INVESTIGATION OF THE COHERENT BEAM-BEAM EFFECTS IN THE ISR

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Summary

Although the beam-beam tune shift is relatively small in the ISR (0.001 per intersection) and below the value where stochastic effects might be expected, coherent beam-beam effects can be important. Each high-intensity ISR beam is close to the transverse single beam stability limit. The addition of the beam-beam force has led to the loss of one of the beams accompanied by coherent oscillations. The change of the betatron frequency distribution due to the other beam has been investigated. The resonance excitation by the non-linear beam-beam force has been measured and seems to influence the transverse stability. Quantitative information about the beam-beam force is obtained by exciting a coherent betatron oscillation in one beam and observing the response of the other beam. This beam-beam transfer function depends on the beam separation in the interaction point and can be used to centre the two beams. At the same time the effective beam height is obtained, which together with the beam currents determines the luminosity.

Introduction

Because of the horizontal crossings, the horizontal tune shift in the ISR vanishes: the vertical tune shift reaches $\Delta Q_{bb} = 0.001$ per intersection, a value significantly below the conventional beam-beam limit; the beam blow-up remains consistent with scattering.

The beam-beam forces do however provoke various phenomena, some of them being fatal for the beams:

- For beam-beam tune shifts above 0.0007, the non-linear resonances are strongly excited ($7Q_y = 62$, etc...); they do not contribute significantly to the beam loss rate unless the tune of the machine is ramped; then a large number of particles cross resonance lines; 1 to 2% of them may be lost against the vertical aperture limit (100), occasionally activating the automatic beam dump which safeguards the machine components.

- The second phenomenon is observed when harmonics of an RF structure on one of the beams overlap betatron frequencies of the second beam, leading to overlap knock-out resonances¹. Such resonances provoke a selective blow-up of part of the beam. In order to weaken these effects, the beams are separated vertically during accumulation and acceleration; the separation of 6σ considerably weakens the non-linear components of the beam-beam forces; the remaining dipolar kicks are made to compensate by a proper choice of the sign of the separation in each intersection.

- Before data-taking, the beam separations must be suppressed. Unlike electron machines, the orbit bump reduction is adiabatic in comparison with the betatron motion.

During this steering process, partial and complete beam losses have occurred.

Along with the investigation of the beam-beam instability, attempts were made to measure the beam-beam forces by transversely exciting one beam and observing the resulting motion of the second beam. The beam-beam transfer function thus defined can be shown to be directly related to the luminosity. This function may also be used to steer the beams precisely for head-on collisions, measure the beam size and the luminosity.

Coherent Beam-Beam Instabilities

Observations during physics runs.

A full or partial loss of one of the two beams has occurred on several occasions when removing the vertical beam separation. This phenomenon occurs at all energy levels when the beam-beam tune shift reaches 0.001 per intersection and when the beam separation is about equal to the rms beam height; it is accompanied by a growing coherent vertical betatron oscillation. The number of losses could be reduced by removing the beam separation in one crossing at a time; a further reduction could be obtained by improving the vertical feedback system and by establishing a betatron frequency distribution of the particles which provides more Landau damping. These losses are now relatively rare. Nevertheless, they compromise the performance of the ISR as they occur after the long filling and acceleration cycle (10 hours).

Observations made during machine experiments

Since during physics fills the described losses occur suddenly and often unexpectedly, observation of the relevant parameters is difficult. For this reason an attempt was made to reproduce the same conditions in machine study runs.

In a first experiment, the vertical separations of two 31 GeV beams of 33 A were reduced gradually in all intersections. At the same time, the vertical single beam transfer function (response of the beam to a vertical excitation) was measured at the betatron side bands around 10 MHz (fig. 1).

Fig. 1 - Beam Transfer Function

Fig. 2 - Stability diagram

From this response the stability diagram can be obtained (fig. 2), which shows the boundary of the stable region. It indicates the maximum additional resistive and reactive wall impedance which can be tolerated before the beam becomes unstable. The beam transfer function is very smooth for large beam separation but shows more and more spikes as the separation is reduced. In particular, the beam response at the 8th-order resonance $8Q_y = 71$ becomes very large for small beam separations. In the stability diagram this manifests itself by the appearance of loops (fig. 2). However, the stable region is only slightly reduced. When the beam separation was reduced below 1σ , the beams were lost; a coherent vertical oscillation was

detected at a frequency corresponding to $8Q_y = 71$.

In a second experiment, the beam-beam tune spread due to the non-linear beam-beam forces was measured, using a special method: the dominating betatron frequency spread in the ISR is caused by the combined effects of the chromaticity and the energy dependence of the revolution frequency. At a specific frequency, these two contributions cancel out; the residual spread due to the non-linear beam-beam force and residual octupole fields can then be measured on a Schottky scan. This has been done for different vertical beam separations $z_1 - z_2$ (fig. 3).

Fig. 3 - Amplitude Dependant Tune Spread

The results agree rather well with the theory² if a residual octupole field of opposite sign is assumed to be present in the ISR. For a certain separation these two contributions nearly cancel each other out and result in a minimum of the betatron frequency spread caused by the amplitude dependence of the frequency.

Interpretations of the observations

Although the results of the experiments described are not completely conclusive, they do allow some likely interpretations to be made. The action of the beam-beam forces results in a growing coherent vertical oscillation; the beam stability diagram must, therefore, be affected by the beam-beam effect either globally or locally. The first and second experiments have shown that the global perturbation (reduction of tune spread) is small and should not alone cause a beam loss. The excitation of non-linear resonances, although spectacular, does not significantly reduce the stable region at 10 MHz. This excitation, which grows rapidly as the beam separation is reduced, is likely to be stronger than observed when the beam loss process starts. In addition, the stable region is smaller at low frequency, due to the large resistive component of the coupling impedance, in spite of the stabilising effect of the feedback system.

The Beam-Beam Transfer Function

Another method was developed to investigate the strength of the beam coupling at crossing points: if one beam is excited externally, the second beam will oscillate as well, with a phase and an amplitude depending on the strength of the excitation, the strength of the coupling at crossing points, and optical and beam parameters: let us assume two coasting beams crossing horizontally at a small angle α_0 , with a vertical separation D (fig. 4). The vertical and horizontal transverse particle distributions are assumed not to be coupled, such that the current density can be written:

$$j_i(x_i, y_i) = I_i g_i(x_i) f_i(y_i)$$

with the normalisation :

$$\int_{-\infty}^{+\infty} g_i(x_i) dx_i = \int_{-\infty}^{+\infty} f_i(y_i) dy_i = 1$$

The index $i=1$ refers to the acting beam I_1 whilst the index $i=2$ refers to the probe beam I_2 ; x, y and s are the transverse and longitudinal coordinates respectively. It is further assumed that x is only determined by the energy deviation and the dispersion; the vertical dispersion, octupolar and higher non-linear effects are neglected.

Fig. 4 - Geometry of the Crossing

Acceleration undergone by a slice of the probe beam at a crossing.

The elementary magnetic field induced by a small current string $d^2 I_1$ at a location (x_2, y_2) is given by:

$$d^2 B = \frac{\mu_0 d^2 I_1}{2\pi\sqrt{u^2+v^2}} = \frac{\mu_0 j_1(x_1, y_1) dx_1 dy_1}{2\pi\sqrt{u^2+v^2}}$$

with $u \approx s \cdot \alpha_0$ and $v = y_2 + D - y_1$ (fig. 4).

The resulting bending of the trajectories is :

$$d^2 \theta_B = \frac{ec}{E} \int_{-L}^{+L} d^2 B \alpha ds_2 = \frac{ec\mu_0 j_1(x_1, y_1) dx_1 dy_1}{\pi E \alpha_0} \arctg \frac{L \alpha_0}{y_2 + D - y_1}$$

where E is the particle energy and L the half length of the crossing. The magnetic deflection due to the acting beam is found by integrating the effect of all current strings:

$$\theta_B = \frac{ec\mu_0 I_1}{\alpha_0 E} \int_0^{y_2+D} f_1(y_1) dy_1 \quad \text{for } L \alpha_0 \gg y_2 + D - y_1$$

In the ultra-relativistic case, the total deflection is twice the magnetic deflection. The deflection of the center of gravity of a slice of particles having the same betatron tune is found by averaging :

$$\langle \theta \rangle = \frac{2ec\mu_0 I_1}{\alpha_0 E} \int_{-\infty}^{+\infty} f_2(y_2) \int_0^{y_2+D} f_1(y_1) dy_1 dy_2$$

The average acceleration G_2 over one turn is given by:

$$G_2 = \frac{c^2}{2\pi R} \langle \theta \rangle, \quad R \text{ being the machine radius.}$$

For small variations of D around D_0 : $D = D_0 + \delta \cdot D$, $\langle \theta \rangle$ can be developed to first order:

$$G_2 = \frac{c^2}{2\pi R} \left[\langle \theta \rangle_{D_0} + \frac{2ec\mu_0 I_1}{\alpha_0 E} \int_{-\infty}^{+\infty} f_2(y_2) f_1(y_2 + D) dy_2 \cdot \delta D \right] \quad (1)$$

The constant part leads to a modification of the closed

orbit and need not be considered in the following analysis.

External excitation of the acting beam:

When subject to an external acceleration $G_1 = G_{10} \exp[-i\omega t]$, the beam centre of gravity position is given by³:

$$Y_1 = -G_{10} \text{TF1} \exp[-in_1\phi_1] \exp[-i\omega t] = \delta D \quad (2)$$

TF1 being the single beam transfer function, which, in the slow wave approximation, is written:

$$\text{TF1} = \frac{1}{2Q_1\Omega_1} \int \frac{g_1(x_1)dx_1}{\omega - \Omega_1(n_1 - Q_1)} \quad (3)$$

with Ω_1 :revolution frequency, Q_1 :betatron tune, n_1 :oscillation mode, $Q_1\phi_1$:betatron phase advance between exciter and place of observation.

The Beam-Beam Transfer Function

The motion of a slice $g_2(x_2)dx_2$ of tune Q_2 of the probe beam is :

$$\ddot{y}_2 + \Omega_2^2 Q_2^2 y_2 = G_2 \quad (4)$$

Using (1), (2) and (4) and integrating over the radial distribution of the probe beam yields the equation of the motion of the beam centre of gravity:

$$Y_2 = Y_{20} \exp[-i\omega t]$$

The beam-beam transfer function is defined by:

$$\frac{I_2 Y_{20}}{G_{10}} = \left\{ \frac{e n_1 c^3}{\pi R \alpha_0 E} \text{TF1} \cdot \text{TF2} \exp[-i(n_1\phi_1 + n_2\phi_2)] \right\}.$$

$$I_1 \cdot I_2 \int_{-\infty}^{+\infty} f_1(y_2) f_2(y_2 + D) dy_2 \quad (5)$$

where TF2 is the transfer function of the probe beam as defined in (3), $Q_2\phi_2$ the betatron phase advance between interaction area and observation point, and n_2 the oscillation mode.

Beam-Beam Transfer Function and Luminosity

The luminosity of two beams crossing at a small angle is proportional to the cross correlation of the vertical distributions⁴:

$$L(D) = \frac{2I_1 I_2}{c e^2 \alpha_0} \int_{-\infty}^{+\infty} f_1(y_2 + D) f_2(y_2) dy_2$$

Replacing in (5) yields:

$$\frac{I_2 Y_{20}}{G_{10}} = \frac{2r_p e c^2}{R\gamma} \text{TF1} \cdot \text{TF2} \exp[-i(n_1\phi_1 + n_2\phi_2)] \cdot L(D)$$

with r_p :classical proton radius, γ :relativistic parameter.

The beam-beam transfer function amplitude is proportional to the luminosity and may thus be used to define the position of head-on collisions; a vertical beam steering will yield the beam effective height; the function TF1 and TF2 may be measured separately to allow a direct measurement of the luminosity.

In case of several crossings, the beam-beam transfer function remains proportional to the total luminosity provided the betatron phase advances between crossings are identical for both beams; the phase

information may be used to study phase advance differences.

Experimental data

Figure 5 represents the beam-beam transfer function of two ISR beams (30A*5A) crossing in one intersection and separated by 6σ elsewhere. A beam misalignment of σ/10 can be detected. The phase rotation for large beam separation is due to the incomplete decoupling in the remaining 7 crossings and to the betatron phase advance difference between short and long arcs of the ISR. Once corrected for this contribution, the beam-beam transfer function amplitude agrees well with the data obtained from scintillation counters (fig. 6).

Fig. 5 - Beam-Beam Transfer Function

Fig. 6

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References

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