PS BEAM MEASUREMENTS AT FLAT-TOP FIELDS NEAR TRANSITION ENERGY

R. Cappi, J.P. Delahaye, K.H. Reich

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R. Cappi, P. Delahaye, K.H. Reif
CERN, CH-1211 Geneva 23, Switzerland

Summary

Beams up to $10^{13}$ protons were kept circulating for more than 100 ms at various magnet field levels near transition and the beam behaviour was observed for several radial positions. These levels corresponded notably to values of $\eta = (\gamma^2 - \gamma_{tr}^{-2})$ in the range from $10^{-2}$ to $< 10^{-6}$. They were measured using i) the synchrotron frequency, ii) the change in revolution frequency with radial position, iii) debunching times, and, close to transition, iv) the onset of the negative mass instability. Results from i), ii) and iv) agree well with standard theory and from iii) with a theory taking into account conditions close to $\gamma_{tr}$. The negative mass instability was observed even when, due to the spread in $\eta$ values, only part of the beam was above transition, so losses and no global instabilities were observed down to $\eta \approx 3 \cdot 10^{-6}$ (below transition). Experimental data and their explanation are presented and the interest of operating beam compressor rings close to $\gamma_{tr}$ is pointed out.

Aim of measurements and questions arising

The measurements reported here were stimulated by the suggestion to operate the proposed A.G. compressor ring IKOR for a German spallation neutron source in an isochronous mode, i.e. close to transition energy. This 1.1 GeV ring of 64 m diameter is designed for multi-turn injection of $2 \cdot 10^{14}$ ppb during 500 ms, and single-turn ejection during 0.56 ms, immediately afterwards. Isochronous operation would maintain the beam and avoid net system an azimuthal void (corresponding to the kicker rise time) until ejection. Apart from its cost, an RF system has the drawback of lowering the transverse space-charge limit because of beam bunching, and has adverse effects on longitudinal beam stability on account of the relatively high coupling impedance. It also creates the conditions for transverse head-tail modes.

From the IKOR point of view, the main questions to be answered were: i) how close can the beam approach $\gamma_{tr}$ in terms of $\eta = (\gamma^2 - \gamma_{tr}^{-2})$ without experiencing loss; ii) by how much would an azimuthal void in the beam shorten in 500 ms for that range of values of $\eta$ (from the PS point of view, further questions were: iii) how to bring the beam best into a flat-top close to transition energy (the beam normally being rushed through transition with the $\gamma_{tr}$-jump); iv) how to measure with precision the values of $\eta$; and what are the conclusions for the IKOR?

Experimental conditions

With the $\gamma_{tr}$-jump off, the beam was accelerated every 1.2 ms and observed during a 500 ms flat-top of the guide field set at levels $\beta$ corresponding mainly to $\eta$ values in the range from $10^{-2}$ to $< 10^{-6}$. To avoid transition crossing during the $\Gamma_\beta$ field overshoot at the beginning of the flat-top, the beam was kept at an inside radial position (via RF steering) during this time (Fig. 1); (this procedure makes use of the $\gamma_{tr}$ dependence on radius discussed below). The RF voltage $V_{RF}$ was then set to a value most suitable for the particular measurement, the beam steered to the radial position chosen and, for the observation of the debunching, the voltage reduced to zero and the cavity gaps short-circuited (taking about 2 ms). Beam intensity was varied in the range from $10^{12}$ to $10^{13}$ ppb. The longitudinal emittance was adjusted between 8 and 22 mmrad ($\epsilon_z = \Delta \phi_{pp} \Delta \phi_y$) by "shaking" the beam on an intermediate flat-top at 1 GeV by means of 200 MHz cavities.

$\gamma_{tr}$ dependence on radial beam position

To calculate the variations of $\gamma_{tr}$ across the momentum aperture, one has to take into account second order terms in the orbit length as a function of $dp/p$.

For a particle with momentum $p = p_0 + dp$, circulating on an orbit of length $L = L_0 (1 + \alpha_1 \beta_1^2 + \alpha_2 \beta_2^4)$, one finds the usual notation (the subscript "0" referring to the nominal value):

$$\gamma_{tr} = \gamma_{tro} \left[ 1 - 0.5 \alpha \delta k^2 \right]$$

$$\Delta R/R_0 = \alpha_1 \beta_1 + \alpha_2 \beta_2^2 \delta_0 \alpha_1 \delta k$$

$$\gamma_{tr} = \gamma_{tro} \left[ 1 - 0.5 \alpha \delta k^2 \right]$$

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$$\alpha_1 \beta_1 = 0$$

$$\alpha_2 \beta_2^2 = -1$$

$$\Delta k = \frac{\delta k}{\alpha_0 \beta_0}$$

$$w_2 = \text{wiggling factor} = \frac{1}{2} \Delta \beta_0 \int_0^L a_z^2 \, ds$$

$$\alpha_0 = \text{dispersion function}$$.

The measurement of $\gamma_{tr}(\Delta R)$ was based on the previous observation that, as expected (see Appendix), the PS beam is stable below transition, but experiences the negative mass instability above, and the present

Fig. 1: Experimental conditions
(see text for explanation)

Fig. 2: 1.5 GHz negative mass instability ($\tau = 0.7$ ms)

Fig. 3: $\gamma_{tr}(\Delta R)$; measured (dots) and calculated (line)
observation that this instability can be observed only when the protons are above a certain value (Fig. 3). Having calculated that standard RF bucket theory works close enough to transition for present purposes, the radial position was moved to the outside until instability was observed and the corresponding \( \gamma_{t_{RF}} \) point determined via eq. (3) (Fig. 3), the mean radial beam position being measured directly.

The slope of the measured curve agrees well with the theoretical value of 1.8 for \( \gamma_{t_{RF}} \). A slight uncertainty remains as to the values of \( \gamma_{t_{RF}} \) measured: 6.10, calculated: 6.121 (program AGS).

**Synchrotron frequency dependence on \( \eta \)**

Longitudinal quadrupole oscillations were excited through a fast (non adiabatic) reduction of the accelerating voltage (Fig. 1). Accurate measurements of the synchrotron frequency \( f \) were made using a spectrum analyser in the "receiver mode" and tuned to the RF accelerating frequency (9.4 MHz). \( \gamma_{t_{RF}} \) values were deduced via the known \( \gamma \) values and the \( \eta \) value determined for each level from the measured \( f \) values (Fig. 2).

**\( \eta \) dependence on the change of the revolution frequency with radius**

Changing the radial beam position and measuring the corresponding variations of the revolution frequency at several energies yielded the curve \( df/dR \) versus \( \eta \), respectively B. Observing that, as for \( f_{RF} (\eta) \) everything was well-behaved, \( \gamma_{t_{RF}} \) was found from each measurement via \( \gamma_{t_{RF}} = \gamma_{t_{RF}} (1 + (df/d\gamma)/(d\gamma/dR))^{1/2} \).

**Fig. 4:** Measured values of \( \gamma_{t_{RF}} \) (calculated value: 6.121)

**Fig. 5:** Bunch shapes 1 to 14 ms after RF off (\( \eta = 10^{-3} \)) (Fig. 4). In spite of a certain scatter of the latter points, both methods yield as before a value of \( \gamma_{t_{RF}} \) close to 6.10.

**Debunching time**

The debunching time \( T_{dB} \) is defined here as the time taken by the head of a bunch to reach the tail of the preceding one, once the RF voltage is off (Fig. 5). Fig. 6 shows the measured points compared to the theoretical values calculated using the simplified formula \( T_{dB} = (2\pi \eta |\Delta p/p|)^{1/2} \) with \( \Delta p/p \) (half total momentum spread) determined as before.

Analysis of the amplitude behaviour of the bunch harmonic \( q \) in time domain was used as a second method to measure debunching time. During debunching, when \( q \) supposedly increases linearly with time, the modulus of the amplitude \( A \) evolves in the present case\(^\text{13} \) for \( q = 20 \) = \( h_{RF} \) as \( q \)

- **Parabolic bunches:** \( A_{20} = \frac{2q}{2q} \left( \frac{1}{\phi} + \frac{1}{2} \right) \left( \sin \theta - \cos \phi \right) \)

- **Triangular bunches:** \( A_{20} = \frac{1}{\phi} \frac{1}{4} \left( 1 - \cos \phi \right) \)

- **Gaussian bunches:** \( A_{20} = \frac{1}{2\sqrt{2\pi}} \exp(-\phi^2/2) \)

with \( f = \) d.c. proton current and \( \theta = \) bunch half length in RF radians.

In the present case the bunch shape was essentially parabolic for \( \eta > 10^{-2} \) (8 to 3200 causes), changing to triangular and then Gaussian (long tails) for \( \eta < 10^{-4} \) (Fig. 7). This behaviour is reversible (bunches became again parabolic when decelerated).

**Fig. 7:** Evolution of \( A_{20} \) for parabolic bunches (a), as calculated \( [A(\phi)] \) (b), and measured \( [A(\phi)] \) (c); idem for triangular bunches (d, e, f).

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For the three shapes the points $\phi = \pi$ (representing the debunching time) correspond respectively to 0.3, 0.41, and 0.29 times the maximum value of $A_{50}$. Values of $T_{db}$ measured with this method mostly agreed with the values obtained by the first one.

High signal-to-noise ratios (narrow-band system) and the possibility to work at very high frequencies are particular advantages of this method. Inversely, it can be used to infer the bunch shape from the shape of $A_{50}(t)$ and to estimate $\Delta p/p$ via $T_{db}$.

Conclusions

i) Bunched beams up to $10^{13}$ ppp (the highest intensity available during the measurements) could be kept stably and practically without loss close to, but below transition energy, for times far exceeding those of interest for the IKOR. While the mean FS intensity was a factor twenty-seven lower than the nominal IKOR intensity, the (local) peak intensities were nevertheless comparable.

ii) During debunching, microwave instabilities appeared (Fig. 5) when the Boussard criterion was met, leading to a 50% beam loss for the highest intensities. It should be possible to minimize such instabilities in the IKOR case through an appropriate design of the ring (low coupling impedance) and its filling procedure (avoidance of high $I/(dp/p)^2$ spots).

iii) Up to the highest intensities no clear "skewing" effect (outward motion of head of beam towards center of their tail, at small $n$ values, due to longitudinal space-charge forces) could be observed during debunching.

iv) If at least part of the beam is above transition energy, the negative mass instability appears (even for $10^{12}$ ppp; rise time $t = 0.7$ ns for the 1.5 GHz mode (Fig. 2)) but no transverse instability was observed.

v) In view of ii) and iv) coupled with the fact that a certain $\gamma_{FS}$ spread is expected on the IKOR ring on account of the transverse space-charge forces, it seems advisable to reduce the $\gamma_{FS}(SR)$ dependence in that ring. This would notably permit to stay more safely close to isotropic isochromat.

vi) For the smallest (loss-free) $n$ values achieved in the PS, the void between bunches shortened by about 5 ns in 500 ns. Operation of the IKOR at larger $n$ values, say of the order of $10^{12}$, should not lead to values larger than 20 ns.

vii) While the present measurements contributed to increase the understanding of PS beam behaviour at transition, the results are clearly encouraging for studying further the use of isochromatic A.G. rings for time compression of beam pulses.

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Appendix

The imaginary part of the effective longitudinal impedance $Z_{//}(n)_{eff}$ is

$$Z_{//}(n)_{eff} = \ln(2m) - \omega_0^2/20\gamma^2 + 2\ln b/a$$

(7)

with $\omega_0 = 1 + 2 \ln b/a$

$$\gamma_0 = (\epsilon + c)/c = 377 \Omega$$

$b$: chamber radius; $a$: beam radius

The first term in the RHS of (7) represents the contribution of chamber discontinuities. Using a "broad-band" model, it can be described by

$$Z_{\infty} = \frac{R}{\infty} \left( \frac{1}{1 + \frac{1}{\epsilon} \left( \frac{f_x}{f_c} \right)^{\epsilon} - \frac{f_x}{f_c} } \right)$$

with $\Omega$ 1 and $f_x$ = cut-off frequency of the vacuum chamber = 1.3 GHz in the PS. Im($Z_{\infty}$) for low frequencies has been measured recently and as $\approx 18 \Omega$ (inductive), leading to $\psi_0 = 69 \Omega$.

The second term in the RHS of (7) is the space-charge contribution. With $\gamma_{FS} = 20 \times 10^{-5} \text{rad} \cdot m$ ($\psi_0 = 6$), its value in the PS near transition is $\approx 21.0$, yielding a total impedance of negative sign for all frequencies. The negative mass instability is hence expected only above transition, modes at frequencies higher than 1.3 GHz growing faster.

A 1.5 GHz structure on the bunch signal was observed as soon as even a part of the beam was above transition. Losses also were proportional to the number of particles above transition.