -1/2, 1) , D(3, 0, -1/3) \\
 \underline{\bar{5}} & : N^c(1, -1/2, 0) , E^c(1, 1/2, 1) , D^c(\bar{3}, 0, 1/3) \\
 \underline{1} & : L(1, 0, 0) \\
 \underline{1} & : \nu^c(1, 0, 0)
 \end{aligned} \tag{2}$$

where ν^c is the member of the 16-plet of $SO(10)$, singlet under $SU(5)$. The Higgs bosons giving mass to the fermions and, at the same time, providing the mechanism of spontaneous symmetry breaking of E_6 down to $SU(3)_C \otimes U(1)_{e.m.}$ ^{*)} are contained in the product :

$$\underline{27} \times \underline{27} = (\underline{27} + \underline{351})_{\text{symm.}} + \underline{351}'_{\text{antisymm.}} \tag{3}$$

We shall only make use of the symmetric part of Eq. (3) (our notation will be : $\underline{27} \rightarrow H$ and $\underline{351} \rightarrow \Phi$). We assume that H develops small ($h \sim M_W$) and super-large (H) vacuum expectation values (VEV's), whereas Φ is allowed to develop only superlarge (Φ) VEV's. This assumption is as "unnatural" as the hierarchy itself : we can only hope some explanation will come from an understanding of the "hierarchy problem". The mass terms then come from the following Yukawa couplings :

$$\begin{aligned}
 \psi \phi \psi \rightarrow & \phi(1) LL + \phi(16) \nu^c L + \phi(126) \nu^c \nu^c + \\
 & + \phi(54, 24) (D^c D - 3/2 E^c E - 3/2 N^c N) + \\
 & \phi(144, 24) (d^c D - 3/2 E^c e - 3/2 N^c \nu)
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 \psi H \psi \rightarrow & H(1, 1) (D^c D + E^c E + N^c N) + \\
 & + H(16, 1) (d^c D + E^c e + N^c \nu) +
 \end{aligned} \tag{5}$$

*) E_6 is the only Lie group with the following features : i) it is automatically anomaly free ; ii) it possesses useful complex representations ; iii) the fermions of each generation belong to an irreducible representation ; iv) all the symmetry breaking can be realized by using only those Higgs representations which give mass to fermions.

$$\begin{aligned}
 & + h (16, \bar{5}) (D^c d + e^c E + N^c \nu^c) + \\
 & + h (10, \bar{5}) (d^c d + e^c e + N^c L) + \\
 & + h (10, 5) (u^c u + \nu^c \nu + L N)
 \end{aligned} \tag{5Ctd}$$

where the number in parentheses gives the transformation properties under $SO(10)$ and $SU(5)$ respectively, and u, d, e and ν indicate a generical $Q = 2/3$, $Q = -1/3$, $Q = -1$ and $Q = 0$ fermion, respectively.

We notice that from (4) and (5), a substantial difference emerges between the charged lepton and $Q = -1/3$ quark sectors on one side, and the $Q = 2/3$ quark sector on the other. Indeed, in the latter, the current eigenstate u coincides with the mass eigenstate u_0 , while in the former the mass eigenstates are given by the mixings :

$$(d_0, d_0^c) = (d, d^c \cos \theta + D^c \sin \theta) \tag{6a}$$

$$(e_0, e_0^c) = (e \cos \theta + E \sin \theta, e^c), \tag{6b}$$

where θ is a function of $\phi(54, 24)$ and $\phi(144, 24)$. As we shall see, the mixings (6a) and (6b) could provide a possible key for the interpretation of the hierarchy $m_c > m_u \sim m_s$.

We restrict the Yukawa Lagrangian to the following couplings :

$$\psi_3 H \psi_3 + \psi_2 \phi \psi_2 + (\psi_1 \psi_3 + \psi_3 \psi_1) \phi \tag{7}$$

where the indices 1, 2 and 3 refer to the u, c and t family, respectively.

The expression (7) can be naturally guaranteed by imposing the global $U(1) \otimes U(1)$ symmetry :

$$\begin{aligned}
 \psi_1 & \rightarrow e^{-i\theta} \psi_1, \quad \psi_2 \rightarrow \psi_2, \quad \psi_3 \rightarrow e^{i\theta} \psi_3, \quad H \rightarrow e^{-2i\theta} H, \quad \phi \rightarrow \phi; \\
 \psi_1 & \rightarrow e^{2i\chi} \psi_1, \quad \psi_2 \rightarrow e^{i\chi} \psi_2, \quad \psi_3 \rightarrow \psi_3, \quad H \rightarrow H, \quad \phi \rightarrow e^{-2i\chi} \phi.
 \end{aligned} \tag{8}$$

The symmetry (8), broken by the VEV's, must also be broken explicitly by soft terms in the Higgs potential, e.g., $HH\phi$.

If the components of $\tilde{\Phi}$, which are non-singlet under $SU(2)_W \otimes U(1)_Y$, do not develop a VEV getting a large positive mass squared, then the only "low energy" Yukawa coupling is $\sim \psi_3 \psi_3 H$, which respects the global symmetry (8) preventing ψ_1 and ψ_2 from getting direct "low" mass terms. Notice that, in fact, the diagrams providing a "low" radiative mass to ψ_1 and ψ_2 respect the original $U(1) \otimes U(1)$ in (8) before symmetry breaking (Fig. 1). Clearly, it is only h that can ultimately give rise to a "low energy" mass term $\psi_2 \psi_2$ (or $\psi_1 \psi_3$). However, $\psi_2 \psi_2 H$ (or $\psi_1 \psi_3 H$) are forbidden by (8). It is actually in this way that the finiteness of the radiative mass diagrams of Fig. 1 is guaranteed, since $\psi_1 \psi_3 HH^+ \tilde{\Phi}$ and $\psi_2 \psi_2 HH^+ \tilde{\Phi}$ are non-renormalizable couplings.

We would like to stress why the gauge boson mixing through H bosons is unavoidable in Fig. 1 : the survival hypothesis ^{*)} compels the internal fermion mass term to be a $SU(3) \otimes SU(2) \otimes U(1)$ singlet, whereas the external leg fermions must exhibit different $SU(2)$ transformation properties. Thus, the two exchanged gauge bosons carry different weak isospin numbers, so that their mixing can be allowed only through the VEV's h of the H .

Two ingredients are required to construct explicitly all the possible diagrams of the kind represented in Fig. 1 : the gauge currents of E_6 and the relevant quantum numbers of the neutral H components.

The 78 gauge currents of E_6 transform under $SO(10)$ as :

$$\underline{78} = \underline{45} + \underline{16} + \overline{\underline{16}} + \underline{1} \quad (9)$$

The 45 currents of $SO(10)$ are well known ⁴⁾, while it can be useful to give the currents in the $\underline{16}$:

$$\begin{aligned} (1,2) \quad & \begin{cases} Q=1 & (J_\mu)_1 = \bar{D} \gamma_\mu u + \bar{N}^c \gamma_\mu e^c + \bar{E} \gamma_\mu \nu^c + \bar{e} \gamma_\mu L \\ Q=0 & (J_\mu)_2 = \bar{D} \gamma_\mu d + \bar{E}^c \gamma_\mu e^c + \bar{N} \gamma_\mu \nu^c + \bar{\nu} \gamma_\mu L \end{cases} \\ (1,1) \quad & Q=0 \quad (J_\mu)_3 = -(\bar{D}^c \gamma_\mu d^c + \bar{E} \gamma_\mu e + \bar{N} \gamma_\mu \nu + \bar{\nu}^c \gamma_\mu L) \end{aligned} \quad (10)$$

^{*)} Same footnote as on p. 1.

$$(1,1) \quad Q=1 \quad (J_\mu)_4 = -(\bar{D}^c_\mu u^c + \bar{N}^c_\mu e + \bar{E}^c_\mu e + \bar{e}^c_\mu L)$$

$$(3,2) \quad \begin{cases} Q=1/3 \quad (J_\mu^i)_5 = \varepsilon^{ijk} \bar{D}^c_j \gamma_\mu u_k - \bar{d}_i \gamma_\mu L + \bar{D}_i \gamma_\mu \nu \\ \quad \quad \quad + \bar{E} \gamma_\mu u^c_i - \bar{N}^c \gamma_\mu d^c_i \\ Q=-2/3 \quad (J_\mu^i)_6 = \varepsilon^{ijk} \bar{D}^c_j \gamma_\mu d_k + \bar{u}_i \gamma_\mu L - \bar{D}_i \gamma_\mu e \\ \quad \quad \quad + \bar{N} \gamma_\mu u^c_i - \bar{E}^c \gamma_\mu d^c_i \end{cases}$$

$$(3,1) \quad Q=-1/3 \quad (J_\mu^i)_7 = \varepsilon^{ijk} \bar{D}^c_j \gamma_\mu u^c_k + \bar{d}^c_i \gamma_\mu L - \bar{D}^c_i \gamma_\mu \nu^c \\ + \bar{N}^c \gamma_\mu d_i - \bar{E}^c \gamma_\mu u_i \quad (10\text{Ctd})$$

where, with (a,b), we have indicated the colour and $SU(2)_W$ numbers of each current.

The H neutral components can be usefully classified according to their $SU(3)_C$, T_{3L} , T_{3R} , Y_L and Y_R quantum numbers :

$$H(1,1) = (1, 0, 0, 1/3, -1/3)$$

$$H(16,1) = (1, 0, -1/2, 1/3, 1/6)$$

$$h(10,5) = (1, -1/2, 1/2, -1/6, 1/6)$$

$$h(10, \bar{5}) = (1, 1/2, -1/2, 1/6, -1/6)$$

$$h(16,5) = (1, 1/2, 0, -1/6, -1/3)$$

(11)

The contribution of the diagrams of Fig. 1 is :

$$\delta m \simeq 3 \frac{\alpha}{\pi} m_F \frac{M_W}{M_a - M_b} \left[\frac{M_a^2}{m_F^2 - M_a^2} \log \left(\frac{m_F}{M_a} \right)^2 - \frac{M_b^2}{m_F^2 - M_b^2} \log \left(\frac{m_F}{M_b} \right)^2 \right] \quad (12)$$

where M_a and M_b indicate the masses of the exchanged gauge bosons.

We shall show in a moment that the leading contribution from (12) turns out to be $\sim O(\alpha/\pi M_W)$. Therefore, the following mass matrix arises for any fermion of definite charge, once the one-loop radiative corrections (12) are taken into account :

$$m = \begin{pmatrix} 0 & 0 & c(\frac{\alpha}{\pi})M_W \\ 0 & b(\frac{\alpha}{\pi})M_W & 0 \\ c(\frac{\alpha}{\pi})M_W & 0 & aM_W \end{pmatrix} \quad (13)$$

with a , b and c numerical factors $O(1)$, dependent upon the charged sector under consideration.

The three eigenvalues of the matrix $\mathcal{M}$ are of order M_W , $\alpha/\pi M_W$ and $(\alpha/\pi)^2 M_W$, thus accounting correctly for the order of magnitude of the masses of the three families.

3. We now discuss in more detail the contributions (12) to the second families in order to get the distinct values m_c , δm_μ and δm_s . The rich gauge structure of E_6 allows for such a quantity of diagrams of the kind represented in Fig. 1, that we need some criterion to decide where the leading contributions come from. We assume $SO(10)$ to be a good intermediate symmetry, so that the symmetry breaking scheme is :

$$E_6 \xrightarrow{M_{E_6}} SO(10) \xrightarrow{M_{SO(10)}} SU(3)_C \otimes SU(2)_W \otimes U(1)_Y \xrightarrow{M_W} SU(3)_C \times U(1)_{em} \quad (14)$$

From (14) and (4), the following splitting in the F masses holds ^{*)} :

$$m_L \sim M_{E_6} \quad ; \quad m_{\nu^c} \sim m_D \sim m_E \sim m_N \sim M_{SO(10)} \quad (15)$$

*) As H couples only to the third generation, the VEV $H(1,1)$ gives mass $\sim O(M_{E_6})$ only to the D , E and N belonging to this generation, whereas the superheavy fermions taking part in the radiative contributions to the charm and up-generation belong to the second and first generations. This justifies why we must simply consider the Yukawa coupling of ϕ_{351} , thus getting the splitting (15).

The other crucial quantity in the evaluation (12) is the mass of the exchanged gauge bosons. A major difference between diagrams whose internal fermion line is a ν^c or a D, E and N comes about : as ν^c belongs to the 16-plet of SO(10), the exchanged bosons belong to the $\underline{45}$ of SO(10) with a mass $\sim O(M_{SO(10)})$, whereas in all the other cases (i.e., with D, E, N and L in the internal line) at least one gauge boson belongs to the $\underline{16}$ (or $\overline{16}$) of SO(10) in Eq. (9) thus being $\sim O(M_{E_6})$. Therefore, the diagrams exchanging D, N or E are suppressed with respect to those exchanging ν^c , as their gauge bosons are heavier, and to those exchanging L as $m_D \sim m_N \sim m_E < m_L$. Indeed, it can easily be seen that the diagrams with L and ν^c are of the same order of magnitude, so that one would be led to conclude that all (and only) the diagrams of Fig. 2 give the leading contribution in (12). Things are not so : the graphs 2c and 2e of Fig. 2 are absent as they involve gauge bosons not present in $E_6(10)$. For example, the boson coming out of the vertex $s^c \nu^c$ should be a triplet of colour and $SU(2)_R$ and a singlet of $SU(2)_L$.

From the remaining graphs of Fig. 2, we can read the following contributions [we make use here of a simplified version of Eq. (12), which is valid when the mass of the internal F is smaller than the gauge boson masses] :

$$\delta m_c \sim \frac{\alpha}{\pi} \left[\alpha m_{\nu^c} \frac{H(16,1)^* h(16,5)}{M_a^2 - M_b^2} \lg \frac{M_a^2}{M_b^2} + \right. \\ \left. + \alpha m_L \frac{h(10,5)^* H(1,1)}{M_6^2 - M_8^2} \lg \frac{M_6^2}{M_8^2} \right] \quad (16a)$$

$$\delta m_s \sim \frac{\alpha}{\pi} \left[\alpha m_L \frac{h(10,5)^* H(1,1)}{M_6^2 - M_7^2} \lg \frac{M_6^2}{M_7^2} \right] \cos \theta \quad (16b)$$

$$\delta m_\mu \sim \frac{\alpha}{\pi} \left[\alpha m_L \frac{h(10,5)^* H(1,1)}{M_1^2 - M_4^2} \lg \frac{M_1^2}{M_4^2} \right] \cos \theta \quad (16c)$$

The numbers put as indices of the masses refer to the classification given in (10), while a and b in (16a) refer to gauge bosons belonging to the $\underline{45}$ of SO(10) and thus are not contained in (10).

Let us consider (16b), for example, in which :

$$M_6^2 - M_7^2 \sim O(M_{E_6} \cdot M_W) \quad \text{and} \quad \lg \frac{M_6^2}{M_7^2} \sim O\left(\frac{M_W}{M_{E_6}}\right)$$

so that we get :

$$m_s \sim \frac{\alpha}{\pi} O(M_{E_6}) \frac{O(M_W) O(M_{E_6})}{O(M_{E_6} \cdot M_W)} \cdot O\left(\frac{M_W}{M_{E_6}}\right) \sim \frac{\alpha}{\pi} O(M_W) \quad (\text{at GUT mass})$$

as we had preannounced just after (12).

However, the expressions (16) tell us more than the simple order of magnitude of the masses, suggesting a possible explanation of the hierarchy :

$$m_c > m_s \sim m_\mu \quad (17)$$

Indeed, two distinct reasons are available :

- i) the suppression of the $\cos\theta$ in (16b-c) with respect to (16a) due to the fact that s and μ must "share" their radiative contributions with their superheavy partners (differently from the quark c) ;
- ii) the presence of an additional term in (16a) due to the different T_{3L} and T_{3R} transformation properties of c with respect to s and μ (causing the existence of diagram 2a and the impossibility of diagrams 2c and 2e).

We notice that even though D and E do not seem to play a decisive rôle as the diagrams where they are involved are suppressed, they are essential in our scheme : the presence of D and E could be the ultimate reason for the hierarchy (17) allowing for the mixings (6).

For the first generation, we must limit ourselves to the correct order of magnitude as provided by the mass matrix (13).

In conclusion we have shown that the mass mechanism of Ref. 1), based on the presence of superheavy fermions, can be successfully implemented in a E_6 model. This adds more interest to the study of the superheavy fermion presence in grand unified theories, which has also been advocated ⁵⁾ in connection with the production of a net baryon asymmetry in the early Universe.

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FIGURE CAPTIONS

- Figure 1 The basic diagram assuring a "radiative" mass for the fermions of the second and first generations.
- Figure 2 Leading diagrams according to the symmetry breaking scheme (14).
Notice that the diagrams c) and e) cannot exist as they involve gauge bosons which do not belong to E_6 .

Fig. 1

Fig. 2