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$N = 4$ SUPERGRAVITY COUPLED TO $N = 4$ MATTER

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ABSTRACT

We present the $N = 1$ supergravity in 10-dimensions obtained by truncating the reduced $N = 1$ supergravity from 11-dimensions. This is further reduced to 4-dimensions to give $SU(4)$ supergravity coupled to six $SO(4)$ vector multiplets. As the reduction is from 10-dimensions, the whole system has a built-in "local" dynamical $SO(6)$ symmetry that can be extended to $U(4)$. The possibility of extending the global symmetry is discussed.

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1. INTRODUCTION

The study of the dual spinor model and interacting closed strings revealed the existence of $N = 1$ supergravity in 10-dimensions and predicted the necessary fields¹⁾. This knowledge was then used in constructing the pure $SU(4)$ supergravity²⁾.

From a different angle, after the establishment of $N = 1$ supergravity in 11-dimensions³⁾, it was argued by Scherk that this can be truncated to give the 10-dimensional theory⁴⁾. It is also known that in four-dimensions, after reduction from ten, the model describes $SU(4)$ supergravity coupled to six $N = 4$ vector matter multiplets. However, neither the theory in ten-dimensions, nor in four-dimensions, are fully worked out.

Our interest in this model stems from the fact that $N = 4$ supersymmetry is the largest one that admits matter representations, with matter defined as consisting of particles of maximum spin 1⁵⁾. Thus the ten-dimensional theory provides an example of an unrestricted scheme coupling supergravity to an arbitrary number of matter multiplets. This may be considered as an advantage over the $N = 8$ supergravity. Other uses of this model for the spinning string theory are advocated by Schwarz⁵⁾.

Taking the $N = 8$ supergravity as an example⁶⁾, it is natural to try to find the hidden symmetries that may be present. One can argue, guided by the work of Cremmer and Julia⁶⁾, that the reduction from ten dimensions will result in a dynamical local $SO(6)$ symmetry, which, like pure $N = 4$ supergravity, can be extended to $U(4)$. The global symmetry for the supergravity sector is $SU(1,1) \times SU(4)$, and it is an open problem whether the presence of the matter scalars will help in enlarging this global symmetry. On the negative side, it was shown in the calculation of the scalar-scalar and photon-photon interaction of $O(4)$ supergravity coupled to self-dual $O(4)$ matter multiplets that the results are not finite, implying the non-vanishing of the β function, although it does vanish for pure $O(4)$ supergravity and pure $O(4)$ matter⁷⁾.

The plan of this paper is as follows. In Section 2, we start with the $N = 1$ supergravity in 11-dimensions and show the consistent truncation that leads to the $N = 1$ theory in 10-dimensions. In Section 3 we reduce the theory to four dimensions and identify the physical fields. As it turns out, the hunt for hidden symmetries is not straightforward, thus we shall only comment on this in Section 4, as we hope to treat this interesting investigation in a future publication.

2. N = 1 SUPERGRAVITY IN 10-DIMENSIONS

We start with the now-familiar N = 1 Lagrangian in 11-dimensions⁶⁾, using the same notation and conventions. The fields are the elfbein e_M^A , the Rarita-Schwinger Majorana field ψ_M , and the antisymmetric potential A_{MNP} . The supersymmetry transformation rules are:

$$\begin{aligned}\delta e_M^A &= -i \bar{\epsilon} \Gamma^A \psi_M \\ \delta A_{MNP} &= \frac{3}{2} \bar{\epsilon} \Gamma_{[MN} \psi_{P]} \\ \delta \psi_M &= D_M(\hat{\omega}) \epsilon + \frac{i}{144} (\Gamma^{NPQR}{}_M - 8 \Gamma^{PQR} \delta_M^N) \epsilon \hat{F}_{NPQR}\end{aligned}\tag{2.1}$$

The definitions of the supercovariant connection $\hat{\omega}_{MAB}$ and field strength $\hat{F}_{NPQR}$ are given in Ref. 6).

Using part of the local SO(1,10) symmetry we can bring the elfbein to the form:

$$e_M^A = \begin{pmatrix} e_\mu^\alpha & B_\mu^{11} \\ 0 & e_{i1}^{11} = \varphi \end{pmatrix}\tag{2.2}$$

where $\mu, \nu, \dots = \dot{0}, \dot{1}, \dots, \dot{9}$ are curved indices, and $\alpha, \beta, \dots = 0, 1, \dots, 9$ are flat indices.

Note that we reserve the dotted numbers for curved indices and the undotted for flat ones. By splitting explicitly the eleventh index, the other fields decompose as:

$$\begin{aligned}A_{MNP} &= (A_{\mu\nu\rho}, A_{\mu\nu i1}) \\ \psi_M &= (\psi_\mu, \psi_{i1})\end{aligned}\tag{2.3}$$

We are not interested in keeping all these fields, as this will correspond to the known N = 8 supergravity in four-dimensions (and N = 2 supergravity in 10-dimensions), and we have to set some fields to zero. Due to the special properties of ten-dimensions, we can impose the Majorana and Weyl conditions simultaneously on the spinors. Thus each of the Majorana spinors ψ_μ, ψ_{i1} and ϵ , split into two Majorana-Weyl ones. We can show that keeping only the left-handed components,

of ψ_μ and ε and the right-handed component of ψ_{11} , and setting B_μ^{11} and $A_{\mu\nu\rho}$ to zero, form a consistent truncation. In other words we now have:

$$\begin{aligned}\gamma_{11} \psi_\mu &= \psi_\mu \\ \gamma_{11} \varepsilon &= \varepsilon \\ \gamma_{11} \psi_{11} &= \psi_{11}\end{aligned}\tag{2.4}$$

where

$$\Gamma^{11} = -\Gamma_{11} = i \gamma_{11}, \quad (\Gamma^{11})^2 = -1, \quad (\gamma_{11})^2 = +1$$

The truncation is consistent because

$$\begin{aligned}\delta B_\mu^{11} &= -i \bar{\varepsilon} \Gamma^{11} \psi_\mu = \bar{\varepsilon} \psi_\mu = 0 \\ \delta A_{\mu\nu\rho} &= \frac{3}{2} \bar{\varepsilon} \Gamma_{[\mu\nu} \psi_{\rho]} = 0\end{aligned}\tag{2.5}$$

and these vanish due to the condition (2.4). Moreover, the handedness of ψ_{11} and ψ_{11} is preserved as the quantities $\hat{\omega}_{\mu\alpha 11}$, $\hat{\omega}_{11\alpha\beta}$, and $\hat{F}_{\nu\kappa\lambda\rho}$ all vanish. We shall write the new transformation rules after the reduction of the theory, as some field redefinitions and scalings are needed. As the dimensional reduction technique is explained in detail in Ref. 6), we shall only indicate the necessary steps so that the reader can keep track of the field redefinitions, and without falling into repetition.

The bosonic Lagrangian in 11-dimensions is:

$$\begin{aligned}-\frac{1}{4} V_{11} R(\omega) - \frac{1}{48} V_{11} F_{MNPQ} F^{MNPQ} \\ + \frac{2}{(12)4} \varepsilon^{M_1 \dots M_{11}} F_{M_1 \dots M_4} F_{M_5 \dots M_8} A_{M_9 M_{10} M_{11}}.\end{aligned}\tag{2.6}$$

To reduce the gravitational sector we define the Weyl scaling for the 10-bein:

$$e_{10\mu}^\alpha = \varphi^{\frac{1}{8}} e_\mu^\alpha\tag{2.7}$$

and we obtain

$$-\frac{V_{11}}{4} R(\omega) = -\frac{V_{10}}{4} R_{10}(\omega) + \frac{1}{2} \left(\frac{3}{4}\right)^2 (\partial_{10\alpha} \log \varphi)^2 \quad (2.8)$$

The remaining parts of (2.6) give

$$\frac{V_{10}}{12} \varphi^{-\frac{3}{2}} g_{10}^{\mu\kappa} g_{10}^{\nu\lambda} g_{10}^{\rho\sigma} F_{\mu\nu\rho ii} F_{\kappa\lambda\sigma ii} \quad (2.9)$$

where

$$F_{\mu\nu\rho ii} = 3 \partial_\mu A_{\nu\rho ii} \quad (2.10)$$

For the fermionic kinetic term

$$-\frac{iV}{2} \bar{\psi}_M \Gamma^{MNP} D_N \psi_P \quad (2.11)$$

we need a Weyl-scaling and a field redefinition to diagonalize the ψ_μ and ψ_{11} contributions. This is achieved with

$$\begin{aligned} \chi_{11} &= e_{11}^{11} \psi_{11} \\ \chi_\alpha &= e_\alpha^\mu \psi_\mu \\ \chi_{10} &= \frac{3}{2\sqrt{2}} \varphi^{-\frac{1}{16}} \chi_{11} \\ \psi_{\mu 10} &= \varphi^{-\frac{1}{16}} e_{10\mu}^\alpha \left(\chi_\alpha - \frac{i}{6\sqrt{2}} \Gamma_\alpha \chi_{11} \right) \end{aligned} \quad (2.12)$$

so that (2.11) gives the normalized form:

$$-\frac{iV_{10}}{2} \bar{\psi}_{10\mu} \Gamma^{\mu\nu\rho} D_\nu \psi_{10\rho} + \frac{iV_{10}}{2} \bar{\chi}_{10} \Gamma^\mu D_\mu \chi_{10} \quad (2.13)$$

To cut the story short, the interaction terms are easily calculated with no further definitions. We then collect all terms and forget about the original theory. As

we are now in ten-dimensions, we can restore the notation that indices at the beginning of the alphabet denote flat co-ordinates, and the later characters curved ones, e.g.,

$$A, B, C, \dots = 0, 1, \dots, 9.$$

$$M, N, P, \dots = \dot{0}, \dot{1}, \dots, \dot{9}. \quad (2.14)$$

Thus the $N = 1$ Lagrangian in ten-dimensions is given by:

$$\begin{aligned} \kappa^2 \mathcal{L}_{10} = & -\frac{V}{4} R(\omega) - \frac{iV}{2} \bar{\Psi}_M \Gamma^{MNP} D_N \left(\frac{1}{2} (\omega + \hat{\omega}) \right) \Psi_P \\ & + \frac{iV}{2} \bar{\chi} \Gamma^M D_M \left(\frac{1}{2} (\omega + \hat{\omega}) \right) \chi + \frac{V}{12} \exp(-2\phi) F_{MNP} F^{MNP} \\ & + \frac{iV}{48} \left[\bar{\Psi}_M \chi^{MNPQR} \Psi_N - i\sqrt{2} \bar{\Psi}_M (\Gamma^{MPQR} - 3g^{M[CP} \Gamma^{QR]}) \chi \right] \\ & \quad \times (F_{PQR} + \hat{F}_{PQR}) \\ & + \frac{V}{2\sqrt{2}} (\partial_M \phi + \hat{D}_M \phi) (\bar{\Psi}^M \chi) \end{aligned} \quad (2.15)$$

where

$$\chi^{MNPQR} = \Gamma^{MNPQR} - 6 g^{M[CP} \Gamma^Q g^{R]N} \quad (2.16)$$

and

$$\phi = \frac{3}{4} \log \varphi \quad (2.17)$$

All the quartic terms are hidden in the supercovariant quantities, and the distinction between ω and $\hat{\omega}$ is a consequence of the second order formalism for which the ω equation of motion is not supercovariant. The connection is:

$$\begin{aligned} \omega_{MAB} = & \omega_{0MAB}(e, \partial e) + \frac{i}{4} \left[-\bar{\Psi}_N \Gamma^{NP}{}_{MAB} \Psi_P - \bar{\chi} \Gamma_{MAB} \chi \right. \\ & + 2 (\bar{\Psi}_M \Gamma_B \Psi_A - \bar{\Psi}_M \Gamma_A \Psi_B + \bar{\Psi}_B \Gamma_M \Psi_A) \\ & \left. - \frac{i\sqrt{2}}{3} (\bar{\Psi}_M \Gamma_{AB} \chi - 2 e_{M[A} \bar{\Psi}_{B]} \chi + \bar{\Psi}_N \Gamma^N{}_{MAB} \chi) \right] \end{aligned} \quad (2.18)$$

while the supercovariant connection is:

$$\hat{\omega}_{MAB} = \omega_{0MAB}(e, \partial e) + i \left[2(\bar{\psi}_M \Gamma_B \psi_A - \bar{\psi}_M \Gamma_A \psi_B + \bar{\psi}_B \Gamma_M \psi_A) - \frac{2i\sqrt{2}}{3} e_{M[A} \bar{\psi}_{B]} \chi - \frac{1}{36} \bar{\chi} \Gamma_{MAB} \chi \right] \quad (2.19)$$

The supercovariant quantities $\hat{F}_{PQR}$ and $\hat{D}_M \phi$ are:

$$\hat{F}_{PQR} = F_{PQR} - \frac{3i}{\sqrt{2}} \exp(\phi) (\bar{\psi}_P \Gamma_Q \psi_R - \frac{i}{\sqrt{2}} \bar{\psi}_P \Gamma_Q \chi - \frac{7}{72} \bar{\chi} \Gamma_{PQR} \chi) \quad (2.20)$$

$$\hat{D}_M \phi = \partial_M \phi + \frac{1}{\sqrt{2}} \bar{\psi}_M \chi \quad (2.21)$$

Finally the new transformation rules take the simple form:

$$\delta e_M^A = -i \bar{\epsilon} \Gamma^A \psi_M$$

$$\delta \phi = -\frac{1}{\sqrt{2}} \bar{\epsilon} \chi$$

$$\delta A_{MN} = \exp(\phi) (i \bar{\epsilon} \Gamma_M \psi_N + \frac{1}{2\sqrt{2}} \bar{\epsilon} \Gamma_{MN} \chi) \quad (2.22)$$

$$\delta \psi_M = D_M(\hat{\omega}) \epsilon + \frac{1}{36} \exp(-\phi) (\Gamma^{PQR}{}_M + 9 \delta_M^P \Gamma^{QR}) \epsilon \hat{F}_{PQR}$$

$$\delta \chi = \frac{i}{\sqrt{2}} \hat{D}_M \phi (\Gamma^M \epsilon) + \frac{i}{12\sqrt{2}} \exp(-\phi) (\Gamma^{PQR} \epsilon) \hat{F}_{PQR}$$

Note that ψ_M and χ preserve the conditions (2.4) under these transformations. Moreover, A_{MN} is subject to the Abelian gauge transformation:

$$\delta A_{MN} = \partial_M \xi_N - \partial_N \xi_M$$

To summarize, we have found the $N = 1$ supergravity Lagrangian and transformation rules in 10-dimensions. The fields are the 10-bein e_M^A , the Majorana-Weyl left-handed gravitino ψ_M and right-handed spinor χ , the antisymmetric tensor A_{MN} , and the scalar ϕ . As mentioned in the introduction, all these fields are predicted by the spinning string model in 10-dimensions¹⁾.

3. REDUCTION TO FOUR DIMENSIONS

We know what to expect in four dimensions: $N = 4$ supergravity coupled to six $N = 4$ vector matter multiplets with a manifest $SO(6) \sim SU(4)$ global symmetry and thus no duality transformations are needed. The only problem is to identify the physical fields, as for the dimensional reduction it is necessary to make many field redefinitions.

Using part of the gauge invariance $SO(1,9)$ we bring the 10-bein to the form:

$$e_M^A = \begin{pmatrix} e_\mu^\alpha & B_\mu^a \\ 0 & e_m^a \end{pmatrix} \quad (3.1)$$

where $m, n, p, \dots = 4, 5, \dots, 9$

$a, b, c, \dots = 4, 5, \dots, 9$

and we are left with the local symmetry $SO(1,3) \times SO(6)$. The Weyl-scaling for the vierbein is:

$$e_{4\mu}^\alpha = \Delta^{\frac{1}{4}} e_\mu^\alpha \quad (3.2)$$

where

$$\Delta = (\det e_m^a) \quad (3.3)$$

The gravitational sector gives

$$\begin{aligned} -V_{10} R = & -\frac{e_4}{4} R_4 + \frac{e_4 \sqrt{\Delta}}{16} g_4^{\mu\rho} g_4^{\nu\sigma} (G_{4\mu\nu}^a G_{4\rho\sigma}^b) \eta_{ab} \\ & - \frac{e_4}{16} g_4^{\mu\nu} (\partial_\mu g_{mn}) (\partial_\nu g^{mn}) + \frac{e_4}{32} g_4^{\mu\nu} (\partial_\mu \log \Delta) (\partial_\nu \log \Delta) \end{aligned} \quad (3.4)$$

where

$$g_{mn} = e_m^a e_n^b \eta_{ab}$$

$$G_{4\alpha\beta}{}^a = 2 e_{4[\alpha}{}^\mu e_{\beta]}{}^\nu \partial_\mu (B_\nu{}^b e_b{}^m) e_m^a \quad (3.5)$$

This describes one graviton, six vector fields B_μ^m , and 21 scalars g_{mn} .

The antisymmetric tensor A_{MN} decomposes into three types: $A_{\mu\nu}$ describing a pseudoscalar¹⁾, six pseudovectors $A_{\mu m}$, and 15 pseudoscalars A_{mn} . Under internal changes of co-ordinates: $x^m = x'^m + \zeta^m(x)$ these fields transform as:

$$\delta A_{\mu\nu} = -2 \partial_\mu \zeta^\mu A_{\nu}{}^m$$

$$\delta A_{mn} = 0$$

$$\delta A_{\mu m} = \partial_\mu \zeta^m A_{mn} \quad (3.6)$$

Also

$$\delta B_\mu^m = \partial_\mu \zeta^m \quad (3.7)$$

The appropriate potentials and field strengths which are invariant under internal changes of co-ordinates are:

$$A'_{\mu m} = A_{\mu m} - B_\mu{}^n A_{nm}$$

$$A'_{\mu\nu} = A_{\mu\nu} + 2 B_{[\mu}{}^m A_{\nu]m} + B_\mu{}^m B_\nu{}^n A_{mn} \quad (3.8)$$

and

$$F_{4\mu\nu a} = e_a^m (2 \partial_{[\mu} A'_{\nu]}{}^m + G_{\mu\nu}{}^n A_{nm})$$

$$F_{4\mu\nu\rho} = 3 (\partial_{[\mu} A'_{\nu\rho]} - G_{\mu\nu}{}^m A'_{\rho]m}) \quad (3.9)$$

The definitions (3.9) are related to the quantities found from $e_A^M e_B^N e_C^P F_{MNP}$ by:

$$F_{\alpha\beta\gamma} = \Delta^{\frac{3}{4}} F_{4\alpha\beta\gamma}$$

$$F_{\alpha\beta a} = \Delta^{\frac{1}{2}} F_{4\alpha\beta a} \quad (3.10)$$

With no further difficulties, the bosonic Lagrangian takes the form:

$$\begin{aligned} \kappa^2 \mathcal{L}_{\text{bosonic}} = & -\frac{e_4}{4} R_4(\omega) + \frac{e_4}{16} \sqrt{\Delta} (G_{4\alpha\beta})^2 + \frac{e_4}{2} (\partial_\mu \phi) (\partial^\mu \phi) \\ & - \frac{e_4}{16} (\partial_\mu g_{mn}) (\partial^\mu g^{mn}) + \frac{e_4}{32} (\partial_\mu \log \Delta) (\partial^\mu \log \Delta) \\ & + \frac{e_4}{12} \exp(-2\phi) \left[\Delta F_{4\mu\nu\rho} F_4^{\mu\nu\rho} + 3\sqrt{\Delta} F_{4\mu\nu m} F_4^{\mu\nu n} g^{mn} \right. \\ & \left. + 3 g^{mp} g^{nq} (\partial_\mu A_{mn}) (\partial^\mu A_{pq}) \right] \end{aligned} \quad (3.11)$$

The field strength $F_{4\mu\nu\rho}$ was shown in Ref. 1) to describe a pseudoscalar. This can easily be seen by writing $F_{\mu\nu\rho} = \varepsilon_{\mu\nu\rho\sigma} B^\sigma$ and introducing a Lagrange multiplier B . This will produce many interaction terms when taking the fermions into account.

Although the $SO(6)$ global symmetry is manifest, we have to identify the six supergravity vectors and the six matter vectors. In reality the $SU(4)$ supergravity has three vectors and three pseudovectors, and similarly for matter, thus we expect the physical field strengths to be a combination of $F_{4\alpha\beta a}$ and $G_{4\alpha\beta a}$. We therefore define the new vector potentials:

$$\begin{aligned} \mathcal{A}_{\mu m} &= \frac{1}{\sqrt{2}} \left(A'_{\mu m} + \frac{1}{2} \exp(\phi) B_{\mu m} \right) \quad (\text{gravity}) \\ \mathcal{B}_{\mu m} &= \frac{1}{\sqrt{2}} \left(A'_{\mu m} - \frac{1}{2} \exp(\phi) B_{\mu m} \right) \quad (\text{matter}) \end{aligned} \quad (3.12)$$

with the corresponding field strengths

$$\begin{aligned} \mathcal{F}_{\mu\nu m} &= \frac{1}{\sqrt{2}} \left(F_{4\mu\nu m} + \frac{1}{2} \exp(\phi) G_{4\mu\nu m} \right) \\ \mathcal{G}_{\mu\nu m} &= \frac{1}{\sqrt{2}} \left(F_{4\mu\nu m} - \frac{1}{2} \exp(\phi) G_{4\mu\nu m} \right) \end{aligned} \quad (3.13)$$

Then the kinetic terms for the vector fields can be written as:

$$\frac{e_4}{4} \sqrt{\Delta} \exp(-\phi) \left(\mathcal{F}_{\mu\nu m} \mathcal{F}^{\mu\nu m} + \mathcal{G}_{\mu\nu m} \mathcal{G}^{\mu\nu m} \right) \quad (3.14)$$

These relations are uniquely determined, as they also should emerge from the requirements that the supergravity fermionic transformations involve $\mathcal{F}_{\mu\nu m}$ while the matter fermionic transformations are proportional to $\mathcal{G}_{\mu\nu m}$, and also from the vector interaction terms.

We now turn our attention to the fermions and we first note that our spinors are 32-components subject to the Majorana-Weyl conditions, and thus every spinor gives four-independent spinors in four dimensions. We can represent the spinors with the $SO(6)$ notation $\psi_{\mu A}$ and χ_A , where A is an $SO(6)$ spinor index taking eight-values and the Weyl conditions (2.4) take the form:

$$\begin{aligned} (\Gamma^{11})_A^B \psi_{MB} &= i\gamma_5 \psi_{MA} \\ (\Gamma^{11})_A^B \chi_B &= -i\gamma_5 \chi_A \end{aligned} \quad (3.15)$$

and the $SO(6)$ generators are the 8×8 matrices $(\Gamma^{ab})_A^B$. Also the $SU(4)$ notation is possible by the following representation:

$$\psi_{MA} = \begin{pmatrix} \psi_{MK} \\ -i\gamma_5 \psi_{MK} \end{pmatrix}, \quad \chi_A = \begin{pmatrix} \chi_K \\ i\gamma_5 \chi_K \end{pmatrix} \quad (3.16)$$

where $K = 1, \dots, 4$ is an $SU(4)$ index.

In the following we shall keep the $SO(6)$ notation with the spinors subject to (3.15), and this can easily be converted to the $SU(4)$ notation by using (3.16) and the explicit representation of the $(\Gamma^a)_A^B$ matrices.

The fermions will involve a two-stage diagonalization, first separating the spin-3/2 fields from the spin-1/2 fields, then taking care of the mixed spin-1/2 fields. We make the following scalings and redefinitions:

$$\begin{aligned} \psi_a &= \psi_M e_a^M \\ \psi_\alpha &= \psi_M e_\alpha^M \\ \psi_{a4} &= \Delta^{-\frac{1}{8}} \psi_a \\ \chi_4 &= \Delta^{-\frac{1}{8}} \chi \\ \psi_{\mu 4} &= \Delta^{-\frac{1}{8}} e_{4\mu}^\alpha \left(\psi_\alpha + \frac{1}{2} \gamma_\alpha \Gamma^a \psi_a \right) \end{aligned} \quad (3.17)$$

where

$$\Gamma^a = \gamma_5 \otimes (\Gamma^a)_A^B$$

Then the fermionic kinetic Lagrangian becomes

$$\begin{aligned} \kappa^2 \mathcal{L}_F^{kin} = & -\frac{ie_4}{2} \bar{\Psi}_{4\mu A} \gamma^{\mu\nu\rho} D_\nu(\omega) \Psi_{4\rho A} \\ & -\frac{ie_4}{2} \left[\bar{\Psi}_{a4} \left(\frac{1}{2} \Gamma^a \Gamma^b + \eta^{ab} \right) \gamma^\nu D_\nu(\omega) \Psi_{b4} - \bar{\chi}_4 \gamma^\mu D_\mu(\omega) \chi_4 \right] \end{aligned} \quad (3.18)$$

Although the χ_4 field is diagonalized, its transformation under supersymmetry is proportional to a mixture of $\mathcal{F}_{\mu\nu m}$ and $\mathcal{G}_{\mu\nu m}$, and thus can neither be identified with the gravity spinor nor with a matter spinor. The spinors that transform properly are χ_a and λ defined by:

$$\begin{aligned} \chi_4 &= \frac{\sqrt{2}}{4} (i \Gamma^a \chi_a - \sqrt{2} \lambda) \\ \Psi_{a4} &= \chi_a - \frac{1}{8} \Gamma_a \Gamma^b \chi_b - \frac{i\sqrt{2}}{8} \Gamma_a \lambda \end{aligned} \quad (3.19)$$

bringing the spin- $\frac{1}{2}$ fields to the form

$$\frac{ie_4}{2} \left[\bar{\chi}_{aA} \gamma^\nu D_\nu(\omega) \chi_{aA} + \bar{\lambda}_A \gamma^\nu D_\nu(\omega) \lambda_A \right] \quad (3.20)$$

The definition of λ is unique, and although the χ_a diagonalization is defined up to an orthogonal transformation of the 24 degrees of freedom, the required transformation for χ_a restricts this freedom.

The interaction terms arise from three sources:

$$\begin{aligned} & -\frac{iV}{8} \bar{\Psi}_M \Gamma^{MNP} \Gamma^{RS} \Psi_P \omega_{NRS} \\ & +\frac{iV}{8} \bar{\chi} \Gamma^{NRS} \chi \omega_{NRS} \\ & +iV \exp(-\phi) F_{PQR} \left[\bar{\Psi}_M \chi^{MNPQR} \Psi_N - i\sqrt{2} \bar{\Psi}_M \left(\Gamma^{MPQR} - 3g^{M[P} \Gamma^{Q]R} \right) \chi \right] \end{aligned} \quad (3.21)$$

This gives a large number of terms, the easiest part of it being the spin 3/2 - spin 3/2 interaction; the other spinors give complicated contributions because of definition (3.19). These, as in the $N = 8$ supergravity, are grouped once the operating symmetry is completely identified. Here we shall only write the gravitinos interaction, hoping to find a compact form for the spinor interactions when dealing with the hidden symmetries.

The spin 3/2 - spin 3/2 interactions are:

$$\begin{aligned}
 & \frac{ie_4}{4\sqrt{2}} \Delta^{\frac{1}{4}} \mathcal{F}_{\kappa\lambda a} \bar{\psi}_{4\mu} (\Gamma^{\mu\nu\kappa\lambda} + 2 g^{\mu\kappa} g^{\nu\lambda}) \Gamma^a \psi_{4\nu} \\
 & - \frac{ie_4}{4} \Delta^{\frac{1}{2}} \exp(-\phi) F_{4\mu\nu\rho} \bar{\psi}^\mu \Gamma^\nu \psi^\rho \\
 & + \frac{ie_4}{8} \exp(-\phi) e_a^m e_b^n (\partial_\rho A_{mn}) \bar{\psi}_{4\mu} \Gamma^{\mu\nu\rho} \Gamma^{ab} \psi_{4\nu} \\
 & + \frac{ie_4}{8} (e_a^m \partial_\rho e_{bm}) \bar{\psi}_{4\mu} \Gamma^{\mu\nu\rho} \Gamma^{ab} \psi_{4\nu}
 \end{aligned} \tag{3.22}$$

The matter scalars interact proportionally to $(\Gamma^{ab})_A^B$ which generates the $SO(6)$ group. We can therefore write the gravitinos kinetic term as:

$$-\frac{ie_4}{2} \bar{\psi}_{4AA} \gamma^{\mu\nu\rho} (\partial_\nu \delta_A^B - Q_{\nu A}^B) \psi_{\rho B} \tag{3.23}$$

where

$$Q_{\nu A}^B = \frac{1}{4} \left[e_a^m \partial_\nu e_{bm} + \exp(-\phi) e_a^m e_b^n \partial_\nu A_{mn} \right] (\Gamma^{ab})_A^B \tag{3.24}$$

This indicates that a dynamical $SO(6)$ symmetry is generated, which is to be expected because pure $N = 4$ supergravity has a dynamical local $SU(4)$ which is extended to $U(4)$. We anticipate that the same thing occurs for the whole system and this is confirmed by the different contributions of (3.21). To conclude this section, we give the supersymmetry transformation rules for the new fields.

The vierbein transforms according to the universal law when the compensating transformation is taken into account:

$$\delta e_{4\mu}^a = -i \bar{\epsilon}_{4A} \gamma^a \psi_{\mu A} \tag{3.25}$$

where

$$\varepsilon_4 = \Delta^{\frac{1}{8}} \varepsilon \quad (3.26)$$

The gravity and matter vectors transformations are:

$$\delta \mathcal{A}_{\mu m} = \Delta^{-\frac{1}{4}} \exp(\phi) \left(-i \bar{\varepsilon}_4 \Gamma_m \psi_{\mu 4} - \frac{1}{\sqrt{2}} \bar{\varepsilon}_4 \gamma_\mu \Gamma^a \lambda \right) \quad (3.27)$$

$$\delta \mathcal{B}_{\mu m} = \Delta^{-\frac{1}{4}} \exp(\phi) \left(\frac{i}{\sqrt{2}} \bar{\varepsilon}_4 \gamma_\mu \chi_m + \dots \right)$$

The transformation rules of ϕ and e_m^a raise a little complication, and it turns out that these should be scaled in order to give the proper transformations. In particular we have to define:

$$\exp(\phi') = \Delta^{-\frac{1}{2}} \exp(\phi) \quad (3.28)$$

$$e'_m{}^a = \exp(\frac{1}{2}\phi) e_m^a$$

giving

$$\begin{aligned} \delta e'_m{}^a &= e'_m{}^b (-i \bar{\varepsilon}_4 \Gamma^a \lambda_b) \\ \delta \phi' &= \frac{\sqrt{2}}{4} \bar{\varepsilon}_4 \lambda \\ \delta A_{mn} &= i e'_m{}^a e'_n{}^b \bar{\varepsilon} [\Gamma_a \lambda_b] \end{aligned} \quad (3.29)$$

Actually the scaling (3.28) will absorb some of the inconvenient factors of $\exp(\phi)$; in particular, (3.24) now reads:

$$Q_{\nu A}{}^B = \frac{1}{4} \left[e'_a{}^m e'_b{}^n \partial_\nu A_{mn} + e'_a{}^m \partial_\nu e'_b{}^m \right] (\Gamma^{ab})_A{}^B$$

Finally the spinors transform as:

$$\begin{aligned} \delta \lambda &= -\frac{i\sqrt{2}}{4} (\gamma^{\mu\nu} \Gamma^a \varepsilon_4) \hat{F}_{\mu\nu a} - \frac{i}{6} \Delta^{\frac{1}{2}} \exp(-\phi) (\gamma^{\mu\nu\rho} \varepsilon_4) \hat{F}_{\mu\nu\rho} \\ &\quad - \frac{i}{2} (\gamma^\mu \varepsilon_4) (\partial_\mu \phi') + \dots \end{aligned} \quad (3.30)$$

$$\begin{aligned}
\delta \chi_a &= \sqrt{2} \Delta^{\frac{1}{4}} \exp(-\phi) \mathcal{G}_{\mu\nu a} (\gamma^{\mu\nu} \varepsilon_4) - \frac{1}{4} \gamma^\mu (\Gamma^b \varepsilon_4) F'_{4\mu ab} \\
&\quad - \frac{1}{4} [e'^m e'_n \partial_\mu g'_{mn}] (\gamma^\mu \Gamma^b \varepsilon_4) + \dots \\
\delta \psi_{\mu 4} &= D_\mu (\hat{\omega}) \varepsilon_4 - \frac{1}{4\sqrt{2}} \exp(-\phi) (\gamma^{\kappa\lambda} \gamma_\mu \Gamma^a \varepsilon_4) \hat{\mathcal{F}}_{\kappa\lambda a} \\
&\quad + \frac{1}{12} \Delta^{\frac{1}{2}} \exp(-\phi) (\Gamma^{\nu\kappa\lambda} \varepsilon_4) \hat{F}_{4\nu\kappa\lambda} \\
&\quad + \frac{1}{4} (\Gamma^{ab} \varepsilon_4) \hat{F}'_{4\mu ab} + \dots
\end{aligned} \tag{3.30}$$

cont.

where

$$F'_{4\mu ab} = e'^m e'_n \partial_\mu A_{mn}$$

4. COMMENTS ON HIDDEN SYMMETRIES AND CONCLUSION

Looking back at the spectrum, we have the $N = 4$ supergravity multiplet with the following fields: the graviton e_μ^α , four gravitinos $\psi_{\mu K}$, six vectors $\mathcal{A}_{\mu m}$, four spinors λ_K , a scalar ϕ' , and a pseudoscalar $A_{\mu\nu}$ (or B). The $N = 4$ matter multiplets give the fields: six vectors $\mathcal{B}_{\mu m}$, 24 spinors χ_{aK} , and 36 scalars and pseudoscalars g_{mn} and A_{mn} respectively. The pure $N = 4$ supergravity is known to have a dynamical local symmetry $U(4)$ and a global $SU(4) \times SU(1,1)$ for the equations of motion with the two scalars parametrizing the coset space $SU(1,1)/U(1)$, $U(1)$ being the dynamical local symmetry extending $SU(4)$ to $U(4)$ ⁶⁾. With the presence of matter, and as seen from some of the interaction terms, a consequence of the dimensional reduction from 10-dimensions is the generation of a dynamical $SO(6) \sim SU(4)$ symmetry for the whole system, and this should be easily extended to $U(4)$, with the global $SU(1,1) \times SU(4)$ as the symmetry of the equations of motion. At this stage it is natural to ask whether such global symmetry can be further extended. A naive extrapolation would be to say that the 36 scalars parametrize the homogeneous space of $G/U(4)$ where G is the required global group. This implies

$$\dim G = \dim U(4) + 36 = 52$$

The only group with rank four and dimension 52 is the exceptional group F_4 ⁸⁾. But $U(4)$ is not the maximally compact subgroup of any one of the F_4 groups ⁹⁾.

Therefore it is questionable whether this is the required extension, if there is an extension at all. Only a detailed analysis can answer this possibility. We conclude by summarizing the results. We have constructed the $N = 1$ supergravity in 10-dimensions in a closed form by consistently truncating the 11-dimensional theory. We made a further reduction to 4-dimensions and identified the physical fields. A detailed analysis is needed to find whether hidden symmetries are present.

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