CERN-EP/80-206
13 November 1980

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

PARAMETRIZATION OF THE LONGITUDINAL DEVELOPMENT OF
HADRONIC SHOWERS IN SAMPLING CALORIMETERS

R.K. Bock

CERN, Geneva, Switzerland

T. Hansl-Kozanecka

Technische Hochschule, Aachen, Fed. Rep. Germany

T.P. Shah

Rutherford Laboratory, Chilton, Didcot, UK

ABSTRACT

A parametrization of the longitudinal development of hadronic showers in calorimeters is presented; this is being used for simulation of showers in the experiment UA1. These calorimeters cover almost 4π solid angle, and consist of stacked electromagnetic (lead) and hadronic (iron) sandwiches. We have used published data of fine granularity together with new test data obtained for our calorimeter prototypes in order to extract the longitudinal shower shape in lead and iron. A satisfactory parametrization for the average longitudinal development of the showers is obtained. An example for fluctuating about this mean is also given for coarsely integrated read-out.

(Submitted to Nuclear Instruments and Methods)

1) Introduction

Calorimetric methods have become increasingly more essential for measuring hadronic energy and, consequently, the parametrization of hadron shower development becomes important for simulation purposes. The calorimeters used in UA1 aim to cover a maximum solid angle and to also give a good performance for electromagnetic showers. Since it is generally not feasible to carry out test and calibration runs for all possible configurations, computer simulation methods serving to 'interpolate' between measurements at fixed settings become a necessary tool in calorimeter design and in the simulation and analysis of events.

The Monte Carlo simulation of the physical phenomena giving rise to showers, both electromagnetic and hadronic, has led to a number of elaborate programs giving satisfactory results (1)-(4). Understandably, such simulations going to the level of individual interactions, are already expensive in computer resources at low energies, and the cost increases at least linearly with energy.

Attempts to describe showers globally by empirical parametrization have been successful for electromagnetic showers (5). The resulting formula for the shape of the longitudinal e.m. shower development is :

$$dE = k s^{a-1} e^{-bs} ds$$

where s is the depth of the calorimeter expressed in radiation length units, and a , b are energy-dependent dimensionless constants describing the shower shape (6). The scale factor k can be chosen to suit the unit in which the shower depositions are read out and depends largely on the scintillator/absorber ratio. It is most convenient to postulate

$$\int_0^{\infty} dE = E_{inc}$$

which leads to :

$$k = E b^a / \Gamma(a)$$

We have found no similarly satisfactory parametrization for hadronic shower development in the literature and we have therefore used various test data taken with fine granularity in sampling calorimeters over a wide range of energies, to obtain an equally simple and efficient means of simulating energy depositions in hadronic calorimeters. We describe in para. 2 the test data used, in para. 3 the ansatz and the fit results. In paragraph 4, a crude way to reproduce the shower fluctuations is discussed.

2) The Test Data used

There exists abundant literature with results from hadronic sampling calorimeter test runs (e.g.(7)-(11)). For our purpose, published curves and numbers do not suffice, however, because they are obtained using quite different setups (trigger, read-out) and include corrections for shower containment (end or side leakage, albedo). They are also not analysed with consistent criteria defining a shower origin, and, if fluctuations are published, they are presented in a necessarily simplified form. We have therefore chosen to go back to the raw data for a selection of these results.

Our analysis refers to showers developing from "origins" distributed according to an exponential law, with an attenuation given by the material's absorption length. It is therefore important to find criteria for the origin which match this distribution as well as possible. Our criterion requires two successive depositions to be above an energy-dependent minimum. In the measured showers, of course, several such origins can sometimes be identified, with little deposition in between. We always refer all depositions to the first identified origin. Also, the later a shower starts, the higher the probability is that energies escaping the origin criteria have been deposited before. We disregard such depositions entirely. All data used are free of containment problems, both lateral and longitudinal.

We restrict ourselves to two data sets with fine granularity, made available to us in all desirable detail, and to test data from our own UA1 setup. These data are described below.

2.1) UA1 Hadronic Test Runs (10), (11)

For calibrating the magnetised iron calorimeter used in the CERN neutrino experiment UA1, a test setup was used consisting of 45 plates of iron of average thickness 5 cm, each plate being followed by 6 mm of plastic scintillator (NE110) viewed individually by PM-s. The data are scaled per scintillator to represent a multiple of the most probable pulse height of single muons, a unit called NEP (number of equivalent particles). The data used were taken at 15, 50 and 140 GeV/c, each sample comprising 500 to 1000 incident pions. An electron contamination in the beam was removed at data taking time by tagging with a differential Cerenkov counter. The published data do not contain showers originating in the first plate (to avoid problems of albedo). We believe that for simulation purposes shower shapes typical for early showers must be included, and have therefore formed averages including these. It was necessary, however, to eliminate a remaining electron contamination (about 10%) among the showers in plate 1, at 15 GeV/c, which could be identified by their containment (95%) in 4 or fewer iron plates. There remains an observed excess of "early" showers compared to what is expected if the origin distribution follows an

exponential law with the absorption length of iron.

2.2) Test Runs for Experiment 379 FNAL (12)

This neutrino production experiment relied on a fine-grain calorimeter to measure the energy lost during interaction by incident protons. A resolution of $\pm 3.5\%$ was achieved at 400 GeV, using suitable cuts and careful calibration. The calorimeter consists of 20 steel plates 3.84 cm (1.5") thick, followed by 25 plates 5.12 cm (2") thick. The plastic scintillators were 0.64 cm thick, and viewed individually. About 4000 incident pions at 375 and 400 GeV incident energy each have been used for our analysis. When referred to a shower origin, depositions had to be interpolated to a constant sampling size of 3.84 cm. Events with shower origin beyond the first 10 plates (2.33 absorption lengths) had been removed from tapes before transmission. Again, when using a cutoff resulting in the correct attenuation for shower origins, an excess of early origins remains.

2.3) Test Data for Experiment UA1 (14), (15)

The present study was done in preparation of the CERN $\bar{p}p$ collider experiment UA1 (13). We therefore could not only have access to detailed test data, but also perform specific tests useful to our purpose.

The UA1 experiment uses throughout the accessible solid angle a combination of electromagnetic/hadronic sampling calorimeters. Most typical is an electromagnetic part consisting of about 24 radiation lengths of lead subdivided into layers of 2-3mm Pb and 1.5-2mm scintillator, and read out in 3 or 4 parts, followed by at least 4 interaction lengths (75 cm) of iron, subdivided into layers of 5 cm Fe and 1 cm scintillator, and read out in two samples. To study the behaviour of such combined calorimeters, extensive tests were performed in pion beams of energies from 5 to 90 GeV/c. More details are reported in (14) and (15). The samples retained for defining a shower shape were taken specifically for this study to ensure a suitable origin resolution. An assembly of 16 modules of lead-scintillator sandwiches (13 layers each) representing 16×0.383 absorption lengths (16×7.53 radiation lengths) was read out over wave length shifter bars on two sides, each being viewed by a PMtube. The sum of the two ADC outputs was used as measure of deposited energy. Due to the short testing time available, a correct normalisation to muons could not be done and some readout problems remained, resulting in a spread larger than necessary. Data were taken at 10, 20, 40 and 60 GeV/c, with 5000 incident pions at each momentum. The observed distribution of origins was found to be in good agreement with that expected from exponential absorption length dependence.

3) The Ansatz and the Fit

3.1) Preliminaries

Our parametrization makes use of the following simplifications :

- When referred to a shower "origin", the fluctuations about an average longitudinal shower development are assumed minimal. The distribution of shower origins follows an exponential law with an attenuation given by the absorption length and independent of energy.
- Only the shower shape is of interest ; any normalisation is to be treated independently, and will have to take into account the specific ratio of visible energy and any calibration constants considered.
- No deposition has to be accounted for before the "origin", thus the ionisation losses of a charged primary, or small/short showers resulting in less deposition than the cut used are simply ignored.
- The average shower development from the origin shows a fast rise towards a maximum, and a slower exponential fall with the indication of a shoulder. We ascribe this to a first interaction giving rise to an electromagnetic shower part from π^0 -s that can be developed according to the electromagnetic shower parametrization : to this has to be added a "hadronic" shower part (in reality more determined by multiple secondary interactions each producing new "electromagnetic" components), which is parametrized according to the same formula, but using the absorption length.

3.2) The Parametrization

This leaves us with an ansatz of the form:

$$dE = k \{ w s^a - 1 e^{-bs} + (1-w) t^c - 1 e^{-dt} \} ds$$

where s = depth from shower origin in radiation lengths = x/X_0

t = depth from shower origin in absorption lengths = x/λ

a, b, c, d = shape parameters to be fitted

$w, 1-w$ = relative weight of the two curves

k = normalisation constant.

All four parameters a, b, c, d are allowed a logarithmic energy dependence:

$$a = a_1 + a_2 \log(E)$$

etc.

3.3) The Fit

In order to find suitable coefficients w, a, b, c, d all test data expressed as average deposition referred to a shower origin were fitted to the above formula. A quadratic form

$$\sum \{(\text{dep}_{\text{fit}} - \text{dep}_{\text{meas}})^2 / \text{dep}_{\text{fit}}\}$$

was used for minimising, using the general minimising code MINUIT. This form gives a reasonable relative weight to depositions in the peak and tail, because the spread of the deposition is assumed of the same order as the deposition itself. A normalisation was performed for each set of data, such that $\sum(\text{dep}_{\text{fit}}) = \sum(\text{dep}_{\text{meas}})$. All measurements were introduced in approximate NEP units, which results in $\sum(\text{dep}_{\text{meas}}) \approx 10 E$ (E is the incident energy in GeV). Whereas the exponential attenuation parameters b, d of the curves can very well be separated, there is strong correlation between the rise-parameters a, c , and the weight factor w , so that we finally imposed $a = c$.

In order to obtain good agreement with data, we had to introduce a "smearing" function taking care of the different granularity in origins. This is particularly relevant for the data taken in pure lead, where showers starting early and late in the same sample will leave two rather differently distributed depositions. The smearing was obtained by crude numerical integration over the sample, assuming shower origins distributed according to e^{-t} .

3.4) The Result

We obtained a reasonable agreement with our input data with the following parameters :

$$\begin{aligned} a &= 0.6165 (\pm 0.004) + 0.3183 (\pm 0.002) \log E \\ b &= 0.2198 (\pm 0.003) \\ c &= a \\ d &= 0.9099 (\pm 0.001) - 0.0237 (\pm 0.003) \log E \\ w &= 0.4634 (\pm 0.008) \\ & (E \text{ in GeV}) \end{aligned}$$

Figs. 1 and 2 show the fits to the input data for iron and lead calorimeters respectively. The quoted errors on the fitted quantities are those obtained by the minimising program and correspond to a change in the minimised function of unity, disregarding correlations. The function value (χ^2) is 57.9, indicating that in average $\langle (\text{dep}_{\text{fit}} - \text{dep}_{\text{meas}})^2 / \text{dep}_{\text{fit}} \rangle = 0.2$.

In our final parametrization, normalisation to $\int dE = E$ is obtained by setting

$$\int_0^x dE = E \{ w G(a, bs) / \Gamma(a) + (1-w) G(c, dt) / \Gamma(c) \}$$

where

$$G(q,p) \equiv \int_0^p x q^{-1} e^x dx$$

is available as library function (partial gamma function). For a normalisation closer to experimental conditions see (16).

This parametrization was subsequently compared with the original averages referring to the calorimeter origin, and with several independent test results obtained for UA1. Fig. 3 shows the agreement after integrating over shower origins for iron calorimeters. The excess of "early showers" in the test data is clearly visible. In Fig. 4 one can see that the fit also describes well the data for combined lead-iron calorimeters. Disagreements at the end of the tail have to do with the read-out and data transmission precision.

3.5) Limitations

Some data taken under different angles of incidence were also compared to predictions from the above parametrization, and satisfactory agreement found. Any situation however, where lateral energy distribution is important, can only be described adding further assumptions. Some attempts in that direction have been described elsewhere (16). We also do not believe that our representation would be useful for extrapolation to other materials or to lower energies.

4) Introduction of Fluctuations

The machinery described above for obtaining calorimeter signals contains as only fluctuation the random generation of shower origins. Further fluctuations are necessary to reproduce the test data also in their statistical behaviour. We find that altogether three types of fluctuations are needed to reproduce our test data adequately:

- i) Fluctuate the shower origin using absorption length values. This is obvious and done by default.
- ii) Fluctuate the incident particle energy to simulate the energy resolution of the calorimeter. For the UA1 calorimeters we use the following resolution values:
 $\text{mean} = E_{inc}$
 $\text{error} = 0.8/\sqrt{E_{inc}}$
- iii) Fluctuate the length of the shower (by scaling the values s and l) such that the mean shower length is tuned to the data and its width reflects that of the data:
 $\text{mean} = c_1 L_{normal}$
 $\text{error} = c_2 \text{ mean}$

With these three parameters randomised, one is able to simulate the relative pulse heights we expect to observe in the UA1 test setup(14). Reasonable agreement with existing data is obtained with $c_1 = 0.78$ and

$c_2 = 0.45$. The evident need to reduce the average shower length compared to what follows from paragraph 3.4 seems to reflect the fact that in the fit, where we have used mean values and Gaussian errors, the most probable pulse height is systematically overestimated as one approaches the end of the shower. The fluctuation on length is purely empirical and would appear to be well suited only for the UA1 setup. Fig. 5 shows a comparison with the test data. It shows that with these three parameters one has a fairly good representation of the UA1 test data. It should be emphasised that the prescription adopted here would almost certainly be inadequate to reproduce distributions in single scintillator readout situations.

5) Acknowledgements

We are greatly indebted to F. Dydak, Heidelberg and S. Wojcicki, SLAC for their help in making their original test data of experiments WA1 (CERN) and 379 (FNAL) available to us, and for fruitful discussions of their interpretation. The team from College de France / UA1 under L. Dobrzynski is thanked for setting up the lead hadron calorimeter.

6) References

- (1) W.V. Jones, Simulation of Nuclear-Electromagnetic Cascades in Ionisation Calorimeters, and A. Van Ginneken, Monte Carlo Simulations of Calorimeter Response, both in : Proceedings of the Calorimeter Workshop, ed. M. Atac, FNAL, May 1975.
- (2) A. Baroncelli, Nucl.Inst.Meth. 118 (1974) 445.
- (3) EGS, A Monte Carlo Program to Simulate Electromagnetic Showers, R.L. Ford, W.R. Nelson, SLAC Report 210 (1978).
- (4) A. Grant, Nucl.Inst.Meth. 131 (1975) 167.
- (5) E. Longo, I. Sestili, Nucl.Inst.Meth. 128 (1975) 283.
- (6) Using Monte Carlo Calculation for Electromagnetic Showers up to 5 GeV, (5) determines :
$$a = 3.0 + 0.43 \log E$$

and
$$b = 0.46.$$

Using test results for electrons in iron and lead from 5 to 92 GeV/c, we obtain :
$$a = 2.2840 + 0.7136 \log E$$

$$b = 0.5607 + 0.0093 \log E$$

with a satisfactory fit at a readout granularity limited to 5 cm, (single scintillator) in iron and to 4.2 cm (for 21 scintillators) in lead.

- (7) B.C. Barish et al., Nucl.Inst.Meth. 116 (1974), 413 and Nucl.Inst.Meth. 130 (1975), 49.
- (8) P.L. Cheshire et al., Nucl.Inst.Meth. 126 (1975), 253.
- (9) B. Friend et al., Nucl.Inst.Meth. 136 (1976), 505.
- (10) M. Holder et al., Nucl.Inst.Meth. 141 (1977), 43 and Nucl.Inst.Meth. 151 (1978), 69.
- (11) Untersuchungen an einem total absorbierenden Hadronenkalorimeter, Th.Flottmann, Diplomarbeit, University of Heidelberg (1978)
- (12) The Production of Neutrinos and Neutrino-like Particles in Proton Nucleus Interactions : J.P. Dishaw, Ph.D. Dissertation, SLAC-Report 216 (1979).
- (13) UA1 Proposal, CERN/SPSC/78-06 (1978).
- (14) M.J. Corden et al., Performance of a UA1 Hadron Calorimeter Prototype, in: Proceedings of the International Conference on Experimentation at LEP, Uppsala, 1980 (to be published in Physica Scripta).
- (15) Results of the Forward Detector Prototype Tests $\bar{p}p$ Tech. Note 79/50, College de France, 3.9.79.
- (16) M. Della Negra, Pattern Recognition in Calorimeters, in: Proceedings of the International Conference on Experimentation at LEP, Uppsala, 1980 (to be published in Physica Scripta).

7) Figure Captions

Fig. 1 Shower shape as function of distance from shower origin in radiation lengths. Data from iron calorimeters in experiments WA1 (CERN) and 379 (FNAL). Superimposed lines correspond to the parametrization discussed in the text.

Fig. 2 Idem for a lead calorimeter, data from UA1 test setup.

Fig. 3 Shower shape as function of depth of calorimeter in radiation lengths. Data from iron calorimeters in experiments WA1 (CERN) and 379 (FNAL). Superimposed lines correspond to the parametrization discussed in the text. The discontinuity in the 400 GeV data shows the change of plate thickness in the calorimeter used.

Fig. 4 Idem for combined lead - iron calorimeter data from UA1 tests.

Fig. 5 Energy fluctuation. Data and simulation for an iron calorimeter (UA1), with pions at 10 GeV.

- a) Deposition in front stack
- b) Deposition in back stack

Fig. 6 Idem for a combined lead - iron calorimeter (UA1), with pions at 40 GeV.

Fig. 1,2,3,4

Fig. 5

Fig. 6