MULTI-CELL SUPERCONDUCTING STRUCTURES FOR HIGH ENERGY $e^+ e^-$ COLLIDERS AND FREE ELECTRON LASER LINACs

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1 INTRODUCTION
Future High Energy Physics $e^+e^-$ experiments\(^1\) will require beams colliding at a center-of-mass energy $E_{cm}$ in the range of 0.5-1 TeV with luminosity of the order of $10^{34}$-10\(^3\)5 cm\(^-2\)s\(^-1\) [1]. For such high-energy beams, a linear $e^+e^-$ collider seems to be the only technically feasible particle accelerators option, which can fulfill these two requirements. Circular accelerators, similar to those used in the past, LEP\(^2\), or still in operation like CESR\(^3\) or HERA\(^4\) e-ring, become impractical due to the enormous energy lost by the accelerated beam in the form of synchrotron radiation. The radiated energy ($U_e$) by a charged particle in one turn in a circular accelerator is [2]:

$$U_e = \frac{q^2}{3\varepsilon_0 \rho_{acc}^2} \left(\frac{E_n}{mc^2}\right)^4$$

(1.1)

Here: $q$, $mc^2$, and $E_n$ are charge, rest energy and energy of a particle respectively, while $\rho_{acc}$ is an average bending radius of an accelerator. Consider a circular accelerator with $\rho_{acc} \approx 5000$ m (~LEP size) storing one electron (positron) bunch of $N_e = 2 \cdot 10^{10}$ particles at 400 GeV. At every turn, the bunch would radiate energy of 1.46 kJ, which has to be imparted to it by an acceleration system. 460 GV of the effective accelerating voltage would be needed to compensate for this energy loss. The revolution frequency of relativistic particles for such a circular accelerator would be about 10 kHz and the total radiated energy/s by one bunch would be 14 MJ/s. It is obvious that a multi-bunch operation of such facility would cause radiation of several hundred megawatts of power from each of the colliding beams. Formula 1.1 shows that the radiated energy increases with the beam energy very rapidly and it is the major limitation in the maximum energy attainable in an $e^+e^-$ circular accelerator. Unlike electrons and positrons, particles with bigger rest mass, like muons or protons, on machines with much smaller bending radii, radiate only a small fraction of what electrons and positrons do. This makes circular accelerators still very attractive for high-energy "heavier" leptons or hadrons. The LHC project presently being commissioning at CERN [3] and Muon Collider studies at FNAL and BNL [4] are good examples of the circular very high energies accelerators.

The luminosity ($L$) of an accelerator colliding head-on bunched beams, with Gaussian cross-sectional density distribution of particles, is:

$$L = \frac{N_e^2 f_c}{4\pi \sigma_x \sigma_y} H_D$$

(1.2)

Here $N_{\alpha}$ as above in the example, is the population of particles in the bunch, $H_D$ is the disruption enhancement parameter\(^5\), $f_c$ is the collision repetition frequency and $\sigma_x$ and $\sigma_y$ are the horizontal and

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\(^1\) The Physics case is not included in this Report. The reader can find it in [1], [10] and [13].

\(^2\) LEP: Large Electron Positron collider, CERN, Geneva, Switzerland.

\(^3\) CESR: Cornell Electron Storage Ring, Cornell University, Ithaca, USA.

\(^4\) HERA: Hadron-Electron Ring Anlage, Deutsches-Elektronen Synchrotron, Hamburg, FRG.

\(^5\) $H_D$ results from the attraction force of the opposite charged interacting beams.
vertical beam dimensions\(^6\) at the interaction point (IP). In case of a pulse operated accelerator \(f_i\) is:

\[
f_i = f_p \cdot n_b
\]

where \(f_p\) and \(n_b\) are the pulse repetition frequency and number of bunches per pulse respectively. A linear collider must be operated with extremely small beam size at the IP to meet the high luminosity requirement. Both colliding beams will be “flat”, with big aspect ratio \(\sigma_x/\sigma_y \gg 1\) as a remedy for the strong beamstrahlung, a phenomenon that takes place at the IP. Colliding particles move on trajectories bent in the electromagnetic (e-m) field of the opposite charged particles of the counter propagating beam. The bending of trajectories causes radiation of hard gammas. The radiated fraction of the energy, \(\Delta_{BS}\) is:

\[
\Delta_{BS} = 0.86 \frac{r_e^3 N_e^2 \gamma}{\sigma_x (\sigma_x + \sigma_y)^2}
\]

(1.3)

Where \(\gamma\) is the relativistic factor, \(r_e\) is the classical electron radius and \(\sigma_z\) is the longitudinal bunch length [5]. Expressions (1.2) and (1.3) show that beams with large aspect ratio enhance luminosity (the product \(\sigma_x \sigma_y\) is small) while the beamstrahlung can be kept low in the range of few percent (when \(\sigma_x + \sigma_y \approx \sigma_z\)). High luminosity implies also high number of colliding particles/s: \(N_{col} = N_e \cdot f_c\). The product: \(E_{cm} N_{col}\) equals to the total beam power \(P_b\) of both colliding beams. We can rewrite (1.2) in the form:

\[
L_e = \frac{P_b N_e}{4\pi\sigma_x\sigma_y E_{cm}} H_D = \frac{\eta P_{ac} N_e}{4\pi\sigma_x\sigma_y E_{cm}} H_D
\]

(1.4)

The expression points out dependence on the overall efficiency \(\eta = P_b/P_{ac}\) of a collider in transferring the ac power \(P_{ac}\) into the beam power. The \(P_{ac}\) budget of linear collider should not exceed 150 MW for the operational cost reasons. Optimization of all subsystems, especially the main linac technology optimization, plays the main part in increasing \(\eta\) and the luminosity. Alternatively, collider with higher efficiency can operate with bigger beam dimensions at the IP, less complicated Beam Delivery System (BDS) \([6, 7]\) and relaxed technical tolerances for the main accelerator (both linacs\(^7\)). A collider based on the superconducting accelerating structures (cavities) provides the highest \(\eta\) value. The idea of the linear collider was first published by M. Tigner over 35 years ago [8]. Twenty-five years after that publication, in July 1990, the first Workshop on the TESLA\(^8\) collider was held at Cornell University [9]. From 1990 to 2004, the worldwide international collaboration, coordinated at DESY was

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\(^6\) The \(\sigma_x\), \(\sigma_y\) and \(\sigma_z\) are the standard Gaussian deviations representing distribution of charge in bunch for all three dimensions. In this report we will follow the common convention of coordinates: \(z\) is the direction of acceleration, \(x\) and \(y\) are horizontal and vertical directions respectively.

\(^7\) Very often both e\(^+\) and e\(^-\) linacs of a collider are called with common name “main accelerator”.

\(^8\)
developing subsystems and components for the TESLA collider and for the X-ray Free Electron Laser facility (XFEL), an integrated part of the collider at that time. The XFEL was designed to utilize the first three kilometers of the TESLA e⁻ linac as the driving accelerator. The scientific XFEL program, which requires a very flexible time structure of the accelerated electron beam, much smaller capital cost of the XFEL project and very different schedule, led to separation of the XFEL facility from the TESLA collider. Since the end of 2001 the XFEL has been proposed as a stand alone European facility with site in Hamburg, with the intention to serve a broad scientific community involved in experiments with short (\(\lambda = 1\ \text{Å}\)) and very intense pulses of coherently emitted X-rays.

After eleven years of R&D programs, the maturity of the superconducting technology for the first, \(E_{cm} = 500\ \text{GeV}\) stage of the linear collider was summarized in the TESLA TDR (Technical Design Report) [12], which was presented to the German Government in March 2001. Further R&D programs have to be continued, with the main goals of: higher gradients needed for the energy upgrade to 1 TeV and significant reduction of the capital and operation costs.

The consensus of the High Energy Physics (HEP) community, presented in 2001 at the American Physical Society Workshop in Snowmass, was that only one linear collider worldwide could be built due to the enormous costs of the facility. Following this, the ICFA\(^9\) created the International Technology Recommendation Panel (ITRP) in 2003, which after a one year study of two proposed options, normal conducting or superconducting RF, recommended in August 2004 to coordinate all worldwide activities towards the superconducting option. The main ITRP arguments leading to this decision were as follows:

- The technical maturity of the TESLA project
- The overall capital and operational costs
- The impact of the further superconducting technology development for many worldwide projects e.g. European XFEL in Hamburg and other coherent and synchrotron light facilities driven by superconducting linacs.

The new name of the superconducting option is: International Linear Collider (ILC) and all worldwide activities are coordinated by Global Design Effort (GDE). The assumed, very ambitious schedule, of the project is shown below in Figure 1.1. The Conceptual Design Report (CDR) was published at the end of 2006. In the CDR the layout of the collider and its main components are presented. The site question is and will stay open for at least the next several years. Technical details of the ILC, published very recently, are briefly discussed in the next chapter.

\(^{8}\) TESLA stands for TeV Energy Superconducting Linear Accelerator. The name has been proposed by W. Hartung.

\(^{9}\) International Committee for Future Accelerators.
In this publication, we will discuss among other topics, superstructures (SST), a novel layout of the superconducting accelerating cavities for the collider, relevant in achieving the mentioned goals. The scheme was first proposed in 1997 during the Tesla Collaboration Meeting in Orsay [10]. In January 1999, after the Epiphany'99 Conference in Krakow [11] when the more elaborated concept was presented, a project group was established at DESY to build a prototype and conduct the “proof of principle” test with the electron beam at the Tesla Test Facility (TTF) linac. The test was scheduled for the second half of 2002. Meanwhile further studies have led to a new version of the layout, seen as even more attractive. In order to progress faster towards the first experimental results the decision has been made to continue with the layout very similar to the later proposed version, but made of elements already built for the first version. Both versions were presented in reference [12] with the hope that before the approval of the 1 TeV upgrade of the superconducting linear collider there will be enough technical experience and experimental justification for using the SST’s in the extension of the main accelerator. The proposed scheme can have a wider application in other superconducting linear accelerators, independently of the experiments they are built to carry out. An example of other application to Energy Recovery Linacs (ERL) driving Free Electron Laser facilities is discussed in details in reference [14].

This Report will consist of six Chapters. After the Introduction (Chapter I), in Chapter II we will briefly discuss options of the linear collider. In the two following chapters, we will present multicell superconducting structures proposed for the implementation in the linear collider and linacs driving FEL facilities. Fundamentals of superconducting TESLA structure and its new alternative are given in Chapter III. Chapter IV presents and reviews the superconducting superstructure concept, its advantages and constrains. In Chapter V we will present an application of short superconducting cavities for superconducting electron sources (SRF-guns). We will focus mainly on the SRF-gun with a superconducting photo-cathode made of lead, which has quantum efficiency sufficient for a 1 mA-class electron source. Finally, in Chapter VI we will briefly summarize this Report.
II $e^+ e^- \text{ LINEAR COLLIDERS; AN OVERVIEW}$
General remarks

Stanford Linear Collider (SLC) at SLAC, which was the first particle accelerator built to serve High Energy Physics experiments ran from 1989 until 1998. The center-of-mass energy of the SLC colliding beams was 100 GeV. The luminosity was $\sim 10^{30}$ cm$^{-2}$s$^{-1}$, almost $10^4$ times lower than the luminosity spec for the new collider. Three collider projects will be discussed here, all of which profit from the design and operating experience of the SLC, which was delivering beams not only for the experiments in High Energy Physics but offered the unique opportunity for many years of Accelerator Physics studies, allowing for milestones in:

- The focusing of the colliding beams (FFTB experiment).
- The operation of accelerator with many traveling wave structures.
- The beam loading understanding and minimization of energy spread.
- The beam emittance preservation.
- The construction and operation of sources delivering many megawatts of the microwave power.

Two technological options had been proposed in early 90’s for the second generation linear collider following the SLC. The first option was NLC/JLC, a joined USA and Japan project utilizing the X-band 11.4 GHz traveling wave normal conducting structures made of copper. The second option, TESLA, was mainly developed in Europe. Its main accelerator was based on the superconducting standing wave niobium structures, which are cooled with superfluid helium for operation at 2 K.

Many novel ideas, technical solutions and a great deal of expertise was developed over fifteen years for both competing collider options. All of this knowledge can be now implemented in the ILC

![Figure 2.1: Layout of the generic linear collider.](image)

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10 Stanford Linear Accelerator Center, Palo Alto, CA, USA.
11 Final Focus Test Bed.
12 CLIC: Compact Linear Collider [15]. This version, with normal conducting, 30 GHz accelerating structures, is not as advanced as two of others. It won’t be discussed in this Report since it belongs to the third generation of linear colliders.
13 NLC/JLC is the abbreviation of Next Linear Collider/Japanese Linear Collider.
project, which from the main accelerator technology point of view is the continuation of the TESLA collider. Figure 2.1 shows a simplified layout of a generic linear collider. Its subsystems and their functions are listed and explained in Table 2.1.

Table 2.1: Common subsystems for second generation of linear colliders

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Function</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>e⁺ injector complex</td>
<td>Generation of positron bunches</td>
<td>e⁺ RF-gun and booster, undulator, high-Z material targets, capturing system for e⁺ beam*</td>
</tr>
<tr>
<td>e⁺ damping ring</td>
<td>Emittance reduction of e⁺ beam</td>
<td>Accelerating sectors*, high field bending magnets* and wigglers*</td>
</tr>
<tr>
<td>e⁺ bunch compressors*</td>
<td>Shortening of bunch length, σₑ</td>
<td>Dipole magnets, quadrupole magnets, dipole chicanes</td>
</tr>
<tr>
<td>e⁺ injector complex</td>
<td>Generation of electron bunches</td>
<td>e⁺ RF-gun and booster</td>
</tr>
<tr>
<td>Two main linacs</td>
<td>Acceleration from few GeV up to several hundreds GeV</td>
<td>Accelerating structures, RF-power sources and distribution system*</td>
</tr>
<tr>
<td>Beam Deliver System*</td>
<td>Collimation, final focusing and beam parameter measurements at IP</td>
<td>Collimators, strong quadrupole magnets</td>
</tr>
</tbody>
</table>

Component is not shown in the following simplified layouts in Fig. 2.1, Fig. 2.2 and Fig. 2.7.
2.2 NLC/JLC

The successful operation of the SLC has led directly to R&D programs for the NLC project at Stanford Linear Accelerator Center. A great deal of theoretical studies and experiments were done to answer the following important questions for the NLC and later for the NLC/JLC project:

- How to design and machining the traveling wave structures at 11.4 GHz, a frequency almost four times higher than the frequency of the SLC traveling wave structures?
- What precautions should be taken in the machining and cleaning of the copper surface to guarantee operation at very high accelerating gradient without electron field emission?
- How to reduce the interaction of the accelerated beam with parasitic resonances of the accelerating structures to avoid the emittance dilution and energy spread along the linac?
- What beam optics should be applied in the Beam Delivery System to produce minimum spot size of both interacting beams?

The layout of NLC (Fig. 2.2) was first presented in a Zero-Design Report [16] and then, with several modifications, in 2001 at American Physical Society Workshop in Snowmass. The design parameters proposed at the Snowmass Workshop for the NLC are listed in Table 2.2. The main change in the 2001 design was 179° angle between both linacs instead of 180°, eliminating the parasitic collisions between bunches which already passed the IP with bunches of the counter-propagating beam before they arrive at the IP. These parasitic interactions happen when the time spacing of bunches is very short and beams after the head-on collisions cannot be deflected fast enough off axis towards the beam dumps. The parasitic collisions enhance background in the detector and cause bending of trajectories of incoming particles before they arrive at the IP.

Fig. 2.2 Simplified layout (not to scale) of the NLC for then energy range of 500 GeV to 1 TeV.

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14 The head-on collisions maximize the luminosity.
### Table 2.2: Parameters of 500 GeV NLC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>$2 \times 10^{34}$</td>
<td>[cm$^{-2}$s$^{-1}$]</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>120</td>
<td>[Hz]</td>
</tr>
<tr>
<td>Particles’ population in one bunch</td>
<td>$7.5 \times 10^9$</td>
<td>-</td>
</tr>
<tr>
<td>Charge of one bunch</td>
<td>1.2</td>
<td>[nC]</td>
</tr>
<tr>
<td>Number of bunches/pulse</td>
<td>190</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_y/\sigma_x$ at IP</td>
<td>$2.7/245$</td>
<td>[nm]</td>
</tr>
<tr>
<td>$\sigma_z$ at IP</td>
<td>110</td>
<td>[$\mu$m]</td>
</tr>
<tr>
<td>Time between two subsequent bunches</td>
<td>1.4</td>
<td>[ns]</td>
</tr>
<tr>
<td>Effective loaded gradient</td>
<td>48</td>
<td>[MV/m]</td>
</tr>
<tr>
<td>Unloaded gradient</td>
<td>70</td>
<td>[MV/m]</td>
</tr>
<tr>
<td>Frequency of accelerating mode</td>
<td>11.424</td>
<td>[GHz]</td>
</tr>
<tr>
<td>Normalized emittance at injection, $\gamma \varepsilon_y/\gamma \varepsilon_x$</td>
<td>$2/300$</td>
<td>$10^{-8}$ [m·rad]</td>
</tr>
<tr>
<td>Normalized emittance at IP, $\gamma \varepsilon_y/\gamma \varepsilon_x$</td>
<td>$3.5/360$</td>
<td>$10^{-8}$ [m·rad]</td>
</tr>
<tr>
<td>Active length of both linacs</td>
<td>10.4</td>
<td>[km]</td>
</tr>
</tbody>
</table>

The proposed nominal time structure of the accelerated beam for NLC is shown in Fig. 2.3 (left diagram). The charge per bunch is rather low to avoid an enhanced excitation of Higher order Modes (HOMs) causing the emittance dilution along the linac. The excitation is strong due to a very small aperture of the accelerating structures, which is a consequence of the high operating frequency and for such a low charge/bunch the collision frequency must be high, $f_c = 22.8$ kHz and the beam dimensions at the IP need to be small to reach the luminosity specification. The dissipation of the stored energy in the structures strongly limits the duty factor to $3.17 \times 10^{-5}$ and thus the time separation of subsequent bunches is only 1.4 ns. The spatial distance between two succeeding bunches is 0.42 m. Consequently, the two closest locations to the IP at which the beams could collide parasitically are only 0.21 m apart downstream and upstream from the IP. The angle of 179° between linacs in the later NLC layout would allow for the radial separation of both beams by 3.7 mm at these locations. We will see later that the parasitic collisions are mitigated in the TESLA collider because the bunch spacing is much bigger and the beam trajectories can be bent enough for the separation outside the IP.

![Figure 2.3: Time structure of the NLC beam (left) and its frequency spectrum, (right).](image-url)
2.2.1 NLC accelerating structures

The first layout of the main NLC accelerator, reported in ZDR, was based on the traveling wave structures with a semi-constant gradient, in which particles are accelerated by the electric field of the electromagnetic wave co-propagating with the beam. The total number of structures required for the 500 GeV stage was 5836. Each structure was 1.8 m long and had 206 cells. The phase advance of the accelerating field per cell was $2\pi/3$. The group velocity (the velocity of energy propagation along the structure) varied from 0.12$c$ at the beginning of the structure to 0.03$c$ at the end. The structure is

![Diagram](image)

**Figure 2.4:** Copper disk used for the 206-cell traveling wave structures of NLC. a. Sketch of the disk, b. cut-away view of the disk (courtesy of SLAC).

![Image](image)

**Figure 2.5:** Input coupler region of the NLC structure with the input line and manifolds for the HOM damping and whole structure on the test stand. (Courtesy of SLAC.)
made of brazed copper disks. The shape of the copper disk is shown in Fig. 2.4. Several first cells of
the structure with input lines, manifolds for the HOM damping and a picture of the whole structure on
the test stand are shown in Fig. 2.5. The excitation of parasitic resonances, HOMs, by the accelerated beam
will be discussed in the TESLA collider chapter. Here we should point out that the short bunches of the
NLC beam, $\sigma_z=110 \, \mu\text{m}$, can excite HOMs of the accelerating structures in a large frequency range up
to 1.5 THz. This can be seen in Figure 2.3, which shows the frequency spectrum of the NLC beam.

Several NLC accelerating structures were manufactured at SLAC and were tested with high
power klystrons. They showed a strong degradation in the performance after a rather short time of
conditioning and operating. This degradation was seen even at accelerating gradients lower than 70
MV/m, which was the nominal unloaded operation gradient of the NLC. The degradation was caused
by the phenomenon called RF-breakdown, which happens when very high electric field on the cavity
wall in the iris region ($E_{\text{surf}}$ in Fig. 2.6) pull electrons from the metal. The emitted electrons ionize
residual gases in the structure, which then generate ions that bombard the metal surface and melt
craters into it. The process is very fast and has an avalanche behavior because the evaporated metal is a
source of new ions. The average conditioning parameters of five tested structures are listed in Table
2.3. The two last rows of the table display the number of affected cells and average phase perturbation
per affected

![Figure 2.6: $E_{\text{surf}}$ and $E_{\text{acc}}$ along the 206-cell structure at the average gradient of 50 MV/m.](image)

![Figure 2.7: Example of the phase perturbation in 206-cell structure after 1000 h of conditioning at 50 MV/m.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditioning time</td>
<td>1000</td>
<td>[h]</td>
</tr>
<tr>
<td>Unloaded gradient</td>
<td>63</td>
<td>[MV/m]</td>
</tr>
<tr>
<td>Maximum electric field on the wall</td>
<td>135</td>
<td>[MV/m]</td>
</tr>
<tr>
<td>Total time of the conditioning (60 Hz pulse repetition), $t_{\text{run}}$</td>
<td>60</td>
<td>[s]</td>
</tr>
<tr>
<td>Number of affected cells in the structure</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>Phase change per affected cell</td>
<td>0.24</td>
<td>[°]</td>
</tr>
</tbody>
</table>
cell respectively (see example shown in Fig. 2.7). The perturbation would lead to the asynchronism between the electromagnetic traveling wave and accelerated particles and thus a significant reduction of the effective accelerating gradient and beam energy gain in the main NLC accelerator. All tested structures were damaged in the high group velocity side, close to the input line [17]. Several solutions to the problem were proposed:

- to build shorter traveling wave structures with a lower group velocity
- to replace the traveling wave structures with standing wave structures
- to reshape the irises to lower the electric field on the metal wall
- to make irises of tungsten

The arguments for the first solution came from the above mentioned observation that only the high group velocity side was damaged. The second solution was motivated by the fact that when the RF-breakdown happens in a standing wave structure its frequency changes significantly, which automatically decouples the structure from the RF-power source and no energy is transferred from the power source to the arcing area. Reshaping of irises and increasing their curvature lowers the electric field in this region but it decreases the cells characteristic impedance and enhances energy dissipation in the metal wall. Finally, the high melting temperature of tungsten would make the irises less sensitive to ion bombardment and to the creation of craters.

All of the above listed modifications required a lot of effort and additional time to implement in the NLC structures, and they had not been fully explored when the ITRP recommended the superconducting option for the collider, and as such were not continued after the recommendation.
2.3 TESLA

The TESLA collider is shown in Fig. 2.8. At first glance the layouts of TESLA and NLC look similar, but the main linac technology is what makes these machines very different. Additionally, in the TESLA layout, the electron beam, before it collides at the IP, is used to generate x-rays in the undulator for the $e^+$ production. Parameters for the first stage of the TESLA collider (500 GeV) as they were proposed in the Technical Design Report (TDR) [12] are listed in Table 2.4. In the design the main TESLA accelerator was made of 20592 superconducting 9-cell standing wave structures. The time structure of the TESLA beam was very different from the NLC beam structure. Small energy dissipation in the superconducting structures, even when they operate at high gradients, made possible operation with 1.3 ms long RF-pulses. Each RF-pulse accommodated 2820 bunches. The time spacing between bunches within a RF-pulse was 337 ns. Two subsequent RF-pulses were spaced by 198.7 ms intervals at 5 Hz repetition rate (Fig. 2.9). In two TESLA interaction points 14100 collisions per second should take place. Even though, the beam dimensions at both IPs were larger by almost factor of 2 and the collision frequency lower by 60% than for NLC, the higher luminosity, $3.4 \times 10^{34}$ cm$^{-2}$s$^{-1}$, was expected due to much higher bunch population (charge/bunch) in the TESLA collider. The bunch spatial separation by 337 ns $\nu = 101$ m enabled sufficient bending of trajectories to avoid the parasitic interactions of TESLA beams which should collide either in the head-on mode or at rather small angle of 34 mrad's. The TESLA bunches were a factor of 3 longer than the NLC bunches. The frequency range of the TESLA beam spectrum is up to 0.5 THz. This and the 70 mm diameter aperture of the TESLA structures led to substantially weaker HOM excitation and according to the beam dynamic simulations, much lower emittance dilution in the main accelerator. On the other hand, longer beam-on time increases the probability of the multi-bunch HOM excitation and special measures have been taken to suppress dangerous parasitic modes. The train of bunches accelerated within a RF-pulse was long in the proposed TESLA collider, with total length $2820 \times 101$ m = 284.8 km and the maximum

![Figure 2.8: Layout (not to scale) of the superconducting linear collider TESLA for the energy range up to 500 GeV.](image-url)
Table 2.4: Parameter set for the 500 GeV TESLA collider.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>3.4·10^{34}</td>
<td>[cm^{-2}s^{-1}]</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>5</td>
<td>[Hz]</td>
</tr>
<tr>
<td>Particles’ population in one bunch</td>
<td>2·10^{10}</td>
<td>-</td>
</tr>
<tr>
<td>Charge of one bunch</td>
<td>3.2</td>
<td>[nC]</td>
</tr>
<tr>
<td>Average beam current in pulse</td>
<td>9.5</td>
<td>mA</td>
</tr>
<tr>
<td>Number of bunches/pulse</td>
<td>2820</td>
<td>-</td>
</tr>
<tr>
<td>(\sigma_y/\sigma_x) at IP</td>
<td>5/553</td>
<td>[nm]</td>
</tr>
<tr>
<td>(\sigma_z) at IP</td>
<td>300</td>
<td>[\mu m]</td>
</tr>
<tr>
<td>Time interval between two subsequent bunches</td>
<td>337</td>
<td>[ns]</td>
</tr>
<tr>
<td>Accelerating gradient</td>
<td>23.5</td>
<td>[MV/m]</td>
</tr>
<tr>
<td>Frequency of accelerating mode</td>
<td>1.300</td>
<td>[GHz]</td>
</tr>
<tr>
<td>Normalized emittance at Injection, (\gamma \epsilon_y/\gamma \epsilon_x)</td>
<td>2/800</td>
<td>10^{-8} [m·rad]</td>
</tr>
<tr>
<td>Normalized emittance at IP, (\gamma \epsilon_y/\gamma \epsilon_x)</td>
<td>3/1000</td>
<td>10^{-8} [m·rad]</td>
</tr>
<tr>
<td>Active length of both linacs</td>
<td>21.8</td>
<td>[km]</td>
</tr>
</tbody>
</table>

Figure 2.9: Time structure of the TESLA beam (left) and its frequency spectrum (right).

compression, limited only by the state-of-the-art kicker design (20 ns pulse cycle), has been implemented to keep the damping rings as short as possible [18]. Finally each of proposed dog-bone rings had almost an 17 km circumference. All their straight sections have been accommodated in the tunnel of the main accelerator, simply for cost reasons, while the arcs required additional tunnels.

From 2002 to 2004, a great deal of effort was done to demonstrate the upgrade potential of the TESLA collider to 800 GeV. Two important steps towards the upgrade were demonstrated in these years.

The first was the “proof of principle” beam test of two superstructure prototypes in 2002. We will discuss later the superstructure concept, noting here that in the TDR that layout was assumed as the technical solution for the TESLA upgrade, improving the performance and saving cost of the collider. The second step was the long term test of a standard 9-cell structure at 35 MV/m, a gradient required to reach 800 GeV [19]. Both experiments confirmed the expectation that the TESLA energy range could be extended beyond 500 GeV if it was required by the HEP experimental program.
After fourteen years of extensive R&D, leading to significant progress in the superconducting radiofrequency (SRF) technology, the TESLA Collaboration became obsolete in August 2004, but was subsequently transformed to the ILC Collaboration. Its achievements, particularly the high accelerating gradients in the multi-cell superconducting structures and successful tests of complete, fully equipped cryomodules in the TTF linac, led to the ITRP decision that it could now be implemented into the ILC design.
2.4 ILC

The layout of the ILC is shown in Figure 2.10. The technical data and nominal parameter set are listed in Tables 2.5 and 2.6 respectively [20]. The superconducting technology, positron production and small, 14 mrad, angle between colliding beams make the ILC similar to the TESLA collider. We will briefly point out differences in both colliders. The assumed ILC operating gradient of 31.5 MV/m is higher than the TESLA gradient by 8.5 MV/m. This led to the lower number of cavities in the ILC main accelerator as compared to the TESLA collider. The increased cryogenic load, which is proportional to \((E_{\text{acc}})^2\), can be partially compensated with the high intrinsic quality factor of the accelerating cavities achievable with todays technology. Both damping rings of ILC are located in the same tunnel in the center of the facility. The long transport lines for both beams are technically challenging. In such a long line, the residual gases ionized by the beam may cause beam loss. Also the wall roughness and all discontinuities will increase the energy spread within a bunch. The ILC main accelerator has two tunnels, each 4.5 m in diameter. The main tunnel houses all of the components of both beam lines: accelerating cryomodules, optics and diagnostics. The second, the service, tunnel contains all other subsystems such as: klystrons, modulators, electronics racks and computer stations. The tunnels are connected with 5-7.5 m long penetrations accommodating the RF-waveguides, cabling and wiring. There are \(~1680\) penetrations with the total length of 10.6 km. The second tunnel makes the collider more expensive, but it increases reliability of the facility allowing for maintenance during the beam operation. The proposed installed total AC power is for the ILC facility is 230 MW and exceeds the AC power for previous colliders by more than 50%.

The beam spot at the IP is slightly larger compared to the TESLA spot size (see Tables 4 and 6). This and less bunches per RF-pulse, make the ILC luminosity lower than the TESLA luminosity, but simplifies the final focus optics. The bunch length is the same for both colliders. The time

![Figure 2.10: Layout (not to scale) of the ILC for the energy range up to 500 GeV](image)

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Table 2.5: Technical data for the ILC collider

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of tunnels for main accelerator</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Total site length</td>
<td>31</td>
<td>[km]</td>
</tr>
<tr>
<td>Total site power consumption</td>
<td>230</td>
<td>[MW]</td>
</tr>
<tr>
<td><strong>Main Linacs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average gradient in accelerating structures</td>
<td>31.5</td>
<td>[MV/m]</td>
</tr>
<tr>
<td>Intrinsic quality factor at the nominal gradient</td>
<td>1·10^{10}</td>
<td>-</td>
</tr>
<tr>
<td>Length of e− main linac</td>
<td>10.917</td>
<td>[km]</td>
</tr>
<tr>
<td>Length of e+ main linac</td>
<td>10.770</td>
<td>[km]</td>
</tr>
<tr>
<td>Active length of both linacs</td>
<td>15.113</td>
<td>[km]</td>
</tr>
<tr>
<td>Injection energy</td>
<td>15</td>
<td>[GeV]</td>
</tr>
<tr>
<td>End energy</td>
<td>250</td>
<td>[GeV]</td>
</tr>
<tr>
<td>Duty cycle</td>
<td>0.005</td>
<td>[%]</td>
</tr>
<tr>
<td>Total power consumption</td>
<td>150</td>
<td>[MW]</td>
</tr>
<tr>
<td><strong>Damping Rings</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam energy</td>
<td>5</td>
<td>[GeV]</td>
</tr>
<tr>
<td>Circumference</td>
<td>6.7</td>
<td>[km]</td>
</tr>
<tr>
<td><strong>Interaction Region</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total length of the BDS for 2 beams</td>
<td>4.5</td>
<td>[km]</td>
</tr>
<tr>
<td>Beam crossing angle at the IP</td>
<td>14</td>
<td>[mrad]</td>
</tr>
<tr>
<td><strong>Superconducting Installation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of 1.3 GHz 9-structure cryomodules</td>
<td>1180</td>
<td>-</td>
</tr>
<tr>
<td>Number of 1.3 GHz 8-structure cryomodules</td>
<td>634</td>
<td>-</td>
</tr>
<tr>
<td>Number of other 1.3 GHz cryomodules</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>Number of 0.65 GHz 8-structure cryomodules</td>
<td>36</td>
<td>-</td>
</tr>
<tr>
<td>Number of 3.9 GHz cryomodules</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Number of 1.3 GHz 9-cell structures</td>
<td>15764</td>
<td>-</td>
</tr>
<tr>
<td>Number of 0.65 GHz structures</td>
<td>36</td>
<td>-</td>
</tr>
<tr>
<td>Number of 3.9 GHz structures</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Equivalent cryogenic plant capacity at 4.5 K</td>
<td>196</td>
<td>[kW]</td>
</tr>
<tr>
<td>Total AC power operating/installed for cryogenics</td>
<td>37/48</td>
<td>[MW]</td>
</tr>
</tbody>
</table>

Table 2.6: Nominal beam parameter set for the ILC collider

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center-of-mass energy</td>
<td>500</td>
<td>[GeV]</td>
</tr>
<tr>
<td>Luminosity</td>
<td>2·10^{34}</td>
<td>[cm^{-2}s^{-1}]</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>5</td>
<td>[Hz]</td>
</tr>
<tr>
<td>Particles’ population in one bunch</td>
<td>2·10^{10}</td>
<td>-</td>
</tr>
<tr>
<td>Charge of one bunch</td>
<td>3.2</td>
<td>[nC]</td>
</tr>
<tr>
<td>Number of bunches/pulse</td>
<td>2625</td>
<td>-</td>
</tr>
<tr>
<td>Beam pulse length</td>
<td>965</td>
<td>µs</td>
</tr>
<tr>
<td>Average beam current in pulse</td>
<td>9</td>
<td>mA</td>
</tr>
<tr>
<td>σ_y/σ_x at IP</td>
<td>5.7/639</td>
<td>[nm]</td>
</tr>
<tr>
<td>σ_y at IP</td>
<td>300</td>
<td>[µm]</td>
</tr>
<tr>
<td>Time interval between two subsequent bunches</td>
<td>369.2</td>
<td>[ns]</td>
</tr>
<tr>
<td>Beam power in each beam</td>
<td>10.8</td>
<td>[MW]</td>
</tr>
<tr>
<td>Normalized emittance at Injection, γε_y/γε_x</td>
<td>0.024/8</td>
<td>[mm mrad]</td>
</tr>
<tr>
<td>Normalized emittance at IP, γε_y/γε_x</td>
<td>0.034/10</td>
<td>[mm mrad]</td>
</tr>
</tbody>
</table>

Spacing between bunches for the nominal ILC operation is longer by 32.2 ns than for TESLA. The beam spectra of both colliders (main accelerators) differ marginally in the spacing between spectral...
lines, which is larger by 0.27 MHz and amplitudes which are smaller by 5% for ILC, but have the same frequency reach as shown in Figure 2.9. The similarity of both beams makes it possible to implement the TESLA accelerating structures in the ILC with their auxiliaries developed for the TESLA collider.

The first ILC cost estimate was published very recently in the Reference Design Report [20]. The total capital cost, including 22 million labor hours and 20% contingency, is close to 9.5 billion USD. All three 31 km long sites discussed in that document and two additional European sites can be extended to 50 km to accommodate the upgrade of the collider to 1 TeV. The upgrade cost has not been included in the presented estimate. Still, much effort can and should be done to lower the price of the facility in the first and second stage. Even a few percent cost reduction will result in substantial money saving.

The main accelerator is the most expensive part of a superconducting collider. This was the case for TESLA and is the same for the ILC facility. Many potential improvements, requiring further R&D effort, were proposed at the American Physics Society Workshop in 2005 in Snowmass [21]. Two of them: the implementation of the large grain niobium and the superstructure concept can substantially reduce the cost of the main accelerator and deserve further R&D. Both will be discussed in the next chapters.
III TESLA SUPERCONDUCTING STRUCTURE AND
ALTERNATIVE STRUCTURES FOR ILC
3.1. General remarks

The application of the superconductivity phenomenon in accelerators originated with pioneer work in the late sixties at the High Energy Physics Laboratory (HEPL) at Stanford University [22]. Starting with the deposition of lead on copper and niobium on copper, by the end of 1968 the HEPL group had developed and tested the first bulk niobium superconducting accelerating cavity [23]. The result was very encouraging and initiated R&D programs at many other laboratories like Brookhaven National Laboratory and Kernforschungszentrum Karlsruhe.

Since then, many superconducting accelerators have been built for the high energy (HEP) and nuclear physics (NP) experiments, with both hadrons and leptons. Table 3.7 shows the largest superconducting (sc) accelerators for this purpose that either have been operated in the past, at present or are being commissioned for the future operation. In the second column (C) stands for circular and (L) for the linear type accelerator, the third column displays the particles being accelerated, while the fourth column lists the final beam energy and the fifth column the number of accelerating cavities installed in an accelerator.

Table 3.7: Large sc accelerators for HEP and NP

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Particles</th>
<th>Energy [GeV]</th>
<th>Number of cavities</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tristan(^\text{15})</td>
<td>C</td>
<td>(e^+) (e^-)</td>
<td>2x2</td>
<td>32</td>
<td>Dismantled</td>
</tr>
<tr>
<td>LEP(^\text{16})</td>
<td>C</td>
<td>(e^+) (e^-)</td>
<td>2x100</td>
<td>288</td>
<td>Dismantled</td>
</tr>
<tr>
<td>CEBAF(^\text{17})</td>
<td>L</td>
<td>(e^-)</td>
<td>6</td>
<td>320</td>
<td>Operating</td>
</tr>
<tr>
<td>HERA(^\text{18})</td>
<td>C</td>
<td>(e^+) or (e^-)</td>
<td>27</td>
<td>16</td>
<td>Operating</td>
</tr>
<tr>
<td>SNS(^\text{19})</td>
<td>L</td>
<td>(H^-)</td>
<td>1</td>
<td>81</td>
<td>Commissioning</td>
</tr>
</tbody>
</table>

The worldwide development of the superconducting technology for the TESLA linear collider led to many new proposed facilities based on superconducting accelerators (Table 3.8). Two big projects ILC and European XFEL have been mentioned already in the previous chapters. Both will operate in the pulse mode with a low duty factor of less than one percent. Potential application of superconducting linear accelerators as drivers for continuous wave operating coherent light sources (Free Electron Lasers) and synchrotron radiation sources results in many other projects worldwide. Three of them are listed in the table. All five future projects are based on the superconducting accelerating cavity designed in 1992 for the TESLA collider [33]. The features of the TESLA cavity will be introduced to the reader in the next section.

\(^{15}\) KEK, Tsukuba, Japan [24]  
\(^{16}\) CERN, Geneva, Switzerland [25]  
\(^{17}\) TJNAF, Newport News, USA [26]  
\(^{18}\) DESY, Hamburg, Germany [27]  
\(^{19}\) ORNL, Knoxville, USA [28]
Table 3.8: Proposed future facilities driven by sc linear accelerators.

<table>
<thead>
<tr>
<th>Name</th>
<th>Particles</th>
<th>Energy [GeV]</th>
<th>Number of cavities</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>ILC</td>
<td>e⁺ e⁻</td>
<td>2x250</td>
<td>15764</td>
<td>Studies</td>
</tr>
<tr>
<td>European XFEL [29]</td>
<td>e⁻</td>
<td>20</td>
<td>928</td>
<td>Proposal</td>
</tr>
<tr>
<td>ERL [30]</td>
<td>e⁻</td>
<td>5.3</td>
<td>310</td>
<td>Proposal</td>
</tr>
<tr>
<td>BESSY FEL [31]</td>
<td>e⁻</td>
<td>2.3</td>
<td>144</td>
<td>Proposal</td>
</tr>
<tr>
<td>4GLS [32]</td>
<td>e⁻</td>
<td>0.9</td>
<td>40</td>
<td>Proposal</td>
</tr>
</tbody>
</table>

3.2. TESLA superconducting structure

Figure 3.1 shows the cross-section and a photograph of the 9-cell TESLA structure. The cavity is made of niobium, a type II superconductor, with a critical temperature \( T_c = 9.2 \) K and a critical magnetic flux \( B_c \approx 190 \) mT. The body of the cavity, 9-cells, which from the RF point of view are nine coupled resonators, is cylindrically symmetric. The accelerated beam traverses the cavity along its symmetry axis. The beam tubes at either end have symmetry perturbations introduced by the attached auxiliaries: Higher Order Mode (HOM) couplers, a fundamental power coupler (FPC) and the pickup probe. The axial length of each cell is equal to half of the free space wavelength of the fundamental (accelerating) resonant mode. At the operating TESLA frequency of 1.3 GHz the cell length is \( \lambda/2 = 115.3 \) mm.

![Cross-section and photograph of the superconducting 9-cell TESLA structure equipped with two HOM couplers, a fundamental power coupler (FPC) port and a pickup probe.](image)

3.2.1. Interaction with accelerated beam

Figure 3.2 shows computed magnetic and electric field contours for the accelerating mode. The structure is a so-called standing wave structure in which particles experience the Lorentz force of the

---

20 The whole material presented in the following subsections applies in general to any superconducting standing wave accelerating structure.
standing electro-magnetic wave. As known, \( n \) coupled resonators can oscillate in \( n \) modes for each resonance field pattern being an eigenvector of a single (uncoupled) cell. The \( n \) modes, which differ in frequency and cell-to-cell phase advance, form the passband. In the TESLA structure the TM\(_{010}\) like\(^{21}\) resonance pattern is utilized for particle acceleration. It is a monopole mode with no angular dependency of the fields inside the cylindrical symmetric part of the cavity and it is the highest frequency mode in the TM\(_{010}\) pass-band. The cell-to-cell phase advance \( \beta_{cc} \) is \( \pi \). When the structure is well tuned, all cells have the same amplitude |\( E \)| of the accelerating electric field on the axis (Fig. 3.3).

![Figure 3.2: Contour of magnetic (a) and electric (b) field in the TESLA structure. On the color scale red means high and blue low value respectively\(^{22}\).](image)

In general, the interaction between the fundamental mode of a cavity and a charged beam may lead to two processes:

- Beam energy gain – acceleration
- Beam energy loss – deceleration, energy recovery process.

Furthermore, the beam may excite other resonances, HOMs of a cavity:

- HOM excitation.

The excited HOMs very often degrade beam quality causing emittance growth and/or energy modulation. Suppression of the HOMs will be discussed later in this chapter.

![Figure 3.3: Computed normalized accelerating electric field on the axis in the TESLA structure.](image)

\(^{21}\) The convention to name field patterns follows the convention commonly used for cylindrical waveguides and cylindrical RF resonators \([34]\) for cylindrical coordinate system (\(\phi, r, z\)). TM stands for transverse magnetic (no \(z\)-component of magnetic field). Indices \(010\) refer to the number of half wave length in all three dimensions. “0” means field has constant value in the direction. “1” means a half wavelength in the direction and so on. The convention, when applied to cylindrical resonators with no openings (irises), is very helpful to describe the field pattern of a mode, but it is slightly misleading in case of cells of an accelerating structure, which fields vary in \(z\)-direction.

\(^{22}\) In all following figures on the color scale red means high and blue low value respectively.
Interaction between the electric field of a mode \( k \) and a point-like charge, having relativistic velocity close to the speed of light \( c \), is described in the frequency domain by the characteristic impedance \((R/Q)_k\)\(^{23}\):

\[
(R/Q)_k = \frac{|V_k|^2}{\omega_k W_k}
\]

where \( \omega_k = 2\pi f_k \) and \( W_k \) are the angular frequency of the mode and its stored energy respectively. \( |V_k| \) is the amplitude of the voltage experienced by the bunch of particles along its trajectory:

\[
|V_k| = \sqrt{\left( \int_{y_0}^{l} \frac{E_k \sin(\frac{\omega_k}{c}(y - y_0))}{\sqrt{1 - \left( \frac{\omega_k}{c}(y - y_0) \right)^2}} dy \right)^2 + \left( \int_{y_0}^{l} \frac{E_k \cos(\frac{\omega_k}{c}(y - y_0))}{\sqrt{1 - \left( \frac{\omega_k}{c}(y - y_0) \right)^2}} dy \right)^2}
\]

\( l \) and \( y_0 \) are the trajectory length and its beginning respectively, \( E_k \) is the electric field along the trajectory.

The higher \((R/Q)_k\) the stronger is the potential energy exchange between the mode and the bunch. The way the voltage \( |V_k| \) is defined\(^{24}\) makes \((R/Q)_k\) independent of the phase a bunch enters the cavity. The net energy exchanged \( \Delta U_k \) with the mode is:

\[
\Delta U_k = |qV_k| \cos(\varphi_k - \varphi_{0,k})
\]

which depends on the entrance phase \( \varphi_k \), the charge of the bunch \( q \) and the mode stored energy \( W_k \) since \( |V_k| \approx \sqrt{W_k} \). The phase \( \varphi_{0,k} \) giving the maximum \( \Delta U_k \) is defined by the equation:

\[
\varphi_{0,k} = \arcsin\left( \frac{y_0}{|V_k|} \right)
\]

Figure 3.4 shows the simplified replacement lumped element circuit for the interaction of a beam with the fundamental mode (beam loading). The LCR circuit represents the accelerating structure with its intrinsic resistance \( R_k \). The beam is represented by the current source \( I_b \).

![Figure 3.4: Replacement circuit for the beam loading](image)

---

\(^{23}\) In this article \((R/Q)\) is defined in the linac convention. For the RF definition one needs to take half of its value.

\(^{24}\) Italic bold characters denote phasors
A current source $I_g$ and its impedance $R_g$ replace the RF power source. The impedances are defined as follows:

$$R_0 = \left( \frac{R}{Q} \right)_{FM} Q_0$$  \hfill (3.5)

$$R_g = \left( \frac{R}{Q} \right)_{FM} Q_{ext}$$  \hfill (3.6)

$(R/Q)_{FM}$ is the characteristic impedance of the fundamental mode (FM). $Q_o$ and $Q_{ext}$ are the intrinsic and external quality factors. The power $P_b$ delivered to or extracted from the beam and power dissipated in the cavity wall $P_0$ are:

$$P_b \equiv |V_g| |I_g| \cos(\Phi_b)$$  \hfill (3.7)

$$P_0 \equiv \frac{|V_0|^2}{R_0}$$  \hfill (3.8)

where $\Phi_b$ is the phase between the beam current $I_b$ and the cavity voltage $V_o$. The phases of the voltages and currents from the replacement circuit are shown in Figure 3.5. At the resonance frequency and without the beam the generator voltage $V_g$ and its current $I_g$ are parallel, which means that the RF source is loaded by real impedance and that energy transfer from the RF source to the load is most effective. In general, when particles traverse the cavity the beam induced voltage $V_b$ “detunes” the cavity [35]. The vector sum:

$$V_o' = V_g + V_b$$  \hfill (3.10)

is not parallel to $I_g$ and a correction of the cavity frequency must take place to rotate this vector by the angle $-\Delta \Phi$:

$$\Delta \Phi \equiv \arctan \left( \frac{|V_b| \sin(\Phi_b)}{|V_g| - |V_b| \cos(\Phi_b)} \right)$$  \hfill (3.11)

Figure 3.5: Phases for the replacement circuit

Practically, in circular accelerators like synchrotrons, $\Phi_b$ varies between $90^\circ$ at the injection and $45^\circ$ when beam is stored for experiments. In linear accelerators $\Phi_b$ is close to 0. The acceleration is “on crest” and $\Delta \Phi$ is 0 or very small.
Figure 3.6: Power distribution scheme in a standing wave structure.

Schematic distribution of the RF power for a standing wave structure accelerating the beam is shown in Fig. 3.6. The input power $P_{in}$ is:

$$P_{in} = P_0 + P_b = P_{for} = P_{ref} = \frac{4\beta_b}{(1 + \beta_b)^2} P_{for}$$  \hspace{1cm} (3.12)

$P_{for}$ and $P_{ref}$ are the incident and the reflected power respectively. $\beta_b$ is the dynamic, beam dependent coupling factor between the cavity and the input line:

$$\beta_b = \frac{\beta_c}{\sqrt{1 + \frac{Q_0}{Q_c}(R/Q)_{FM} \cos(\Phi_b)}}$$  \hspace{1cm} (3.13)

$\beta_c$ is the intrinsic wall loss dependent coupling factor:

$$\beta_c = \frac{Q_0}{Q_{ext}}$$  \hspace{1cm} (3.14)

The amplitudes of $V_g$ and $V_b$ are given by:

$$|V_g| = \frac{4\beta_c}{(1 + \beta_c)^2} P_{for} (R/Q) Q_0$$  \hspace{1cm} (3.15)

$$|V_b| = |I_b| (R/Q) \frac{Q_0}{(1 + \beta_c)}$$  \hspace{1cm} (3.16)

The optimum external quality factor at which the operation is reflection-free ($P_{ref}=0$) is achieved when $Q_{ext}$ fulfils the following condition:

$$Q_{ext} = \frac{Q_0 (R/Q)_{FM} \cos(\Phi_b)}{1 + \frac{Q_0 |I_b| (R/Q)_{FM} \cos(\Phi_b)}{|V_0|}}$$  \hspace{1cm} (3.17)

### 3.2.2. Intrinsic wall losses

A time-dependent RF magnetic field applied to a superconductor dissipates energy in a thin layer (the penetration depth). This is different from the DC case, where currents flow without losses. The dissipation is due to the temperature dependent BCS resistance $r_{BCS}$ and due to impurities in the superconductor resulting in the residual resistance $r_r$. The total surface resistance $r_s$ for niobium in the superconducting state at the temperature $T$ and the frequency $f$ is:

$^{25}$ BCS stands for J. Bardeen, L. Cooper and R. Schrieffer. Their theory explains phenomenon of superconductivity.
\[ r_s = r_r + r_{BCS} = r_r + \frac{0.0002}{T} \left( \frac{f [GHz]}{1.5} \right)^2 \exp(-\frac{\Delta}{k_B T}) \]  

where \( k_B \) is the Boltzmann constant and \( \Delta \) is the energy gap. In the case of the TESLA cavity \( r_{BCS} \) is \(~10\, \text{n}\Omega\) at \( T=2\,\text{K} \). For a very pure niobium e.g. big grain or single crystal material, for which \( r_r \) is of the order of \( 1\,\text{n}\Omega \) an operation at \( 1.8\,\text{K} \) or even \( 1.7\,\text{K} \) can be economically justified, even though it requires more complex refrigeration. The dissipated power \( P_k \) by mode \( k \) in the cavity wall is:

\[ P_k = \frac{r_s}{2} \int_S \overline{|H_k|^2} \, ds \]  

where \( \overline{H_k} \) is the magnetic field of mode \( k \) on the wall and \( S \) denotes the inner surface of a cavity.

The geometric factor \( G_k \):

\[ G_k = \frac{\omega_k \cdot W_k}{\frac{1}{2} \int_S \overline{|H_k|^2} \, ds} \]  

is commonly used to compare various cell shapes with respect to their cryogenic losses. The higher \( G_k \) the lower the dissipation \( P_k \) at the given stored energy \( W_k \) and surface resistance \( r_r \). The above definition of \( G_k \) yields to the relation:

\[ G_k = Q_k \cdot r_s = \frac{\omega_k \cdot W_k \cdot r_s}{P_k} \]  

Consequently, higher \( G_k \) of a mode \( k \) means higher intrinsic quality factor \( Q_k \).

The dissipated power \( P_0 \) for the fundamental mode at the accelerating voltage \( |V_0| \) can now be expressed by the formula:

\[ P_0 = \frac{r_s \cdot |V_0|^2}{G_{FM} \cdot (R/Q)_{FM}} \]  

which indicates that cell shapes with high product \( G_{FM} \cdot (R/Q)_{FM} \) are preferable due to the lower dissipation.

### 3.2.3. Maximum fields on the cavity wall

The magnetic flux on the cavity wall has to be smaller than \( B_c \), a critical value at which superconductivity breaks and niobium goes to the normal-conducting state. The ratio:

\[ \eta_B = \frac{B_{peak}}{E_{acc}} \]  

of the peak magnetic flux \( |B_{peak}| \) on the wall to the accelerating gradient \( |E_{acc}| \):

\[ |E_{acc}| = \frac{|V_0|}{l_{active}} \]  

where \( l_{active} \) is the axial length of all cells, shows the ultimate limit in the achievable gradient for the shape.
The ratio of the peak electric field on the wall $|E_{\text{peak}}|$ to $|E_{\text{acc}}|$:

$$\eta_E \equiv \frac{|E_{\text{peak}}|}{|E_{\text{acc}}|}$$

has practical meaning. The fields on the wall can be as high as 100 MV/m. Such a strong field withdraws electrons from sharp edges of grain boundaries, welds or residual particulates on the surface. When these electrons accelerated by the electric field they either bombard the superconducting wall of the cavity or get captured into synchronic acceleration forming dark currents bombarding surface of other cavity in a cavity chain. Both processes lead to x-rays which may cause a quench, a local loss of superconductivity transferring the whole stored energy into heat load. The design of the TESLA cavity in 1992 assumed this phenomenon as the potential limit in the performance and a great deal of effort went into keeping this factor close to two.

Very recently, due to significant progress in the surface preparation methods over the past fifteen years, the optimization of the ILC cavity shape with respect to $\eta_E$ was proposed. Two proposed shapes will be briefly discussed later but both have $\eta_E > 2$ and seem to be more sensitive to residual impurities on the surface.

### 3.2.4. Cell-to-cell coupling

The cell-to-cell coupling $k_{\text{cc}}$ for the fundamental mode in a $N$-cell structure plays an important role for the field profile sensitivity to cell frequency errors. The sensitivity factor $a_{ji}$ is defined in the following way [36]:

$$a_{ji} = \frac{N^2}{k_{\text{cc}}}$$

The relative sensitivity of the field amplitude $A_i$ in cells is given by the equation:

$$\frac{\Delta A_i}{A_i} = a_{ji} \frac{\Delta f_i}{f}$$

where $\Delta f_i$ is the frequency error of the cell number $i$. $k_{\text{cc}}$ increases with radius of the iris but this makes $\eta_E$ and $(R/Q)_{\text{TM}}$ less favorable.

### 3.2.5. Cell shape optimization

In general, the shape optimization is a difficult process because an improvement in one parameter usually causes another parameter to get worse [37]. One varies the geometric parameters (GP) shown in Fig. 3.7 to shape the inner cells. The most common criteria for optimization and the RF-parameters, which have to be optimized for a given criterion, are listed in the first and second column of Table 3.3. In the table (D↓) and (I↑) indicate decreasing and increasing value respectively. The third column shows required change in an RF-parameter. The iris radius $r_i$ is the most effective geometric parameter for each criterion (see last column). Other GPs listed in the last column are used for fine tuning.
Figure 3.7: Geometrical parameters of an elliptical cell.

Table 3.3: Criteria and RF-parameters

<table>
<thead>
<tr>
<th>Criterion</th>
<th>RF-parameter</th>
<th>Opt.</th>
<th>GP</th>
</tr>
</thead>
<tbody>
<tr>
<td>High gradient</td>
<td>( \eta_B ) and ( \eta_E )</td>
<td>( \downarrow )</td>
<td>( r_i \leftrightarrow (D \downarrow) )</td>
</tr>
<tr>
<td>Low cryogenic load</td>
<td>( \frac{R}{Q}<em>{FM} ) ( G</em>{FM} )</td>
<td>( \uparrow )</td>
<td>( r_i \leftrightarrow (D \uparrow) )</td>
</tr>
<tr>
<td>Low HOM impedance</td>
<td>( \frac{R}{Q}_{HOM} )</td>
<td>( \downarrow )</td>
<td>( r_i \leftrightarrow (\uparrow) )</td>
</tr>
</tbody>
</table>

3.2.6. Parameters of the TESLA and ILC structures

The RF-properties of a multi-cell cavity are usually dominated by those of the inner cells. This is the case found in the TESLA cavities. Tables 3.4 and 3.5 show RF-parameters of the inner cell and the whole TESLA structure respectively. In addition, in Table 5 the nominal operation parameters for the ILC are displayed in the last four rows. These parameters are still under discussion and in the end may differ from those displayed here. The inner cell shape is shown in Figure 3.8 along with two alternative shapes proposed recently (Table 3.6). The first, Low Loss (LL) shape was first proposed in 2002 for the 12-GeV upgrade of the CEBAF accelerator [38] and was later scaled for the ILC frequency [39]. The iris of the LL cell is 60 mm in diameter as compared to 70 mm of the original TESLA cell. This allows for lower \( \eta_B \) and enhances the limit in the achievable gradient to 51 MV/m from 44 MV/m for the TESLA shape. High \( \frac{R}{Q}_{FM} \) and \( G_{FM} \) lead to significantly higher combined product which results in a \( \sim 20\% \) reduction in the cryogenic load in comparison to the TESLA cell. On the other hand smaller \( k_{\alpha} \) and higher \( \eta_E \) made that shape more sensitive to cell frequency errors and increased probability of the electron emission phenomenon. Similarly, the Re-entrant cavity shape allows for higher gradients [40, 41]. It has more favorable \( \eta_E \) and \( k_{\alpha} \) as compared to the LL shape, but technical difficulty in acid and water removal after the cleaning in a multi-cell assembly might limit its applications.
### Table 3.4: Parameters of the inner cell

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{FM}$</td>
<td>1300 MHz</td>
</tr>
<tr>
<td>$k_{cc}$</td>
<td>1.98 %</td>
</tr>
<tr>
<td>$\eta_E$</td>
<td>- 1.98 %</td>
</tr>
<tr>
<td>$\eta_B$</td>
<td>4.15 mT/(MV/m)</td>
</tr>
<tr>
<td>$(R/Q)_{FM}$</td>
<td>113.8 Ω</td>
</tr>
<tr>
<td>$G_{FM}$</td>
<td>271 Ω</td>
</tr>
<tr>
<td>$(R/Q)<em>{FM}G</em>{FM}$</td>
<td>30840 Ω²</td>
</tr>
</tbody>
</table>

### Table 3.5: Parameters of the TESLA cavity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{FM}$</td>
<td>1300 MHz</td>
</tr>
<tr>
<td>$(R/Q)_{FM}$</td>
<td>1012 Ω</td>
</tr>
<tr>
<td>$G_{FM}$</td>
<td>271 Ω</td>
</tr>
<tr>
<td>$l_{active}$</td>
<td>1038 mm</td>
</tr>
<tr>
<td>$a_f$</td>
<td>- 4091</td>
</tr>
<tr>
<td>$Q_{opt}$</td>
<td>$10^9$ 3.4</td>
</tr>
<tr>
<td>$Q_0$ at $E_{acc} = 31.5$ MV/m</td>
<td>$10^{10}$ 1</td>
</tr>
<tr>
<td>Nominal gradient $</td>
<td>E_{acc}</td>
</tr>
<tr>
<td>$P_{in}$</td>
<td>312 kW</td>
</tr>
<tr>
<td>$I_b$</td>
<td>9.5 mA</td>
</tr>
<tr>
<td>$\Phi_b$</td>
<td>0 °</td>
</tr>
</tbody>
</table>

### Figure 3.8: Magnetic field contour\(^2\) of the fundamental mode in inner cells proposed for the ILC, a) TESLA, b) Low Loss, c) Re-entrant.

### Table 3.6: Alternative inner cell shapes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>LL</th>
<th>RE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{FM}$</td>
<td>MHz</td>
<td>1300</td>
<td>1300</td>
</tr>
<tr>
<td>$k_{cc}$</td>
<td>%</td>
<td>1.52</td>
<td>1.8</td>
</tr>
<tr>
<td>$\eta_E$</td>
<td>-</td>
<td>2.36</td>
<td>2.21</td>
</tr>
<tr>
<td>$\eta_B$</td>
<td>mT/(MV/m)</td>
<td>3.61</td>
<td>3.76</td>
</tr>
<tr>
<td>$(R/Q)_{FM}$</td>
<td>Ω</td>
<td>133.7</td>
<td>126.8</td>
</tr>
<tr>
<td>$G_{FM}$</td>
<td>Ω</td>
<td>284</td>
<td>277</td>
</tr>
<tr>
<td>$(R/Q)<em>{FM}G</em>{FM}$</td>
<td>Ω²</td>
<td>37970</td>
<td>35120</td>
</tr>
</tbody>
</table>
3.2.7. Vertical test

The economically justified and therefore commonly used fabrication technique of elliptical Nb cavities by deep-drawing of metal sheets and electron beam welding of half-cells has a severe drawback. The shapes of actual fabricated cavities deviate from their RF theoretical models and the frequencies of individual cells in a multi-cell cavity have to be corrected to equalize the field profiles of modes. Unfortunately, this is possible only for the accelerating mode and is done by means of plastic deformation of cells. In general, frequencies of HOMs stay perturbed but luckily, many of the high impedance parasitic modes have strong cell-to-cell coupling and their field profiles still allow for sufficient damping. Very few high (R/Q) passbands have small cell-to-cell coupling and sensitive field profiles. In this case, the mode suppression needs more attention and can be corrected for example by a dedicated adjustment of HOM couplers.

The tuning of the FM field profile has to be done very carefully. The number of plastic deformations must be kept as low as possible because they change mechanical properties of niobium. The difficulty is that in a multi-cell cavities cells stay electromagnetically coupled and modifying the frequency of one cell changes the field profile in all cells. A method based on frequency and field profile measurement for all modes of the FM passband was successfully implemented for the first time in the late 80’s [42, 43]. All sixteen HERA cavities and more than seventy TESLA cavities have been tuned up to now using that method and no mechanical changes have been observed.

In preparation for the test in a vertical cryostat, after tuning and chemical cleaning, the cavity is equipped with a coaxial input antenna providing very high $Q_{ext}$, close to an expected $Q_0$ of the cavity at the chosen test temperature. Usually the input antenna, attached to one of the beam tubes, allows for limited coupling adjustment during the test (variable antenna). In the case of a structure built for the beam operation the port for FPC and HOM coupler output ports are typically blanked off. Both, test cavities and those for beam operation are equipped with field (pickup) probe. In the vertical cryostat the whole cavity under test is immersed in liquid helium. One should note that in the vertical tests the 3dB resonance width is often a fraction of a Hz. To ensure field stability for the measurement, the RF-source has to be locked to the frequency of the tested cavity, which is modulated by mechanical resonances and helium pressure variation. This is different from the beam operation, when all cavities in an accelerator must have the same frequency and proper phase, and therefore all are locked to the master oscillator.

The main aim of the vertical tests is to determine the $Q_0$ value as a function of $|E_{acc}|$. When the input antenna and pickup probe cables are calibrated at the cryogenic temperature the next step is to measure their $Q_{ext}$. For this, one investigates transmitted and reflected signals that are a response of the tested cavity to a rectangular RF pulse. The amplitude $A(t)$ of the reflected power is shown in Fig. 3.9. Both solid curves show real reflected signals for an over-critical coupling (red curve) and an under-
critical coupling (green curve). Both blue lines show signals displayed by a spectrum analyzer or a scope when the negative sign is dropped. The dotted-line shows the displayed signal for the under-critical coupling and the dash-line for the over-critical coupling. When no non-linear phenomena take place (like multipacting, quench, field emission…) the measured signals agree very well with the theoretical model. The functions $f_1(t)$ and $f_2(t)$ in Fig. 3.9 can be found by means of the Laplace transformation for the previously discussed replacement circuit without the beam. They are defined by the following equations:

$$f_1(t) = \frac{1 - \beta_L}{1 + \beta_L} - \frac{2 \beta_L}{1 + \beta_L} e^{-\frac{\omega_0 t}{2 Q_L} S(t)} \quad \text{for} \quad t < \tau_p$$

$$f_2(t) = f_1(t) + f_1(t - \tau_p) S(t - \tau_p) \quad \text{for} \quad t < \tau_p$$

where $S(t)$ is the unit step function, $\tau_p$ is the pulse duration, $Q_L$ and $\beta_L$ are loaded quality factor and coupling factor of the input antenna respectively. From the theoretical model, one can compute $\beta_L$ knowing reflected signal values at time: $t = 0$, $t = \tau_p$, and $t = \tau_p +$. Three formulae can be used to define $\beta_L$:

$$\beta_L = \frac{A(0) - A(\tau_p-)}{A(0) + A(\tau_p-)}$$

$$\beta_L = \frac{A(\tau_p+)}{2A(0) - A(\tau_p+)}$$

$$\beta_L = \frac{A(\tau_p+)}{2A(\tau_p-) + A(\tau_p+)}$$

Practically, all three values are computed to find the mean value of $\beta_L$ with the smallest error.

Figure 3.9: Time depended response to a rectangular RF pulse. Theoretical function $f_1(t)$ describes the amplitude of the reflected power for the pulse duration, $f_2(t)$ after it has been switched off.
\( Q_L \) is measured by means of the decay of the field probe signal (transmitted signal, \( P_{\text{tran}} \)) right after the RF pulse is switched off. The cavity stored energy decays exponentially due to the power dissipation and radiation via all active ports:

\[
W(t) = W(\tau_{\text{p}+})e^{-\frac{\Omega_0(t-\tau_{\text{p}+})}{Q_L}}
\]

(3.33)

The transmitted power, which is proportional to \( W(t) \), when measured at a chosen time interval allows for the determination of \( Q_L \). In addition to \( Q_L \), \( \beta_L \) and \( P_{\text{tran}} \), the input power \( P_{\text{in}} \) needs to be measured to finally determine the intrinsic quality factor \( Q_0 \):

\[
Q_0 = Q_L(1 + \frac{P_{\text{tran}}}{P_{\text{in}} - P_{\text{tran}}})(1 \pm \beta_L)
\]

(3.34)

3.2.8. Test examples; single-cell cavities

The single-cell 1.3 GHz cavities of the LL and RE shape fabricated and tested at KEK in Japan reached the highest accelerating gradient in niobium cavities. The measured curves \( Q_0 \) as a function of \( |E_{\text{acc}}| \) at 1.68 K and 2 K for the LL-shape are shown in Fig. 3.10. The cavity reached 46.7 MV/m.

Figure 3.10: LL single cell cavity as tested at KEK. Test was performed at 2 K and 1.68 K. Courtesy K. Saito.

The highest accelerating gradient of 50.9 MV/m was achieved for the single-cell RE shape cavity. The test was done at 2 K and the intrinsic \( Q_0 \) at that gradient was \( 7 \cdot 10^9 \). One should note that in that test the peak magnetic flux on the wall was very close to its critical value for niobium and the peak electric field reached 112 MV/m.

The original TESLA single-cell cavities demonstrated gradients on the order of 40 MV/m, with peak electric fields up to 82 MV/m and peak magnetic fluxes up to 185 mT. Figure 3.11 displays the

---

26 The error analysis for the measurements \( Q_0 \) as function of \( |E_{\text{acc}}| \) is given in Appendix III
test result obtained recently at DESY for the single-cell TESLA type cavity made of the poly-crystal niobium. This test was performed at 2 K.

![Figure 3.11: Test result of the single-cell TESLA shape cavity made of the poly-crystal niobium.](image)

All single-cell cavities discussed in this subsection were electropolished (EP). The procedure was first applied in early 70’s by the Siemens Company in collaboration with the Kernforschungszentrum Karlsruhe group (P. Kneisel and co-workers). In the mid 80’s the method was applied to the TRISTAN cavities (K. Saito) and since 1998 to the single-cell and multi-cell cavities at all laboratories working on high gradients in elliptical structures (TJNAF, KEK, DESY, CEA-Saclay). The advantage of the EP treatment in the case of the poly-crystal niobium is a smoothing of the surface roughness to around 1 µm. This allows for better removal of field emitting residual particulates by the high pressure water rinsing (HPR) procedure which follows the chemical treatment.

3.2.9. Test examples; 9- cell cavities

The excellent single-cell results do not simply reproduce for the multi-cell structures even though they have very similar shape. The difficulties come from the application of the preparation techniques: chemical treatment and HPR to much larger and convoluted surface. In addition, a large surface of a multi-cell structure has a much higher probability of containing defects and contaminations leading to quenching at high gradients.

The Buffered Chemical Polishing (BCP) was the chemical treatment applied to the first, second and third production of the TESLA cavities, many of which still operate in the Tesla Test Facility linac at DESY. The test results for the best BCP treated cavities from the third production are shown in Fig. 3.12. Their performance met the very first (1997) specification of the TESLA-500 GeV collider, which was 25 MV/m and $5 \times 10^9$ for the accelerating gradient and intrinsic quality factor respectively.
The test results for the best performing four poly-crystal TESLA cavities at DESY, which have been cleaned with the EP method are displayed in Fig. 3.13. As expected, these cavities reached higher gradients and exhibited less radiation, a sign of field emission, as compared to the BCP treated cavities, for which $Q_0$ drops due to the heavy electron emission above 25 MV/m. Still one should note that the BCP process is much simpler and less expensive than the EP treatment. This has a significant impact on the capital cost of large accelerators like ILC and the European XFEL linac.

3.2.10. Test examples; large grain single-cell cavities

The breakthrough in the cost saving is expected to come from the very recently proposed (TJNAF group) implementation of the large grain niobium (LG-Nb). Slices for deep drawing were cut directly from the ingot to build several single-cell cavities at higher frequency. The surface of LG-Nb (and single crystal Nb, SC-Nb) samples showed very small roughness, below 30 nm, after the BCP
treatment. The very smooth surface together with less grain boundaries could potentially lead to a reduction of the residual resistance. Investigations of a SC-Nb 1.3 GHz TESLA cavity fabricated at DESY are continuing at TJNAF to find minimum thickness of the total removed layer (minimum BCP) which provides high gradient and high quality factor. The preliminary, but very promising, result obtained at TJNAF is shown in Fig. 3.14. The single cell SC-Nb TESLA cavity was tested at 2K after only a 80 µm layer was removed by the BCP treatment [44]. Both the gradient of 35.7 MV/m and $Q_0=10^{10}$ fulfill new spec of the ILC accelerator. This result compares favorably to the average removal of 280 µm needed to reach such a gradient and $Q_0$ in the TESLA cavities with the standard EP treatment.

![Figure 3.14: Test result for the single-cell LG-Nb TESLA cavity after the 80 µm layer was removed by BCP. Courtesy P. Kneisel.](image)

Figure 3.14: Test result for the single-cell LG-Nb TESLA cavity after the 80 µm layer was removed by BCP. Courtesy P. Kneisel.
3.3. Suppression of HOMs

Suppression of Higher Order Modes plays an important role in the operation of accelerators based on the superconducting technology [45]. Very high values of the intrinsic quality factor due to the superconducting state of the cavity wall, which is the advantage over normal conducting cavities in case of the accelerating mode, makes beam impedances of HOMs also very high. This may lead to a strong beam–cavity interaction causing growth of the emittance, bunch-to-bunch energy spread and/or additional cryogenic load.

The synchronic or near-synchronic multi-bunch excitation of a parasitic mode is well characterized in the frequency domain by the mode beam impedance. The impedance $Z_k$ of a parasitic mode $k$ is:

$$Z_k = \frac{(R/Q)_k \cdot Q_{L,k}}{1 + iQ_{L,k} \cdot (f_n - f_k)/f_k}$$

It depends on the mode’s characteristic impedance $(R/Q)_k$ as defined by expression (3.1), its loaded quality factor $Q_{L,k} = (1/Q_{o,k} + 1/Q_{ext,k})^{-1}$ and the relative detuning of the mode resonant frequency $f_k$ with respect to the beam spectral lines $f_n$. $Q_{o,k}$ and $Q_{ext,k}$ are the intrinsic and external quality factor of the mode respectively. Fig. 3.15 shows an example of the highest impedance parasitic monopole mode $(R/Q)_{18} = 200 \Omega$ of the TESLA structure and how it overlaps with the spectral line No 768 of the nominal ILC beam. This spectral line contributes mostly to the total power induced by the beam.

$Z_k$ must be kept low to mitigate the above mentioned phenomena. While $(R/Q)_k$ values depend only on the geometry of an accelerating structure, low $Q_{ext,k}$ and thus low $Q_{L,k}$ result from the HOM energy dissipation in external devices. The beam induced energy in an accelerating structure is coupled out by HOM couplers or radiates via beam tubes towards the beam line absorbers. The dissipation causes...
an exponential decay of the HOM stored energy $W_k(t)$:

$$W'_k(t) = W'_k(0) \cdot e^{-\frac{\omega_k \cdot t}{\tau_k}}$$

(3.36)

where $\tau_k$ is the decay time of a mode. When $\tau_k$ is several times shorter than the time $t_b$ between bunches, every bunch passes through the HOM free cavity (single pass excitation). However, when $\tau$ is longer than $t_b$ the multi-bunch excitation of a HOM takes place and every bunch passing the cavity interacts with HOMs excited by all preceding bunches.

In the single pass excitation the beam loses the smallest amount of energy to a HOM. This is the lowest energy deposited by a point-like charge $q$:

$$\Delta W_k = \frac{\omega_k}{4} \cdot \frac{(R / Q)_k}{q^2}$$

(3.37)

The power loss by the beam to a higher order mode in the multi-bunch excitation is:

$$P_k = Re[Z_k] \cdot I_b^2$$

(3.38)

where $I_b$ is the beam current.

3.3.1. HOM trapping inside cavity

In multi-cell superconducting accelerating structures HOM couplers must be placed at the beam tubes. Experience shows that HOM couplers attached to the cells limit the performance of a cavity to a low accelerating gradient. Unfortunately, some HOMs have very little stored energy in the end-cells thus making the suppression of these modes difficult. The phenomenon is commonly called: trapping inside the cavity and it is caused by the difference in mode frequency between the end- and the inner cells. It is similar to the field-flatness problem as discussed in the Section 3.2.4, however here the amplitude sensitivity dependence on the frequency perturbation does not need to follow equation 3.27 for the $\pi$-mode. A computed example is shown in Fig. 1.16. In a 13-cell TESLA-like structure the 2.4 GHz monopole mode has very little stored energy in the end cells. The difference in frequency between end- and inner cells for this field pattern is 30 MHz. Since the cell-to-cell coupling is 3% only for this passband the end cells cannot resonate “together” with the inner cells [46].

![Contour of the electric field amplitude](Image)

Figure 3.16: Contour of the electric field amplitude; Example of mode trapping in a 13-cell cavity. End-cells and inner-cells have different frequencies for this resonant pattern.
3.3.2. Trapping inside cryomodule

A similar phenomenon happens in a multi-structure cryomodule, a vacuum vessel housing several accelerating structures. The frequency difference (due to fabrication errors) between neighboring structures causes trapping of some propagating modes. Fig. 3.18 displays a computed example for the 8.8784 GHz monopole mode in a part of the TTF like eight-structure cryomodule, schematically shown in Fig. 3.17.

Figure 3.17: Schematic cross-section of the TTF-like cryomodule.

The beam in the middle cavity excites the mode. It does not propagate towards a beam line absorbers located between cryomodules because in the outer cavities the frequency of this mode higher by $\Delta f = 32 \text{ MHz}$, which is the statistic value (see section 3.3.6) measured for the first three cavity productions for the TTF linac [47]. The intrinsic quality factor of this mode, even though the wall surface resistance scales with frequency as $f^2$ (see equation 3.19), is expected to be $\sim 5 \cdot 10^9$ hence it has very high geometric factor of the order of 2000. Marginal energy dissipation in the non superconducting walls of bellows and flanges in both cavity interconnections reduce the quality factor to $\sim 2 \cdot 10^7$. With this $Q$ value the synchronic excitation by the ILC beam will cause an additional average cryogenic load of 8 W at 2K for the middle structure, which is twice as much as the dynamic cryogenic load of a complete normally operating cryomodule at 31.5 MV/m. The additional voltage interacting with the beam will modulate the energy of bunches in the range of $\pm 0.36 \text{ MeV}$.

Figure 3.18: Contour of the electric field amplitude; Computed example of trapping in cryomodule. The high frequency monopole mode having $2\Omega$, a non negligible value of $(R/Q)$ can not be suppressed due to the large frequency difference between the three cavities.

The general opinion that larger HOM frequency spread leads to less coherence and therefore is better for the emittance preservation along the main accelerator may turn out to be wrong. The additional bunch-to-bunch energy spread and cryogenic load can limit performance of the accelerator. The trapping phenomenon does not take place when tighter mechanical tolerances reduce $\Delta f$ to half of its
current statistic value. The ideal case when all three cavities have the same mode frequency is shown in Fig. 3.19.

\[ f = 8.8784 \, \text{GHz} \]

Figure 3.19: Contour of the electric field amplitude. An improvement in the presented above example when the frequency spread is zero

3.3.3. Measures against trapping

3.3.3.1. Number and matching of cells

Fewer cells in a structure reduce trapping probability. The mode in the first example (Fig.3.16) demonstrates less trapping (stronger fields in the end-cells) when the number of cells is reduced to nine or five (see Fig. 3.20). This obvious advantage of short cavities needs to be weighted against their main disadvantage, namely fast cost increase of an accelerator based on shorter structures.

Figure 3.20: Contour of the electric field amplitude. Shorter structures make trapping less probable.

The second way to minimize trapping probability is by means of matching the end- and inner-cells and enlarging the irises to enhance cell-to-cell coupling for HOMs. The 5-cell 704 MHz structure for the electron cooling at RHIC\(^{27}\) (Fig. 3.21) fulfills all mentioned features to provide excellent mode damping and avoids trapping [48]. Computer simulations showed that this structure can accelerate, for the proposed RHIC cooling scheme and optics, beams up to 2A (forty times more than the specification). The HOM energy radiates out of the cavity and is dissipated in beam line absorbers. There is no need to attach HOM couplers. We should note that the shape of end-cells is very similar to the inner-cells. The niobium prototype of the structure was built by Advance Energy Systems in 2006 and will be tested soon at Brookhaven National Laboratory. This kind of design is suitable for short accelerators operating at moderate gradients since enlarging of irises leads to strong reduction of the intrinsic impedance for the fundamental mode and to an enhanced cryogenic load.

\(^{27}\) Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, NY, USA

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To avoid partial HOM trapping in longer multi-cell structures, designed for high gradient operation, one can use two different shaped end-cells (asymmetric cavity). The method has been applied to TESLA structure as a remedy against trapping of the highest impedance mode in the third dipole passband (TE121-like), which has been found to be trapped in all 4-cell LEP structures. The modification of end-cell geometry in the TESLA structure resulted in increased stored energy in the dipole mode, improving its damping, but led to less suppression of the highest impedance monopole mode (TM011). The two applied end-cells in the TESLA structure compromise damping of both passbands energy and have the same amplitudes for the accelerating mode passband (TM010) and first two dipole passbands (TE111, TM110). The inner half-cells (cups) of both end-cells are identical with the cups of the inner cells (Fig. 3.22a). All geometric parameters (see also Fig. 3.7) of all three TESLA cups are listed in Table 3.7. The normalized amplitudes for all mentioned passbands are shown schematically in Fig. 3.22b.

Figure 3.22: Asymmetry in the TESLA structure (a) and the normalized amplitudes of the accelerating mode passband and HOM passbands.
Table 3.7: Geometric parameters of three TESLA cups, all in [mm]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inner-cup</th>
<th>End-cup 1</th>
<th>End-cup 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equator ellipse r-half axis: $H_r$</td>
<td>42</td>
<td>42</td>
<td>40.34</td>
</tr>
<tr>
<td>Equator ellipse z-half axis: $H_z$</td>
<td>42</td>
<td>42</td>
<td>40.34</td>
</tr>
<tr>
<td>Equator ellipse center point r-coordinate: $R_o$</td>
<td>61.305</td>
<td>61.305</td>
<td>62.965</td>
</tr>
<tr>
<td>Equator ellipse center point z-coordinate: $Z_o$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Iris ellipse r-half axis: $b_r$</td>
<td>19</td>
<td>12.8</td>
<td>13.5</td>
</tr>
<tr>
<td>Iris ellipse z-half axis: $b_z$</td>
<td>12</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Iris ellipse center point r-coordinate: $r_o$</td>
<td>54</td>
<td>51.8</td>
<td>52.5</td>
</tr>
<tr>
<td>Iris ellipse center point z-coordinate: $z_o$</td>
<td>57.692</td>
<td>56.85</td>
<td>55.8</td>
</tr>
<tr>
<td>Length of the cup</td>
<td>57.692</td>
<td>56.85</td>
<td>55.8</td>
</tr>
<tr>
<td>Iris radius: $r_i$</td>
<td>35</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>Equator radius: $R_e$</td>
<td>103.305</td>
<td>103.305</td>
<td>103.305</td>
</tr>
</tbody>
</table>

3.3.3.2. Weakly coupled structures

The other way of preventing the HOM trapping in a long high gradient structure, which keeps its cost advantage (one fundamental coupler supplying many cells), is splitting it into short subunits connected by $\lambda/2$-long tube(s). The length of the interconnections ensures synchronism between the beam and the accelerating mode enabling energy flow between weakly coupled subunits and offers space for the attachment of HOM couplers. We will discuss this layout in the next chapter in more details. Its tested version of 2x7-cells showed a very good HOM damping without mode trapping. The computed example (Fig. 3.23) demonstrates how splitting of a 14-cell TESLA like structure in two 7-cell subunits helps avoiding the trapping of modes. The displayed TM011-like mode, trapped in the fourteen-cell structure, can be easily damped by HOM couplers attached to the tube connecting two weakly coupled subunits.

![Space to attach HOM couplers](image)

Figure 3.23: Contour of the magnetic field of the TM011-like monopole mode trapped in the 14-cell structure. The mode can be damped by HOM couplers located at the interconnecting tube of the 2x7-cell structure. On the color scale red ad blue indicate high and low value respectively.
3.3.4 HOM couplers

3.3.4.1 Coaxial line couplers

HOM couplers based on the coaxial line technique were proposed in 1985 and applied for the first time few years later to 500 MHz 4-cell HERA cavities and 352 MHz 5-cell LEP cavities. All forty-eight HERA couplers were in operation till June 2007 [49]. Their construction and location (couplers are immersed in the helium bath) allowed for the continuous wave (cw) operation at moderate gradients. The HERA couplers provide excellent suppression of dangerous modes to $Q_{ext} < 1000$ ensuring stable operation with electron and positron beams up to 45 mA.

The 1.3 GHz TESLA HOM couplers (Fig. 3.24) are similar to the HERA HOM couplers. Their RF-design has been simplified due to rather moderate damping specification, $Q_{ext} \leq 2 \cdot 10^5$, for the dipole modes with high characteristic impedances [50]. The couplers are located outside the helium vessel to minimize cost of the production and assembly. This positioning is possible because of a negligible small heating in the coupler due to the low, less than 1%, duty factor of the TESLA (ILC) collider and driving linac of the European XFEL facility. The RF performance of the coupler and role of its parts can be better seen with help of the replacement circuit shown in Fig. 3.25.

![Fig. 3.24: Cross-section of the TESLA HOM coupler and picture of its can and inner part, both made of bulk niobium.](image)

![Fig. 3.25: Replacement circuit of the TESLA HOM coupler and its transfer function.](image)

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28 The spec has been relaxed in comparison to the values in the TESLA TDR.
The stray capacitor and short piece of coaxial line penetrating the beam tube represent the electric antenna tip. The two welded stubs connecting the inner conductor and the can form inductors substantially suppressing the out-coupled power of the fundamental mode. The RLC series circuit represents the output line with its capacitive coupling to the inner conductor, residual inductance and 50 Ω termination. The end coaxial line and end capacitor form the additional FM filter, positioning the minimum of the electric standing wave at the location of the output antenna. The design ensures that HOM couplers, when properly tuned, have $Q_{\text{ext}}$ for the FM higher than $5 \cdot 10^{11}$ (see notch in the transfer function) simultaneously providing six orders of magnitude lower values of $Q_{\text{ext}}$ for the parasitic modes, even for those having frequency only 25% higher that the accelerating mode.

Both HERA and TESLA couplers can effectively suppress HOM modes, which are below cut-off frequency of the beam tubes and thus having the e-m fields decaying exponentially but stable at either cavity end. The frequency range of these modes depends on their field pattern and size of the beam tubes, but usually it does not exceed two times the accelerating mode frequency. Fortunately, all modes with high beam impedance are included in this frequency range.

3.3.4.2. Waveguide couplers

The HOM couplers based on the waveguide were first designed at Cornell University in 1982 for the 1.5 GHz cavities (Fig. 3.26). 160 of these cavities operate very successfully in the CEBAF accelerator. Low nominal beam current of CEBAF and very small amount of the HOM power allow termination of the HOM couplers with waveguide loads inside cryomodules. The experience with the CEBAF waveguide couplers showed that they do not have the discussed heating problems when they operate cw but their mechanical dimensions strongly influenced the size of cryomodules. In the case of higher beam current, the terminating loads must be located outside the cryomodule. This would be mechanically rather complicated and would cause additional heat leakage and shielding problems.

G. Wu [51] and R. Rimmer [52] proposed new applications of waveguide HOM couplers for structures operating with high current at JLab. The first structure, shown in Fig. 3.27, can accelerate beams up to 100 mA, the second one shown in Fig. 3.28 is meant for 1 A class accelerators.

Figure 3.26: Pair of 5-cell 1.5 GHz CEBAF cavities with HOM waveguide couplers.
3.3.5 Beam-line absorbers

3.3.5.1 Room temperature absorbers

Single cell superconducting structures in B-factories, synchrotrons or light sources operate with high currents and therefore need strong HOM suppression. The common idea is to make all HOMs propagating into the beam tubes and dissipate their whole energy at room temperature outside the cryomodule. While operating gradients are rather low for these cavities, a low intrinsic impedance of their fundamental mode, a consequence of the enlarged irises, does not cause large cryogenic load.

Four 500 MHz cavity designs: KEK-B in Tsukuba, CESR at Cornell [53], Taiwan Light Source and BEP-II in Beijing utilize ferrite beam-line absorbers to dissipate several kilowatts of the HOM power. The CESR cavity absorber is shown in Fig. 3.29. Similar absorber will be used for the RHIC electron cooling 5-cell cavity.

Figure 3.29: Ferrite beam line absorber used for CESR cavities.
3.3.5.2. Low temperature absorbers

The ERL at Cornell (project in the study stage) will operate with nominal beam current up to 100 mA. The stable operation of the linac requires suppression of about 130 W of the HOM power per 7-cell cavity. For this, beam line absorbers will be installed between cavities inside the cryomodule. The type of material (ferrite or ceramic) to be used for the absorption is still under investigation. The absorbers will be thermally connected to the liquid nitrogen line keeping their working temperature near 80 K for the nominal current operation. Two bellows will match the length of the absorber and isolate it thermally from the 2 K environment. Fig. 3.30 shows the present design of the absorber [54].

A high frequency part of the HOM spectrum of the European XFEL facility will be dissipated in the beam-line absorbers installed between 8-cavity cryomodules [55]. The propagating HOM power will be 5.4 W/cryomodule for the operation with 40000 bunches/s and the nominal bunch charge of 1 nC. CERADYNE ceramic rings will be used for the absorption (Fig. 3.31). Dissipated power will be transferred to the liquid nitrogen line by the copper stub brazed directly to the ceramic. The stub holds the ring in the stainless steel vacuum chamber. The heat capacity of the absorber is more than 100 W. This extra margin is introduced for the future upgrade of the facility. Manufacturing of the prototype is almost finished and it will be tested in the TTF linac in 2007.

Figure 3.30: Ferrite beam line absorber as proposed at present for the ERL at Cornell; Absorbing insert (Left), cross-section of the absorber (right).

Figure 3.31: Beam line absorber for the European XFEL facility.
3.3.6. **HOM frequency and damping statistics for TESLA structures**

Two HOM couplers attached to each TESLA 9-cell structure fulfill the TESLA (ILC) and European XFEL damping specification for the three lowest passbands TE111, TM110 and TM011 with very few exceptions. The damping statistics for 50 TESLA structures fabricated for the TTF linac and measured at 2K is shown in Figures 3.32-3.34. The diagrams for non-monopole modes show statistics for both polarizations\(^{29}\). The TE111 dipole passband (\(f\) range 1620-1790 MHz) is well damped. Its standard frequency deviation \(\sigma_f \approx 5\) MHz is almost the same for all 18 modes, which

---

\[29\] All non-monopole modes (dipoles, quadrupole…) have two polarizations due to perturbation in cylindrical symmetry in real cavities. Both polarizations of a mode differ slightly in frequency and have the same field pattern shifted angularly by 90°.
indicates that all modes of this passband are equally sensitive to the fabrication errors. The situation is different for the next dipole passband TM110 where the frequencies are from 1790 MHz to 1890 MHz. Here $\sigma_f$ varies from 1 MHz to 6 MHz and modes with lower frequency are more sensitive to the machining errors. The damping of highest impedance modes fulfills the specification.

![Damping statistics for TM110 dipole passband (900 modes) and standard deviation for mode frequencies. Red bars indicate computed (R/Q) values for individual modes.](image)

The damping specification for the lowest HOM monopole passband TM011, with a frequency range from 2370 MHz to 2450 MHz, is dictated by the protection of the coupler output line and its power capability. When the $Q_{ext}$ of two highest impedance modes stays below $2 \cdot 10^5$, the nominal TESLA or ILC beam will deposit about 15 W/coupler, which is well within the output line power capability. Similarly, to TE111, this passband is equally sensitive to the fabrication errors demonstrating
of very close to 9 MHz for all modes. The damping specification has not been achieved for five cavities from the very early production in which some repaired cells were welded twice at the equator. The relative frequency spread for this passband can be as high as \( \pm 0.375\% \). This number justifies our assumption on the cavity-to-cavity frequency spread in the mode trapping discussion in Section 3.3.2.

![Graph showing damping statistics for TM010 monopole passband (450 modes) and standard deviation for mode frequencies. Red bars indicate computed (R/Q) values for individual modes.](image)

**Figure 3.34**: Damping statistics for TM010 monopole passband (450 modes) and standard deviation for mode frequencies. Red bars indicate computed (R/Q) values for individual modes.

The damping of the third dipole passband TE121 required correction in the loop orientation of the HOM coupler that was located at the end without the input coupler. The damping statistics for 60 modes, prior to the correction, is shown in Fig. 3.35. For all modes the higher frequency polarization is much better suppressed than the lower one, which in several cases exceeds the specification limit. The measurements were made in the horizontal test cryostat, which can house only a single 9-cell
cavity. The highest impedance (frequency) mode, which propagates in the beam tubes, could not be measured properly due to presence of the vacuum flanges at both cavity ends. The beam test in the TTF linac performed in 1999 and 2000 also showed that for this mode the damping of the higher polarization did not fulfill the specification. The new coupler arrangement was tested on copper models and will be tested on superconducting cavities with beam at the TTF linac by the end of 2007.

As mentioned already this passband was trapped ($Q_{ext} > 10^7$) in the LEP structures which only had four cells. The discussed asymmetry in the end-cells of the 9-cell TESLA structure led to considerable improvement in the damping. The asymmetry combined with rotation of one coupler will allow $Q_{ext}$ to be kept below $10^5$.

![Graph](image1)

![Graph](image2)

Figure 3.35: Damping statistics for TE131 dipole passband (60 modes) and standard deviation for mode frequencies. Red bars indicate computed (R/Q) values for individual modes.
Determination of actual HOMs parameters: frequencies, quality factors, beam impedances and polarizations, requires dedicated measurement methods, especially when modes overlap, couple weakly to the HOM couplers or have sensitive field pattern to the cells frequency errors (due to the small cell-to-cell coupling). We will discuss some of these methods in Chapter IV presenting the superstructure beam experiment.

3.4. Transient state in multi-cell structures

An energy flow along multi-cell standing wave accelerating structures operating in the π-mode was investigated in the early 90’s in the frame of the TESLA collaboration. The question was the stability of fields in individual cells during the acceleration process and its sensitivity to the cell-to-cell coupling $k_{cc}$. One of the approaches to answer that question, proposed in 1993, was based on the Laplace transformation method applied to the lumped element replacement circuit representing a multi-cell structure (Fig. 3.36). The method and its experimental benchmarking are discussed widely in [56].

![Figure 3.36: Replacement circuit for an N-cell standing wave structure with the capacitive coupling.](image)

The main conclusion drawn in this publication was that the energy flow in the 9-cell TESLA cavity, where cell-to-cell coupling $k_{cc}=1.98\%$, is sufficient to keep the mean value of the accelerating field in the structure stable. Further result was that beating of all passband modes still takes place at the beam arrival time, $\sim 500 \mu s$ after the RF-pulse is switched on. The beating depends on the RF-pulse rise time $\tau_{RF}$ and can be suppressed when $\tau_{RF}$ is larger than 10 $\mu s$. The main contribution to the beating comes from the closest neighboring mode (8π/9-mode), whose frequency is 750 kHz lower than $f_{FM}$.

Later this method was used for other types of standing wave structures with various numbers of cells and coupling strength. Figure 3.37 shows an example of cell voltages in a 7-cell cavity, having the same $k_{cc}$ as the TESLA cavity, for two intervals of time $t = <0, 100 \text{ ns}>$ and $t = <0, 10 \mu s>$. After the initial delay (up to 35 ns in cell No. 7), all signals rise simultaneously on the larger time scale, which
means that the energy flow compensates for the dissipation of stored energy. In that particular case, the assumed dissipation ($R_k$ values) in the replacement circuit was equal to the power, which would be transferred by this cavity to the TESLA nominal beam at 25 MV/m (matched loaded quality factor of the accelerating mode $Q_{L, FM}=3.4 \cdot 10^6$).

![Figure 3.37: Voltages $V_k(t)$ in cells ($k=1 \ldots 7$) of the 7-cell structure for two time intervals.](image1)

Four of these structures (subunits) were used in the transient state and energy flow modeling for the first superstructure, a chain of weakly coupled multi-cell subunits, which is discussed in the next chapter. The subunit-to-subunit coupling was only 0.036%. The result shown in Fig. 3.38, illustrates the case when $Q_{L, FM}$ is matched for the nominal TESLA operation. Although the coupling is very small and causes significant delay at the beginning of the charging process (left diagram), the energy flow is sufficient to keep the same slope of all four signals in the time of acceleration (right diagram), which means that the mean stored energy is the same in all subunits. The bold red line depicts the average value of four signals. Its relative modulation is smaller than $1 \cdot 10^{-4}$. This result, obtained in early 1996, was the very first indication that weakly coupled structures could be used for acceleration. This was later confirmed with more advanced modeling methods and beam experiment at TTF linac in 2002.

![Figure 3.38: Voltages $V_n(t)$ in four ($n=1 \ldots 4$) weakly coupled 7-cell subunits; (left) signals instantaneous after the RF-source is switched on, (right) signals in the acceleration time interval.](image2)
The bunch-to-bunch energy stability requires special attention when superconducting cavities operate in the pulse mode (e.g. TESLA, ILC and XFEL). The method presented above does not include the direct interaction with bunched beams. In addition, its accuracy is limited for large number of cells with weak coupling. Even for a TESLA-like cavity, the computation had to be performed with extended precision complex numbers. The energy stability problem was addressed in a more advanced way in [57, 61]. In both publications, authors apply the modal analysis to the electric field in the structure. Frequencies, RF-parameters and field patterns for all modes from the accelerating passband need to be computed by means of numerical codes solving the Helmholtz equation in the frequency domain. This may need some numerical effort too, particularly for a long chain of cavities with weak coupling, as was mentioned in [57] where the authors use the finite difference solver on a rectangular mesh. The modes couple to a driving RF-source and the accelerated beam, both represented by currents interacting with their electric field on axis. The numerical code HOMDYN presented in [61] together with the finite element method (FEM) solver employing the third order approximation [62] were used for studies on superstructures and electron superconducting sources discussed in the following chapters. Figure 3.39 shows the bunch-to-bunch energy modulation $\delta E/E$ computed with these two codes for the first nominal TESLA beam $^{30}$ of 1130 bunches (3.6 nC each) accelerated at 25 MV/m by the TESLA structure [1]. The modulation is smaller than $\pm 5 \cdot 10^{-4}$ and thus fulfills the specification for the collider. Electric fields on axis of nine modes from the fundamental passband used in this computation are shown in Figure 3.40. For comparison, the field amplitudes in cells are displayed for the same stored energy $W = 80$ J. Table 3.8 contains RF-parameters and loaded quality factors $Q_L$ resulting from the matching condition for the accelerating mode ($\pi$-mode). The dispersion diagram, mode frequency vs. cell-to-cell phase advance $\beta_{cc}$, for all modes is shown in Fig. 3.41.

$^{30}$ The nominal parameters of the TESLA collider were changed to those shown in Table 2.4 in 1999.
Figure 3.40: $E_z(0,z)$ on axis for all nine modes at 80 J stored energy. This energy is stored in the accelerating $\pi$-mode when the TESLA structure operates at 25 MV/m.
Figure 3.41: Dispersion curve, $f$ vs. $\beta_{cc}$, for the 9-cell TESLA structure, computed with the FEM code and formula (3.40).

Table 3.8: Parameters of the fundamental passband modes

<table>
<thead>
<tr>
<th>No</th>
<th>$\beta_{cc}$</th>
<th>$f$ [MHz]</th>
<th>$(R/Q)$</th>
<th>$Q_L$ [$10^6$]</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi/9$</td>
<td>1276.416</td>
<td>0.00</td>
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<tr>
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<td>6.0</td>
</tr>
<tr>
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<td>$4\pi/9$</td>
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<td>0.00</td>
<td>3.6</td>
</tr>
<tr>
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<td>0.02</td>
<td>2.6</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.03</td>
<td>1.7</td>
</tr>
<tr>
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<td>0.16</td>
<td>1.5</td>
</tr>
<tr>
<td>9</td>
<td>$\pi$</td>
<td>1300.000</td>
<td>1011.51</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The amplitudes $A_{kj}$ in cells and frequencies of modes $f_j$ are well approximated by the following formulas:

$$A_{kj} = X_j \cdot \sin(\beta_{cc,j} (k - 0.5))$$  \hspace{1cm} (3.39)

$$f_j = f_\pi \cdot \sqrt{1 - \frac{k_{cc}}{1-k_{cc}} (1+\cos \beta_{cc,j})}$$  \hspace{1cm} (3.40)

where indices $k$ and $j$ denote the cell- and mode number respectively, $f_\pi$ is the resonant frequency of an inner cell with the magnetic short at both irises and $X_j$ is a factor proportional to $(W)^{-0.5}$. The frequency difference between the $\pi$-mode and $\pi/9$-mode gives a good estimation for $k_{cc}$. One should note that $\partial f / \partial \beta$ at $\beta_{cc} = \pi$ is 0, which means that the group velocity for $\pi$-mode is 0 too.
IV SUPERCONDUCTING SUPERSTRUCTURES
4.1. General remarks and motivation

A superstructure (SST), chain of superconducting multi-cell cavities (subunits) connected by $\lambda/2$ long tube(s) and supplied with RF-power via only one FPC coupler, has been proposed as an alternative layout for the TESLA main accelerator [58, 59]. The axial length of the interconnection equals the length of a cell, which ensures synchronism between the accelerating field and beam. There are two main advantages of the layout in comparison to the standard one, based on 9-cell cavities. The first, an economic advantage, is that structures made of more cells will reduce the number of FPCs in the linac. Even though many intensive R&D programs have been conducted worldwide, the cost of a FPC continues to be very high due to the complex fabrication process, independent of its working frequency. The goal is to reduce the present cost by more than factor of three, to $12000$/piece. Consequently with less FPCs, the number of all auxiliaries needed to distribute the RF power, such as waveguides, bends, circulators, 3-stub transformers, loads, etc., can be reduced too. In addition, the layout reduces the number of electronics controlling phase and amplitude of cavities in the linac and simplifies the design of the cryomodules due to fewer openings for the FPCs. Figure 4.1 shows the RF-power distribution systems for the 108-cell TDR cryomodule based on the standard TESLA structures and on the newer type II of SST [12], which is made of two weakly coupled 9-cell structures.

![RF-power distribution system for the TESLA cryomodule housing twelve 9-cell cavities (upper drawing) and one housing six 18-cell cavities (lower drawing).](image)

31 The estimated total cost savings would be above 180 millions USD, assuming the mentioned FPC price [21].
32 The first proposed superstructure SST-I was built of four 7-cell units will be discussed later.
The second advantage is the increased filling of the linac tunnel with accelerating structures, because the distance between subunits is only $\lambda/2$, much shorter than for the standard layout. The space saving can be significant. In the case of versions of SSTs discussed later, it was $\sim 1.8$ km for TESLA and it is $\sim 1.4$ km for ILC. When the final energy of the collider is fixed one has a choice to keep the nominal gradient and to make the accelerator shorter or to keep its length and lower the nominal gradient. The best way to use this additional space is a trade-off between the investment cost and the performance of the collider.

### 4.2. Superstructure concept

Figure 4.2 illustrates difference between the standard layout and SST-II. Two standard accelerating structures are separated by the long interconnection (286 mm for ILC, 346 mm for XFEL), preventing coupling between their accelerating $\pi$-modes whose amplitudes and phases are

![Diagram of Standard Layout](image1)

![Diagram of SST-II Layout](image2)

*Figure 4.2: (Upper drawing) Two TESLA cavities with two FPCs separated by a long interconnection and their uncoupled but synchronized accelerating modes ($|E_{acc}|$ on axis); (Lower drawing) the superstructure SST-II with one FPC only, the short interconnection allowing for the FM energy flow and the coupled accelerating $\pi$-0 mode.*
adjusted by means of the 3-stub waveguide transformers installed in the input lines (Fig. 4.1). Each structure is equipped with a cold tuner adjusting its frequency by varying the length of a structure. The resolution of the cold tuners is typically of the order of few Hz. In the SST-II layout the short interconnection ($\lambda/2 = 115$ mm at 1.3 GHz) couples the structures together. The energy flowing via the interconnection re-fills cells of the structure without FPC and thus only one FPC is required for the proper beam operation. The stability of the accelerating field in the SST layout was one of the main objectives of the beam test that we will discuss later.

There are several differences, easily observed, between standard long multi-cell structures (e.g. 56-cell HEPL structure) and superstructures. In a superstructure, unlike a standard long multi-cell structure, the accelerating mode is the $\pi$-0 mode. It has $\pi$ cell-to-cell phase advance within a unit ($\pi$-mode as in a standard structure) and 0 subunit-to-subunit phase advance (all units are in phase). The second difference from a standard long multi-cell structure is the very weak coupling of subunits, $k_s$. It is smaller roughly by a factor of 50 than cell-to-cell coupling $k_c$. Although the $k_s$ coupling is very weak, we can control the stored energy (gradient) in each subunit by means of a cold tuner, taking advantage of its high frequency resolution. A further and very important difference is as follows. The $\pi$-0 mode is below cut-off in the interconnecting tube(s). This makes possible to attach the HOM coupler(s) at the interconnection(s), in the “middle” of the long multi-cell structure. By doing this, good damping of parasitic modes can be maintained as it is in the standard cavities with the same number of cells as the subunit.

The very first experiment on the transient state and wave propagation along the 4x9-cell superstructure [60], made of the existing Cu models of the TESLA structure, had been performed before the 7-cell subunits (Cu and Nb) for the first version (SST-I) was built in 1998. The experiment benchmarked HOMDYN [61] and FEM [62] for very weakly coupled resonators, therefore both codes have been used extensively to study the RF properties and bunch-to-bunch energy modulation modeling in all versions of the SST layout proposed later.

![Figure 4.3: Measured field profile on axis for the pre-prototype of the SST made of four Cu models of the TESLA structure](image)
Fig. 4.3 shows the measured field profile of the π-0 mode for the assembly after the cavities have been tuned and when the location of the input and output antennae prevented excitation of neighboring modes, which otherwise would strongly overlap with the fundamental mode. The computed and measured voltages in the cells as function of time, after the driving generator supplying power to cell No.1 was switched on, agreed very well. Two examples are shown in Fig. 4.4. Small differences in the transient and delay result from geometry perturbations of cells in a real cavity. One should note that the rise time, in the microseconds range, is due to low loaded quality factor (~15000) for copper models loaded with a matched antenna.

![Figure 4.4: Measured (red, bold line) and computed (black line) voltages vs. time in the superstructure pre-prototype made of the Cu TESLA structure models.](image)

4.3. RF properties of SST-I and SST-II

Cross-sections of two proposed superstructures are shown in Fig. 4.5. The first superstructure SST-I was meant to be made up of four 7-cell cavities. We have built a Cu model of this version and six Nb 7-cell subunits, of which the best performing four would be assembled in the superstructure. The assembly would be done by means of Nb gaskets, which unfortunately did not perform reliably in the superconducting state. Meanwhile, the second version SST-II has been studied and found to be more attractive for the collider. This version keeps the same fill factor of the tunnel as the first one.

![Figure 4.5: Cross-sections of SST-I and SST-II.](image)
SST-II is shorter and its production, cleaning and handling should be easier. In addition, in case the present R&D program at TJNAF on the superconducting connection is not successful, subunits of the SST-II can be welded together. One technical challenge for the SST-II will be in the final chemical treatment and high pressure rinsing of welded subunits, as the whole assembly is 2 m long. These final preparation procedures are not technically feasible for the SST-I when their four subunits are welded together. Savings in the investment cost are similar for both superstructures. The first version reduces the number of FPCs to 33% of the number needed for the standard layout. Since subunits are shorter (7 cells not 9 cells) it demands more liquid He vessels and cold tuners. The second version, SST-II, reduces the number of FPCs to 50% but it keeps the same number of He vessels and tuners as the standard layout. RF parameters of both superstructures are listed in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>SST-I</th>
<th>SST-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells in subunit</td>
<td>-</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Number of subunits</td>
<td>-</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$\pi$-0 mode frequency</td>
<td>[MHz]</td>
<td>1300</td>
<td>1300</td>
</tr>
<tr>
<td>$(R/Q)_{IM}$ per subunit</td>
<td>[Ω]</td>
<td>732</td>
<td>985</td>
</tr>
<tr>
<td>$\eta_{E}$</td>
<td>-</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\eta_{B}$</td>
<td>[mT/(MV/m)]</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>$l_{active}$</td>
<td>[m]</td>
<td>3.23</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Dispersion diagrams ($f$ vs. cell-to-cell phase advance) for two SSTs are shown in Fig. 4.6. SST-I has seven groups of modes (due to seven cells in each subunit) and four modes within each group (due to four subunits in the chain). SST-II has nine groups each with two modes; 0 when both subunits are in phase and $\pi$ when they oscillate in counter-phase. One should note that frequency differences

![Figure 4.6: Dispersion curves of both superstructures; (left) SST-I seven groups with four modes each, (right) SST-II nine groups with two modes each. Red-dotes mark accelerating $\pi$-0 modes.](image)
between modes 0 and π in both diagrams vary from group to group. This is due to the coupling between subunits ($k_s$) which depends on the stored energy in the end-cells and thus is different for different groups. Fig. 4.7 and 4.8 show $k_s$ and $|E_z|$ on axis for modes $\pi/7-0$ and $6\pi/7-0$ of the SST-I respectively. Low $k_s$ value for first three groups leads to uncoupled oscillations of individual subunits when fabrication shape errors are too big in real structures. Fortunately $(R/Q)$'s of these modes are close to zero and even diminutive suppression keeps their beam impedance negligible.

![Figure 4.7: Coupling $k_s$ for seven mode-groups of SST-I.](image)

![Figure 4.8: Field profile for modes $\pi/7-0$ and $6\pi/7-0$ in SST-I illustrating the difference in the coupling $k_s$ for $\pi/7$- and $6\pi/7$-group of modes.](image)
4.3.1. Bunch-to-bunch energy modulation

As was already mentioned, the normalized bunch-to-bunch energy modulation $|\delta E/E|$ within a train of 2820 nominal TESLA (ILC) bunches has been specified to be less than $5 \times 10^{-4}$ according to the physics experiments requirement. In the frame of the theoretical analysis, we performed the energy gain simulations for the new TESLA nominal beam accelerated by each of two proposed SSTs. For this modeling, subunits of each SST were identical (theoretical case). The result of simulations (Fig. 4.9) showed that both versions fulfill this requirement with a large safety margin (almost an order of magnitude). Fundamental passband modes of both superstructures and their RF-parameters can be found in Appendix-I.

```
Figure 4.9: Computed energy modulation normalized to the energy gain for the nominal TESLA (ILC) bunch train in two discussed superstructures.
```

4.3.2. Adjustment of the accelerating field

The FM field profile in subunits of a superstructure has to be balanced for proper operation in an accelerator. Before the final chemical treatment and high pressure water rinsing the standard field adjustment method [42], utilizing bead-pull perturbation technique, can be applied to balance the field of each subunit. For this, a subunit has to be terminated at $\lambda/4$ distance from the end irises with electric shorts, creating the electrical boundary conditions identical to those in the superstructure. After the final chemical polishing and rinsing it is no longer possible to use the bead-pull technique to measure and to adjust the fields. There is always a risk of contamination of a cleaned superconducting surface by moving a perturbing bead and its string through the interior of a cavity. Still, one can apply the perturbation method to balance the mean gradient in subunits after cool-down to cryogenic temperature, using the cold tuners instead of a bead to perturb the e-m fields. The frequency and field adjustment after cool-down are usually necessary because unequal shrinkage of subunits leads to differences in frequency. When subunits have the same stored energy (and thus mean gradient) the
same perturbation causes an identical frequency shift $\Delta f$. For the adjustment, the stepping motor of the cold tuner of each subunit should be moved by a fixed number of steps and the frequency change $\Delta f$ of the $\pi$-0 mode should be measured. Then final positions of the tuners are chosen to maintain the exact operating frequency of the $\pi$-0 mode and simultaneously to ensure that all subunits show the same $\Delta f$ when their tuners are moved by the same number of steps.

4.3.3. Copper models

The Institute for Nuclear Studies (INS) in Świerk (Poland) built copper models of both superstructures in 1998/1999 for the verification of computer modeling and further optimization of their designs. Fig. 4.10 shows model of SST-I. Both models were tuned in the way described in the previous paragraph. Three threaded rods and two aluminum discs on each subunit substituted for the cold tuner, allowing for the frequency and field profile adjustment. The measured $\pi$-0 mode field profile is displayed in Fig. 4.11. For this measurement, the input and output antenna were located at the first and last cell respectively. This led to excitation of modes overlapping with the accelerating mode and a slight perturbation of its profile. The peak-to-peak field flatness factor was still very good, close to 0.94. Next, a model of the fundamental power coupler was tested to establish its coupling strength with the tuned assembly of SST-I. The result for two FPC locations, 35 mm and 45 mm apart from the first iris (Fig. 4.12) confirms that its $Q_{ext}$ can be adjusted by means of the inner conductor penetration depth ensuring the reflection free operation of SST-I in a large range of voltages and currents.
Figure 4.12: Cross section of the FPC attached to the beam tube. Coupling strength ($Q_{ext}$) for the FPC model attached to the SST-I vs. the penetration depth of the inner conductor (antenna tip) into the end beam tube.

Figure 4.13: $Q_{ext}$ for three dipole passbands of SST-I achieved with five HOM couplers. Red bars indicate ($R/Q$).

Figure 4.14: $Q_{ext}$ for the TM011 passband of SST-I achieved with five HOM couplers. Red bars indicate ($R/Q$).
Finally, the SST-I copper model was used for optimization of the HOM suppression. With five TESLA type HOM couplers, three attached to the interconnections and two at both end-tubes, we could define their angular positions and loop orientations providing the damping spec for the TESLA collider. Fig. 4.13 displays the $Q_{ext}$ values for the three lowest dipole passbands TE111, TM110 and TE121. The suppression of the monopole TM011 passband is shown in Fig. 4.14.

Shape imperfections in cells (manufacturing errors) lead to frequency differences for modes from the FM passband, even though the accelerating $\pi$-0 mode has the same frequency for all subunits after the tuning. The frequency differences in the SST-I model, shown in Fig 4.15, caused the $\pi/7$ and $2\pi/7$ groups to have a $k_{ss}$ substantially smaller than the $\pi$-group, as subunits behave like uncoupled cavities (as it was discussed in the Section 4.3). The influence of the de-coupling on bunch-to-bunch energy spread turned out to be rather marginal. The worst case, when all potentially de-coupled modes have $Q_{L}=1\cdot10^9$ can be found in Appendix-I.

![Figure 4.15: Frequency differences in the fundamental mode passband for the subunits of the SST-I model.](image)

The copper model SST-II was assembled and tuned at DESY after it was manufactured at INS. Its final field profile of the accelerating mode (Fig. 4.16) had field flatness factor of 96%. The model had four attached HOM couplers of the TESLA type, two at the interconnection and two at the

![Figure 4.16: Field profile $|E_z|$ of the accelerating mode of the SST-II model.](image)
end beam tubes. The HOMs suppression scheme was almost identical to that of the standard 9-cell TESLA cavity. The achieved $Q_{ext}$ values were therefore very similar to those presented in Section 3.3.6.

Here we also observed that the cell shape imperfections led to the subunits frequency differences (Fig. 4.17) causing uncoupled oscillations for modes from the fundamental passband, which have small $k_{ss}$. Field profiles of two modes from the $\pi/9$-group demonstrating this phenomenon are shown in Fig. 4.18a and b. The coupling strength for the FPC vs. antenna penetration depth follows

![Graph showing frequency differences](image1)

**Figure 4.17:** Frequency differences in the fundamental mode passband for two subunits of the SST-II model.

![Graph showing field profiles](image2)

**Figure 4.18:** Two uncoupled modes from the $\pi/9$-group; a) $\pi/9$-0 mode: $f = 1275.933$ MHz and $(R/Q) = 0.02\Omega$, b) $\pi/9$. $\pi$ mode: $f = 1276.077$ MHz and $(R/Q) = 0.001\Omega$
two curves measured for the SST-I (Fig. 4.12). $Q_{ext}$ values for the SST-II must be scaled with factor 18/28, the ratio of number of cells in both superstructures. For each penetration depth, $Q_{ext}$ for SST-II equals to 64% of its value for SST-I. The factor results from differences in the storage energy in each superstructure at the given field strength.

4.4. Beam test of two Nb prototypes

The cold and beam test evaluating the superstructure layout and verifying all the experiments carried out at 300K as well as the computer simulations was expected to answer the following three questions:

- How good can we balance the gradient in subunits at 2 K?
- How stable is the energy gain?
- How good is damping of the HOMs?

The test of the SST-I prototype in the TTF linac was scheduled for the second half of 2002. Unfortunately, at the beginning of that year the performance of superconducting connection needed for the assembly was still unreliable. We could not reach stable accelerating gradients testing subunits in the vertical cryostat, even though the results for a 2-cell test cavity were encouraging. To keep the schedule we proceeded with two prototypes of 2x7-cell. Subunits of 2x7-cell prototypes could be welded together. The infrastructure for cavity preparation at DESY was able to accommodate 2.08 m long cavities without major changes. The second argument to split SST-I was the similar computed bunch-to-bunch energy modulation of the 2x7-cell and 2x9-cell versions. For all bunches in the TESLA macro-pulse the computed modulation is very small, $\pm 5 \cdot 10^{-5}$ for the 2x9-cell (see Fig. 4.9) and $\pm 3 \cdot 10^{-5}$ for the 2x7-cell version. The conclusion was that the beam test of the existing

![Computed energy modulation in the 2x7-cells superstructure for the nominal TESLA beam.](image_url)

Figure 4.19: Computed energy modulation in the 2x7-cells superstructure for the nominal TESLA beam.

33 The modeling was performed for the identical subunits. The field profiles and RF data can be found in Appendix I.
7-cell subunits assembled in two 2x7-cell prototypes would tell us more about the favorable SST-II version, will benchmark our beam energy modulation computation and finally give two times more statistics for the measured results [63].

4.4.1. Preparation of Nb prototypes

After subunits of both prototypes had been welded, the pairs underwent the final BCP treatment (20µ removal). The final welding slightly perturbed the field profiles (Fig. 4.20). The field flatness factor was 92% and 94% for the prototype No 1 (P1) and No 2 (P2) respectively and could not be improved further because helium vessels had to be welded on the subunits prior to welding of subunits into pairs; therefore, cells were not accessible after the final welding. Field profiles of other modes and dispersion curves are shown in Appendix-II.

![Figure 4.20](image)

Figure 4.20: Measured at 300K final field profiles of the accelerating mode for two 2x7-cells prototypes.

4.4.2. Linac preparation and field profile adjustment at 2 K

Both 2x7-cell superstructures (see Fig. 4.21) were assembled into a spare cryomodule and installed in the TTF linac next to the injector. Many components limiting the aperture of the beam line, which could influence our test, were removed from the linac. Additional beam position monitors (BPMs) were installed behind and at the front of the dipole magnet next to the injector to measure the energy of the beam coming out of the injector. The energy measurement at the end of the linac was performed by means of a spectrometer dipole with two BPMs at its front and one BPM behind it. With this arrangement, collective betatron motion could be extracted from the displacement measured behind the dipole. The highest estimated accuracy was $2 \cdot 10^{-4}$ and it was limited by the resolution of BPMs. It was a factor of 3 better than for the operation of standard 9-cell structures observed during the entire time of experimental operation at the TTF linac in previous years. Due to a very intense experimental program at the TTF linac in 2002, a second cryomodule, housing eight 9-cell cavities, was installed along with the superstructures for a long-term performance test. The presence of this
cryomodule had consequences for the test of the superstructures, as we will see later. Both cryomodules were cooled down in May 2002. The configuration of the TTF linac is shown schematically in Fig. 4.23.

Before the beam was accelerated, we crosschecked the field profiles in both prototypes. For each cold prototype, the fundamental passband frequencies were compared with the frequencies measured at room temperature when the bead-pull method showed the best achievable field profile. The deviation from an ideal linear shift of frequencies is very sensitive to the relative detuning of subunits and/or frequency perturbation of individual cells (due to possible non-uniform chemistry or non-uniform shrinkage). This is a very good indicator of any change in the field profile. The conclusion drawn from the test of this method and its application to the standard 9-cell cavities was that when the relative deviation of frequencies was below $10^{-5}$, the change in the field flatness was less than 2%. The deviation we measured for both prototypes (Fig. 4.22) was below $8 \times 10^{-6}$, so we concluded that field deviations were acceptable.

![Figure 4.21: Arrangement of two prototypes in the cryomodule and photograph of the cryomodule insert before the installation in the cryomodule.](image)

![Figure 4.22: Relative deviations (zigzag lines) from ideal linear spectrum shifts (straight lines) as measured for eight highest modes of both prototypes.](image)
Figure 4.23: Schematic drawing (not to scale) of the TTF linac arrangement for the beam test of two SST prototypes.

- RF gun + capture cavity
- Superstructures
- Beam dump
- Dipole
- $E = 15.5\,\text{MeV}$
- $E = 63\,\text{MeV}$
- Eight 9-cell cavities detuned by 200 kHz
- $P_{in} \leq 200\,\text{kW}$
- $\Delta E/E$ measurements with accuracy: $2.5\div3.5 \times 10^{-4}$ (rms)
profiles remained unchanged after the final chemistry, high pressure water rinsing, and assembly in the cryostat, and after the cool-down procedure. Table 4.2 displays frequencies and loaded $Q$, for both prototypes after the tuning at 2K. $Q_L$ value of the accelerating mode was matched to the beam current and gradient in the TTF experiment.

Table 4.2: Nb prototype. Measured RF-parameters of modes from the FM passband

<table>
<thead>
<tr>
<th>No</th>
<th>$\beta_c$</th>
<th>$\Omega$</th>
<th>$f$ [MHz]</th>
<th>$Q_L$ [10^6]</th>
<th>$f$ [MHz]</th>
<th>$Q_L$ [10^6]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi/7$</td>
<td>0.18</td>
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<td>26.5</td>
<td>1276.569</td>
<td>27.8</td>
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<tr>
<td>2</td>
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<td>1276.802</td>
<td>500.0</td>
<td>1276.995</td>
<td>628.0</td>
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<tr>
<td>3</td>
<td>$3\pi/7$</td>
<td>0.05</td>
<td>1280.181</td>
<td>7.5</td>
<td>1280.198</td>
<td>7.3</td>
</tr>
<tr>
<td>4</td>
<td>$4\pi/7$</td>
<td>0.35</td>
<td>1280.297</td>
<td>61.0</td>
<td>1280.431</td>
<td>142.3</td>
</tr>
<tr>
<td>5</td>
<td>$5\pi/7$</td>
<td>2.47</td>
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<td>3.7</td>
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<td>1285.382</td>
<td>17.9</td>
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<td>7</td>
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<td>1299.136</td>
<td>1.5</td>
</tr>
<tr>
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<td>1300.000</td>
<td>4.1</td>
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<td>4.1</td>
</tr>
<tr>
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<td>$\pi$</td>
<td>0.18</td>
<td>1300.324</td>
<td>5.2</td>
<td>1300.323</td>
<td>4.9</td>
</tr>
</tbody>
</table>

4.4.3. Bunch-to-bunch energy modulation test

This experiment was the “proof of principle” test. As was already mentioned, the main concern was the energy flow via very weak coupling between subunits. The stability of the energy gain for all bunches in the train means that the cells’ stored energy is refilled in time between two consecutive bunches. The test was performed in two parts. First, we examined the prototypes slow decay of the stored energy during the acceleration. In the second part, we directly measured the bunch-to-bunch energy modulation at the end of the linac. In both parts of this test neither prototype nor injector were pushed to their performance limits to keep the operation of the linac very stable. Both prototypes could be operated very reliably at a gradient of 15 MV/m (the subunits had not been thermally treated at 1400 C). The TTF injector was designed in 1997, when the TESLA beam parameters were different from those optimized later and proposed in TDR in 2001. The operation of the injector, with the smallest charge fluctuation of 2.8 % within the macro-pulse, was possible when the bunch charge did not exceeded 4 nC. The bunch spacing of $t_b = 1 \mu$s (the same as for the very first TESLA beam) had been chosen to meet the highest sampling rate of the implemented BPMs’ electronics.

The HOMDYN prediction for this TTF beam was that bunch-to-bunch energy variation is in the same range as shown in Fig. 4.19. The reflection-free operation, when the beam current is 4 mA
and the gradient is 15 MV/m, demanded \( Q_L \) of the FPCs to be \( 4.2 \times 10^6 \). The rise time of e-m fields resulting from this \( Q_L \) value was 790 \( \mu \)s and the longest beam on time was limited to 530 \( \mu \)s (klystron pulse length was in total 1.32 ms). Each prototype was equipped with four field probes, one placed near each end-cell. They were used to monitor the field strength during the acceleration. An example of the measured signals sampled in synchronization with the accelerating beam is presented in Fig. 4.24. Without the energy re-filling the beam would take almost 70% of the stored energy in cells and the voltage would drop by 45%. No such phenomenon was observed. Both diagrams confirm the expectation that energy flow via weak coupling is sufficient. Even the signals of probes No. 3 and No. 4 (located at the subunit without FPC) in both prototypes did not decay during the acceleration. All signals had some noisy fluctuations. The strongest oscillation was at 250 kHz. It was caused by down-converters of the low level RF-system controlling the phase and the amplitude of accelerating fields. Zoomed signals, sampled at 250 kHz rate (without synchronism with the beam) in the time range 160-260 \( \mu \)s after the beam was switched on, are shown in Fig. 4.25. All signals in this figure are normalized: \( (V' \cdot <V'>)/<V'> \). They showed the oscillation at 250 kHz (4 \( \mu \)s period) which is superimposed on the other oscillations, well seen in the signal of probes No. 4 and No. 1 of the P1 and P2 prototype respectively.

![Figure 4.24: Signals from the field probes (P1 upper diagram, P2 lower diagram) measured during the acceleration of 530 bunches with \( q = 4 \) nC and spacing \( t_b = 1 \) \( \mu \)s at 15 MV/m. The sampling was synchronic with the beam.](image-url)
Figure 4.25: Signals from the field probes (P1 upper diagram, P2 lower diagram) measured during the acceleration as in the figure above for 101 succeeding bunches. The 250 kHz sampling rate was not synchronous with the beam.

The 250 kHz modulation was also seen during operation of the standard 9-cell cavities. We found, in the second part of the experiment, six more oscillations, which were caused by the feedback loops.

The second part of this experiment was devoted to measuring directly the bunch-to-bunch energy variation at the end of the linac. The Fourier transformation of three signals (from the BPM behind the dipole), measured for three different gains in the feedback loop, is shown in Fig. 4.26. One can see 15 oscillations in total, which reacted to the gain change in very different ways. Peaks No. 1, 2, 12 and 13 increased when the loop gain increased. Peaks No. 14 and 15 decreased with the gain. All other peaks remained unchanged, including the 250 kHz one, which we discussed above. Seven peaks were due to the feedback loops; eight (No. 3-10) were caused by the second cryomodule. All eight cavities of this cryomodule were detuned from the 1.3 GHz frequency by roughly 200 kHz and no power was delivered to these cavities during the entire energy gain test. Still, the beam induced voltage in these cavities modulated the energy of bunches. We could change the frequency of an individual peak by driving the cold tuner of the corresponding cavity. Finally, the conclusion from the energy stability test was that no slow gradient decay and no modulation caused by superstructure prototypes was seen within the accuracy limit in the measurement of $2 \cdot 10^{-4}$\cite{64}. One should mention here that this result proves that superstructures fulfill the linear collider specification for bunch-to-bunch energy variation, which must be below $5 \cdot 10^{-4}$.
4.4.4. HOM damping test

Three methods were applied to measure the frequency and impedance, \( Z = (R/Q) \cdot Q_{\text{ext}} \) of HOMs. At first, we measured the modes’ frequency and \( Q_{\text{ext}} \) by means of network analyzers. For both prototypes, we measured modes up to 3.2 GHz including the five dipole passbands and the lowest passbands of monopole, quadrupole and sextupole modes. In total, 420 modes have been measured. The method gives the impedance when one assumes that actual \((R/Q)\) equals its computed value. The method is limited to well isolate modes. The error in frequency measurements increases when \( Q_{\text{ext}} \) of a mode gets lower and when neighboring modes overlap. The frequency errors were misleading for the search of some modes we wanted to check with two other methods.

The second method, applied for the first time ever, was the active mode excitation [65]. Modes with potentially high impedance were excited via one of the HOM couplers by means of an amplifier. Controlling the reflected power and the power coupled out by two other HOM couplers one can find the modes’ stored energy and relate the deflection of on axis injected beam to the mode impedance. The deflection was measured in the BPM, downstream from the cryomodule. The method (Fig. 4.27 shows the setup) can potentially give all parameters of an excited mode: impedance \( Z \) and the polarization if deflection is measured in both x and y direction. One can apply this method to modes, $P_{\text{HOM}}$ is the forward power delivered by an amplifier. BPM measures position of the beam both in the vertical (y) and in horizontal (x) direction.
which do not propagate (undefined RF-power leakage into the beam tubes) and couple well to the HOM couplers. Forty-seven modes were measured with this active method. An example of measured BPM signals for 10 macro-pulses with 32 bunches in each, is shown in Fig. 4.28. In this particular case lower polarization of the highest impedance dipole (\(R/Q = 27 \Omega/cm^2\)) at \(f = 2573.971\) MHz was excited with 20 W forward power. The damping of this mode was very good. Its \(Q_{ext}\) was only \(2.1 \cdot 10^4\).

The deflection was measured in both directions. The signals without the mode excitation show that within a macro-pulse, the trajectories were generally stable (9 of 10), with bunch-to-bunch deflections smaller than \(\pm 0.1\) mm. When the mode was excited, bunches within a macro-pulse pass the superstructure in a random phase and thus some are deflected at the BMP location. The maximum observed deflection amplitude was the measure of the transverse force having the electric and magnetic components proportional to square root of the mode stored energy. Bunch trajectory was sensitive to the beam optic elements (quadrupole and sextupole magnets) between the test cryomodule and BPM. The mode was measured six times, for various settings of the optics elements and for various HOM couplers used to transfer RF power into the cavity (Fig. 4.29). The differences were mainly due to the optics whose setting should minimize its influence on the trajectory. The estimation of the beam deflection (\(R_c\)) was done with the assumption that the beam drifts between the cryomodule and BPM. The discussed mode was above cut-off remaining confined by beam tubes with smaller diameter (63 mm) at the both ends of the cryomodule. The mean value of the measured deflections was \(<R> = 1.8\) mm and its computed value was \(R_c = 1.7\) mm, which confirmed that the optics influence on average was minimized. Fig. 4.30 shows the polarization of the mode. The mean value, which was found for the cold prototype, was \(73^\circ \pm 10^\circ\) (angle measured clockwise from the y-direction). The differences were mainly due to calibration errors of the BPM signals in both planes. The polarization at 300K (black dot) was defined with limited accuracy due to the strong overlapping of modes. Nevertheless, the measured deflection gave the estimation of \(Z\), which in the worst case would be two...
times higher than expected from the network analyzer measurements and which still would be harmless to the TESLA beam. The polarization measurement showed that this mode, when excited by the accelerated beam, would deflect it almost horizontally.

![Figure 4.29](image1.png)
**Figure 4.29:** Deflection measured for the different optics setting. Yellow triangle indicates the computed value.

![Figure 4.30](image2.png)
**Figure 4.30:** Mode polarization measured by the BPM 15m downstream from the P1 prototype. The cold measurements are compared to the polarization measured at 300K (black dot) by means of the bead-pull technique.

Other dipoles of the P1 prototype, measured in a similar way are shown in Fig 4.31. In general, the computed deflections were either close or higher than their measured values. For the first group of modes (<1900 MHz), which was under cut-off, smaller than computed impedances resulted from the perturbed field profiles in the actual subunits and thus smaller than computed \((R/Q)\)'s. For the higher frequency dipoles above the cut-off frequency (>2500 MHz) this can take place too but additionally the radiated RF-power reduces their stored energy, which leads to the deflecting force weaker than computed with the assumption that the whole measured energy remains in the subunits.

The third method applied to measure Z was based on the HOM excitation by the accelerated beam when it passes the cavity off axis. The charge of the beam was modulated to hit modes with sideband spectral lines generated in this way. This method was previously applied twice to the standard cavities and is discussed in more detail in [66].

---

34 Dipoles of the P2 prototype have been measured too, but they are not presented in this report.
Figure 4.31: Computed ($R_c$) and measured amplitudes of the beam deflection resulting from the excitation of selected dipoles of the P1 prototype.

All three methods proved very good damping of HOMs. A summary of the damping of dipoles modes with $(R/Q) > 1 \, \Omega/cm^2$ for both prototypes is shown in Fig. 4.32. All modes relevant for the superconducting collider, up to 2.58 GHz, are damped by a factor 5 to 100 better than the specification ($Q_{ext} \leq 2 \cdot 10^5$). The damping of dipoles with $(R/Q) < 1 \, \Omega/cm^2$ and other transverse modes is also good (Fig. 4.33). Only a few modes were found (in the 5th passband, ~3.08 GHz), among 420 measured modes, with $Q_{ext} \approx 10^7$-2$\cdot 10^8$. They were located in the beam tube between two prototypes (see Table 4.3). Their $(R/Q)$s are almost zero and thus they cannot degrade the quality of the collider beam. The group of quadrupoles (~2.3 GHz) and sextupoles (~2.7 GHz and ~3.06 GHz) with $Q_{ext} \approx 10^6$ are also irrelevant for the beam emittance.

Figure 4.32: Both prototypes. Damping of dipoles with $(R/Q) \geq 1 \, \Omega/cm^2$. 

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Figure 4.33: Both prototypes. Damping of modes with \((R/Q) < 1 \Omega/cm^2\).

Table 4.3.: Transverse modes found between both prototypes

<table>
<thead>
<tr>
<th>(f) [MHz]</th>
<th>SST1</th>
<th>SST2</th>
<th>(Q_{\text{ext}}) [(\Omega/cm^2)]</th>
<th>(R/Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3247.353</td>
<td>o</td>
<td>o</td>
<td>(X)</td>
<td>2.1(\times)10^7</td>
</tr>
<tr>
<td>3076.263</td>
<td>o</td>
<td>o</td>
<td>(X)</td>
<td>1.4(\times)10^7</td>
</tr>
<tr>
<td>3076.154</td>
<td></td>
<td>(X)</td>
<td></td>
<td>1.2(\times)10^7</td>
</tr>
<tr>
<td>3063.370</td>
<td>o</td>
<td>o</td>
<td>(X)</td>
<td>3.0(\times)10^8</td>
</tr>
</tbody>
</table>

x-indicates that mode has not stored energy at position of the HOM coupler, o-indicates that mode has not stored energy at position of the HOM coupler.

4.5. Summary of the experiment and future improvements

The test of both prototypes has confirmed that one can use weakly coupled structures for acceleration. Neither beam energy modulation, slow gradient decay nor insufficient HOMs damping, resulting from the coupling of two subunits, has been observed. The stability of the bunch-to-bunch energy gain was measured within the limit of the beam diagnostics in the TTF linac. Although the accuracy of the energy gain measurement has not reached the level of the theoretical estimation, the experiment showed that the TESLA specification has already been fulfilled.

The experiment proved that the electronics for phase and amplitude control, used routinely to operate standard 9-cell cavities in the TTF linac, could be applied for superstructures. Improved electronics should provide better suppression of the modulations coming from the control system itself.

The production of superstructures is more challenging than for standard cavities, but obviously not impossible, especially if the infrastructure for welding, pre-tuning and cleaning is planned
from the very beginning to accommodate longer structures. The mechanical tolerances should be
tighter to avoid decoupling of subunits for other than the accelerating group of modes. The final
preparation is more complicated too. The breakthrough may be expected if the current R&D program
at TJNAF succeeds with a reliably operating superconducting interconnection. The final preparation of
subunits will then be very similar to that of standard 9-cell cavities without any especially dedicated
infrastructure. Progress in the R&D program on superconducting interconnection is very encouraging.
One should mention here that a very poor performance of the superconducting joint for the SST-I
prototype subunits, whose flanges were made of the NbTi alloy, is now understood. The main reason
was a very low heat conduction of the alloy at 2K (at least 30 times lower than for Nb), which at that
time was believed, accordingly to some publications, to be marginally different from the conduction of
the pure niobium. A single–cell cavity has been built from that alloy and cold tests confirmed the
DESY result that at 10 mT that material quenches. Two new materials: NbZr and NbN, which
demonstrate high hardness, are at present under study at TJNAF. Figure 4.34 shows preliminary test
results at 2K for the single-cell cavities made of NbZr and NbTi. Obviously, the performance of
NbZr is superior and sufficient for the superconducting interconnection, which will be built in near
future.

\[
\begin{array}{c}
1.E+10 \\
1.E+09 \\
1.E+08 \\
\end{array}
\]

\[
\begin{array}{c}
0 & Q_0 \\
10 & Q_{en} \\
20 & Q_{en} \\
30 & Q_{en} \\
40 & Q_{en} \\
50 & Q_{en} \\
\end{array}
\]

\[
\begin{array}{c}
B_{peak} [mT] \\
0 & 1.E+08 \\
10 & 1.E+09 \\
20 & 1.E+10 \\
\end{array}
\]

Figure 4.34: Cold test results for two single-cell cavities made of NbTi and NbZr. One should note that the NbZr cavity
reached \( B_{peak} \) of 40 mT, which is higher than field needed for the superconducting connection (~30 mT).

The superconducting gasket for the interconnection will be made of a pure soft niobium. The chosen
material will finally be tested on 2x5-cell superstructure at 1.5 GHz built at TJNAF for that purpose
(Fig. 4.35).

\[35\] All results obtained for new materials will be presented in [67].
Figure 4.35: 2x5-cells SST for the superconducting interconnection test. The subunits are welded for the first cold test to determine RF-performance of the assembly without the sc-interconnection. After this test, the interconnecting tube will be cut to place the superconducting joint between the subunits.

The beam experiment on 2x7-cells proved that the SST layout can be a challenge for production, but no fundamental physical problems due to the very weak coupling are expected. The measurements on Cu models of SST-II, with the optimized positions of four HOM couplers, showed good damping of all parasitic modes. Therefore, the next step after the cold interconnection test will be the cold test of the SST-II prototype, which for now is neither scheduled nor funded. Meanwhile, a new application of the superstructure concept has been proposed and was studied in 2003/2004 at TJNAF [68]. The very good HOM damping makes superstructures with fewer cells an attractive alternative to standard accelerating structures for a high current linac driving FELs. The next application to energy recovery linacs, for example the cw-upgrade of the European XFEL facility, is discussed in more details in [14].
V SUPERCONDUCTING PHOTO INJECTOR
5.1. General remarks

In this Chapter we will discuss application of the superconducting cavities for the generation of low emittance electron bunches. The cavities used for this purpose are much shorter. They are made of up to 3.5 cells, which operate at moderate gradients to deliver electron beams up to 10 MeV.

Remarkable improvement in RF performance of superconducting cavities over the past decade has made continuous wave (cw) and near-cw operations of superconducting electron linacs at high accelerating gradients feasible. Several future projects assume these types of operation, for example superconducting accelerators proposed as drivers for FEL facilities [14, 70, 71] and the electron cooling facility proposed for the Relativistic Heavy Ion Collider at Brookhaven National Laboratory [72]. Both operating modes require injectors operating at cw or near-cw, providing low emittance electron beams. An example of such injector (so called split injector) is discussed in [14, 73]. The most demanding component of a cw injector is cw operating RF-gun, delivering highly populated (~1 nC) low emittance bunches. RF-guns, both working at room temperature and superconducting, when they generate highly populated low emittance bunches have to be operated at high accelerating gradients to suppress space charge effects diluting emittance. Normal conducting RF-guns face difficulties in meeting this requirement in the cw or near-cw mode. Their copper walls dissipate many kilowatts of power to fulfill the high gradient condition even when they operate at low pulse repetition rate (low duty factor). It is a technical challenge to significantly increase the duty factor of normal conducting RF-guns while having sufficient cooling and keeping them thermally stable during operation.

Superconducting RF-guns (SRF-guns) dissipate orders of magnitude less power than the normal conducting devices. They can be easily operated at high duty factor but the challenge here is integration of a non super-conducting photo-cathode material with a superconducting cavity [74] while preserving its original high intrinsic quality factor $Q_0$ (small cryogenic losses). One possible solution to this problem, based on choke filter, is investigated at Forschungszentrum Rossendorf [75, 76]. Another approach, which is very attractive and technically feasible for milliampere-class SRF-guns, is to use superconducting material as the photo-cathode. In this case, difficulty arises from low quantum efficiency ($\text{QE}$) of the superconducting materials. Low $\text{QE}$ must be compensated with shorter wavelength and higher energy pulses from the illuminating laser. This has been studied first on Nb cathode at BNL [77]. A complementary approach, with lead is discussed in the next section [78].

5.2. Lead-Niobium SRF-Gun

Low quantum efficiency of niobium was the motivation for testing alternative superconductors as photo-emitters to continue the BNL concept of an all superconducting RF-gun. Several Institutions: DESY, BNL, Stony Brook University, TJNAF, INS and SLAC agreed on the common R&D effort to
investigate feasibility of a hybrid Pb-Nb SRF-gun. Lead, which a critical temperature of 7.2 K is not very different from niobium, is a commonly used superconductor in accelerators. Its critical magnetic flux $B_c$ is 70 mT.

5.2.1. Quantum efficiency of Lead

Lead demonstrates much higher $QE$ than niobium. Tests results of lead $QE$ at room temperature were reported by the collaboration in [79]. Fig. 5.1 shows the summary of $QE$ measurements vs. photon energy $E_p$ for niobium, bulk lead and lead samples deposited with various techniques. The highest $QE$ of 0.55% has been measured for the arc-deposited sample illuminated with 6.5 eV photons. For 5.8 eV photons (fifth harmonics of 1064 nm infrared laser) $QE$ of that sample was still 0.25%. $QE$ of the electroplated and magnetron deposited samples at this photon energy is 0.17%. Emitting a 1 nC bunch at this $QE$ will require only 3.4 µJ energy per pulse. For cw operation of European XFEL, as it was proposed in [14], the average laser power of 3.4 W at 214 nm will be required to generate 1 nC bunches at 1 MHz repetition rate.

![Figure 5.1: Measured QE of lead deposited with various coating methods. Bulk Pb and Nb data are displayed for comparison.](image)

Very recently, in February and March 2007, several experiments at cryogenic temperatures 2 K, 4 K and 6 K were performed at TJNAF, confirming that there is no change in $QE$ when the emitting lead is in the superconducting state.

5.2.2. RF design

In this hybrid Pb-Nb gun, the small emitting spot of lead ($O<3$ mm) will be located in the center of back wall of the half-cell of 1.6-cell cavity (Fig.5.2) which will be made of high purity niobium. The cavity is equipped with two HOM couplers, an input coupler and pickup probe. For operation, the
cavity will be assembled in a small dedicated cryostat. The gun is designed to be implemented in the split injector. A solenoidal static B field will be used for the emittance growth compensation. Tables 5.1, 5.2 and Fig. 5.3 show RF-parameters and the HOM suppression ($Q_{\text{ext}}$) for the present design respectively. The good HOM suppression in a superconducting RF-gun is crucial for the beam quality that can be diluted by the interaction between non-relativistic electrons (in the first half-cell) with deflecting dipole modes. The assumed maximum operating electric field on the lead cathode is $E_{\text{peak}} = 60 \, \text{MV/m}$. At this field the spot will be exposed to $B = 4 \, \text{mT}$, which is much smaller than the lead critical field $B_c$ (see Fig. 5.4).

![Figure 5.2: 1.6-cell SRF-gun cavity with two HOM couplers and the input coupler.](image)

---

**Table 5.1: RF-parameters of the 1.6-cell SRF gun**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi)-mode frequency</td>
<td>[MHz]</td>
<td>1300</td>
</tr>
<tr>
<td>0-mode frequency</td>
<td>[MHz]</td>
<td>1286.5</td>
</tr>
<tr>
<td>Cell-to-cell coupling</td>
<td>-</td>
<td>0.015</td>
</tr>
<tr>
<td>Active length 1.6(\lambda/2)</td>
<td>[m]</td>
<td>0.185</td>
</tr>
<tr>
<td>Nominal (E_{\text{cath}}) at cathode</td>
<td>[MV/m]</td>
<td>60</td>
</tr>
<tr>
<td>Energy stored at nominal (E_{\text{cath}})</td>
<td>[J]</td>
<td>20</td>
</tr>
<tr>
<td>Nominal beam energy</td>
<td>[MeV]</td>
<td>6</td>
</tr>
</tbody>
</table>

**Table 5.2: HOMs of 1.6-cells**

<table>
<thead>
<tr>
<th>Mode</th>
<th>(f) [MHz]</th>
<th>(R/Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole: TE111-1a</td>
<td>1641.8</td>
<td>1.85 [Ω/cm²]</td>
</tr>
<tr>
<td>Dipole: TE111-1b</td>
<td>1644.9</td>
<td>1.30 [Ω/cm²]</td>
</tr>
<tr>
<td>Dipole: TM110-1a</td>
<td>1883.5</td>
<td>10.1 [Ω/cm²]</td>
</tr>
<tr>
<td>Dipole: TM110-1b</td>
<td>1884.0</td>
<td>9.99 [Ω/cm²]</td>
</tr>
<tr>
<td>Dipole: TM110-2a</td>
<td>1957.0</td>
<td>3.90 [Ω/cm²]</td>
</tr>
<tr>
<td>Dipole: TM110-2b</td>
<td>1957.1</td>
<td>3.85 [Ω/cm²]</td>
</tr>
<tr>
<td>Monopole: TM011</td>
<td>2176.5</td>
<td>43.2 [Ω]</td>
</tr>
</tbody>
</table>

\(36\) The first cell is 0.6\(\lambda/2\) long. This length provides the lowest emittance growth.
5.2.3. RF-cold tests

Two types of half-cell cavities have been built to measure lead $Q_E$ at 2 K and to test performance of Nb-Pb cavities illuminated by laser. Both types are shown in Fig. 5.5. The left one was built at TJNAF. This cavity has an opening in the center of the back wall, which is vacuum-sealed with a niobium plug and an indium gasket. The advantage of this cavity type is that plugs can be easily coated with emitting materials to test various coating methods and various superconductors as photo-emitters. It can be an alternative solution to the second cavity type, built at DESY (shown in Fig. 5.5 right), in which technically difficult coating is done directly on the back wall. An additional difficulty in this version is that the emitting spot must withstand cleaning procedures. Two of the cleaning steps could degrade $Q_E$: chemical treatment and high pressure water rinsing. Both procedures are essential for good performance of sc cavities. Two features make this version very attractive. The smooth back wall does not locally enhance the electric field near the cathode and there is no electric contact in high field region, which may reduce intrinsic $Q_o$ of the cavity. Both types of cavity were tested recently (Fig. 5.6), showing improved performance but still did not demonstrate the expected $E_{peak}$ close to 60 MV/m.
The TJNAF cavity reached $E_{\text{peak}}$ of 38 MV/m with lead coated plug (electro-plated spot, $\Omega=7$ mm). The intrinsic $Q$ at this field was $2 \cdot 10^9$ mainly due to insufficient cooling of the plug. The DESY cavity without coating reached $E_{\text{peak}}$ of 60 MV/m. With the lead coating (arc-deposited spot, $\Omega=4$ mm) maximum achieved $E_{\text{peak}}$ was 39.5 MV/m and $Q_0 = 2 \cdot 10^9$ with heavy electron emission and radiation, indicating insufficient cleaning of the surface. The lead protecting fixture makes it difficult to perform the proper chemical treatment and high pressure water rinsing of this cavity.

Figure 5.5: Test half-cell cavities built at TJNAF (left) and at DESY (right) assembled on inserts for the RF-cold test.

Figure 5.6: Test result $Q_0$ vs. $E_{\text{peak}}$ for half-cell cavities: TJNAF type with 7 mm electro-plated lead spot on the plug and DESY type with 4 mm arc-deposited lead spot.
One of the objectives in the recent cold tests at TJNAF was to find out how fast the quasi-particles generated by the laser irradiation, which breaks the Cooper pairs, recombine back into the pairs. Fig. 5.7 shows the relaxation time for both superconductors as it was theoretically estimated in [77]. The conclusion from the diagram is that the process has a tendency to stabilize for $T < T_c$ since relaxation takes shorter time for higher $T$. The diffusion process without external RF-fields is fast and during the half RF-period when electric force pushes quasi-particles into material they spread uniformly in the irradiated layer. From that moment on the relaxation process follows the diagram as a relaxation process starting at $\sim 5$K since it is mainly phonons escaping from the excited volume that determine the duration time of the relaxation and RF-losses are negligible during that time.

The 248 nm excimer laser used for the cold tests generated 10 ns long pulses at up to 250 Hz repetition frequency. The maximum energy per pulse was 5.5 mJ. Half of it could be transferred to the superconducting cavity for the $QE$ and relaxation time experiments. The maximum available peak power of $2.75\text{mJ}/5.7\text{ns} = 0.482 \text{MW}$ at the cathode was two times higher than the power needed for the nominal operation of the gun when 1 nC bunches are generated within 20 ps at 1 MHz repetition rate.

The preliminary relaxation time experiment was performed with the TJNAF type half-cell, which operating frequency was 1.42 GHz (Fig. 5.8). For that test, the cavity had to be attached to a straight vacuum tube oriented vertically upward, to make possible direct irradiation with the laser light via sapphire window installed at the top-plate of the vertical cryostat. The 3 m long vacuum tube contaminated the cavity with particulates, which led to degradation in the performance. At 3 MV/m cavity had intrinsic quality factor of $2.25\times10^9$ only, almost three times lower than $Q$ measured at this gradient in the RF-performance test (see Fig. 5.6). When the Nb wall was irradiated with the maximum

![Figure 5.7: Relaxation time in lead and niobium vs. T.](image)
available laser power, $Q_o$ dropped to $\sim 1.6\cdot 10^9$, but the cavity did not quench and still behaved very stable. The additional power dissipation, due to the locally broken Cooper pairs in the irradiated area, was 5.2 mW. For the nominal operation at high gradients, this additional power dissipation will contribute marginally to the total cryogenic load. The surface resistance, $r_s$, of the irradiated area increased during the laser pulse by factor of 630. The BCS part of $r_s$ (see formula 3.18) for both superconductors is shown in Fig. 5.9. The ratio of $r_s$ before and after the irradiation indicated that the concentration of quasi-particles rose to their equilibrium concentration at 8 K. At this temperature, their relaxation time is shorter than 100 ps (see Fig. 5.7). The lead cathode of the TJNAF cavity was not exposed to high intensity laser light because we wanted to ensure its surface was unchanged for further RF-performance tests. The relaxation time experiment for lead is scheduled for July 2007. It will be carried out at TJNAF with the second DESY type cavity, which was coated at INS in April 2007. The setup for that test will be modified to prevent the cavity from contamination. This shall enable the relaxation time test at high gradients.

Figure 5.8: Computed contours of the electric field and magnetic flux in the TJNAF type half-cell. On the color scale red and blue depict high and low value respectively.

Figure 5.9: The BCS part of the surface resistance at 1.42 GHz for lead and niobium vs. T.
5.3. Summary and perspectives

Up to now, the theoretical and experimental results prove that a low emittance superconducting electron source with superconducting cathode is technically feasible. The encouraging results are gaining attention in the SRF community. The project will receive more financial support in the future (from DESY, TJNAF, BNL and MIT). The Cu model of the SRF-gun is currently under fabrication at TJNAF to verify the computed RF properties of the coupling scheme and HOM suppression. The first Nb prototype will be ready by the end of 2007. The most challenging part of the ongoing R&D program is the arc-deposition of lead. The main technical difficulty comes from the fact that the distance between the plasma source and back wall of a 1.6-cell cavity is significantly larger than in the prototype half-cell gun. A new plasma guiding system and improved conditions of the process will be required to provide high quality coating, comparable to that on the samples made for the QE tests. One this challenge is overcome, one could proceed with the DESY type cavity having unperturbed cylindrical symmetry of the electric field in the cathode region, which is essential for the beam quality. The TJNAF cavity type requires tight tolerances on the plug positioning to mitigate the field distortions.

Alternative superconducting material for photo-cathodes; MgB$_2$ and YBaCo (low work function $\sim$2.9 eV) are now under investigation at BNL, with a hope that the present SRF-gun version can be extended to higher currents. This will be possible when a superconductor with substantially higher $QE$ can be used for the photoemission.
VI FINAL REMARKS
State-of-the-art

Progress in the performance of superconducting cavities over the last fifteen years is remarkable. The gradients in multi-cell cavities rose from several MV/m in early 90’s to 20-25 MV/m achievable at present, mainly due to the careful material inspection eliminating inclusions and better preparation techniques. Few 9-cell TESLA cavities (at DESY, TJNAF) have reached fields above 35 MV/m. Many single-cell cavities worldwide performed close to the theoretical limit for niobium. Still, the main ILC goal to routinely reach an $E_{\text{acc}}$ of 35 MV/m needs many years of R&D before the technical expertise can be transferred to industry for the fabrication of ~16000 structures.

There are several alternatives to the standard TESLA structure with improved shapes for higher than 50 MV/m ultimate gradients (LL, RE). The new shapes presented here are in a premature state and their implementation for multi-cell structures is challenging. On the other hand, their potential high gradient makes them very attractive for long accelerators and it is the motivation for further R&D programs continued at present at KEK, Cornell University and Michigan State University.

Presently there is no alternative material to bulk niobium that could promise to reach high gradients (~35 MV/m) at high intrinsic quality factor ($\sim 10^{10}$) for use in superconducting accelerating structures. The large grain niobium currently being investigated at TJNAF, DESY and KEK seems to be the only material improvement, which will hopefully allow the ILC and European XFEL gradient specification to be met with the BCP treatment. This will lead to significant investment cost reduction for these facilities.

Status of the SST concept

The SST concept was proven experimentally in 2002. The experiment, which was carried out for several months at the TTF linac, confirmed all computed RF-properties of weakly coupled structures. This novel layout of superconducting structures allows for a significant increase in the number of cells fed by a single FPC, thus simplifies, and reduces the cost of the RF-system of an accelerator. The drawback of the layout is the final chemical treatment and high-pressure water rinsing of a long assembly. Both procedures are essential for high gradients and often give expected results for short (single-cell) structures but not always so for the multi-cell structures. Obviously, the difficulties rise with number of cells and a superconducting interconnection between subunits provides a method to mitigate these difficulties, as demonstrated, for 9-cell TESLA structures. The R&D program on the sc interconnection at TJNAF recently gave very promising results, which provides data that a sc joint operating up to 30 mT can be built in the near future and tested with the 2x5-cell superstructure.
Status of the SRF gun

The presented approach to build a low emittance 1 mA class SRF-gun with superconducting photocathode progresses well. Both the RF-performance of niobium cavities with the lead coating and the lead QE at cryogenic temperatures are encouraging for further tests with 1.6-cell structures. Two of these guns will be built by spring and tested by the end of 2008. The coating setup at INS is under modification to accommodate longer structures, since the deposited spot is located at longer distance from the ion source. The spot shielding for the BCP treatment and high pressure water rinsing is under reconstruction for longer structures and will be ready for preliminary tests and implementation by the end of this year. Alternative superconductors (MgB$_2$ and YBaCo) are under QE test. Since the RF-design is ready, we are building a copper model of the gun to verify the computed RF-parameters. We expect to finish the R&D phase early in 2009 and will be ready at that time to build the first prototype.

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Finally, I want thank Dr. A. Burrill of BNL for reading the manuscript and his many valuable corrections.


## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$a_{ff}$</td>
<td>field flatness factor</td>
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<tr>
<td>$B$</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>$B_c$</td>
<td>critical magnetic flux</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light</td>
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<tr>
<td>$e$</td>
<td>electron</td>
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<tr>
<td>$e^+$</td>
<td>positron</td>
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<tr>
<td>$e^-$</td>
<td>electron charge</td>
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<tr>
<td>$E_p$</td>
<td>particle energy</td>
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<tr>
<td>$E_m$</td>
<td>center of mass energy</td>
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<tr>
<td>$E, E, E$</td>
<td>electric field amplitude, phasor, vector</td>
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<tr>
<td>$E_z$</td>
<td>electric field component in z-direction</td>
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<tr>
<td>$f$</td>
<td>frequency</td>
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<tr>
<td>$f_c$</td>
<td>collision frequency</td>
</tr>
<tr>
<td>$f_{FM}$</td>
<td>fundamental (accelerating) mode frequency</td>
</tr>
<tr>
<td>$f_p$</td>
<td>pulse repetition frequency</td>
</tr>
<tr>
<td>$G$</td>
<td>cavity geometric factor</td>
</tr>
<tr>
<td>$H, H, H$</td>
<td>magnetic field amplitude, phasor, vector</td>
</tr>
<tr>
<td>$H_0$</td>
<td>beam disruption factor</td>
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<td>beam current</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
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<td>$k_{cc}$</td>
<td>cell-to-cell coupling</td>
</tr>
<tr>
<td>$k_{ss}$</td>
<td>subunit-to-subunit coupling</td>
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<tr>
<td>$k_u$</td>
<td>cell-to-cell coupling</td>
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<td>$L$</td>
<td>luminosity</td>
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<td>$m$</td>
<td>mass of electron</td>
</tr>
<tr>
<td>$N$</td>
<td>number of cells</td>
</tr>
<tr>
<td>$N_e$</td>
<td>bunch population, number of particles in bunch</td>
</tr>
<tr>
<td>$N_{coll}$</td>
<td>number of colliding particles</td>
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<td>$q$</td>
<td>charge</td>
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<td>intrinsic quality factor</td>
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<td>$Q_{ext}$</td>
<td>external quality factor</td>
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<tr>
<td>$Q_L$</td>
<td>loaded quality factor</td>
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<td>$P_{ac}$</td>
<td>ac power of an accelerator</td>
</tr>
<tr>
<td>$P_b$</td>
<td>beam power</td>
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<tr>
<td>$P_c$</td>
<td>cavity power</td>
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<tr>
<td>$P_g$</td>
<td>generator power</td>
</tr>
<tr>
<td>$P_{in}$</td>
<td>input power</td>
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<tr>
<td>$P_0$</td>
<td>dissipated power for the fundamental (accelerating) mode</td>
</tr>
<tr>
<td>$P_r$</td>
<td>reflected power</td>
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<tr>
<td>$r_{BCS}$</td>
<td>BCS part of the surface resistance</td>
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<tr>
<td>$r_s$</td>
<td>residual surface resistance</td>
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<tr>
<td>$r_s$</td>
<td>surface resistance</td>
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<td>$(R/Q)_{FM}$</td>
<td>characteristic beam impedance of the fundamental (accelerating) mode</td>
</tr>
<tr>
<td>$(R/Q)_k$</td>
<td>characteristic beam impedance of mode $k$</td>
</tr>
<tr>
<td>$R_0$</td>
<td>beam impedance of the fundamental (accelerating) mode</td>
</tr>
<tr>
<td>$R_k$</td>
<td>generator impedance</td>
</tr>
<tr>
<td>$U_r$</td>
<td>radiated energy</td>
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<tr>
<td>$V_p, V_k$</td>
<td>voltage amplitude of mode $k$, phasor</td>
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<tr>
<td>$W_k$</td>
<td>stored energy of mode $k$</td>
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<tr>
<td>$t$</td>
<td>time</td>
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<tr>
<td>$T$</td>
<td>temperature</td>
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<tr>
<td>$T_c$</td>
<td>critical temperature</td>
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</table>
\( Z_k \) - complex impedance of mode \( k \)

**Greek Symbols**

\( \beta_{cc} \) - cell-to-cell phase advance  
\( \beta_b \) - coupling factor with beam  
\( \beta_c \) - coupling factor  
\( \gamma \) - Lorentz factor  
\( \delta \) - incremental change of a value  
\( \Delta \) - energy gap  
\( \Delta_{AS} \) - relative energy loss in the beamstrahlung process  
\( \varepsilon_0 \) - vacuum permittivity  
\( \varepsilon_x \) - emittance in x-direction  
\( \varepsilon_y \) - emittance in y-direction  
\( \eta \) - efficiency  
\( \eta_{E} \) - cavity electric fields ratio  
\( \eta_{B} \) - cavity magnetic to electric field ratio  
\( \eta_b \) - number of bunches per pulse  
\( \Phi_p \) - phase  
\( \lambda \) - wavelength  
\( \rho_{acc} \) - average radius of a circular accelerator  
\( \sigma_x \) - bunch size in x-direction  
\( \sigma_y \) - bunch size in y-direction  
\( \sigma_z \) - bunch size in z-direction  
\( \tau_p \) - pulse duration  
\( \tau_k \) - decay time of mode \( k \)  
\( \omega \) - angular frequency

**Abbreviations**

BDS - Beam Delivery System  
ERL - energy recovery linac  
FEM - Finite Element Method  
FPC - Fundamental Power Coupler  
HOM - Higher Order Modes  
\( QE \) - Quantum Efficiency  
RF - Radio-Frequency  
sc - superconducting  
SST - superstructure  
SRF - Superconducting Radio Frequency  
TDR - Technical Design Report  
TE\(.xxx \) - Transverse Electric mode type  
TM\(.xxx \) - Transverse Magnetic mode type
APPENDIX I

Field patterns and RF-parameters for the accelerating passbands

Field profiles on axis at $W = 1$ J and RF-parameters for modes from the fundamental passband computed with FEM-code and used for the bunch-to-bunch energy spread modeling with HOMDYN. All subunits in a superstructure are here identical, fabrication errors are not taken in to account.

**SST-I**

*Group $\pi/7$*

![Graph of Ez(0,z) for Group π/7](image)

*Group $2\pi/7$*

![Graph of Ez(0,z) for Group 2π/7](image)

*Group $3\pi/7$*

![Graph of Ez(0,z) for Group 3π/7](image)

*Group $4\pi/7$*

![Graph of Ez(0,z) for Group 4π/7](image)
Table AI.1: SST-I. RF-parameters of modes from the FM passband

<table>
<thead>
<tr>
<th>No</th>
<th>Group $\beta_0$</th>
<th>$f$ [MHz]</th>
<th>$(R/Q)$</th>
<th>$Q_L$ [10$^6$]</th>
<th>$Q_{LR}$ [10$^6$]</th>
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</thead>
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<tr>
<td>1</td>
<td>$\pi/7$</td>
<td>1277.143</td>
<td>0.34</td>
<td>32.0</td>
<td>1000.0</td>
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<td>2</td>
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<td>1277.172</td>
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<td>1279.905</td>
<td>0.07</td>
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<td>1000.0</td>
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<tr>
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<td>0.09</td>
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<td>1000.0</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>1280.000</td>
<td>0.55</td>
<td>32.0</td>
<td>1000.0</td>
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<tr>
<td>9</td>
<td>$3\pi/7$</td>
<td>1285.714</td>
<td>3.49</td>
<td>3.0</td>
<td>1000.0</td>
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<tr>
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<td>1000.0</td>
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<td>1285.852</td>
<td>0.01</td>
<td>4.0</td>
<td>1000.0</td>
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<td>22.0</td>
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<td>17</td>
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<td>2.0</td>
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<td>8.0</td>
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<td>1.5</td>
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<td>2928.80</td>
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<td>26</td>
<td>$\pi$</td>
<td>1300.148</td>
<td>0.00</td>
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<td>2.0</td>
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<td>27</td>
<td>$\pi$</td>
<td>1300.342</td>
<td>0.41</td>
<td>8.0</td>
<td>8.0</td>
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<tr>
<td>28</td>
<td>$\pi$</td>
<td>1300.468</td>
<td>0.00</td>
<td>27.0</td>
<td>27.0</td>
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</tbody>
</table>

The last column of the table lists $Q_L$ when the lowest 16 modes are decoupled and no suppression by the FPC takes place. The bunch-to-bunch energy spread for this case (Fig. AI.1) is marginally different from the case when all subunits of SST-I are identical (Fig. 4.9).
Figure AI.1: Computed energy modulation normalized to the energy gain for the nominal TESLA bunch train for the SST-I with decoupled 16 modes from the $\pi/7$, $2\pi/7$, $3\pi/7$ and $4\pi/7$ group.

**SST-II**

**Group $\pi/9$ and $2\pi/9$**

**Group $3\pi/9$ and $4\pi/9$**

**Group $5\pi/9$ and $6\pi/9$**
Likewise the SST-I, the last column of Table AI.2 lists $Q_l$ when the lowest 8 modes are decoupled and no suppression by the FPC takes place. The bunch-to-bunch energy spread for this case (Fig. AI.2) is marginally different from the case when both subunits of SST-II are identical (Fig. 4.9).
Table AI.2: SST-II. RF-parameters of modes from the FM passband

<table>
<thead>
<tr>
<th>No</th>
<th>Group $\beta$</th>
<th>$f$ [MHz]</th>
<th>($R/Q$)</th>
<th>$Q_L [10^6]$</th>
<th>$Q_{LR} [10^6]$</th>
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<tr>
<td>1</td>
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<td>1276.412</td>
<td>0.04</td>
<td>40.8</td>
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<td>$\pi/9$</td>
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<td>1.63</td>
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<td>1300.330</td>
<td>0.16</td>
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</table>

Figure AI.2: Computed energy modulation normalized to the energy gain for the nominal TESLA bunch train for the SST-II with de-coupled 8 modes from the $\pi/9$, $2\pi/9$, $3\pi/9$ and $4\pi/9$ group.

**Nb prototype; 2x7-cells**

*Group $\pi/7$ and $2\pi/7$*
Table AI.3: Theoretical Nb prototype. RF-parameters of mode from the FM passband as used for the modeling shown in Fig. 4.19.

<table>
<thead>
<tr>
<th>No</th>
<th>Group $\phi_{0}$</th>
<th>$f$ [MHz]</th>
<th>$R/Q$ [Ω]</th>
<th>$Q_L$ $[10^6]$</th>
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<td>1295.413</td>
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<td>14</td>
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<td>1300.354</td>
<td>0.18</td>
<td>7.8</td>
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</table>
APPENDIX II

Actual field profiles and frequencies measured for two Nb prototypes

Fig. AII.1 shows frequency differences between the subunits in both prototypes as measured before welding. Measured field profiles after welding and the bulk chemical treatment are displayed in eight following figures, the first four for the prototype P1 and next four for the prototype P2.

Figure AII.1: Frequency differences in the fundamental mode passband for subunits of the Nb prototypes; P1 (left) and P2 (right).

Prototype P1
Prototype P2
All diagrams presented here confirm that the frequencies of subunits from the prototype P1 were closer to each other, therefore these subunits decoupled less than in the prototype P2. The P1 subunits have obviously been manufactured with smaller shape errors than the other two.
APPENDIX III

Generic setup for the vertical test and error analysis

Figure AIII.1 presents a generic setup for the vertical cavity test \( Q_0 \) vs. \( E_{\text{acc}} \). The cavity is immersed in liquid helium in the vertical cryostat. Both mechanical cryostat vibrations and helium pressure variation modulate the frequency of the cavity. In the simple scheme shown here, the signal of a mixer is used to adjust the VCO frequency to the actual frequency of cavity. The phase shifters in the scheme allows for maximizing the RF-power transferred to the cavity. A scope and crystal diodes are used to determine \( \beta_L \) from the cavity response to a rectangular pulse and \( Q_L \) from the decay time. Calibration of the cables’ attenuation \( a_{k,j} \) and coupling factors \( c_{f_{k,j}} \) of directional couplers are done routinely prior to testing to minimize the influence of the room temperature and humidity on the result.

Figure AIII.1: Generic setup for the vertical test (not to scale).
Other potential error sources like deterioration in the directivity of couplers, change in diode characteristics, nonlinearity of amplifiers and power meters are examined occasionally. In the following, we will denote the attenuation, coupling and directivity between points \( k \) and \( j \) by \( a_{kj} \), \( cf_{kj} \) and \( d_{kj} \) respectively. For example, \( d_{4,8} \) and \( d_{1,9} \) are the directivities of the dual coupler DC1 for ports 4-8 and 1-9 correspondingly. \( P(k) \) denotes the power at port \( k \). In order to simplify the error analysis for this generic setup, we assume that the microwave power meters PM1-3, the phase shifters, crystal detectors and directional couplers all have matched ports with negligible reflection. This should be carefully verified for the actual setup before each test. In all of the following formulas, impedances of all RF-cables equal 1.

Usually the test is performed in two steps. In the first step, one determines \( \beta_L \) (formulae 3.30-3.32) and \( Q_L \) (formula 3.33) by investigating the response of the cavity to a rectangular input pulse. With these two values, we can determine \( Q_{ext,1} \) of the input antenna. This is usually done at low gradients before the VCO is switched to the cw mode. Measuring the out-coupled power at port 2 and the cavity power at port 1, we can find out \( Q_{ext,2} \) of the pickup probe (field probe). Having \( Q_{ext,1} \) and \( Q_{ext,2} \) we can proceed with the second step, measurement of \( Q \) vs. \( E_{acc} \) in cw mode.

### Error analysis for the step one

\( P_{ref}(1) \) as it is measured at the input of the crystal detector D1, \( P_{Di} \) is given by the equation:

\[
P_{Di} = \left[ P_{ref}(1) \cdot \frac{d_{4,8} + a_{1,4}}{10} + 2 \cos(\phi_4) \right] \sqrt{P_{ref}(1) \cdot P_{for}(4) \cdot \frac{2 \cdot d_{4,8} + d_{8,10} + a_{1,4}}{10} + }
\]

\[
+ P_{for}(4) \cdot \frac{d_{4,8} + d_{8,10} + a_{1,4} \cdot a_{10,11} + a_{11,16}}{10} \cdot j \cdot \frac{a_{8,10} + d_{10,11} + a_{11,16}}{10}
\]

(AIII.1)

where \( \phi_4 \) is the phase between forward and reflected power at port 4. The crystal detector voltage response when operating in the square-law regime is given by:

\[
A_{Di} = \alpha_{Di} \cdot \sqrt{P_{Di}}
\]

(AIII.2)

where \( \alpha_{Di} \) is the sensitivity of the crystal detector D1. At beginning of the RF-pulse, the entire power is reflected by the cavity, which means that:

\[
P_{ref}(1) = P_{for}(4) \cdot (1 - \frac{d_{4,8} + d_{4,8}}{10} \cdot j \cdot \frac{a_{1,4}}{10})
\]

(AIII.3)

Consequently the amplitude \( A_{Di}(0) \) in Figure 3.9 is:

\[
A_{Di}(0) = \alpha_{Di} \cdot \sqrt{P_{for}(4)} \cdot \frac{d_{4,8} + 2a_{1,4} + a_{10,11} + a_{11,16}}{20} \cdot \frac{d_{4,8} + d_{4,8}}{10} \cdot \frac{a_{1,4}}{10} \frac{j^{0.5}}{10} + 2 \cos(\phi_4) \sqrt{(1 - \frac{d_{4,8} + d_{4,8}}{10})}.
\]

(AIII.4)

The amplitude reaches minimum and maximum values at \( \phi_4 = 0 \) and \( \phi_4 = \pi \) respectively:
Both equations indicate that the directivity of the DC1 coupler plays a crucial role for the test accuracy, and hence it is very important to keep the errors as small as possible. The reflected wave at port 1 arriving at port 8 has an amplitude depending on the attenuation \( a_{1,4} \), which “worsens” the directivity effect as we see in the right most exponents in both formulas. Therefore, this attenuation should be low.

The steady state reflection amplitude \( A_{D1}(\tau_p) \) (see Fig. 3.9) is even more sensitive to \( a_{1,4} \) and \( d_{4,8} \), because usually a substantial fraction of the forward wave energy enters the cavity. The reflected power \( P_{ref}(1) \) at \( t = \tau_p \) is:

\[
P_{ref}(1) = \frac{(1 - \beta_L)^2}{(1 + \beta_L)^2} \cdot P_{for}(4) \cdot \left(1 - \frac{d_{4,8} + d_{4,8}}{10}ight) \cdot \frac{a_{1,4}}{10} \tag{AIII.7}
\]

and the measured signal is:

\[
A_{D1}(\tau_p) = \alpha_{D1} \cdot \sqrt{\frac{P_{for}(4) 10}{20}} \cdot \left[ \left(1 - \frac{\beta_L}{1 + \beta_L}\right)^2 \left(1 - \frac{d_{4,8} + d_{4,8}}{10}\right) + 2 \cos(\phi_4) \frac{|1 - \beta_L|}{1 + \beta_L} \right]^{0.5} \tag{AII.8}
\]

Similar to \( A_{D1}(0) \), the signal \( A_{D1}(\tau_p) \) reaches its minimum and maximum value for \( \phi_4 = 0 \) and \( \phi_4 = \pi \) respectively:

\[
A_{D1,min}(\tau_p) = \alpha_{D1} \cdot \sqrt{\frac{P_{for}(4) 10}{20}} \cdot \left[ \left(1 - \frac{\beta_L}{1 + \beta_L}\right)^2 \left(1 - \frac{d_{4,8} + d_{4,8}}{10}\right) - \frac{d_{4,8} - 2a_{1,4}}{10} \right] \tag{AII.9}
\]

\[
A_{D1,max}(\tau_p) = \alpha_{D1} \cdot \sqrt{\frac{P_{for}(4) 10}{20}} \cdot \left[ \left(1 - \frac{\beta_L}{1 + \beta_L}\right)^2 \left(1 - \frac{d_{4,8} + d_{4,8}}{10}\right) + \frac{d_{4,8} - 2a_{1,4}}{10} \right] \tag{AII.10}
\]

When \( \beta_L \to 1 \) then \( P_{ref}(1) \to 0 \) and the full power arriving at \( D1 \) is induced by the forward wave in the DC1 directional coupler. Both expressions fulfill following relations:

\[
F(\beta_L) = F(\frac{1}{\beta_L})
\]

\[37\] Sensitivity for commonly used Wiltron and Agilent detectors when they are terminated with matched resistors is \(-0.1\) mV/\(\sqrt{\mu\text{W}}\).
which means that there is no difference between an under- and over-coupled case. The amplitude $A_{D1}(\tau_p^+)$, measured right after the RF-pulse is switched off, is not influenced by the directivity. At $t = \tau_p^+$ the forward wave is off and $P_{\text{in}}(4) = 0$, consequently $A_{D1}(\tau_p^+)$ is:

$$A_{D1}(\tau_p^+) = \alpha_{D1} \cdot \sqrt{P_{\text{emit}}(1)} \cdot 10^{-\frac{g_{4,8} + g_{1,4}}{10}} \cdot 10^{-\frac{g_{8,10} + g_{11,16}}{20}}$$  \hspace{1cm} \text{(AIII.11)}$$

when $P_{\text{emit}}$ is the emitted power from the cavity via port 1.

In addition to the directivity, nonlinearity of $\alpha_{D1}$ vs. $P$ and errors in the measurements of $a_{k,j}$ and $c_{k,j}$ contribute to the uncertainty in all three amplitudes discussed above. Uncertainty in $\alpha_{D1}$ does not exceed a few percent for a detector operating with its matched load. In general, error for $a_{k,j}$ and $c_{k,j}$ stay within $\pm 0.1\text{dB}$, when they are measured with a calibrated Network Analyzer. Fortunately, the ratios of the amplitudes as they appear in formula 3.30-3.32 are less sensitive to these errors. The error in formula 3.30 can be estimated for the worst case “scenario”. The $\beta_L$ value, when calculated from $A_{D1,\text{min}}(0)$, $A_{D1,\text{max}}(0)$, $A_{D1,\text{min}}(\tau_p^+)$ and $A_{D1,\text{max}}(\tau_p^+)$ stays in the range: $\beta_{L,\text{min}} \leq \beta_L \leq \beta_{L,\text{max}}$, where:

$$\beta_{L,\text{min}} = \frac{A_{D1,\text{min}}(0) - A_{D1,\text{max}}(\tau_p^-)}{A_{D1,\text{min}}(0) + A_{D1,\text{max}}(\tau_p^-)}$$  \hspace{1cm} \text{(AIII.12)}$$

and

$$\beta_{L,\text{max}} = \frac{A_{D1,\text{max}}(0) - A_{D1,\text{min}}(\tau_p^-)}{A_{D1,\text{max}}(0) + A_{D1,\text{min}}(\tau_p^-)}$$  \hspace{1cm} \text{(AIII.13)}$$

In a practical case, $g_{4,8} \geq 20 \text{dB}$ and $d_{4,8} \geq 20 \text{dB}$. Therefore term:

$$\sqrt{1 - \frac{d_{4,8} + g_{4,8}}{10}} \approx 1$$

This allows for simplification in all formulas containing that term. Figure AIII.2 shows how the range of uncertainty in the $\beta_L$ changes, when it is computed from the formula 3.30.

![Figure AIII.2: $\beta_{L,\text{min}}, \beta_{L,\text{max}}$ vs. $(d_{4,8} - 2a_{1,4})$ for three values of $\beta_L$.](image-url)
Figure AIII.3: Normalized uncertainty vs. \((d_{4,8} - 2a_{1,4})\) for three values of \(\beta_1\).

The relative error \((\beta_{L,max} - \beta_{L,min})/\beta_L\) is displayed in Figure AIII.3. The diagram indicates that the smallest uncertainty is for the reflection-free case, which justifies implementation of an adjustable input antenna.

The emitted power \(P_{\text{emit}}(t)\) at \(t = \tau_p^+\) can be expressed by:

\[
P_{\text{emit}}(t) = \frac{\omega W}{Q_{\text{ext}}} = \left(\frac{2}{1 + \beta_L}\right)^2 P_{\text{for}}(4) \cdot 10^{-\frac{a_{1,4}}{10}}
\]

(AIII.14)

where \(P_{\text{for}}(4)\) is, as above, the forward power for \(t \leq \tau_p\). Merging AIII.11 and AIII.14 leads to:

\[
A_{\text{Di}}(\tau_p^+) = \alpha_{\text{Di}} \cdot \frac{2}{1 + \beta_L} \sqrt{P_{\text{for}}(4)} \cdot 10^{-\frac{a_{1,4}}{20}} \cdot 10^{-\frac{a_{1,16}}{20}}
\]

(AIII.15)

The minimum and maximum values of \(\beta_{L}\), when computed with formula 3.31, are:

\[
\beta_{L,\text{min}} = \frac{\beta_L}{|1+10^{-\frac{a_{1,4}}{20}}|} = \frac{\beta_L}{|1+10^{-\frac{a_{1,16}}{20}}|}
\]

(AIII.16)

\[
\beta_{L,\text{max}} = \frac{\beta_L}{|1+10^{-\frac{a_{1,4}}{20}}|} = \frac{\beta_L}{|1+10^{-\frac{a_{1,16}}{20}}|}
\]

(AIII.17)

The normalized error \((\beta_{L,max} - \beta_{L,min})/\beta_L\) as a function of \((d_{4,8} - 2a_{1,4})\) in this case is shown in Figure AIII.4.
Finally, the error analysis for formula 3.32 yields the same expressions AIII.16 and AIII.17 and the identical diagram as shown in Figure AIII.4.

The conclusion is that for good commercially available directional couplers, with directivity $d_{4,8}$ as high as 30dB, and for properly chosen coaxial cable for the input line, with attenuation $a_{1,4}$ typically below 1 dB, the uncertainty $|\delta \beta_L|/\beta_L \leq 12\%$ when $0.5 \leq \beta_L \leq 2$. The error range is the same for the intrinsic quality factor $Q_0$ (see formula 3.34) and the transmitted power of the pickup antenna.

**Error analysis for step two**

The calibration factor $\alpha_{\text{Trans}}$ of the pickup probe is:

$$\alpha_{\text{Trans}} = \frac{E_{\text{acc}}}{\sqrt{P_{\text{Trans}(2)} \cdot 10^{-a_{2,15}+a_{1,5,3}}}} = \frac{E_{\text{acc}}}{\sqrt{P_{\text{Trans}(3)} \cdot 10^{-a_{2,15}+a_{1,5,3}}}} \tag{AIII.18}$$

It has to be determined at the beginning of each vertical test for the cw measurement of $Q_0$ as a function of the accelerating gradient $E_{\text{acc}}$. For the calibration, one has to measure $\beta_L$ and $Q_L$ in the pulse mode and $P_{\text{for}(4)}$ and $P_{\text{Trans}(3)}$ in the cw mode. The accelerating gradient is given by the expression:

$$E_{\text{acc}} = \frac{1}{I_{\text{active}}} \sqrt{Q_L \cdot (R/Q)_{\text{FM}} \cdot \left( \frac{4 \beta_L}{1+\beta_L} \cdot P_{\text{for}(5)} \cdot 10^{-a_{1,4}+a_{5,4}} - P_{\text{Trans}(3)} \cdot (1+\beta_L) \cdot 10^{-a_{2,15}+a_{1,5,3}} \right)} \tag{AIII.19}$$

and the calibration factor $\alpha_{\text{Trans}}$ is:

$$\alpha_{\text{Trans}} = \frac{1}{I_{\text{active}}} \sqrt{Q_L \cdot (R/Q)_{\text{FM}} \cdot \left( \frac{4 \beta_L}{1+\beta_L} \cdot \frac{P_{\text{for}(5)}}{P_{\text{Trans}(3)}} \cdot 10^{-a_{1,4}+a_{5,4}} - (1+\beta_L) \cdot 10^{-a_{2,15}+a_{1,5,3}} \right)} \tag{AIII.20}$$
The transmitted power is usually much smaller than the forward power, by at least an order of magnitude, because $Q_{ext}$ of the pickup probe is at least an order of magnitude higher than $Q_{ext}$ of the input antenna. When $0.5 \leq \beta_L \leq 2$, the forward power $P_{for}(5)$ can be measured with high accuracy (the error resulting from the $DC1$ directivity because only $\sim 10\%$ power is reflected). In this case, the main contribution to the uncertainty in $\alpha_{Trans}$ comes from the errors in $\beta_L$ appearing in the first term of AIII.20. We can simplify this equation for estimation of the $\alpha_{Trans}$ uncertainty:

$$\alpha_{Trans} \cong C \cdot \frac{P_{for}(5)}{P_{Trans}(3)} \cdot \frac{\beta_L}{1+\beta_L}$$  

(AIII.21)

where $C$ is:

$$C = \frac{1}{l_{active}} \sqrt{Q_L \cdot (R/Q)_{FM} \cdot \frac{a_{1,4+4,5A}}{10}}$$  

(AIII.22)

The uncertainty $\partial \alpha_{Trans}$ can be estimated as follows:

$$\alpha_{Trans} + \partial \alpha_{Trans} \cong \alpha_{Trans} \cdot \left(1 + \frac{\beta_L + \partial \beta_L}{1+\beta_L + \partial \beta_L}\right) = \frac{\beta_L \cdot (1 + \frac{\partial \beta_L}{\beta_L})}{(1 + \beta_L)(1 + \frac{\partial \beta_L}{\beta_L})} = \alpha_{Trans} \cdot \left(1 + \frac{1}{2} \cdot \frac{\beta_L}{1+\beta_L} \cdot \partial \beta_L \right)$$

(AIII.23)

Finally, one obtains:

$$\frac{\partial \alpha_{Trans}}{\alpha_{Trans}} = \frac{1}{2} \cdot \frac{1}{1+\beta_L} \cdot \frac{\partial \beta_L}{\beta_L}$$  

(AIII.24)

The analysis showed that results of vertical tests have the following uncertainties resulting from the directivity of $DC1$:

$$\left| \frac{\partial E_{acc}}{E_{acc}} \right| \leq 0.04$$  

(AIII.25a)

$$\left| \frac{\partial Q_0}{Q_0} \right| \leq 0.12$$  

(AIII.25b)

when $0.5 \leq \beta_L \leq 2$ and the directivity of the $DC1$ coupler is close to 30 dB.