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LIGHT QUARK MASSES ^{*)}

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ABSTRACT

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1. INTRODUCTION

A) The mass pitfalls of the old quark model

I will start by recalling the quark mass scenario of the early sixties. Then, one thought of quarks as heavy objects and of physical hadrons as tightly bound states of them. The quark interactions, as yet unspecified, were assumed independent of flavour, so all differences among hadrons (in particularly mass differences) were attributed to Clebsch-Gordan coefficients, to QED corrections, and to differences among the masses of the various quarks. A particularly satisfactory outcome of this picture was the explanation of the success of the Gell-Mann-Okubo mass formula and, to a certain extent, of the mass differences among hadrons in the same isospin multiplet by postulating that

$$m_s - m_d \sim 150 \text{ MeV}, \quad m_d - m_u \sim 4 \text{ MeV}. \quad (1.1)$$

There is, however, another way to estimate quark masses or mass ratios. Using current algebra and PCAC it is found that¹⁾⁻³⁾

$$\frac{m_s}{m_d} \sim 20, \quad \frac{m_d}{m_u} \sim 2. \quad (1.2)$$

Even if one assumes a mass as small as possible for the u quark as allowed by the bound state character of baryons, say $m_u \sim 300 \text{ MeV}$, (1.2) gives $m_d \sim 600 \text{ MeV}$, $m_s \sim 12 \text{ GeV}$, utterly incompatible with (1.1). Moreover, current algebra calculations²⁾ tended to give very small masses:

$$m_u \sim 4 \text{ MeV}, \quad m_d \sim 8 \text{ MeV}, \quad m_s \sim 160 \text{ MeV}. \quad (1.3)$$

Before continuing with the discussion, I will pause to present a proof^{1),3),4)} of (1.2). Let A be the axial currents, ϕ the pseudoscalar ones:

$$A_{ij}^\mu(x) = \bar{q}_i(x) \gamma^\mu \gamma_5 q_j(x),$$

$$\phi_{ij}(x) = (m_i + m_j) \bar{q}_i(x) \gamma_5 q_j(x)$$

the indices i, j label quark flavours. Consider the function

$$\int d^4x \, e^{iq \cdot x} \langle T A_{ij}^\mu(x) A_{ij}^\nu(0)^+ \rangle_0.$$

Multiplying by $q_\mu q_\nu$, using that $\partial_\mu A_{ij}^\mu = \phi_{ij}$ and letting $q \rightarrow 0$ you have the Ward identity

$$\int d^4x e^{iq \cdot x} \langle T \phi_{ij}(x) \phi_{ij}(0)^+ \rangle_0 = -i \int d^4x e^{iq \cdot x} \langle [A_{ij}^0(0), \phi_{ij}(x)^+] \rangle_0, q \rightarrow 0.$$

In the chiral limit $M_\pi^2 \rightarrow 0$ the left-hand side is exactly given by the pion pole contribution (for, say $i = u, j = d$):

$$\frac{M_\pi^4 f_\pi^2}{M_\pi^2 - q^2} \Big|_{q \rightarrow 0},$$

while one can use current algebra on the right-hand side to get

$$- (m_u + m_d) \langle \bar{q}_u q_u + \bar{q}_d q_d \rangle.$$

Hence for, say, the pion we find

$$M_{\pi^+}^2 f_{\pi^+}^2 = - (m_u + m_d) \{ \langle \bar{q}_u q_u \rangle_0 + \langle \bar{q}_d q_d \rangle_0 \}. \quad (1.4)$$

Let us assume that the VEV's $\langle \bar{q}_i q_i \rangle_0$ are approximately flavour symmetric. Repeating the argument for π^0, K^+, K^0 , subtracting the QED contributions to the meson masses⁵⁾ and taking ratios to get rid of the $\langle \bar{q}q \rangle_0$, we find the result (1.2).

Coming back to the discussion, the incompatibility of (1.1), (1.2) and (1.3) led to the distinction between constituent masses of $\gtrsim 300$ MeV, for which (1.1) holds, and currents masses verifying (1.2). Since no dynamical theory was available at the time, further clarification of the puzzle was impossible except in the context of the free quark model (Melosh transformation). Nowadays we have a good candidate for a strong interaction theory so we should reformulate the question in the light of QCD.

B) Perturbative and non-perturbative masses in QCD

A first remark that has to be made is that, in any renormalizable theory, masses depend on the energy at which they are defined. Thus (and this is a fact that is often forgotten) it makes no sense to say "quark masses are so and so" except if one specifies at which Q^2 .

Let us write the quark propagator as $i(A\not{p} - B)^{-1}$; we may then define the running mass as

$$\bar{m}_i(Q^2) = A_i(p^2)/B_i(p^2), \quad p^2 = -Q^2.$$

In perturbation theory, $\bar{m}_i(Q^2) = Z_{im}^{-1} m_{i0}$, where m_0 is the bare mass that appears in the bare Lagrangian. $\bar{m}$ obeys a renormalization group equation that allows us to write, for $Q^2 \gg \Lambda^2$ with $\Lambda \sim 0.5$ GeV,

$$\bar{m}_i(Q^2) = \hat{m}_i \frac{1}{(\log Q/\Lambda)^d}, \quad (1.5)$$

$$d = \frac{12}{33-2n_f}.$$

As is well known,

$$\frac{\bar{m}_i(Q^2)}{\bar{m}_j(Q^2)} \xrightarrow{Q^2 \rightarrow \infty} \frac{m_{i0}}{m_{j0}}. \quad (1.6)$$

The masses $\bar{m}$ are what we should use in, e.g., parton model calculations^{6),7)}.

However, this is not the whole story. Because chiral symmetry appears to be spontaneously broken we expect the VEV $\langle \bar{q}q \rangle_0$ to be different from zero. Therefore, the full mass gets a non-perturbative contribution and Eq. (1.5) is replaced by⁷⁾

$$m_{i,all}(Q^2) = \bar{m}_i(Q^2) + 4\pi \frac{16}{9} \alpha_s(Q^2) \frac{v_i(Q^2)}{Q^2}, \quad (1.7)$$

where $\bar{m}_i$ is still given by (1.5) and $v_i(Q^2)$ equals $-\langle \bar{q}_i q_i \rangle_0$ renormalized at Q^2 . Note that, at large Q^2 , $m_{i,all} \sim \bar{m}_i$; the v only matters at low Q^2 . Because v has dimensions of $[\text{mass}]^3$ we may write

$$v_i(Q^2) = [\mu(Q^2)]^3,$$

and I have used the flavour invariance of the VEV's $\langle \bar{q}q \rangle$ to define a common spontaneous mass μ . Incidentally, we can now reinterpret (1.4) to mean^{*)}

*) Note that the Q^2 dependence of the $\bar{m}_i$, μ^3 are opposite so the right-hand side of (1.8) is actually independent of Q^2 as it should be.

$$M_\pi^2 f_\pi^2 = 2 \{ \bar{m}_u(Q^2) + \bar{m}_d(Q^2) \} [\mu(Q^2)]^2; \quad (1.8)$$

this will allow us later on to eliminate the $\langle \bar{q}q \rangle$ in favour of the running masses and the known M_π, f_π .

2. RUNNING MASSES IN QCD

A) Masses from spectral sum rules

We will next try to get a handle on the running masses in QCD. A possibility of doing so is to use the old Weinberg spectral sum rules⁸⁾ following a method pioneered by Weissberger⁹⁾ and developed in Refs 10) and 11). One considers the combinations of two-point functions

$$\Delta_{ij}(k^2) = \frac{k^\mu k^\nu}{k^4} \int d^4x e^{ik \cdot x} \{ \langle T V_{ij\mu}(x) V_{ij\nu}(0)^+ \rangle_0 - \langle T A_{ij\mu}(x) A_{ij\nu}(0)^+ \rangle_0 \}, \quad (2.1a)$$

and

$$R_{ijl}(k^2) = \Delta_{ij}(k^2) - \frac{m_i}{m_l} \Delta_{il}(k^2). \quad (2.1b)$$

Here $V_{ij}^\mu = \bar{q}_i \gamma^\mu q_j$. Because R is analytic in the k^2 plane we may write a Cauchy integral for it:

$$k^2 R_{ijl}(k^2) = \frac{1}{\pi} \int_0^{Q^2} dt t \operatorname{Im} R_{ijl}(t) + \frac{1}{2\pi i} \int_\Gamma dt \frac{t}{t-k^2} R_{ijl}(t).$$

Γ is an open circle about the origin with radius Q^2 . If Q^2 is large we expect that R on Γ will be well approximated by its perturbative QCD expression. Since the combinations (2.1) were selected to give a convergent $\sum R(z)$, we hope that the $\int^{Q^2}$ integral may be approximated by the contribution of the π and K poles (for $ijl = uds$). With $Q^2 = 1.44 \text{ GeV}^2$ we find,

$$M_\pi^2 f_\pi^2 \frac{\hat{m}_s}{\hat{m}_d} - M_K^2 f_K^2 = (0.23) \hat{m}_u \hat{m}_d (\hat{m}_s^2 - \hat{m}_d^2). \quad (2.2)$$

Even with very large $\hat{m}$ ($\hat{m}_s \sim 600$ MeV, $\hat{m}_u \sim 15$ MeV), the right-hand side above is $\sim 2 M_\pi^2 f_\pi^2$. Hence, we get

$$\frac{\hat{m}_s}{\hat{m}_d} \simeq 19 \text{ to } 16.$$

This is in agreement with (1.2) within expected errors. Incidentally, the size of these errors makes it dubious that one can reverse (2.2), that is to say, to take ratios as given in (1.2) and use (2.2) to predict the scale. If anyway we go ahead we obtain $\hat{m}_u \simeq 13$ MeV, $\hat{m}_d \simeq 26$ MeV, $\hat{m}_s \simeq 520$ MeV. This is larger than (1.3) but not unreasonable in the light of the bounds to be discussed now.

B) Lower bounds on running masses, and
upper bounds on spontaneous masses¹²⁾

Consider the two-point function

$$\begin{aligned} \Psi_{ij}^5(k) &= i \int d^4x e^{ik \cdot x} \langle T \partial_\mu A_{ij}^\mu(x) \partial_\nu A_{ij}^\nu(0)^+ \rangle_0 \\ &= i(m_i + m_j)^2 \int d^4x e^{ik \cdot x} \langle T J_{ij}^5(x) J_{ij}^5(0)^+ \rangle_0, \end{aligned} \quad (2.3)$$

where $J_{ij}^5 = \bar{q}_i \gamma_5 q_j$. To all orders in perturbative theory (we will work consistently in the $\overline{MS}$ renormalization scheme¹³⁾) the function

$$F_{ij}(Q^2) = \frac{\partial^2}{\partial(k^2)^2} \Psi_{ij}^5(k^2), \quad k^2 = -Q^2,$$

satisfies an unsubtracted dispersion relation. We need to calculate F_{ij} and its derivatives^{*)} at large $Q^2 \gg \Lambda^2$ taking into account perturbative QCD and the leading non-perturbative effects [wrongly neglected in a previous version of Ref. 12)]. The last are associated to the fact that the operators

$$\bar{q}(0)q(0), \quad x^\alpha \bar{q}(0)\partial_\alpha q(x) \Big|_{x=0}, \quad G^2 = \sum_a G_{a\mu\nu} G_a^{\mu\nu}$$

have non-zero VEV's. The effect of the first two may be estimated with the help of Eq. (1.8); it is $(2\pi^2/9) f_\pi^2 m_\pi^2/Q^4$ relative to the perturbative contribution. Since, in the chiral limit $m_\pi^2 \rightarrow 0$, we will neglect it. The last gives a relative contribution of $(2\pi/3) \langle \alpha G^2 \rangle_0/Q^4$, i.e., it is non-zero in the chiral

^{*)} The trick relating the problem to a moment problem through use of derivatives was first employed in this context in Ref. 14). See also Ref. 15).

limit and therefore should be kept. Indeed, we will see that it actually sets the scale for the quark masses. Note also that we write $\langle \alpha_s G^2 \rangle_0$ because the product $\alpha_s G^2$ is renormalization group invariant^{(12), (16)}.

The first result, written directly for the function

$$F_{ij}^{(N)}(Q^2) = \frac{2}{(N+2)!} (-1)^N (t_0 + Q^2)^N \frac{\partial^N}{\partial (Q^2)^N} F_{ij}(Q^2) \quad (2.4)$$

(where t_0 is the threshold for the continuum, see below) is, to leading order in chiral symmetry breaking,

$$\begin{aligned} F_{ij}^{(N)}(Q^2) = & \frac{2}{(N+1)(N+2)} \cdot \frac{3}{8\pi^2} \cdot \left\{ \frac{(\bar{m}_i + \bar{m}_j)^2}{Q^2} \right\} 1 \\ & + \left(\frac{11}{3} + 0.81 - 0.82 \log \log \frac{Q^2}{\Lambda^2} \right) \frac{\alpha_s(Q^2)}{\pi} \\ & + 2S_1(N) \frac{\alpha_s}{\pi} + \left(4 + \frac{33-2N}{12} \right) S_1(N)^2 \left(\frac{\alpha_s}{\pi} \right)^2 \\ & + \frac{\pi}{3} (N+1)(N+2) \frac{\langle \alpha_s G^2 \rangle_0}{Q^4} \}. \end{aligned} \quad (2.5)$$

Here $S_1(N) = \gamma_E + \psi(N+1)$ behaves as $\log N$ for $N \rightarrow \infty$, and we have taken into account the terms $O(\log^2 N)$ to second order in α_s/π .

The reason for the peculiar factors in (2.4) is that if we write a dispersion relation for $F^{(N)}$, separating the pole term:

$$F_{ij}^{(N)}(Q^2) = \frac{2 M_{ij}^4 f_{ij}^2}{(M_{ij}^2 + Q^2)^3} \left(\frac{t_0 + Q^2}{M_{ij}^2 + Q^2} \right)^N + \sigma_N(Q^2) \quad (2.6)$$

then by an appropriate change of variables σ_N may be written as

$$\sigma_N(Q^2) = \int_0^1 dx \, \rho(x; Q^2, t_0) x^N, \quad \rho \sim \text{Im } \psi^5. \quad (2.7)$$

ρ is positive because so was $\text{Im } \psi^5$. This allows us to write positivity constraints on the σ_N ,

$$\sigma_N > 0, \quad \sigma_{N-1} \sigma_{N+1} > (\sigma_N)^2, \text{ etc.}, \quad (2.8)$$

which, because of (2.5) and (2.6) translate immediately into lower bounds for $\bar{m}_1 + \bar{m}_j$. An important point to watch is the terms in $S_1(N)\alpha_s/\pi$, $(S_1(N)\alpha_s/\pi)^2$ in (2.5): they tell us that, for a given Q^2 , we cannot take too many derivatives. Indeed, for large N , Q^2 ,

$$S_1(N) \frac{\alpha_s(Q^2)}{\pi} \simeq \frac{12}{33-2n_f} \cdot \frac{\log N}{\log Q^2/\Lambda^2},$$

so if N is too large the error will make the calculation meaningless. Neglect of this fact makes very dubious the conclusions drawn in Ref. 15) for the quark masses using techniques similar to ours: their forgotten errors $O(S_1(N)\alpha_s)$, $O(S_1^2(N)\alpha_s^2)$ are, respectively, of 50%, 90%.

To write the bounds implied by (2.9) it is convenient to write the $\bar{m}$ in terms of the $\hat{m}$ using Eq. (1.5). We find, for the linear constraint, and neglecting terms of $O(\alpha_s(\log N)^0)$, $O(1/N)$,

$$\begin{aligned} \hat{m}_i + \hat{m}_j &> \frac{4\pi M_{ij}^2 f_{ij}}{\sqrt{3}} \cdot \frac{N}{Q^2} (\log Q/\Lambda)^d \\ \times \left\{ 1 - \left[\frac{\log N}{\log Q^2/\Lambda^2} d + \frac{1}{2} \frac{\log^2 N}{\log^2 Q^2/\Lambda^2} d + 2 \frac{\log^2 N}{\log^2 Q^2/\Lambda^2} d^2 \right] \right. \\ &\left. - \frac{\pi N^2}{6 Q^4} \langle \alpha_s G^2 \rangle_0 \right\}. \end{aligned} \quad (2.9)$$

The best way to get a bound is to expand the right-hand side of (2.9) in powers of $d^{(*)}$, and then optimize with respect to the ratio N/Q^2 . In this way we find^{**)}, with $Q^2 = 20 \text{ GeV}^2$, $\Lambda = 0.45 \text{ GeV}$,

$$\hat{m}_i + \hat{m}_j > \sqrt{\frac{2\pi}{3}} \cdot \frac{8 M_{ij}^2 f_{ij}}{3 \langle \alpha_s G^2 \rangle_0^{1/2}} \{ 1 - (0.7)d - (0.9)d^2 + \dots \}. \quad (2.10a)$$

If we had used the quadratic constraint in (2.8) the leading term would have been the same, but the corrections would have become $-(0.35)d - (0.16)d^2$.

^{*)} Note that, for two flavours, $d = 0.41$.

^{**)} For the people interested in expansions in the number of colours, n_c , I will remark that, for arbitrary n_c the leading term in (2.10a) is $\sqrt{2\pi/n_c}$. $8 M_{ij}^2 f_{ij} / 3 \langle \alpha_s G^2 \rangle_0^{1/2}$. Now, $\alpha_s G^2$ is proportional to n_c , so we get a result homogeneous in n_c if, as expected, $M_{ij} \rightarrow \text{constant}$ for $n_c \rightarrow \infty$, and $f_{ij} \sim n_c$.

With the numerical values for $\langle \alpha_s G^2 \rangle_0$ given in Ref. 17), $\langle \alpha_s G^2 \rangle_0 \approx 0.04 \text{ GeV}^4$, (2.10) gives the numerical bounds

$$\begin{aligned} \hat{m}_u + \hat{m}_d &> (30 \pm 6) \text{ MeV} \\ \hat{m}_u + \hat{m}_s &> (500 \pm 100) \text{ MeV}, \end{aligned} \quad (2.11)$$

with reasonable error allowances.

By virtue of Eq. (1.8) this at once imposes upper bounds on the spontaneous masses. If we let⁷⁾

$$\begin{aligned} \mu(Q^2) &= \hat{\mu} (\log Q/\Lambda)^{1/3}, \\ \text{then} \quad \hat{\mu} &< \left(\frac{3}{2\pi} \right)^{1/6} \frac{(3f_\pi)^{1/3} \langle \alpha_s G^2 \rangle_0^{1/6}}{2}. \end{aligned} \quad (2.12)$$

numerically,

$$\hat{\mu} < (170 \pm 35) \text{ MeV}. \quad (2.13)$$

The bounds on the $\hat{m}_i$ are larger than the current algebra estimates²⁾, $m_u^{c.a.} + m_d^{c.d.} \approx 11 \text{ MeV}$. Likewise, (2.13) is lower than constituent masses, $\mu^c \approx 350 \text{ MeV}$. In the absence of a clear relation between the $\hat{m}$ and the $m^{c.a.}$, or $\hat{\mu}$ and μ^c we do not regard this disagreement as troublesome. Also, the values of $\langle \alpha_s G^2 \rangle$ used could well be off by a large percentage.

3. ISOSPIN AND SU(3) VIOLATIONS FROM THE QUARK MASS MATRIX IN RADIATIVE MESON DECAYS

All the previous arguments for large mass matrix, (relatively) large running masses and small spontaneous ones boiled down to comparing quark masses to meson masses. In this section I will discuss the possibility that the large mass ratios found may show in other quantities as violations of isospin or flavour SU(3). We have to look for processes where the only relevant masses are quark masses. A possibility, as discussed by Weinberg³⁾ are isospin violations in the πN scattering lengths; here I will discuss radiative decays of mesons^{18),19)}.

Consider for example the amplitude for $\pi^+ \rightarrow e^+ \nu \gamma$. It contains a piece where the weak current is axial, rather insensitive to quark masses (see below) and the piece where the weak current is vector. If q is the momentum of the pion, p that of the $e + \nu$ pair, and k that of the photon, it is given by

$$T^{\mu\nu}(p, q) = \int d^4x d^4y e^{-ip \cdot x} e^{iq \cdot y} \langle T \pi(x) V_{ud}^\mu(y)^\dagger J^\nu(0) \rangle_0, \quad (3.1)$$

where J is the e.m. current and π is proportional to the pion field:

$$\pi(x) = (\bar{u}_u + \bar{u}_d) \gamma_5 q_d(x).$$

As in the standard discussion of the axial anomaly^{20),21)} it is convenient to consider the three-current function,

$$R^{\mu\nu\lambda}(p, q) = \int d^4x d^4y e^{-ip \cdot x} e^{iq \cdot y} \langle T A_{ud}^\lambda(x) V_{ud}^\mu(y)^\dagger J^\nu(0) \rangle_0. \quad (3.2)$$

We still have $\partial_\mu J^\mu = 0$; if $m_u \neq m_d$, we can no more assume $\partial_\mu V^\mu = 0$ but rather

$$i\partial_\mu V^\mu(y) = \sigma(y), \quad \sigma(y) = (m_d - m_u) \bar{q}_u(y) q_d(y).$$

By writing the most general expression for R and contracting with k_ν, p_μ, q_λ we find to lowest order in p, q

$$T^{\mu\nu}(p, q) + S^{\mu\nu}(p, q) = a^{\mu\nu}(p, q), \quad (3.3)$$

where we define

$$S^{\mu\nu}(p, q) = \int d^4x d^4y e^{-ip \cdot x} e^{iq \cdot y} \langle T \sigma(x) A_{ud}^\mu(y)^\dagger J^\nu(0) \rangle_0. \quad (3.4)$$

$a^{\mu\nu}$ is the anomaly,

$$a^{\mu\nu}(p, q) = \frac{-i}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta, \quad (3.5)$$

which, as is known²¹⁾, is independent of masses or mass ratios and unrenormalized by strong or e.m. interactions. If m_{ou} equalled m_{od} , we would have

$S^{\mu\nu} = 0$ and (3.3) would tell us that the vector part of $\pi^+ \rightarrow e^+ \nu \gamma$ equals, up to known factors, the amplitude for $\pi^0 \rightarrow \gamma \gamma$. If $m_{ou} \neq m_{od}$ we would expect a different result. As we are in the soft limit, the only masses that enter are the quark masses; furthermore, to lowest order the process is infra-red finite so we may hope that confinement would not spoil this. In fact, there is one particular situation where one can say something exact to all orders of perturbation theory. Consider the transformation

$$\begin{aligned} q_u &\rightarrow \gamma_5 q_u ; m_{ou} \rightarrow -m_{ou} ; \\ q_d &\rightarrow q_d ; m_{od} \rightarrow m_{od} . \end{aligned} \quad (3.6)$$

If $m_{ou} = 0$ this is a symmetry of the QCD Lagrangian. Under it, $V_{ud}^\mu \rightarrow A_{ud}^\mu$, $\pi \rightarrow \sigma$, $J^V \rightarrow J^V$, and so we get $T^{\mu\nu} = S^{\mu\nu}$: therefore, (3.3) gives $T^{\mu\nu} = \frac{1}{2} a^{\mu\nu}$. This is a factor 1/2 in the amplitude, 1/4 in the rate, and hence we expect that, if m_{od}/m_{ou} is large substantial violations of isospin will appear. The same is true for $K^+ \rightarrow e^+ \nu \gamma$ decay and m_{os}, m_{ou}^*). Unfortunately the experimental data are not yet precise enough to allow a clear separation of the axial and vector parts of radiative π or K decays. It is possible²²⁾, by making reasonable smoothness assumptions, to relate the axial piece to the charge radii of the π or K . The (preliminary) results seem to support the existence of sizeable isospin and $SU(3)$ violations.

Another possibility is to consider the decays $K^* \rightarrow K, \rho \rightarrow \pi \gamma$. Here we do not have to estimate an unexisting axial piece; but vector meson dominance has to be used to extrapolate to the soft limit, $p_V = p_P = k_\gamma = 0$. If you believe this (after all, it works nicely for the $\omega \rightarrow \pi \gamma$)^{19), 23)} you get that, if you had $m_{os} = m_{od}$ then²³⁾ $\Gamma(K^{0*} \rightarrow K^0 \gamma) = 213 \pm 14$ KeV; if you take $m_{od}/m_{os} = 1/20 \sim 0$ then you predict 1/4 of this or $\Gamma(K^{0*} \rightarrow K^0 \gamma) = 53 \pm 4$ KeV. Experimentally²⁴⁾, $\Gamma_{exp}(K^{0*} \rightarrow K^0 \gamma) = 75 \pm 35$ KeV. For $\rho^+ \rightarrow \pi^+ \gamma$ there also seems to be some violation of isospin¹⁹⁾; but I regard the evidence as inconclusive because the experimental figure²⁴⁾ has gone up recently by three standard deviations - just what still separates it from the value you would get if $m_{ou} = m_{od}$.

*) I am aware that (3.6) may not be a symmetry of the vacuum and thus the prediction could be spoiled by non-perturbative contributions.

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