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IMPLICATIONS OF CHIRAL DYNAMICS IN THEORIES OF TECHNICOLLOUR

II. - THE MASS OF THE P^+

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A B S T R A C T

In technicolour models, a charged pseudo-Goldstone boson P^+ receives no mass from order α electroweak effects ; in order α^2 , however, a finite mass is generated. We compute this order α^2 electroweak contribution, using chiral perturbation theory, in models with $SU(N) \times SU(N)$ and $SU(2N)$ chiral symmetry. In the latter case, part of the P^+ mass arises from the non-linear interactions of pseudo-Goldstone bosons with one another.

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1. - INTRODUCTION

The spectrum of quark and lepton masses exhibits large mass ratios in an apparently hierarchical pattern. It is not unlikely that the Higgs bosons, when they are found, will also exhibit a hierarchical spectrum of masses. In weak interaction theories with fundamental scalars, it is difficult to arrange that such hierarchies are natural, since the masses and couplings of the Higgs mesons are adjustable parameters which must be severely constrained. If, however, the weak interaction symmetry is broken dynamically, by the effects of a new set of strong interactions ¹⁾, the Higgs meson couplings are generated by these strong interactions. Strong interaction theories of gauge mesons and fermions contain chiral symmetries which are only weakly broken ; these symmetries can constrain the Higgs couplings and produce mass hierarchies in which the ratio of masses is the dimensionless parameter of chiral symmetry breaking. For the case of the Higgs spectrum, this phenomenon is, in fact, generic : Eichten and Lane ²⁾ have shown that, under rather general conditions, a pair of charged pseudo-Goldstone bosons arising from the strong interactions which break the electroweak symmetry receive mass from their electroweak interactions not in order α , but in order α^2 . More recently, models of fermion mass generation through dynamical symmetry breaking have been constructed which exhibit similar order α hierarchies ³⁾.

When such hierarchies appear in conventional perturbation theory, it is generally straightforward to compute those masses which appear to leading order in α , but the analysis of masses at the lower rungs of the hierarchy becomes increasingly complex. In theories of dynamical symmetry breaking, the situation is, at first sight, still worse. Even the computation of order α masses requires the evaluation of strong interaction effects (or, at least, the phenomenological parametrization of these effects). One would expect to find effects appearing in higher orders in α to depend on new phenomenological parameters, so that one could not predict the magnitudes of such effects.

In this paper, we will show how this problem may be evaded. We will follow a strategy proposed some time ago by Li, Langacker, and Pagels ⁴⁾ for application to the usual strong interactions. These authors noted that corrections to the predictions of current algebra may involve processes which are infra-red sensitive and that such processes are, in general, enhanced by a power of $\log(1/\epsilon)$, where ϵ is the dimensionless parameter of explicit chiral symmetry breaking. In the usual strong interactions, $\epsilon = m_\pi^2/m_\rho^2$; a term involving $\log(m_\rho^2/m_\pi^2)$ might arise from an infra-red divergence which is cut off by the pion mass. Li,

Langacker, and Pagels proposed that one could estimate corrections to current algebra by retaining only terms with this logarithmic enhancement. The coefficients of such logarithms depend only on the parameters of current algebra ; no new phenomenological parameters are required.

This methodology extends straightforwardly to the case of weak interaction dynamical symmetry breaking induced by a new strong interaction which we will refer to as technicolour. We will use it to compute, in some specific models of technicolour, the electroweak contributions to the masses of the charged pseudo-Goldstone bosons identified by Eichten and Lane. In the models we will discuss, the parameter of the symmetry breaking is α ; the effects we will compute are of order $\alpha^2 \log(1/\alpha)$.

In isolating the infra-red sensitive contributions to these masses, we will make use of an effective Lagrangian for technicolour dynamics which describes the low energy couplings of Goldstone bosons ⁵⁾. The characteristic mass scale of this Lagrangian is the scale m_T of pseudo-Goldstone boson masses, related to the scale of technicolour strong interactions M_{TC} by $m_T^2 \sim \alpha M_{TC}^2$. Contributions to the strong interaction dynamics which are infra-red sensitive, with logarithmically divergent integrals extending downward to m_T , appear in the perturbation theory of the effective Lagrangian as logarithmic ultra-violet divergences which must be cut off at the scale M_{TC} . Our analysis will be carried out by isolating such logarithmic ultra-violet divergences from the perturbation theory of the effective Lagrangian.

The construction of effective Lagrangians for the Goldstone bosons of technicolour was the subject of the first paper of this series ⁶⁾ ; we will refer to this paper henceforth as I. For the analysis given here we will make use only of the results of I and not of the details of the formalism developed there. These results will be reviewed in this paper as they are needed. In our previous paper, however, we have also reviewed the general elements of technicolour theories and defined specific models for our investigation. We advise the reader to study this discussion, Section 2 of I, before reading the present paper, since we will draw liberally here on concepts and terminology introduced there. We will also use, without a detailed review, the group theoretical conventions set out in the Appendix of I.

We will begin the analysis of this paper, in Section 2, by considering technicolour models in which the chiral symmetry group is $SU(N) \times SU(N)$, spontaneously broken to $SU(N)$. We will first review the result of Eichten and Lane

that, in such models, a pair of charged pseudo-Goldstone bosons $P^\pm$ may not acquire mass in order α . We will then identify the diagrams of chiral perturbation theory which can produce P mass terms of order α^2 , enhanced by a logarithm of $1/\alpha$. Most of these diagrams, however, will be seen to be zero by symmetry arguments ; the remainder are trivially evaluated.

Among the diagrams which vanish by symmetry in this special situation are all of those which contain interactions among Goldstone bosons only. These diagrams, however, do not vanish for more general patterns of symmetry breaking. When they do contribute, their group theoretic structure is interesting if complex ; we will devote the remainder of the paper to its analysis. In Section 3, we will review the form of the Goldstone boson interactions which follow from the effective Lagrangian of I introducing, in the process, a notation which illuminates the group theory factors contained in these vertices. In Section 4, we will compute the diagrams involving these vertices which contribute to the $P^\pm$ mass, reducing them formally to a set of group theory weights. The evaluation of these weights may, in turn, be simplified by the deft use of symmetry. This simplification will be illustrated in Section 5 for a toy model ; in Section 6, it will be applied to compute the $P^\pm$ mass to order $\alpha^2 \log(1/\alpha)$ in a realistic model due to Dimopoulos ⁷⁾.

We should make clear that, although our computation will accurately estimate the electroweak contribution to the $P^\pm$ mass, there may be other, comparable, contributions to the mass of this particle. A realistic technicolour theory must contain interactions which link technifermions to ordinary fermions, the so-called extended technicolour or sideways interactions. The contribution of these interactions to the $P^\pm$ mass has been estimated to be of order 10 GeV, roughly roughly the same size as the electroweak contribution, with a large uncertainty ^{2),8),9)}. This uncertainty clouds the connection between our calculations and phenomenology.

2. - $P^\pm$ MASSES IN MODELS WITH $SU(N) \times SU(N)$ CHIRAL SYMMETRY

In this section, we will compute the electroweak contribution to the mass of the lightest charged Higgs meson in technicolour models for the relatively simple case of $SU(N) \times SU(N)$ chiral symmetry. The arguments of this section, which provide the basis of our more general analysis, have been sketched earlier in Ref. 10).

Our argument begins from a general result of Eichten and Lane ²⁾ : in technicolour models with several weak doublets of technifermions, there are at least two neutral mesons which receive no mass from the electroweak interactions and a pair of charged mesons which receive mass only in order α^2 . Eichten and Lane refer to these mesons as P^0 , P'^0 , $P^\pm$, and we will follow that notation in this paper. For a theory with L weak doublets of technifermions (and, therefore, $2L$ technifermions in all), choose an $L \times L$ matrix b which commutes with weak hypercharge and has the properties

$$tb = 0 \quad , \quad tb^2 = 1 \quad . \quad (2.1)$$

Then these four light mesons are pseudo-Goldstone bosons of the technicolour theory corresponding (in the notation of I) to the four matrices

$$X = \left(\begin{array}{c|c} g & 0 \\ \hline 0 & g^T \end{array} \right) , \quad (2.2)$$

with $2L \times 2L$ matrices g given by the direct products

$$g = \tau_a \otimes b \quad , \quad g = \frac{1}{2} 1_2 \otimes b \quad . \quad (2.3)$$

For the SU and O models defined in Section 2 of I, b is proportional to the matrix B , Eq. (2.6) of I.

To prove the result of Eichten and Lane for the neutral mesons, consider the right-handed $SU(2L)$ transformations generated by the matrices

$$\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1_2 \otimes b \end{array} \right) \quad , \quad \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \tau_3 \otimes b \end{array} \right) \quad . \quad (2.4)$$

These chiral symmetries leave invariant the terms coupling the electroweak bosons to technifermions ; they are exact symmetries of the technicolour theory including the electroweak perturbation. [This may be checked explicitly for the couplings given in (2.5) of I.] The spontaneous breaking of these chiral symmetries implies the existence of two mesons P^0 , P'^0 which remain massless after accounting for the effects of the electroweak interactions. [These mesons may receive mass from the sideways interactions ²⁾ not included in (2.5) of I.]

For the $P^\pm$, the situation is slightly more complex. The left-handed chiral symmetries generated by

$$\left(\begin{array}{c|c} \tau_\pm \otimes b & 0 \\ \hline 0 & 0 \end{array} \right), \quad \tau_\pm = \tau_1 \pm i\tau_2, \quad (2.5)$$

leave invariant the coupling of the $U(1)$ gauge bosons B_μ . The right-handed chiral symmetries generated by

$$\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \tau_\pm^T \otimes b \end{array} \right) \quad (2.6)$$

leave invariant the coupling of the $SU(2)$ gauge bosons W_μ^a . Neither set of symmetries preserves the whole coupling of technifermions to $SU(2) \times U(1)$, but the two together ensure that neither W_μ^a nor B_μ exchange alone can give mass to the $P^\pm$. The electroweak contributions to the P mass m_P must then involve both of these bosons. If the W_μ^a and B_μ coupling constants are given by g and g' , respectively, let

$$\alpha_W = \frac{g^2}{4\pi}, \quad \alpha'_W = \frac{g'^2}{4\pi}. \quad (2.7)$$

Then m_P^2 begins in order $\alpha_W \cdot \alpha'_W$.

Let us now isolate the contributions to m_P of order $\alpha_W \cdot \alpha'_W$, enhanced by a power of $\log(1/\alpha)$. The graphs of chiral perturbation theory which might contribute to m_P in order α^2 are shown in Fig. 1. The vertices in the diagram of class III are the Goldstone boson couplings of the effective Lagrangian. These will be exhibited in Section 3; for the moment we should recall that an n boson vertex contains a factor $(f_\pi)^{2-n}$, where f_π is the technicolour analogue of the pion decay constant. To confirm our estimate of these diagrams, note that each diagram must give a contribution of dimension $(\text{mass})^2$, this contribution must be built from masses of particles in the diagram and factors of f_π . Then the three classes give contributions of order

$$\begin{aligned} \text{class I} &\sim \alpha m_W^2, \\ \text{class II} &\sim \alpha m_\pi^2, \\ \text{class III} &\sim m_T^4 / f_\pi^2, \end{aligned} \quad (2.8)$$

times an infra-red logarithm. In (2.8), m_T is the mass of a generic technicolour pseudo-Goldstone boson (we henceforth abbreviate this to "technion"); (m_W^2/f_π^2) , (m_Z^2/f_π^2) , and (m_T^2/f_π^2) are all formally of order α .

Classes I and II are most easily discussed in the Landau gauge, where the diagrams (a) give no contribution at zero external momentum. The Feynman rule for the $2W$ vertex, given in Fig. 1 of I, indicates that diagram Ib - and, therefore, the full contribution of $W^\pm$ exchange - vanishes. This is, in fact, necessary : the charged W contains only W_μ^a and not B_μ , so its coupling involves only α_W , not α'_W . In models with a chiral symmetry $SU(N) \times SU(N)$, one can show just as easily that the diagrams of class III vanish : the information that the chiral symmetries (2.5), (2.6) are broken is communicated to these diagrams through the mass matrix for the technions in the loop and through the vertices associated, in the chiral Lagrangian formalism, with this mass term. However, we showed in Eq. (5.20) of I that, for $SU(N) \times SU(N)$ models, these symmetry breaking terms receive contributions only from bosons with both left- and right-handed couplings. Since the couplings of the W_μ^a are purely left-handed, the graphs of class III are independent of α_W and so can contribute nothing to m_P . With some effort, one can check this conclusion explicitly by making use of the results of Section 4.

Now only the diagram IIb remains. This diagram is readily evaluated using the Feynman rules given in Fig. 1 of I. For the P^+ $I_3 = 1$ and $Q = 1$, so we find

$$m_P^2 = -ie^2 g_{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \left(\frac{1}{k^2 - m_Z^2} - \frac{1}{k^2} \right). \quad (2.9)$$

The apparent quadratic divergences cancel ; this cancellation accords with Eq. (5.1) of I, where we showed that the quadratic divergences of diagrams with one gauge boson exchange have the form of the order α technion mass matrix. The logarithmic divergence of (2.9) has the form :

$$m_P^2 = \frac{3\alpha}{4\pi} m_Z^2 \log \frac{\Lambda^2}{m_Z^2}. \quad (2.10)$$

The cut-off Λ should be set as some scale characteristic of the technicolour theory.

Our reduction of the expression for m_p^2 to the contribution from Z^0 exchange has essentially justified the original estimate of the electroweak contribution to m_p^2 made by Eichten and Lane²⁾. Taking Λ to vary between 250 GeV and 1 TeV, and setting $m_Z \approx 90$ GeV, we obtain their estimate of the electroweak contribution to m_p :

$$(m_p^2)_{EW} \cong (5-8 \text{ GeV})^2. \quad (2.11)$$

This estimate of the Z^0 exchange contribution to m_p has been refined by several authors^{9),11)} ; however, these authors do not include in their values for m_p the effects of hard strong interaction contributions, of order α^2 but not logarithmically enhanced.

3. - GOLDSTONE BOSON INTERACTIONS

The graphs of class III in Fig. 1 were found to be zero, in the previous section, by the use of an argument peculiar to models with $SU(N) \times SU(N)$ chiral symmetry. In more general patterns of chiral symmetry breaking, the contribution to the P^+ mass from these diagrams with virtual technion exchanges may be comparable to that from Z^0 exchange. We will devote the rest of this paper to simplifying and estimating this contribution. In this section, we will begin that analysis by recalling the forms of the technion mass matrix and vertices constructed in I.

The formalism of I produced a low energy effective Lagrangian for the Goldstone bosons associated with the spontaneous breaking of a group G of chiral symmetries. The various terms of this Lagrangian were given in terms of the structure constants of G . For example, the quartic interaction of Goldstone bosons contained in the G symmetric term of the Lagrangian was found to be [Eq. (3.17) of I]

$$\mathcal{L}_{4\pi} = \frac{1}{6f_\pi^2} (t_{axy} \pi_x \partial_\mu \pi_y)^2, \quad (3.1)$$

where f_π is the pion decay constant of the technicolour strong interactions and t_{abc} is the structure constant defined by

$$[G_a, G_b] = t_{abc} G_c. \quad (3.2)$$

The final form of the class III diagrams of Fig. 1 will involve a complex tangle of such structure constants. To simplify this form, we will introduce a diagrammatic notation for these objects. This notation generalizes a group theoretic notation developed by Cvitanovic¹²⁾. Let t_{abc} be represented by a vertex

$$t_{abc} = \begin{array}{c} b \\ | \\ a \text{ --- } \text{--- } c \end{array} \quad (3.3)$$

Cvitanovic has labelled such vertices "bird tracks". Since t_{abc} is totally antisymmetric, one should take care to read vertices of the type (3.3) clockwise. The chiral Lagrangian formalism often requires us to partition the generators of G into those which are and are not spontaneously broken. We will denote the projectors onto broken and unbroken generators by marking the links connected to vertices

$$\text{---} \bullet \text{---} \quad \text{and} \quad \text{---} \ominus \text{---} , \quad (3.4)$$

respectively. In I, we divided gauged generators of G into broken and unbroken pieces by the notation :

$$G_A = X_{Z(A)} + T_{I(A)} \quad (3.5)$$

Diagrammatically, we may express

$$X_{Z(A)} = A \text{ ---} \bullet \text{---} , \quad T_{I(A)} = A \text{ ---} \ominus \text{---} . \quad (3.6)$$

If the symmetry breaking follows a symmetric space pattern [preserving a parity operation as in (2.1) of I], the structure constants are restricted so that only the two types of vertices

$$\begin{array}{c} \oplus \\ | \\ \oplus \text{ --- } \oplus \end{array} \quad \text{and} \quad \begin{array}{c} \bullet \\ | \\ \oplus \text{ --- } \bullet \end{array} \quad (3.7)$$

are non-vanishing.

This notation allows us to express rather compactly the vertices derived in I. The term (3.1), for example, takes the form

$$\mathcal{L}_{4\pi} = -\frac{1}{6f_\pi^2} \begin{array}{c} \partial^\mu \pi \\ \pi \end{array} \text{---} \bigcirc \text{---} \begin{array}{c} \partial_\mu \pi \\ \pi \end{array} \quad (3.8)$$

The Goldstone bosons π should be understood to be associated with broken generators. Equation (3.8) is the only vertex we will require from the G symmetric terms in the effective Lagrangian. This Lagrangian, however, also contains a counter-term induced by one gauge boson exchange which does not respect the full G invariance. This counter-term contains several elements which will enter our analysis. The first of these is the order α mass matrix of pseudo-Goldstone bosons, given by [Eq. (5.11) of I] :

$$m_{xy}^2 = \frac{M^2}{4\pi} \left(x \begin{array}{c} A \\ \bigcirc \end{array} \text{---} \bullet \text{---} \begin{array}{c} A \\ \bigcirc \end{array} y - x \begin{array}{c} A \\ \bullet \end{array} \text{---} \bigcirc \text{---} \begin{array}{c} A \\ \bullet \end{array} y \right) \quad (3.9)$$

The counter-term also includes three- and four-boson vertices [Eqs. (5.14) and (5.15) of I] :

$$\delta \mathcal{L}_{3\pi} = i \frac{M^2}{6\pi f_\pi} \begin{array}{c} \pi \\ \pi \\ \pi \end{array} \begin{array}{c} A \\ \bigcirc \end{array} \text{---} \bullet \text{---} \begin{array}{c} \pi \\ \pi \\ \pi \end{array} \quad (3.10)$$

$$\delta \mathcal{L}_{4\pi} = \frac{M^2}{24\pi f_\pi^2} \left(\begin{array}{c} \pi \\ \pi \\ \pi \\ \pi \end{array} \begin{array}{c} A \\ \bigcirc \end{array} \text{---} \bullet \text{---} \begin{array}{c} \pi \\ \pi \\ \pi \\ \pi \end{array} \begin{array}{c} A \\ \bigcirc \end{array} - \begin{array}{c} \pi \\ \pi \\ \pi \\ \pi \end{array} \begin{array}{c} A \\ \bullet \end{array} \text{---} \bigcirc \text{---} \begin{array}{c} \pi \\ \pi \\ \pi \\ \pi \end{array} \begin{array}{c} A \\ \bullet \end{array} \right) \quad (3.11)$$

In (3.9)-(3.11), a sum over gauged generators G_A should be understood. For a fixed G_A , each of these terms carries the quantum numbers associated with the exchange of the corresponding gauge boson. We will refer to these terms simply as the contribution of that gauge boson (of, e.g., W_μ^a exchange), even though only the matrices associated with that boson appear explicitly.

The counter-term may also yield a term linear in the π fields. This term, however, must vanish if the vacuum is correctly aligned^{10),11)}; this requirement yields a condition on the vacuum alignment [Eq. (5.17) of I] which may be written :

$$\begin{array}{c} \gamma \\ | \\ \bullet \\ | \\ \text{---} \bigcirc \text{---} \bullet \text{---} A \end{array} = 0 \quad . \quad (3.12)$$

Diagrams contributing to m_P may be built from the vertices (3.8), (3.10), and (3.11). In Fig. 1, we have denoted these diagrams by IIIa, IIIb, and IIIc, respectively.

4. - DIAGRAMS WITH VIRTUAL TECHNIIONS

In this section, we will analyze the diagrams which give the contribution of virtual Goldstone bosons to the P^+ mass - class III of Fig. 1 - for the symmetry breaking pattern in which $SU(2N)$ chiral symmetry breaks to $O(2N)$. Our basic goal will be to isolate the logarithmic divergences of these diagrams and, thus, their order $\alpha^2 \log(1/\alpha)$ contribution to the P^+ mass. It will be convenient, however, to work out these diagrams somewhat more generally, with Goldstone bosons of arbitrary quantum numbers on the external legs. This will allow us to check, among other things, that the more dangerous divergences of these graphs do not introduce interactions of new structure.

Let us therefore compute the contribution of this class of graphs to the pseudo-Goldstone boson mass matrix. We begin with the diagram IIIa, which involves the four-point vertex of the G symmetric Lagrangian. Using the form of this vertex given in (3.8), and evaluating the diagram at zero external momentum, we find the contribution of this diagram to be

$$\delta m_{xy}^2 \Big|_{IIIa} = \frac{i}{3f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} (k^2 - m^2)^{-1}_{wz} \cdot k^2. \quad \begin{array}{c} w \quad \gamma \\ | \quad | \\ \text{---} \bigcirc \text{---} \\ | \quad | \\ x \quad y \end{array} \quad (4.1)$$

Here the matrix $(m^2)_{wz}$ is the order α pseudo-Goldstone boson mass matrix (3.9). The integral is, superficially, quartically divergent. This divergence, however, is physically unwanted, since it would be a mass term for Goldstone bosons

independent of the chiral symmetry breaking parameters in $(m^2)_{WZ}$. Indeed, when one performs the integral using dimensional regularization, the pole at $d = 0$ is cancelled by a factor d from the k^2 in the numerator. The final result for this diagram is shown in Fig. 2.

Of the two diagrams which involve vertices associated with the technion mass counter-term, IIIb is superficially quadratically divergent, while IIIc is only logarithmically divergent. By evaluating IIIb and combining it with IIIa, we can resolve the form of the quadratic divergence in the diagrams of class III. To compute IIIb, one must contract the vertex (3.11) with itself in all possible ways. The result, after the integral has been performed using dimensional regularization, is shown in Fig. 3. The quadratic divergences we seek are then given by the poles at $d = 2$ contained in Figs. 2 and 3. At $d = 2$, the group theory factor in Fig. 2 takes the form

$$\frac{M^2}{4\pi} \left(x \text{---} \text{---} y - x \text{---} \text{---} y \right) \quad (4.2)$$

The first diagram here cancels the first diagram of Fig. 3. The second diagram is almost, but not quite, of the form of the second diagram of Fig. 3. To bring it into this form, we may use the Jacobi identity, which reads, in this diagrammatic notation ¹²⁾,

$$c \text{---} \text{---} d - c \text{---} \text{---} d = c \text{---} \text{---} d \quad (4.3)$$

Using (4.3), we may rewrite :

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \quad (4.4)$$

The last term makes the cancellation. The remaining diagrams in (4.4) and in the $d \rightarrow 2$ limit of Fig. 3 may be simplified greatly by the use of a set of Lie algebraic identities listed in the Appendix. These terms combine to the final form :

$$\delta m_{xy}^2 \stackrel{d \rightarrow 2}{\sim} - \frac{1}{3f_\pi^2} \frac{M^2}{4\pi} \frac{\Gamma(1-d/2)}{4\pi} \times (16L+6) \times \left(x \begin{array}{c} A \quad A \\ \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \hline x \quad y \end{array} - x \begin{array}{c} A \quad A \\ \bullet \quad \bullet \\ | \quad | \\ \circ \quad \circ \\ \hline x \quad y \end{array} \right) \quad (4.5)$$

Here L is, as above (2.1), the number of weak doublets of technifermions, so that the chiral symmetry group of the model is $SU(4L)$. The divergence (4.5) has precisely the structure of the mass term (3.9) and may be absorbed by a renormalization of the coefficient M^2 of this mass counter-term.

The logarithmic divergences of IIIa and IIIb can also be extracted from Figs. 2 and 3, as the poles of these expressions at $d = 4$. To combine the group theory factors which result into as simple a form as possible, we convert each factor into a sum of diagrams of the form of rings with six external lines by the use of the Jacobi identity (4.3). We may, for example, recast a contribution to the last diagram in Fig. 3 :

$$\begin{aligned} & \begin{array}{c} B \quad B \\ \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \hline A \quad A \end{array} \\ &= \begin{array}{c} B \quad B \\ \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \hline A \quad A \end{array} - \begin{array}{c} B \quad B \\ \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \hline A \quad y \end{array} \\ &- x \begin{array}{c} B \quad B \\ \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \hline A \quad y \end{array} + x \begin{array}{c} B \quad B \\ \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \hline A \quad A \end{array} \quad (4.6) \end{aligned}$$

It is then possible to combine terms and represent the complete logarithmic divergence of the graphs with quartic vertices in the form given in Fig. 4.

The graph IIIc is superficially logarithmically divergent. Since we wish only to extract the logarithmically divergent piece of IIIc, we may, in evaluating this graph, ignore factors of m^2 in the internal propagators. Still, the result of computing IIIc looks complex ; the vertex (3.10), contracted with itself in all possible ways, produces a sum of 18 group-weight diagrams. These may be simplified greatly, however, by rewriting them as ring diagrams in the manner of (4.6). Almost all of these ring diagrams assemble into pairs which may be seen to vanish by the use of the Jacobi identity (4.3) and the stability condition (3.12). For example, one may assemble

$$\begin{array}{c}
 \text{Diagram 1} - \text{Diagram 2} \\
 = \text{Diagram 3} = 0
 \end{array} \quad (4.7)$$

The result of these cancellations is the compact expression given in Fig. 5. The sum of Figs. 4 and 5 gives the order $\alpha^2 \log(1/\alpha)$ contribution from the diagrams of class III to the technion mass matrix.

5. - THE P^+ MASS IN A TOY MODEL

The group theory weights represented by Figs. 4 and 5 are straightforward but extremely tedious to evaluate directly. The task of evaluating them may, however, be simplified by making use of the various symmetry restrictions on the appearance of a P^+ mass which we have demonstrated in Section 2. It is easiest to understand this reduction of the diagrams of Figs. 4 and 5 by examining it first in a simplified scheme of gauge couplings to technifermions. We devote this section to the analysis of such a simplified model.

Our arguments in Section 2 require that we explore a model whose chiral symmetry is larger than $SU(N) \times SU(N)$. Let us, then, consider a model of a single weak doublet of technifermions (U, D) which transform under a real representation

of the technicolour group. The chiral symmetry group G of this model is $SU(4)$; we assume that this group is spontaneously broken to $O(4)$. For the coupling of the model to gauge fields, we assume the following : if ψ_0 is the four-component vector of left-handed fermions

$$\psi_0 = \left(U_L, D_L, (U_R)^c, (D_R)^c \right), \quad (5.1)$$

analogous to (2.3) of I, we choose the covariant derivative acting on ψ_0 to be

$$D_\mu \psi_0 = \left[\partial_\mu - i \left\{ g W_\mu \begin{pmatrix} \tau_3 & 0 \\ 0 & 0 \end{pmatrix} + g' B_\mu \begin{pmatrix} 0 & 1 \\ 1 & -\tau_3 \end{pmatrix} \right\} \right] \psi_0. \quad (5.2)$$

Chiral symmetry breaking combines W_μ and B_μ into linear combinations representing massless and massive mesons A_μ and Z_μ . We will use the notation (2.7) to refer to the two coupling constants of (5.2).

To discuss the fate of the Goldstone bosons in this model, let us first divide the generators of G into two classes, calling the generators of the included $SU(2) \times SU(2)$ [or, more generally, the included $SU(N) \times SU(N)$] "diagonal" and the remainder "off-diagonal". For models in which $SU(2N)$ chiral symmetry is broken to $O(2N)$, the off-diagonal generators take the block form

$$G = \begin{pmatrix} & A \\ A^\dagger & \end{pmatrix}. \quad (5.3)$$

For unbroken generators, A is antisymmetric ; for broken generators, A is symmetric¹⁰⁾. Note that these matrices come in pairs since, in either case, A may be taken either real or pure imaginary. In our model there are three pairs of broken off-diagonal generators, corresponding to the three symmetric 2×2 matrices. The corresponding Goldstone bosons receive mass from one W or one B exchange ; these masses may be readily computed from (3.9)^{*)}.

The model also contains three Goldstone bosons corresponding to diagonal generators ; these are analogous to the π^+ , π^- , π^0 of the usual strong interactions. The π^0 is the fictitious Goldstone boson which combines with the Z_μ to give it mass. The $\pi^\pm$, which correspond to the generators

*) One (mass)² is always negative. This pathology of our toy model does not affect the computation done in this section ; it is repaired in the realistic model discussed in the next section.

$$X_y = \left(\frac{\tau_y}{\tau_y^T} \right), \quad y = 1, 2, \quad (5.4)$$

eventually receive mass from gauge boson exchanges, but an argument analogous to that of (2.5) and (2.6) indicates that they cannot receive mass from a graph which does not involve both W_μ and B_μ . These particles are, then, simpler analogues of the $P^\pm$. They receive mass for the first time in order α^2 : one contribution to their mass is precisely (2.10). A second contribution comes from the graphs in class III of Fig. 1; that is, from the diagrams of Figs. 4 and 5. As we discussed in Section 2, this contribution would have been absent if the chiral group had been $SU(2) \times SU(2)$, since the W_μ coupling is purely left-handed. It is not zero, however, in the present case of an $SU(4)$ chiral group.

To compute this contribution, we may begin by noting that each diagram of Figs. 4 and 5 contains a closed loop indicating a sum over all broken and unbroken generators, diagonal and off-diagonal. But all of the generators of G associated with the external lines in these diagrams are of the diagonal type. The commutator of a diagonal generator with an off-diagonal generator yields another off-diagonal generator, and the commutator of two diagonal generators yields a third such generator; hence, each indicated sum may be divided into two pieces, in which only diagonal or only off-diagonal generators, respectively, flow around the closed loop. The diagonal generators are the generators of the $SU(2) \times SU(2)$ subgroup of $SU(4)$; our formalism has been constructed in such a way that restricting a trace over generators to diagonal generators only produces precisely the chiral perturbation theory that would have been obtained if we had begun with an $SU(2) \times SU(2)$ chiral symmetry group. The diagrams of class III, however, vanish in this case. In computing the diagrams of Figs. 4 and 5, then, we need sum only over off-diagonal generators in the loops. Note that, in the $SU(4)$ chiral group, there is only one pair of unbroken off-diagonal generators; this makes these sums particularly simple.

From this point, it is not difficult to evaluate Figs. 4 and 5 directly. The G_A which appear in these diagrams are those of (5.2):

$$\begin{aligned} \text{for } W_\mu: \quad G_A &= g \left(\frac{\tau_3}{0} \right), \\ \text{for } B_\mu: \quad G_A &= -g' \left(\frac{0}{\tau_3} \right). \end{aligned} \quad (5.5)$$

It is convenient to express these as linear combinations of

$$\begin{aligned} T_3 &= \left(\frac{\tau_3}{- \tau_3} \right) = 3 \text{ --- } \bigcirc \text{ --- } \text{Y} , \\ X_3 &= \left(\frac{\tau_3}{\tau_3} \right) = 3 \text{ --- } \bullet \text{ --- } \text{Y} . \end{aligned} \quad (5.6)$$

Then Figs. 4 and 5 may be converted to diagrams whose external lines correspond to these generators and those of (5.4). The $SU(4)$ structure constants yield a number of identities which, if we let y denote a $\pi^\pm$ generator (5.4), and assume that unlabelled generators are off-diagonal, we may write as follows :

$$\begin{aligned} \text{---} \bigcirc \text{ ---} \uparrow \text{---} 3 &= 0 , & \text{---} \bigcirc \text{ ---} \uparrow \text{---} 3 \text{ ---} \bullet \text{ ---} \uparrow \text{---} y \text{ ---} \bigcirc \text{ ---} &= 0 , \\ \text{---} \bullet \text{ ---} \uparrow \text{---} 3 \text{ ---} \bullet \text{ ---} \uparrow \text{---} 3 \text{ ---} \bigcirc \text{ ---} &= 0 . \end{aligned} \quad (5.7)$$

The identities (5.7) simplify Figs. 4 and 5 to the form :

$$\begin{aligned} (\delta m_{xy}^2)_{\Pi a+b} &\sim \frac{1}{3f_\pi^2} M^2 \left(\frac{\alpha_W + \alpha'_W}{4} \right)^2 \frac{\Gamma(2-d/2)}{(4\pi)^2} \times \\ &\times \left[7 \begin{array}{c} \text{Diagram 1: Circle with 6 vertices, 3 external lines labeled 3, 3, 3.} \\ \text{Diagram 2: Circle with 6 vertices, 3 external lines labeled 3, 3, 3.} \end{array} - \begin{array}{c} \text{Diagram 3: Circle with 6 vertices, 3 external lines labeled 3, 3, 3.} \\ \text{Diagram 4: Circle with 6 vertices, 3 external lines labeled 3, 3, 3.} \end{array} + \right. \\ &\quad \left. + 6 \begin{array}{c} \text{Diagram 5: Circle with 6 vertices, 3 external lines labeled 3, 3, 3.} \end{array} \right] , \end{aligned} \quad (5.8)$$

$$\begin{aligned}
 (\delta m_{xy}^2)_{IIIc} &\sim -\frac{4}{f_\pi^2} M^2 \left(\frac{\alpha_W - \alpha'_W}{4} \right)^2 \times \\
 &\times \frac{\Gamma(2-d/2)}{(4\pi)^2} \times \text{Diagram} \quad . \quad (5.9)
 \end{aligned}$$

The remaining diagrams may be evaluated straightforwardly ; for example,

$$\begin{aligned}
 \text{Diagram} &= 2 \times \text{tr} \left([[[[[[T_k, X_x], T_3], T_3], \right. \\
 &\quad \left. X_y], X_3], X_3] T_k \right) \quad . \quad (5.10)
 \end{aligned}$$

where T_k is either one of the two conserved off-diagonal generators, normalized to $\text{tr}(T_k)^2 = 1$. Each of the four diagrams turns out to equal $(2\delta_{xy})$. Then (5.8), (5.9) become :

$$\begin{aligned}
 (\delta m_{xy}^2)_{IIIa+b} &\sim \frac{1}{2f_\pi^2} M^2 (\alpha_W + \alpha'_W)^2 \frac{\Gamma(2-d/2)}{(4\pi)^2} \delta_{xy} \quad , \\
 (\delta m_{xy}^2)_{IIIc} &\sim -\frac{1}{2f_\pi^2} M^2 (\alpha_W - \alpha'_W)^2 \frac{\Gamma(2-d/2)}{(4\pi)^2} \delta_{xy} \quad . \quad (5.11)
 \end{aligned}$$

The terms in α_W^2 and $\alpha'_W{}^2$ cancel, as they must ; the non-vanishing contribution is proportional to $\alpha_W \alpha'_W$. The factor $\Gamma(2-d/2)$ represents a logarithm which reaches upward from the mass of the technions which appear in the loops of Fig. 1 III to an ultra-violet cut-off of the order of the technicolour mass scale. The technions in the loops are of the off-diagonal type ; they have $(\text{mass})^2$ of order $\alpha_W M^2$ or $\alpha'_W M^2$, given by (3.9). If $\alpha_W > \alpha'_W$, we may set

$$\Gamma(2-d/2) \longrightarrow \log \left(\frac{\Lambda^2}{\alpha_W M^2} \right) \quad . \quad (5.12)$$

Then we have

$$\delta m^2|_{III} = \frac{\alpha_W \alpha'_W}{8\pi^2 f_\pi^2} M^4 \log\left(\frac{\Lambda^2}{\alpha_W M^2}\right)$$

as the contribution of the class III diagrams of our toy model to the $\pi^\pm$ masses.

The various cancellations which occurred in the last step of this computation might guide us in simplifying δm^2 in a more involved model. Since we knew in advance that $\delta m^2 \sim \alpha_W \alpha'_W$, we need have computed only the term of this structure. But we can simplify further : consider the consequence of converting the right-handed current to which B_μ couples in (5.2) to a left-handed current :

$$g' B_\mu \left(\begin{array}{c|c} 0 & \\ \hline & -\tau_3 \end{array} \right) \longrightarrow g' B_\mu \left(\begin{array}{c|c} \tau_3 & \\ \hline & 0 \end{array} \right) . \quad (5.13)$$

On the one hand, this change merely multiplies the broken part of the B_μ current by (-1). That introduces an extra factor of (-1) into the terms of Fig. 5 proportional to $\alpha_W \alpha'_W$; otherwise, it leaves Figs. 4 and 5 unchanged. On the other hand, this modification yields a gauge coupling which is completely invariant to the transformations (2.6) ; this ensures that the $\pi^\pm$ remain massless. Thus, after the modification (5.13), the sum of Figs. 4 and 5 must vanish. These two observations are consistent only if

$$(\delta m^2)_{III} = 2 \times (\text{the } \alpha_W \alpha'_W \text{ terms of Fig. 5}) \quad (5.14)$$

This conclusion is quite apparent in the form of (5.11). The use of (5.14) as a starting point drastically simplifies the analysis of more complex gauge coupling schemes.

It is worth exploring an extension of this toy model to extract one more feature of our formalism. Let us modify (5.2) by gauging the whole left-handed $SU(2)$:

$$g W_\mu \left(\begin{array}{c|c} \tau_3 & \\ \hline & 0 \end{array} \right) \longrightarrow g W_\mu^a \left(\begin{array}{c|c} \tau_a & \\ \hline & 0 \end{array} \right) . \quad (5.15)$$

This modification makes $\pi^\pm$, as well as π^0 , fictitious Goldstone bosons which must be kept precisely massless if the Higgs mechanism is to give mass to the $W^\pm$. In particular, the graphs of Fig. 1 III represent the mass increment given to the $\pi^\pm$ in a model with $SU(4)$, as opposed to $SU(2) \times SU(2)$, chiral symmetry. These graphs must vanish after the modification (5.15), since then the $\pi^\pm$ must have zero mass in both cases. One may check readily, using (5.14), that this is true : the components $a = 1, 2$ of the W_μ^a exchange add new contributions proportional to $\alpha_W \alpha'_W$ to Fig. 5, and these cancel precisely the term of this form due to W_μ^3 exchange.

6. - THE P^+ MASS IN $O(16)$

The simplifications which we encountered in the previous section, in our discussion of the toy models based on chiral $SU(4)$, may be readily extended and incorporated into more complex theories. In this section, we will apply these insights to compute the electroweak contribution to m_P in a realistic technicolour theory, the O model defined in Section 2 of I. In this model, as in the previous one, there are two contributions to m_P : the first is given by (2.10), the second by Fig. 1 III. To compute this latter contribution, one must evaluate the diagrams of Figs. 4 and 5 using the set of G_A given in Eq. (2.5) of I and summing the generators in the closed loops over the unbroken and broken generators of the chiral symmetry group $SU(16)$. It is hardly more difficult than before, however, to reduce this computation to a manageable scale.

We observe, first, that the arguments leading to (5.14) apply in this model as in the previous one. This means, in particular, that the generators G_A associated with the colour gluons do not directly enter the evaluation of m_P . (Colour exchanges do enter in an indirect fashion, as we will see in a moment.) The conclusion that one need sum only over off-diagonal generators in the loops also extends to this case.

We should separate the generators G_A which couple to W_μ^a and B_μ in (2.5) of I into unbroken and broken pieces. Then we can write the order $\alpha_W \alpha'_W$ contributions to Fig. 5 as diagrams having, on their external legs, the generators :

$$\begin{aligned}
 T_B &= \left(\begin{array}{c|c} 1_2 \otimes B & \\ \hline & -1_2 \otimes B \end{array} \right) = B \text{ --- } \bigcirc \text{ --- } \text{Y} , \\
 T_a &= \left(\begin{array}{c|c} \tau_a \otimes 1_4 & \\ \hline & -\tau_a^T \otimes 1_4 \end{array} \right) = a \text{ --- } \bigcirc \text{ --- } \text{Y} , \\
 X_a &= \left(\begin{array}{c|c} \tau_a \otimes 1_4 & \\ \hline & \tau_a^T \otimes 1_4 \end{array} \right) = a \text{ --- } \bullet \text{ --- } \text{Y} . \quad (6.1)
 \end{aligned}$$

with $a = 1, 2, 3$ and the P meson generators (2.2), which we will denote by indices x, y . None of the diagrams in this class has two T_B legs, since one leg corresponding to each gauge boson exchange is always associated with a broken generator. Diagrams with one T_B leg may be seen to vanish : these diagrams have the structure shown in Fig. 6. The value of the diagram must be isospin invariant. But T_B is an isosinglet, so the only possible invariant with the indicated indices is ϵ_{xyz} . But the mass matrix δm_{xy}^2 is necessarily symmetric under interchange of x and y ; hence, there is no possible invariant form which could be the value of Fig. 6.

We must still evaluate the diagrams with T_a and X_a only, by summing over off-diagonal generators in the loops. To classify these generators, we may choose the matrix A of (5.3), an 8×8 matrix in this case, to have one of the following three block forms under decomposition of the eight-component fermion field to six techniquark and two technilepton components :

$$A \in \left\{ \left(\begin{array}{c|c} a & \\ \hline & 0 \end{array} \right), \left(\begin{array}{c|c} 0 & \\ \hline & b \end{array} \right), \left(\begin{array}{c|c} & c \\ \hline \pm c^T & \end{array} \right) \right\} . \quad (6.2)$$

(In the last form, one must choose the $+$ sign for a broken generator and the $-$ sign for an unbroken generator.) The Goldstone bosons corresponding to the broken generators of these three types have diquark, dilepton, and quark-lepton quantum numbers, respectively. The division into these three classes is preserved by commutation with the matrices (2.2) and (6.1).

Now we can apply the last of our simplifications : only matrices of the third class contribute to m_P . To see this, consider first reducing our SU and O models to models with a techniquark doublet only. In either case, there is only one pair of charged colour-singlet Goldstone bosons ; these are the fictitious

Goldstone bosons which combine with the $W^\pm$ through the Higgs mechanism. They must be exactly massless. When the SU case is extended to the O case, the diagrams of Fig. 1 III provide an additional contribution to the mass of these particles ; for consistency, that contribution must vanish. This phenomenon is the same one that we observed, in a simpler model, at the end of the previous section. But the matrices of the first class of (6.2) are those which involve only techni-quarks ; the contribution of these matrices to the mass of the P^+ in the original O model is just the same (up to normalization factors) as the full contribution from Fig. 1 III to the mass of the fictitious Goldstone boson in the reduced model. This contribution to m_P therefore vanishes. The same argument, applied to the reduction to a model with technileptons only, shows that the contribution to m_P from generators of the second type also vanishes.

It is now straightforward to compute the remaining contributions from Fig. 5, those with only generators of the third class of (6.2) in the loop. If one represents these generators by writing c as a direct product :

$$c = m \otimes \xi , \quad (6.3)$$

where m is a 2×2 matrix on isospin indices and ξ is a three-component vector on colour indices, the factors ξ are undisturbed by the indicated commutators. Summing over the four choices for m and over the orientations of ξ , one finds

$$(\delta m_P^2)_{III} \sim \frac{\alpha_W \alpha'_W}{4\pi^2 f_\pi^2} M^4 \Gamma(2-d/2). \quad (6.4)$$

The Goldstone bosons whose appearance in the loops of Fig. 1 III gives rise to this divergent term are those with quark-lepton quantum numbers. These are colour triplets and have masses dominated by colour-gluon exchange

$$m^2 = \frac{4}{3} \alpha_A M^2 . \quad (6.5)$$

We may thus identify

$$\Gamma(2-d/2) \longrightarrow \log\left(\frac{\Lambda^2}{\alpha_A M^2}\right) . \quad (6.6)$$

Inserting (6.6) into (6.4), combining this result with (2.10), and using the relation

$$\alpha_W \alpha'_W f_\pi^2 = \frac{\alpha}{4\pi} m_Z^2, \quad (6.7)$$

we find for the complete electroweak contribution to m_P in the 0 model :

$$(m_P^2)_{EW} = \frac{3\alpha}{4\pi} m_Z^2 \left[\log\left(\frac{\Lambda^2}{m_Z^2}\right) + \frac{M^4}{12\pi^2 f_\pi^4} \log\left(\frac{\Lambda^2}{\alpha_s M^2}\right) \right]. \quad (6.8)$$

A numerical estimate of this contribution may be made using the values ¹⁰⁾ $f_\pi = 120$ GeV, $M \approx 500$ GeV, and $\alpha_s \approx 0.1$ at the technicolour scale. The same range of Λ that we used in (2.11) yields in this case

$$(m_P^2)_{EW} = (8 - 14 \text{ GeV})^2. \quad (6.9)$$

The numerical contribution of the diagrams involving Goldstone boson interactions is, in this case, substantial.

In this section, then, we have computed an effect of hierarchical mass generation to which strong interaction effects make an important contribution. The computation required was, in the final analysis, a quite simple one. But it was made possible, and then made straightforward, only through the systematic analysis of the couplings generated by chiral dynamics.

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APPENDIX - SOME USEFUL IDENTITIES IN DIAGRAMMATIC NOTATION

If one normalizes the generators of $SU(N)$, in the fundamental representation, to

$$t_a (G_a G_b) = \delta_{ab} \quad , \quad (A.1)$$

the $SU(N)$ structure constants obey the well known relation

$$t_{abc} t_{dbc} = - 2N \delta_{ad} \quad . \quad (A.2)$$

If t_{abc} is represented diagrammatically as in (3.3), this identity reads

$$a \text{ --- } \bigcirc \text{ --- } d = 2N \times a \text{ --- } d \quad . \quad (A.3)$$

It is useful, however, to generalize (A.3) to the situation in which some links carry the projectors (3.4) onto broken or unbroken generators.

For the case in which $SU(N)$ is broken to $O(N)$, (A.3) is replaced by the three identities :

$$\text{---} \bigcirc \text{---} = (N-2) \text{ ---} \bigcirc \text{---} \quad , \quad (A.4)$$

$$\text{---} \bigcirc \text{---} = (N+2) \text{ ---} \bigcirc \text{---} \quad , \quad (A.5)$$

$$\text{---} \bigcirc \text{---} = N \text{ ---} \bullet \text{---} \quad . \quad (A.6)$$

From these relations and the Jacobi identity (4.3), one can derive the additional relations :

$$\text{---} \bigcirc \text{---} = \frac{1}{2} (N-2) \text{ ---} \bigcirc \text{---} \quad , \quad (A.7)$$

$$\begin{array}{c} \text{Diagram 1} \end{array} = \frac{1}{2} (N+2) \begin{array}{c} \text{Diagram 2} \end{array}, \quad (\text{A.8})$$

$$\begin{array}{c} \text{Diagram 3} \end{array} = \frac{1}{2} (N-2) \begin{array}{c} \text{Diagram 4} \end{array}, \quad (\text{A.9})$$

$$\begin{array}{c} \text{Diagram 5} \end{array} = \frac{1}{2} (N+2) \begin{array}{c} \text{Diagram 6} \end{array}. \quad (\text{A.10})$$

For the case in which $SU(N)$ is broken to $Sp(N)$, one can construct the analogous set of identities by exchanging $(N-2)$ and $(N+2)$ in the relations given above.

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FIGURE CAPTIONS

- Fig. 1 : Diagrams contributing to the mass of the P^+ in chiral perturbation theory. T denotes a pseudo-Goldstone boson.
- Fig. 2 : Contribution to the technion mass matrix from the diagram IIIa of Fig. 1. The notation involving an insertion of m^2 in brackets denotes the order α mass matrix m^2 raised to the power $d/2$.
- Fig. 3 : Contribution to the technion mass matrix from the diagram IIIb of Fig. 1. Each term involves m^2 raised to the power $(d/2 - 1)$.
- Fig. 4 : Logarithmically divergent contributions to the technion mass matrix from the diagrams IIIa and IIIb of Fig. 1.
- Fig. 5 : Logarithmically divergent contributions to the technion mass matrix from the diagram IIIc of Fig. 1.
- Fig. 6 : Structure of order $\alpha_w \alpha'_w$ contributions to Fig. 5 involving the matrix T_B only once.

Fig. 1

$$\delta m^2_{xy} \big|_{\text{III a}} = +i \text{ = \frac{1}{3f_\pi^2} \frac{\Gamma(1 - \frac{1}{2}d)}{(4\pi)^{d/2}} \left(\text{

Fig. 2$$

$$\delta m_{xy}^2|_{\text{IIIb}} = i \cdot \cancel{\otimes}^T$$

$$= -\frac{1}{6f_{\pi}^2} \left(\frac{M^2}{4\pi} \right) \frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}$$

$$\begin{aligned}
 & \times \left\{ \begin{array}{l} \text{Diagram 1} - \text{Diagram 2} \\ + \text{Diagram 3} - \text{Diagram 4} \\ + 2 \text{Diagram 5} - 2 \text{Diagram 6} \\ + 2 \text{Diagram 7} - 2 \text{Diagram 8} \end{array} \right\} \\
 & \quad + (x \leftrightarrow y) \}
 \end{aligned}$$

Fig. 3.

$$\begin{aligned}
& \delta m^2_{xy} \Big|_{\text{III}a+b} \quad \widetilde{d \rightarrow 2} \\
& \frac{1}{6f_\pi^2} \left(\frac{M}{4\pi} \right)^2 \frac{\Gamma(2-d/2)}{(4\pi)^2} \\
& x \left\{ \begin{aligned}
& \begin{array}{c} \text{Diagram 1: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: B. Left: A, right: B. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& \text{Diagram 2: Circle with 6 vertices. Top-left: B, top-right: B, bottom-left: A, bottom-right: A. Left: A, right: A. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& \text{Diagram 3: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: A. Left: A, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& \text{Diagram 4: Circle with 6 vertices. Top-left: A, top-right: A, bottom-left: B, bottom-right: B. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.}
\end{array} \\
& +6 \begin{array}{c} \text{Diagram 5: Circle with 6 vertices. Top-left: B, top-right: B, bottom-left: A, bottom-right: A. Left: A, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& -8 \begin{array}{c} \text{Diagram 6: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: A. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& + \begin{array}{c} \text{Diagram 7: Circle with 6 vertices. Top-left: A, top-right: A, bottom-left: B, bottom-right: B. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.}
\end{array} \\
& -2 \begin{array}{c} \text{Diagram 8: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: B. Left: A, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& -6 \begin{array}{c} \text{Diagram 9: Circle with 6 vertices. Top-left: B, top-right: B, bottom-left: A, bottom-right: A. Left: A, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& -6 \begin{array}{c} \text{Diagram 10: Circle with 6 vertices. Top-left: B, top-right: B, bottom-left: A, bottom-right: A. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.}
\end{array} \\
& +8 \begin{array}{c} \text{Diagram 11: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: B. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& +8 \begin{array}{c} \text{Diagram 12: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: A. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& - \begin{array}{c} \text{Diagram 13: Circle with 6 vertices. Top-left: A, top-right: A, bottom-left: B, bottom-right: B. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& - \begin{array}{c} \text{Diagram 14: Circle with 6 vertices. Top-left: A, top-right: A, bottom-left: B, bottom-right: B. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.}
\end{array} \\
& + \begin{array}{c} \text{Diagram 15: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: B. Left: A, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& +6 \begin{array}{c} \text{Diagram 16: Circle with 6 vertices. Top-left: B, top-right: B, bottom-left: A, bottom-right: A. Left: A, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& -8 \begin{array}{c} \text{Diagram 17: Circle with 6 vertices. Top-left: A, top-right: B, bottom-left: A, bottom-right: A. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.} \\
& + \begin{array}{c} \text{Diagram 18: Circle with 6 vertices. Top-left: A, top-right: A, bottom-left: B, bottom-right: B. Left: x, right: y. Vertices: 3 black, 3 white. Edges: 3 solid, 3 dashed.}
\end{array}
\end{aligned} \right. \\
& + (x \leftrightarrow y)
\end{aligned}$$

Fig. 4

$$\delta m^2_{xy} \Big|_{\text{III c}} \quad \widetilde{d \rightarrow 2}$$

$$\frac{4}{f_{\pi}^2} \left(\frac{M}{4\pi} \right)^2 \frac{\Gamma(2 - d/2)}{(4\pi)^2}$$

$$x \left\{ \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right\}$$

The diagrams show two Feynman-like diagrams for a loop process. Each diagram consists of a circle with eight vertices. Four vertices are marked with solid black dots, and four are marked with open circles. External lines are attached to these vertices: two labeled 'A' (top-left and top-right), two labeled 'B' (bottom-left and bottom-right), and two labeled 'x' and 'y' (top and bottom). In the first diagram, the 'x' and 'y' lines are attached to open circles, while 'A' and 'B' lines are attached to solid dots. In the second diagram, the 'x' and 'y' lines are attached to solid dots, while 'A' and 'B' lines are attached to open circles. A minus sign is placed between the two diagrams, and they are enclosed in large curly braces with a 'x' to the left.

Fig. 5

Fig. 6