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FLAVOUR AND SUPERUNIFICATION. II

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A B S T R A C T

We consider the possibility of a non-trivial embedding of $(\underline{10} + \underline{\bar{5}})$ $SU(5)$ families of spin $\frac{1}{2}$ left-handed fermions in a combination of irreducible massless supermultiplets of N extended supersymmetry. We demand for the whole spectrum of spin $\frac{1}{2}$ states to be anomaly-free with respect to $SU(N)$. This turns out to be a necessary condition for the absence of anomalies at the $SU(5)$ level. We find two classes of models, with spin $\frac{1}{2}$ fermions in $SU(N)$ representations associated to one- and two-column Young-tableaux, respectively, in which each irreducible massless multiplet occurs at most once. These two classes of models lead to a non-trivial family generation due to supersymmetry. For $N = 8$ extended supersymmetry, they give at most three and five families, respectively. The first class of models is more natural in the way it excludes $SU(5)$ exotics. The same analysis is extended to the massless multiplets that can be obtained from bilinear composite fields of the (preonic) elementary fields of N extended supergravity. We prove that the generation of families requires the repetition of massless multiplets and that $(\underline{10} + \underline{\bar{5}})$ $SU(5)$ families can only be generated in pairs. General properties of multilinear composite operators of the preonic fields are given and the rôle of massive representations to classify towers of operators with definite spin is pointed out.

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1. INTRODUCTION

Although the original motivation of N extended supergravities¹⁾⁻⁵⁾ was the final unification of all fundamental interactions, it is only recently that an apparent reconciliation of these theories with present particle phenomenology has been possible^{6),7)}. In fact, under certain assumptions to be reviewed later, Ellis, Gaillard, Zumino (EGZ)⁷⁾ have been able to extract a minimal Grand Unified Theory (GUT) based on $SU(5)$ ^{8),9)} from the conjectured non-perturbative dynamics of $N = 8$ supergravity. This has been made possible after the breakthrough due to Cremmer and Julia⁵⁾ who have shown that N extended supergravities reveal hidden local $SU(N)$ symmetries and have further suggested that the non-dynamical $SU(N)$ gauge fields, which are present in the classical Lagrangian, may actually propagate through non-perturbative quantum effects. The discovery of a local $SU(8)$ gauge symmetry in $N = 8$ supergravity was crucial for the inclusion in the theory of the non-gravitational gauge interactions. In fact, previous phenomenological attempts¹⁰⁾ based on a gauged $SO(8)$ symmetry, described by the elementary fields of the supergravity Lagrangian, failed since $SO(8)$ does not contain the minimal gauge group $SU(3) \times SU(2) \times U(1)$ of strong and electroweak interactions. A generalization of this approach to $SO(9)$ has been considered¹¹⁾. However, irrespective of the fact that no dynamical theory is known to exist in this case, the spin $1/2$ fermions obtained in this way have wrong assignments with respect to the $SU(2)$ weak interaction group.

Let us first review the results obtained by EGMZ^{6),7)} in their phenomenological analysis of $N = 8$ supergravity. The scenario for the non-gravitational interactions comes from the hypothesis that their effective Lagrangian is realized in terms of composite fields of the elementary fields of the basic supergravity Lagrangian. The only analogous example which realizes this conjecture occurs in the CP^N models in two dimensions¹²⁾. Although it is completely unclear whether such a phenomenon can also occur in four-dimensional field theory, it seems, however, tempting to assume that this is indeed the case. N extended supergravities have in fact many similarities with σ models⁵⁾, and the way in which a local symmetry emerges in these theories is so non-trivial that it is not unreasonable to assume that it will trigger some dynamical effects.

The starting point of the analysis of EGMZ^{6),7)} is to assume that the known fermions of low energy phenomenology belong to the same supermultiplet which contains the $SU(8)$ gauge particles. It is quite natural to suppose that this "supercurrent" multiplet¹³⁾ has the following structure:

$$\left(\frac{3}{2}\right)^A, \left(1\right)_B^A, \left(\frac{1}{2}\right)_{BC}^A, \dots, \left(\frac{3}{2} - \frac{k}{2}\right)_{B_1 \dots B_k}^A, \dots, \left(-\frac{5}{2}\right)^A \quad (1)$$

+ PCT conjugate

where the symbol $(\lambda)_{B_1 \dots B_J}^{A_1 \dots A_K}$ denotes a massless state of helicity λ in the product $[k] \times [N-J]$ of totally antisymmetric representations of $SU(N)$. For $N = 8$, this multiplet has the following $SU(8)$ representation content for left-handed (negative helicity) spin 1/2 fermions:

$$\begin{aligned} \lambda = -1/2: \quad & (\underline{70} \times \underline{8}) + (\underline{28} \times \overline{\underline{8}}) \\ & = \underline{504} + \overline{\underline{56}} + \overline{\underline{216}} + \underline{8} \end{aligned} \quad (2)$$

In order to extract an effective GUT from the composite states which are supposed to emerge from the dynamics of supergravity, EGZ have made the following hypotheses:

- i) All states with $\lambda \geq 1$ are unbound, except for the gauge bosons of the effective theory. The fermions in the "trace part" $\overline{\underline{56}} + \underline{8}$ of Eq. (2) are assumed to be unbound.
- ii) All the particles of the effective gauge theory, including the spin 1/2 states, are contained in the supercurrent multiplet, Eq. (1). Among the states in (2), only the $\underline{504}$ and $\overline{\underline{216}}$ representations are retained.
- iii) Out of the spin 1/2 states (in $\underline{504} + \overline{\underline{216}}$) one selects the maximal anomaly free set, such that colour and electric charge are vectorlike; all the other states complex under $SU(5)$ are also assumed to be unbound.

The last hypothesis provides a first criterion for the choice of the effective GUT gauge group: EGZ⁷⁾ suggest that one considers the largest group G such that the fermion states fulfilling the hypothesis (iii) are in a complex representation of G . In this way they find $G = SU(5)$ with precisely the three $(\underline{10} + \overline{\underline{5}})$ families needed to accommodate all the left-handed fermions required in present phenomenology.

The above authors use another argument beyond the breaking of $SU(8)$ by anomalies to choose $SU(5)$ as the maximal unbroken gauge subgroup G of $SU(8)$:

they consider the scalar particles in the supermultiplet given by Eq. (1), which are in the $\underline{420} + \overline{\underline{420}}$ representation of $SU(8)$ (after having assumed that the trace part $\underline{28} + \overline{\underline{28}}$ is also unbound). Then they observe that the vacuum expectation value of the $\underline{420}$ has directions which leave $SU(5)$, but not $SU(6)$ or $SU(7)$, unbroken^{*)}.

Part of these hypotheses may be discussed on more general grounds; Veltman¹⁴⁾ and others¹⁵⁾ have argued that states whose interactions are relevant in an effective theory at energies much lower than their inverse size, must have only renormalizable interactions; this could justify hypothesis (i) namely the non-binding of high spin states and of anomalous spin $1/2$ fermions, or at least their irrelevance for the effective GUT. On the other hand, such an argument cannot be invoked to exclude the states in the "trace part" $\overline{\underline{56}} + \underline{8}$ from the effective theory. However, if these "traces" are allowed to bind, the third hypothesis leads to, at most, one $\underline{5} + \underline{10}$ family.

Attempts to find a solution without any of the hypotheses (i) to (iii) have been shown to be hopeless: one seems to need an infinite number of supermultiplets¹⁶⁾ to give masses to all high spin states, except to those states which have to appear in the effective GUT. It is evident that one can always find a trivial way to give masses to the states of the supercurrent multiplet. This is achieved by a massive $N = 8$ supermultiplet with $J_{\text{MAX}} = 6$, with states classified by the symplectic group $USp(16)$ ¹⁷⁾. This solution is clearly unacceptable because it would imply the possibility of giving an $SU(8)$ invariant mass to all composites. It has recently been argued¹⁸⁾ that under certain simplifying assumptions, colour reality actually implies $SU(8)$ reality, and moreover, the only way higher helicity states can become massive is by enlarging a massless supermultiplet into a massive one with the consequent embedding of real $SU(8)$ representations in $USp(16)$ representations.

^{*)} As a matter of fact, the little group of any $SU(5)$ invariant direction in the $\underline{420}$ representation of $SU(8)$ is $SU(5) \times SU(2) \times U(1)$. If the "trace", i.e., the $\underline{28}$ representation, is also added, this little group can be reduced to $SU(5) \times U(1)$. In any case, the extra $U(1)$ factor cannot be gauged without introducing extra anomalies for any choice of the complex $SU(5)$ fermion multiplets in Eq. (2). Therefore, the scalars in the supermultiplet cannot be used to fully determine the effective theory by a kind of Higgs mechanism; this would always result in a real fermion representation without any chiral family. In order to obtain non-trivial solutions, one has to assume furthermore that the $U(1)$ factor is not gauged in the effective GUT. Notice, however, that the largest anomaly free group which satisfies the EGZ criteria is indeed $SU(5) \times SU(2)$, the three fermion families transforming as $(\underline{10} + \overline{\underline{5}}, \underline{1} + \underline{2})$ and $SU(2)$ acting as a "horizontal" gauge group. This would lead to new (both flavour conserving and non-conserving) neutral currents, coupled to gauge bosons which could be relatively light if one associates the heavier families to the $SU(2)$ doublet. The scalars in the "trace part" $\underline{28} + \overline{\underline{28}}$, if included, may acquire v.e.v.'s which can either break the gauge group down to $SU(5)$, or conserve the horizontal symmetry above.

Because of the difficulties listed above, we have abandoned the idea of describing all physical states in a single supermultiplet [hypothesis (ii)] and we have tried to explore alternative situations in which additional irreducible supermultiplets are included ^{*)}. In Section 2, we consider the embedding of left-handed spin - families in supermultiplets of N extended supersymmetry, each occurring at most once and with its maximal helicity state in a fundamental representation of $SU(N)$. This set of supermultiplets is selected by the requirement that the whole spectrum of spin $\frac{1}{2}$ states participates into a renormalizable and hence anomaly-free $SU(N)$ effective gauge theory. In order to avoid exotics in a natural way, we require the spin $\frac{1}{2}$ states to belong to a reducible $SU(N)$ representation whose complex (chiral) part contains only totally antisymmetric representations (single-column Young-tableaux).

In Section 3 we extend the previous analysis to solutions in which the chiral part of the $SU(N)$ representation of the spin $\frac{1}{2}$ states contains, in addition, double-column Young-tableaux. We then require that these solutions do not have $SU(5)$ exotics in the chiral part of the spectrum.

In Section 4 we consider the symmetry restrictions which emerge if we assume that the supermultiplets which give rise to the spin $\frac{1}{2}$ $SU(5)$ families are realized as conventional bound states of the fundamental (preonic) fields of the N extended supergravity Lagrangian. We show that bound states corresponding to multilinear products of elementary fields of N extended supergravity are uniquely classified in terms of certain massive representations of N extended supersymmetry.

If the massless supermultiplets considered in Sections 2 and 3 are supposed to be obtained from these bound states, then we show that it is not possible to obtain $SU(5)$ families without allowing for repetition of these supermultiplets. This implies that several massive representations and hence multilinear products of fields are needed in order to obtain a sensible solution. Examples of these solutions will be given in the same section.

^{*)} For instance, if one adds to the supercurrent multiplet in Eq. (1) the graviton supermultiplet, its spin $1/2$ left-handed states in a $\underline{56}$ of $SU(8)$ will combine with the "trace part" of the $\underline{70} \times \underline{8}$ in Eq. (2) to give Dirac particles, so that one again obtains three families, without making the drastic hypothesis that the "trace part" in Eq. (2) is unbound. However, the additional Higgs particles in the graviton supermultiplet, which are in the $\underline{70}$ of $SU(8)$, are not able to solve the problem raised in the previous note since any v.e.v. would break $SU(5)$. Indeed, the maximal stability group of the $\underline{70}$ is $SU(4) \times SU(4)$. Unfortunately, this last group cannot be used as the effective GUT gauge group because any choice of its $SU(3) \times SU(2) \times U(1)$ subgroup gives wrong quantum numbers for the light left-handed spin $1/2$ states.

Let us introduce some notational conventions. The symbol $[k]_N$ will denote a k fold antisymmetric $SU(N)$ representation. We will often use the fact that $[k]_N = [N-k]_N$. A massless irreducible supermultiplet of N extended supergravity will be denoted by its maximal helicity λ_{MAX} and the $SU(N)$ representation of this helicity state: $\{\lambda_{MAX}, R\}$. PCT conjugate states are always added; this means that $\{\lambda_{MAX}, R\} = \{(N/2) - \lambda_{MAX}, \bar{R}\}$. PCT self-conjugate supermultiplets are those for which $\lambda_{MAX} = N/4$ and $R = \bar{R}$; in that case the PCT doubling is not necessary. A given massless multiplet $\{\lambda_{MAX}, R\}$ therefore contains the following collection of helicity states:

$$\begin{aligned} \{\lambda_{MAX}, R\} = & (\lambda_{MAX}, R) + (\lambda_{MAX} - \frac{1}{2}, [\bar{1}]_N \times R) + (\lambda_{MAX} - 1, [\bar{2}]_N \times R) \quad (3) \\ & + \dots + (\lambda_{MAX} - \frac{k}{2}, [\bar{k}]_N \times R) + \dots + (\lambda_{MAX} - \frac{N}{2}, R) \\ & + \text{PCT conjugate} \end{aligned}$$

Note that unless R is the identity representation $[0]_N$, the lower helicity states in (3) will belong to reducible representations of $SU(N)$. We will often denote $SU(N)$ representations by their dimensionality d . For example, in $N = 8$ supergravity the graviton supermultiplet is PCT self-conjugate with helicity states

$$\{2, 1\} = (2, 1) + (\frac{3}{2}, \bar{8}) + \dots + (2 - \frac{k}{2}, \bar{\binom{8}{k}}) + \dots + (-2, 1) \quad (4)$$

2. AN EFFECTIVE RENORMALIZABLE $SU(N)$ GAUGE THEORY

We now state our main assumption, namely that the non-perturbative dynamics of N extended supergravity should be responsible for an effective renormalizable $SU(N)$ gauge theory^{*)},^{**) . In this scenario, assumption (iii) is automatically avoided and, as we shall see, a very limited number of effective}

^{*)} Earlier attempts to consider $SU(N)$ as a full gauge group have been discussed by Curtright and Freund [Ref. 19)].

^{**) We note that for $N \neq 8$ the effective gauge group could, in principle, be $U(N)$ rather than $SU(N)$ ⁵⁾. However, in general, one expects that anomalies would forbid the gauging of the $U(1)$ factor.}

GUT's which include the whole spin $\frac{1}{2}$ spectrum of states will emerge from the models described below. What we are really looking for is a non-trivial embedding of $(\underline{10}+\underline{5})$ $SU(5)$ families for spin $\frac{1}{2}$ left-handed states in massless supermultiplets of N extended supergravity. Our first class of models for superunification is defined by the following three hypotheses :

- (I) The left-handed spin $1/2$ fermions are in a complex anomaly free reducible representation of $SU(N)$ which is obtained from a superposition of massless irreducible supermultiplets $\{\lambda_{MAX}, R\}$ of N extended supersymmetry, each occurring at most once and with maximal helicity λ_{MAX} in a totally antisymmetric representation R of $SU(N)$.
- (II) The complex part of the $SU(N)$ representation of the left-handed fermions contains only totally antisymmetric representations. This implies for the $SU(5)$ content a sequence of $(\underline{10}+\underline{5})$ families, the remaining part being real under $SU(5)$.
- (III) All states with $\lambda \geq 1$ are neglected, with the exception of the $\lambda = 1$ states in the adjoint representation of $SU(N)$.

We now comment on these hypotheses. The anomaly-free condition stated in hypothesis (I) for the $SU(N)$ representations is not only sufficient, but also necessary for having an anomaly-free gauge theory at the $SU(5)$ level. Indeed, in the decomposition $SU(N) \supset SU(n)$ with $N \rightarrow n + (N-n)\underline{1}$ ($N > n \geq 3$), the $SU(N)$ and the $SU(n)$ anomalies of any $SU(N)$ representation coincide. The massless multiplets with λ_{MAX} in fundamental representations of $SU(N)$ are those which occur in the decomposition of a massive supermultiplet with an $SU(N)$ singlet as maximum spin state. We shall see in Section 4 that these massive representations classify all composite operators which are bilinear in the preonic fields of N extended supergravity. The multiplicity assumption for each multiplet to occur at most once is natural in order to have a unique definition for the number of $SU(5)$ families. It also corresponds to embed all massless supermultiplets in a single massive supermultiplet. The second hypothesis is the natural way to avoid exotics at the $SU(N)$ level without any reference to the $SU(5)$ subgroup²⁰⁾. The assumption (III), which is less obvious, is again advocated because of the previously discussed Veltman theorem¹⁴⁾ and is mainly due to our ignorance of the dynamics of high spin particles.

From these three hypotheses for superunification, we obtain the following results (see Table I).

1) The only extended supergravity which accommodates at most three $(10+\sqrt{2})$ SU(5) families of left-handed spin 1/2 fermions is $N = 8$ supergravity. Remarkably, we find with great satisfaction that among the supermultiplets which participate in the effective SU(8) gauge theory, are the supercurrent $\{3/2, 8\}$ and the graviton $\{2, 1\}$ supermultiplets. These are the multiplets which contain all the gauge particles of a superunified gauge theory. Surprisingly, in our approach, the very existence of these states is required to avoid anomalies from the spin 1/2 sector !

2) There are at most two families in $N = 5$ supergravity, one family in $N = 6$ and two families in $N = 7$ supergravity^{*)}.

3) From our group theoretical analysis we can also explore $N > 8$ cases where no underlying dynamical theory is known to exist, and we find that we can accommodate three families for $N = 9$, four families for $N = 10$ and seven families for $N = 11$. This analysis can be generalized to any N .

The proof of our results is based on some general properties of massless supermultiplets of N extended supersymmetry. Given any supermultiplet $\{\lambda_{\text{MAX}}, R\}$, the states with helicity λ will belong to the (reducible) representation of SU(N)

$$\overline{[2(\lambda_{\text{MAX}} - \lambda)]_N} \times R + [2(\lambda_{\text{MAX}} + \lambda)]_N \times \overline{R} \quad (5)$$

[unless $\{\lambda_{\text{MAX}}, R\}$ is PCT self-conjugate, in which case only the first term in (5) occurs]. In particular, one has for the left-handed spin 1/2 states:

$$\{\lambda_{\text{MAX}}, R\} \supset \overline{[2\lambda_{\text{MAX}} + 1]} \times R + [2\lambda_{\text{MAX}} - 1] \times \overline{R} \quad (6)$$

Hereafter, the symbol $\supset$ denotes the SU(N) content of the $\lambda = -1/2$ states in the supermultiplets. Only supermultiplets with $\lambda_{\text{MAX}} \leq (N+1)/2$ can contribute to the spin 1/2 sector. However, the expression in Eq. (6) does not

^{*)} It is often assumed that $N = 7$ and $N = 8$ supergravities are the same theory. However, the irreducible supermultiplets of $N = 7$ and $N = 8$ supersymmetries, other than the graviton supermultiplet, are generally different. So we may also consider SU(7) on its own, in which case two, rather than three, families exist which fulfil our hypotheses.

satisfy our hypothesis (II) because the direct product of two antisymmetric representations will contain, in general, complex $SU(N)$ representations which are not totally antisymmetric (corresponding to two-column Young tableaux) and thus have to be systematically avoided. This can be done in only three ways:

a) For $\lambda_{MAX} = (N+1)/2$, one has

$$\left\{ \frac{N+1}{2}, \bar{R} \right\} = R \quad (7)$$

which contains only totally antisymmetric representations in view of our hypothesis (I).

b) For $R = \underline{1}$, the right-hand side of Eq. (6) corresponds to two totally antisymmetric $SU(N)$ representations, so that one can consider combinations

$$\begin{aligned} \sum_{\lambda_{MAX}} a(\lambda_{MAX}) \{ \lambda_{MAX}, \underline{1} \} \\ = \sum_{\lambda_{MAX}} a(\lambda_{MAX}) \{ [2\lambda_{MAX}+1] + [2\lambda_{MAX}-1] \} \end{aligned} \quad (8)$$

with $a(\lambda_{MAX}) = 0, 1$ according to our hypothesis. In some cases, for a given R , one can construct a combination such that

$$\sum_{\lambda_{MAX}} a(\lambda_{MAX}) \{ \lambda_{MAX}, \underline{1} \} = R + \text{real} \quad (9)$$

c) By noticing that, for a given R , the complex conjugates of the $SU(N)$ representations in Eq. (6) are only contained in the supermultiplets $\{\lambda_{MAX}-1, R\}$ and $\{\lambda_{MAX}+1, R\}$, we will show that there is, at most, one combination of these supermultiplets, other than the previous ones in Eqs (7) and (8), such that the spin $1/2$ sector has only totally antisymmetric representations in its complex part.

We consider three separate cases: $N = 2n + 1$, $N = 4n + 2$ and $N = 4n$.

$$\underline{N = 2n + 1}$$

As a consequence of PCT doubling, we have just to consider integral values of λ_{MAX} , $\lambda_{MAX} = 0, 1, \dots, (N+1)/2$. Then, the $\lambda = -1/2$ content of the sum of supermultiplets is

$$\sum_{\lambda_{\text{MAX}}=0}^{(N-1)/2} \{ \lambda_{\text{MAX}}, R \} \supset R + \text{real} \quad (10)$$

$$\underline{N = 4n + 2}$$

By PCT, only the values $\lambda_{\text{MAX}} = N/4, N/4 + 1/2, N/4 + 1, \dots, (N+1)/2$ have to be considered. Then one has the following properties

$$\sum_{\lambda_{\text{MAX}} = \frac{N}{4} + \frac{1}{2}}^{N/2} \{ \lambda_{\text{MAX}}, R \} \supset \left[\frac{N}{2} \right] \times \bar{R} + \text{real} \quad (11)$$

$$\sum_{\lambda_{\text{MAX}} = \frac{N}{4}}^{(N-1)/2} \{ \lambda_{\text{MAX}}, R \} \supset R + \left[\frac{N}{2} - 1 \right] \times \bar{R} + \text{real} \quad (12)$$

We see that Eq. (11) is useless for our purposes, while in Eq. (12) one has to neutralize the term $\left[(N/2) - 1 \right] \times \bar{R}$ which violates our assumption (II). However, this is not possible for a complex $\left[(N/2) - 1 \right]$ representation because of the PCT invariance of the supermultiplets (this is to be compared with the case $N = 4n$ below).

$$\underline{N = 4n}$$

By PCT, the values $\lambda_{\text{MAX}} = N/4, N/4 + 1/2, \dots, (N+1)/2$ have to be considered. One has

$$\sum_{\lambda_{\text{MAX}} = \frac{N}{4}}^{N/2} \{ \lambda_{\text{MAX}}, R \} \supset \left[\frac{N}{2} - 1 \right] \times \bar{R} + \text{real} \quad (13)$$

$$\sum_{\lambda_{\text{MAX}} = \frac{N}{4} + \frac{1}{2}}^{(N-1)/2} \{ \lambda_{\text{MAX}}, R \} \supset R + \left[\frac{N}{2} \right] \times \bar{R} + \text{real} \quad (14)$$

We have again that Eq. (13) cannot be used. If Eq. (9) holds, then we can construct

$$\sum_{\lambda_{\text{MAX}} = \frac{N-1}{2}}^{\frac{(N-1)/2}{2}} \{ \lambda_{\text{MAX}}, R \} + \sum_{\lambda_{\text{MAX}}} a(\lambda_{\text{MAX}}) \{ \lambda_{\text{MAX}}, \left[\frac{N}{2} \right] \} \quad (15)$$

$$= R + \text{real}$$

Notice that this construction only works because Eq. (9) is satisfied and $\left[\frac{N}{2} \right]$ is real.

Next, we have to look for all reducible complex $SU(N)$ representations of the form^{21),22)}

$$F_i = \sum_k b_i(k) [k]_N \quad (16)$$

with integer coefficients $b_i(k)$, which satisfy our hypothesis (II). In this context, we take the simplest embedding $SU(5) \subset SU(N) : \underline{N} = \underline{5} + (N-5)\underline{1}$. Incidentally, this is the only possible embedding in extended supergravity, where $N \leq 8$. The complex, anomaly free representations F_i with $b_i(k) = 0, 1$ have been classified by Georgi²¹⁾ in the context of GUTs. However, this is not enough for our purposes. Indeed, as we have discussed above, there are, at most, three different ways of constructing massless (in general, reducible) supermultiplets, such that the complex part of the $SU(N)$ representation of the $\lambda = -1/2$ states is a given totally antisymmetric representation. Therefore, one has to look for all possible representations given by Eq. (16) with $b_i(k)$'s up to three.

We now have a systematic way of investigating all possible superpositions of N extended supermultiplets fulfilling our hypotheses (I) and (II), which we call superfamilies. Notice that to each superfamily there corresponds a $SU(N)$ representation, F_i , as in Eq. (16) for the left-handed spin $1/2$ states. The number of $SU(5)$ families in the $\lambda = -1/2$ sector of a superfamily will then be given by the number of $(\underline{10} + \underline{\bar{5}})$'s in the decomposition of the corresponding F_i .

We now apply our procedure to extended supergravities ($N = 5, 6, 7, 8$) and to some higher N cases ($N = 9, 10, 11$). We first consider odd N 's. For $N = 5$ we obtain two superfamilies, each containing one family, from Eqs (7) and (10) with $R = \underline{10} + \bar{5}$. Instead, Eq. (8) does not give any new family because

$$\{3, \underline{1}\} \supset \underline{1}, \{2, \underline{1}\} \supset \underline{1} + \underline{10}, \{1, \underline{1}\} \supset \underline{10} + \underline{5}, \{0, \underline{1}\} \supset \bar{5}$$

and there is no combination of supermultiplets occurring at most once which contains $\underline{10} + \bar{5}$. For $N = 7$, there is no complex anomaly free set of antisymmetric representations of $SU(7)$ without repetition. However, we can obtain two superfamilies, each containing one family, by combining the supermultiplets in Eqs (7), (8) and (10). For $N = 9$, since $R = [\underline{4}] + [\underline{7}] = \underline{126} + \bar{36} \supset (\underline{10} + \bar{5}) + \text{real}$, we get two superfamilies (two families) from Eqs (7) and (10) and a third superfamily from Eq. (9). For $N = 11$, $R = [\underline{10}] + [\underline{9}] + [\underline{8}] + [\underline{4}] \supset 3(\underline{10} + \bar{5}) + \text{real}$ we can use both Eqs (7) and (10) to obtain two superfamilies, each one with three families. One extra family is obtained by noticing that $2[\underline{5}] + [\underline{7}] \supset (\underline{10} + \bar{5}) + \text{real}$, and the corresponding superfamily can be constructed by applying both Eqs (7) and (10) again.

For $N = 4n + 2$, only Eqs (7) and (9) can be used. In the case $N = 6$, there is just one F_1 as defined in Eq. (16), namely, $2[\underline{5}] + [\underline{2}] \supset (\underline{10} + \bar{5}) + \text{real}$, and (because of the repetition) one can only combine Eqs (7) and (9) to get one superfamily with one family. For $N = 10$, among several solutions, a maximum of four families is obtained from the anomaly free representations of $SU(10)$, $2[\underline{8}] + [\underline{4}] + 2[\underline{9}] \supset 3(\underline{10} + \bar{5}) + \text{real}$, and $[\underline{4}] + [\underline{7}] \supset (\underline{10} + \bar{5}) + \text{real}$ by the repeated use of Eqs (7) and (9).

For $N = 8$, the following $SU(8)$ complex representation contains one $(\underline{10} + \bar{5})$ family

$$[\underline{3}] + [\underline{6}] + [\underline{7}] = \underline{56} + \bar{28} + \bar{8} = \underline{10} + \bar{5} + 2(\underline{10} + \bar{10}) + 3(\underline{5} + \bar{5}) + 7(\underline{1}) \quad (17)$$

Then we get, in a unique way, three superfamilies (three families) by taking

$$\begin{aligned} f_1: \left\{ \frac{9}{2}, \underline{56} + \bar{28} + \bar{8} \right\} &\supset \underline{56} + \bar{28} + \bar{8} \\ f_2: \left\{ 2 + 4 + \frac{7}{2}, \underline{1} \right\} &\supset \underline{56} + \bar{28} + \bar{8} + \underline{1} \end{aligned} \quad (18)$$

$$f_3: \left\{ \frac{7}{2} + \frac{5}{2}, \underline{56} + \underline{28} + \underline{8} \right\} + \left\{ 2 + 4 + \frac{7}{2}, \underline{70} \right\} = \underline{56} + \underline{28} + \underline{8} + \underline{70} + (\underline{28} + \underline{70}) \times (\underline{56} + \underline{8} + \underline{28}) + (\underline{28} + \underline{70}) \times (\underline{56} + \underline{8} + \underline{28}) \quad (18)$$

These three families, which are a unique feature of $N = 8$ supergravity, require 15 irreducible supermultiplets.

Remarkably enough, the superfamilies f_2 and f_3 contain the graviton and the supercurrent supermultiplets^(6),7).

$$\{2, \underline{1}\}, \quad \left\{ \frac{3}{2}, \underline{8} \right\} = \left\{ \frac{5}{2}, \underline{8} \right\}. \quad (19)$$

These multiplets contain, among others, the following helicity states

$$(\lambda = \pm 2, \underline{1}), \quad \left(\lambda = -\frac{3}{2}, \underline{8} \right), \quad (\lambda = \pm 1, \underline{63}), \quad (20)$$

which are the gauge quanta of gravitation (graviton), local supersymmetry (gravitini) and local $SU(8)$ symmetry (gauge vector bosons). Note that although the existence of the first superfamily f_1 is manifest, the existence of f_2 and f_3 is highly non-trivial and is a peculiar property of $N = 8$ supermultiplets. Moreover, the existence of f_3 , which requires the existence of f_2 , gives a kind of complementarity property between the two gauge supermultiplets given in Eq. (19). We should stress, however, that for $N \neq 8$ the supercurrent multiplet $\{3/2, \underline{1}_N\}$ which provides the gauge fields for the effective GUT is not automatically included in our solutions. For $N = 6$ or 10 , one should add to the solution in the Table a combination of supermultiplets giving $SU(N)$ real spin $1/2$ states and containing $\{3/2, \underline{1}_N\}$. For $N = 6$ one may add $\{3/2, \underline{1}\} + \{5/2, \underline{1}\} + \underline{5}$, and for $N = 10$, $\{3/2, \underline{1}\} + \{9/2, \underline{9}\} + \underline{5}, \underline{6}\} + \{11/2, \underline{9}\}$ is convenient. Also, for $N = 5$ or 9 the solutions which include the supercurrent multiplet are not minimal with respect to the number of supermultiplets (in particular, for $N = 9$, one has to substitute the third family in the Table by a larger one in order to comprise the supercurrent multiplet). Instead, for $N = 7$ or 11 , the supercurrent multiplet is already contained in the solution with a maximum number of $(\underline{10} + \underline{5})$ families. The requirement that the supercurrent multiplet be included could be used as a further criterion to select the physical solution, since in all the above

cases several solutions are possible which have the same number of $(\underline{10}+\underline{\bar{5}})$ families. It is remarkable that this can be done consistently with our hypotheses I and II, but it is also satisfactory to notice that for $N = 8$ this further requirement is not needed since the solution in the Table is unique !

We now comment upon our results. What appears to be a striking feature of our construction is the fact that $N = 8$ supersymmetry happens to have the right minimal supermultiplet structure to give rise to an effective anomaly free $SU(8)$ gauge theory with precisely three $(\underline{10}+\underline{\bar{5}})$ $SU(5)$ families of spin $1/2$ left-handed fermions. In this scenario, the most remarkable fact is that these three families are embedded in a unique combination of supermultiplets which contain all the gauge particles of local interactions, including local supersymmetry and gravitation ^{*)}.

3. FERMION FAMILIES WITH TWO-COLUMN YOUNG-TABLEAUX

In the preceding section we have assumed that the representation F of the spin $1/2$ left-handed fermions obtained from our set of supermultiplets had a non-real part which was only an anomaly-free combination of antisymmetric representations $[k]_N$; this is our hypothesis (II). This assumption gives a simple and natural way of obtaining the usual families of 15 fields classified in the $(\underline{10}+\underline{\bar{5}})$ representation of the $SU(5)$ subgroup. It is, however, possible to admit more complicated representations in the complex part of F ; this is the purpose of this section.

According to hypothesis (I), we consider only supermultiplets with maximal helicity in a totally antisymmetric representation $[k]_N$ of $SU(N)$. This kind of supermultiplets contains helicity $-1/2$ states in representations whose Young-tableaux have at most two columns [see Eq. (6)]. The multiplet of spin $1/2$ left-handed fields will then be a combination

$$F = \sum_{j,k} a_{jk} [j]_N \times [k]_N + \sum_j b_j [j]_N \quad (21)$$

*) One might wonder whether the graviton and the gravitini should participate in the effective gauge theory since they give rise, in any case, to non-renormalizable interactions. At present, it indeed remains unclear whether the helicity states of the $\{2,1\}$ supermultiplet should be identified with the $\lambda \geq 1$ gauge quanta.

with conditions (for instance, the cancellation of anomalies) imposed on the coefficients a_{jk} and b_j . From the helicity $-1/2$ content, Eq. (6), of our supermultiplets, we see that a given product $[j]_N \times [k]_N$ will be found in

$$\left\{ \frac{j+1}{2}, [N-k]_N \right\} \text{ and } \left\{ \frac{k+1}{2}, [N-j]_N \right\}$$

If we assume [hypothesis (I)] that each supermultiplet may appear at most once, we obtain

$$\begin{aligned} a_{jk} &\leq 2, \quad j \neq k \\ a_{jj} &\leq 1 \end{aligned} \tag{22}$$

But in fact we have stronger conditions : suppose, for instance, that F contains a given product $[j]_N \times [k]_N$ obtained from the supermultiplet $\{(j+1)/2, [N-k]_N\}$ which has $\lambda = -1/2$ states in

$$[j]_N \times [k]_N + [N-j-2]_N \times [N-k]_N$$

Then the non-real part of F can contain at most one product $[j+2]_N \times [k]_N$, since $a_{(j+2)k} \leq 2$. This kind of condition will, in fact, strongly restrict the possible F 's we have to consider since for light fermions only its non-real part is of interest. Moreover, F has to be anomaly-free.

All the above conditions were formulated using hypothesis (I) ; we will now try to enlarge the assumption (II) which forbids the appearance of products $[j]_N \times [k]_N$ in the complex part of F . Constructing combinations of supermultiplets satisfying only hypothesis (I) will, in general, provide unacceptable solutions, containing coloured or charged massless fermions. In fact we must impose the reality of F under $SU(3)_C \times U(1)_{em}$ and moreover forbid the appearance of states receiving their mass from a breaking of $SU(2)_L$ by a triplet of Higgses. These conditions are conveniently stated in terms of the $SU(5)$ subgroup of $SU(N)$ (in this section we consider $N \leq 8$), and they lead to the following hypothesis :

- (II') The complex $SU(N)$ representation of the left-handed spin $1/2$ fermions, when decomposed under the $SU(5)$ subgroup, must contain only a sequence of $(\underline{10} + \underline{\bar{5}})$ families, the remaining part being real under $SU(5)$.

This formulation is slightly weaker than our previous hypothesis (II). We stress, however, that it implies that the dynamics of the model is able to distinguish the $SU(5)$ subgroup of the $SU(N)$ gauge group. On the contrary, all our previous assumptions were stated at the level of $SU(N)$, giving no particular importance to $SU(5)$.

We have four different complex irreducible representations with two-columns Young-tableaux in $SU(5)$: $\underline{15}$, $\underline{40}$, $\underline{45}$, $\underline{50}$ (and their complex conjugates). Thus we impose four $SU(5)$ reality conditions on the representation of the helicity $-1/2$ states, in addition to the cancellation of anomalies. A systematic study of the solutions gives the following results.

$N = 6$: Since $SU(6)$ has six independent complex products $[j]_6 \times [k]_6$, we obtain only one solution. It contains two $(\underline{10} + \underline{\bar{5}})$ families and the supercurrent multiplet $\{3/2, \underline{6}\}$ is included.

$N = 7$: We have four solutions. The first one can accommodate four $(\underline{10} + \underline{\bar{5}})$ families, but it cannot contain the supercurrent multiplet $\{2, \underline{7}\}$. On the contrary, the other solutions include the supercurrent, but we obtain only two families.

$N = 8$: In this case we have twelve complex products $[j]_8 \times [k]_8$ and we obtain numerous solutions, with a maximal number of five $(\underline{10} + \underline{\bar{5}})$ families. The most interesting solution is the following set of supermultiplets :

$$\begin{aligned}
 \mathcal{F} &= \{9/2, [6]\} + \{4, [1] + [2]\} + \{7/2, [1] + [2] + \\
 &+ [4] + [7]\} + \{3, [2] + [3] + [5] + [6] + [7]\} + \\
 &\{5/2, [0] + [2] + [7]\} + \{2, [1] + [2] + [3] + [4]\} \\
 &= 2(\underline{28} \times \underline{70}) + (\underline{56} \times \underline{70}) + 2(\underline{8} \times \underline{56}) + (\underline{8} \times \underline{70}) \\
 &+ (\underline{8} \times \underline{56}) + (\underline{8} \times \underline{28}) + (\underline{8} \times \underline{28}) + (\underline{8} \times \underline{8}) + 3(\underline{28}) \\
 &+ SU(8)\text{-real} = 3(\underline{10} + \underline{\bar{5}}) + SU(5)\text{-real} \quad (23)
 \end{aligned}$$

The helicity $-1/2$ content of F is anomaly-free and contains three $(\underline{10}+\underline{5})$ families in its $SU(5)$ complex part. Adding the two solutions F_1 and F_2 of Table I, we obtain five families. The supercurrent multiplet $\{5/2, [\underline{7}]\}$ is included in F .

Since the solution should include the supercurrent multiplet which contains the $SU(N)$ gauge fields, the only remaining possibility is then to consider $N = 8$. In this case, we can obtain up to five families instead of the three we had in Section 2. These new families are, however, generated in a rather complicated and unnatural way.

4. COMPOSITES IN $N = 8$ SUPERGRAVITY THEORY

We now consider the possibility that the composite states be conventional bound states of the elementary fields (preons) in the N extended supergravity Lagrangian. Then the preons are in the massless supermultiplet,

$$\{2, [0]\} = (2, [0]) + (3/2, [N-1]_N) + (1, [N-2]_N) + \dots \quad (24)$$

In this context, only $N = 6$ and $N = 8$ supergravities have to be considered, since $N = 7$ supergravity is believed to be dynamically equivalent to $N = 8$ supergravity as far as they contain the same preonic fields and have the same local invariance ^{*)}. Then one notices ^{**)} that, for even N , the elementary massless supermultiplet in Eq. (24) has the property that the $SU(N)$ representations of the integral spin (half-integral spin) states correspond to an even (odd) number of $SU(N)$ indices (i.e., of boxes in their Young-tableaux). Therefore, a similar rule must apply to (conventional) bound states of any number of these preons, namely :

"Integral (half-integral) spin composites must belong to $SU(N)$ representations contained in even (odd) products of the fundamental $(\underline{N})$ representation."

^{*)} The case of $N = 5$ extended supergravity will be discussed at the end of this section.

^{**)} This observation is due to J. Ellis, M.K. Gaillard and B. Zumino.

In terms of massless supermultiplets, $\{\lambda_{\text{MAX},R}\}$, R must be a $SU(N)$ representation whose Young-tableau has an even (odd) number of boxes if λ_{MAX} is an integer (half-integer). With this restriction it is straightforward to prove that, for either $N = 6$ or $N = 8$, there are no solutions which fulfil both hypotheses (I) and (II') (even in the weaker version of Section 3, the stronger form in Section 2 being trivially excluded). A possible way out is to relax the multiplicity condition in hypothesis (I) and to allow for repetitions of the irreducible supermultiplets. But, even in this case, there are still no solutions for $N = 6$, because there are no combinations of $SU(6)$ representations with one and two columns and an odd number of boxes in their Young-tableaux whose complex parts have only the $(\underline{10}+\underline{5})$ representation of $SU(5)$.

For $N = 8$, instead, there are three and only three independent anomaly-free combinations of $SU(8)$ representations whose Young-tableaux have an odd number of boxes in one and two columns which have no $SU(5)$ exotic states in their complex part. These combinations have the property that they all display an even number of $(\underline{10}+\underline{5})$ families; this property can be traced back to the fact that the $SU(5)$ complex part of the most general combination of $SU(8)$ representations with odd numbers of boxes in two-column Young-tableaux, which has no $SU(5)$ exotic states, is always a multiple of $4(\underline{10})+12(\underline{5})$. Hence any solution has a spin $1/2$ content that can be written as

$$F = \sum_i n_i F_i \quad (25)$$

where the n_i 's are integers and the F_i 's are three independent combinations. Two of the F_i 's can always be chosen to be real under $SU(5)$ while the third combination contains two $(\underline{10}+\underline{5})$ families. A possible choice for the F_i 's is :

$$\begin{aligned} F_1 = & ([1] \times [2]) + 3([1] \times [4]) + 5([1] \times [6]) + \\ & ([5] \times [6]) + 3([3] \times [6]) + ([3] \times [4]) + \\ & 2([3]) + 6[1] = SU(5) \text{ real} \end{aligned}$$

$$\begin{aligned} F_2 = & 11([1] \times [2]) + 24([1] \times [4]) + 25([1] \times [6]) + \\ & 9([5] \times [6]) + 21([3] \times [6]) + 8([3] \times [4]) \\ & = SU(5) \text{ real} \end{aligned} \quad (26)$$

$$F_3 = [3] + 5 [7] = 2(10 + \overline{5}) + SU(5)\text{-real}$$

With this choice, any solution will contain $2n_3$ families.

If multiplicities remain unconstrained, the combinations given by Eqs. (25) and (26) can be embedded in $N = 8$ supermultiplets in several ways. For example, the embedding of F_3 is straightforward due to the general procedure outlined in Section 2. Also, notice some characteristics of the expressions in Eqs. (25) and (26) which have to be respected in their embedding into sets of supermultiplets, namely, that the number of $[1] \times [6]$ is a multiple of 5, while the numbers of $[1] \times [4]$ and $[3] \times [6]$ are multiples of 3. In view of the analysis in the previous sections, one realizes that high multiplicities are required for the massless supermultiplets. For instance, some multiplicities have to be ≥ 3 in order to obtain more than two families, as in the following example which has four families (together with the supercurrent multiplet) :

$$\begin{aligned} & 5\{4, [0]\} + \{4, [4]\} + \{4, [6]\} + 2\{2, [0]\} + \\ & 3\{9/2, [1]\} + 2\{7/2, [7]\} + \{5/2, [7]\} \\ & \supset 2(\underline{56}) + 10(\underline{8}) + SU(8)\text{-real} = 4(10 + \overline{5}) + SU(5)\text{-real} \quad (27) \end{aligned}$$

It is worth emphasizing that, without any restriction on supermultiplet multiplicities, the concept of family generation remains obscure. But, whenever the multiplicities are restricted by some dynamical criteria, the solutions have still to be of the general form in Eqs. (25) and (26). Therefore, independently of any dynamical consideration, the general property which emerges from the above anomaly-free solutions constructed with massless supermultiplets $\{\lambda_{MAX}, [k]\}$, such that $k = 2\lambda_{MAX} \pmod{2}$ according to the above rule, is that families can only be obtained in pairs.

We now exploit some kinematical properties of multilinear bound states which will be useful to classify all possible composites we may encounter in $N = 8$ supergravity. Since the basic preonic fields are assigned to the fundamental PCT self-conjugate massless representation $\{2, 1\}$, if we make bilinear products of this massless supermultiplet we get a sequence of irreducible massive representations which all have the property that their maximum spin state $J_{MAX} = 4, 6, 8, \dots$ is a singlet under $SU(8)$. Indeed the statistics for the

Clifford vacuum of the PCT self-conjugate preon supermultiplet requires J_{MAX} to be even, and since the bilinear bound states correspond to $SU(8)$ representations with at most two columns in their Young-tableaux, the maximum spin state of the corresponding massive supermultiplets have to be $SU(8)$ singlets. These massive representations, in the zero-mass limit contain precisely the massless supermultiplets considered in this paper, namely, with λ_{MAX} in a totally antisymmetric representation of $SU(N)$. More generally, multilinear products of n preonic fields behave under $SU(8)$ as representations whose Young-tableaux have at most n columns. Therefore these multilinears belong to massive representations with maximum spin state in $SU(8)$ multiplets associated to Young-tableaux with up to $(n-2)$ columns.

As stated above, the composite fields defined by bilinear operators in the preonic fields, which are the simplest bound states we can deal with, belong to massive multiplets, $[J_{\text{MAX}}, \underline{1}]$, characterized by their maximum spin ($J_{\text{MAX}} = 4, 6, 8, \dots$) state being a $SU(8)$ singlet. The lowest spin multiplet with $J_{\text{MAX}} = 4$, is the fundamental massive representation of $N = 8$ supersymmetry. Its "disintegration" into massless supermultiplets gives the result:

$$[J_{\text{MAX}} = 4, \underline{1}] = \{4, [0]\} + \{7/2, [1]\} + \{3, [2]\} + \{5/2, [3]\} + \{2, [4]\} \quad (28)$$

All other massive representations are obtained from this one by changing the spin of the Clifford vacuum. The massive states of definite spin in the representation $[J_{\text{MAX}} = 4, \underline{1}]$ are associated to conserved bilinear products of the preonic field of $N = 8$ supergravity. The spin J ($J = 0, 1/2, \dots, 4$) states correspond to the $(8-2J)$ -fold traceless antisymmetric representation of $USp(16)$ ¹⁷⁾. Their $SU(8)$ assignments are obtained from the $SU(8)$ decomposition of these $USp(16)$ multiplets, as reported in Table II. In particular, we notice that the only bilinear operator which behaves as the adjoint representation of $SU(8)$ has spin $J = 3$. The only way to obtain a spin $J = 1$ bilinear in a $\underline{63}$ of $SU(8)$ is to assign spin $J = 2$ or $J = 4$ to the Clifford vacuum. So we conclude that there are precisely two irreducible massive supermultiplets ($J_{\text{MAX}} = 6, 8$) which contain a spin $J = 1$ state in the adjoint representation. In the zero-mass limit, the $\lambda = \pm 1$ components of this massive $J = 1$ states in a $\underline{63}$ of $SU(8)$ precisely go into the massless supermultiplet $\{3/2, \underline{8}\}$ considered in Ref. 6). Then we have a proof that if the $\phi \overset{\leftrightarrow}{\partial}_\mu \phi$ is part of a composite operator in a $\underline{63}$ which

would give rise to massless bound states, then these states must necessarily belong to the $\{3/2, \underline{8}\}$ massless supermultiplet of $N = 8$ supersymmetry (the ϕ 's are the scalar preonic fields)*).

We now consider the possibility that the massless spin 1/2 fermions are composite states associated to bilinear operators of the preonic fields of $N = 8$ supergravity that belong to an anomaly-free combination of $SU(8)$ representations. These are to be embedded into massive supermultiplets of $N = 8$ extended supersymmetry associated with the bilinear operators. Since the only anomaly-free, non-exotic combinations are given by (25) and (26), if we assume the hypothesis (II) as stated in Section 2, we are only left with the combination $\underline{56} + 5(\underline{8})$ which contains two $(\underline{10} + \underline{5})$ families. From Eq. (28), one easily verifies that the massive supermultiplets with $J_{MAX} = 4, 6$ and 8 contain all massless supermultiplets $\{\lambda_{MAX}, [k]\}$ with $\lambda_{MAX} = 2, 5/2, \dots, 9/2$ and $k = 0, 1, \dots, 7$. In particular, $[J_{MAX} = 8, \underline{1}]$ includes all the massless representations considered in Sections 2 and 3. However, with these three massive supermultiplets one can accommodate at most one $SU(8)$ superfamily of the type $\underline{56} + 5(\underline{8})$ and, therefore, only two $SU(5)$ families. In order to have at least four $SU(5)$ families without repetition of massive multiplets one has also to include $J_{MAX} = 10$. The solution in Eq. (27) can then be embedded in these four massive supermultiplets. Optional issues would be to consider, for instance, the repetition of the lower values of J_{MAX} or include composite supermultiplets corresponding to n multilinear composite fields with $n > 2$. However, any choice of these algebraic solutions should be justified by additional dynamical features of extended supergravity.

Finally, we mention the $N = 5$ case. Because N is odd, bosons and fermions can transform according to the same representations of $SU(5)$. Since the preonic supermultiplet is $\{2, \underline{1}\} = \{1/2, \underline{1}\}$ we obtain two lowest massive supermultiplets which are bilinear in the preonic fields, one (complex) with $J_{MAX} = 5/2$ and another with $J_{MAX} = 4$. If we assume that massless supermultiplets of bound states exist which are constructed in terms of these bilinear products, we can accommodate two $(\underline{10} + \underline{5})$ families of spin 1/2 left-handed fermions.

) The present considerations apply to composite operators that can be expressed in terms of the gauge invariant (preonic) supermultiplet on the zero-mass shell. In $N = 8$ supergravity such a multiplet has a scalar field $\phi^{ABCD} = \phi_{ABCD}^$ as the lowest component²³⁾. We are aware that gauge variant composite operators exist as well. However, their inclusion in our analysis is rather delicate due to difficulties of the kind considered in Ref. 24) and will not be discussed here.

5. CONCLUSIONS

In the present work we have explored several possibilities for an embedding of $SU(5)$ families of spin $1/2$ left-handed fermions in an effective non-Abelian gauge field theory in which the observed spectrum of quarks and leptons is supposed to be part of some supermultiplets of N extended supersymmetry. We have assumed that the gauge group is $SU(N)$ in N extended supersymmetry, as suggested by the dynamics of supergravity.

Since this $SU(N)$ is chiral, the anomaly-free condition, which is needed in order to have an effective renormalizable gauge theory, gives very strong constraints on the possible solutions. Moreover the condition of absence of anomalies has been shown to be not only sufficient but also necessary in order to have an effective anomaly-free gauge theory for a subgroup $SU(n) \subset SU(N)$ under the decomposition $\underline{N} \rightarrow \underline{n} + (N-n)\underline{1}$.

We have first obtained a set of algebraic solutions which give rise to $SU(N)$ families constructed only in terms of single-column Young-tableaux. These are the solutions which avoid $SU(5)$ exotics in a natural way. We have found that $N = 8$ supersymmetry has precisely the right multiplet structure to give in a unique fashion three $SU(8)$ families, containing three $(\underline{10} + \underline{\bar{5}})$ $SU(5)$ families, from $N = 8$ massless supermultiplets without repetitions.

Since massless supermultiplets include at least double-column Young-tableaux, we have considered a second set of models in which the spin $1/2$ spectrum is in a reducible $SU(N)$ representation whose complex part includes double-column Young-tableaux. We have then demanded for these solutions both the absence of anomalies and the absence of $SU(5)$ exotics in the complex part of the spectrum. We have found solutions for this class of models with a suitable combination of massless supermultiplets without repetition and with at most five $SU(5)$ $(\underline{10} + \underline{\bar{5}})$ families for $N = 8$ supersymmetry.

In the final part of the paper, we have considered the restrictions which arise for the massless supermultiplets participating in the effective gauge theory when they are supposed to be realized as conventional bound states of N extended supergravity. For even N (in particular $N = 8$), fermions must belong to $SU(N)$ representations with an odd number of boxes. This has the consequence that we cannot find anomaly-free solutions without exotics for $N = 6$ involving only massless supermultiplets with maximal helicity in a single-column Young-tableau. For $N = 8$ we have obtained solutions only by allowing for the repetition of

some massless supermultiplets and we have found the general result that $SU(5)$ $(10 \pm \frac{5}{2})$ families can only be generated in pairs. It follows that the algebraic solutions found in Sections 2 and 3 cannot be obtained in terms of conventional bound states. This also applies to the solutions of Ref. 19). We have also exploited some properties of composite multilinear operators constructed in terms of the massless elementary supermultiplet of N extended supergravity. We have shown that these operators are classified by massive representations of N extended supersymmetry. Bilinear operators in $N = 8$ supergravity are classified by those massive representations of $N = 8$ supersymmetry with an $SU(8)$ singlet J_{MAX} . This, in particular, allows us to uniquely identify those representations which can have a conserved vector current in the adjoint representation of $SU(8)$. In the limit of zero mass these representations include the massless supermultiplet which has massless spin 1 particles in the adjoint representation of $SU(8)$ which are believed to be the gauge particles of the effective $SU(8)$ [or $SU(N)$] gauge theory.

It is worth mentioning that the algebraic properties we use to classify the bound states have been derived in free field theory. The supersymmetry properties and the $SU(8)$ assignments of the composites have been extracted from operator products of massless free fields. In the interacting theory, where a global E_7 symmetry is also present, one could probably get more restrictions from the E_7 assignment of the composite operators.

In this paper we have completely neglected helicity states with $|\lambda| \geq 1$ with the exception of the graviton, the eight gravitinos and the vector bosons which are in the adjoint representation or singlets under $SU(8)$. Of course, our analysis is inconclusive for helicity states $\lambda > 1/2$ due to our inability to give an $SU(8)$ or even $SU(5)$ invariant mass to these states. However, we must stress that even if such a generalized Higgs effect were possible, it would require for these higher helicity states real $SU(N)$ [or $SU(5)$] representations. In particular, the spin 1/2 families given by Eq. (18), which occur in complex anomaly-free $SU(5)$ representations, could never participate in this Higgs effect which is only sensitive to the real part of the superfamilies. This might suggest that it is not impossible that our results could survive even if one tries to give an invariant mass through an infinite chain of supermultiplets¹⁶⁾ to all higher spin states.

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n	Number of supermultiplets	Number of superfamilies	Number of SU(5) families	Superfamilies	$\lambda = -1/2$ states: SU(N) content	SU(5) families
5	8	2	2	$F_1: (3, [1], [3])$ (5) (10) $F_2: (2+1+0, [2], [4])$ (10) (5)	$[2] + [4]$ (10) (5) $[2] + [4] + \text{real}$ (10) (5)	$= (10, \bar{5})$ $= (10, \bar{5}) + \text{real}$
6	≥ 3	1	1	$F_1: (7/2, [1], [4]) + (3, [0])$ (6) (15) (1)	$2[5] + [2]$ (6) (15)	$= (10, \bar{5}) + \text{real}$
7	≥ 14	2	2	$F_1: (3+2, [0]) + (4, [1], [2], [4])$ (11) (7) (21) (35) $F_2: (0, [0]) + (3+2+1+0, [3], [6])$ (11) (35) (7)	$2[3] + [5] + [6] + \text{real}$ (35) (21) (7) $2[6] + [3] + \text{real}$ (7) (35)	$= (10, \bar{5}) + \text{real}$ $= (10, \bar{5}) + \text{real}$
8	15	3	3	$F_1: (9/2, [1], [2], [5])$ (8) (28) (56) $F_2: (2+4+7/2, [0])$ (11) $F_3: (7/2+5/2, [7], [6], [3]) + (2+4+7/2, [4])$ (8) (28) (56) (70)	$[7] + [6] + [3]$ (8) (28) (56) $[7] + [6] + [3] + \text{real}$ (8) (28) (56) $[7] + [6] + [3] + \text{real}$ (8) (28) (56)	$= (10, \bar{5}) + \text{real}$ $= (10, \bar{5}) + \text{real}$ $= (10, \bar{5}) + \text{real}$
9	≥ 16	3	3	$F_1: (5, [2], [5])$ (36) (126) $F_2: (4+2+1+0, [0])$ (11) $F_3: (4+3+2+1+0, [4], [7])$ (126) (36)	$[4] + [7]$ (126) (36) $[4] + [7] + \text{real}$ (126) (36) $[4] + [7] + \text{real}$ (126) (36)	$= (10, \bar{5}) + \text{real}$ $= (10, \bar{5}) + \text{real}$ $= (10, \bar{5}) + \text{real}$
10	≥ 8	2	4	$F_1: (11/2, [4], [3]) + (5/2, [0])$ (210) (120) (1) $F_2: (11/2, [6], [2], [1]) + (5+9/2, [0])$ (210) (45) (10) (1)	$[4] + [7] + \text{real}$ (210) (120) $[4] + 2[8] + 2[9] + \text{real}$ (210) (45) (10)	$= (10, \bar{5}) + \text{real}$ $= 3(10, \bar{5}) + \text{real}$
11	≥ 36	3	7	$F_1: (6, [1], [2], [3], [7])$ (11) (55) (165) (330) $F_2: (5+4+3+2+1+0, [10], [9], [8], [4])$ (11) (55) (165) (330) $F_3: (6, [4], [6]) + (5+4+3+2+1+0, [5])$ (330) (462) (462)	$[10] + [9] + [8] + [4]$ (11) (55) (165) (330) $[10] + [9] + [8] + [4] + \text{real}$ (11) (55) (165) (330) $2[5] + [7] + \text{real}$ (462) (330)	$= 3(10, \bar{5}) + \text{real}$ $= 3(10, \bar{5}) + \text{real}$ $= (10, \bar{5}) + \text{real}$

- TABLE I -

J	SU(8) Representation
4	1
7/2	$8 + \bar{8}$
3	$63 + 28 + \bar{28}$
5/2	$56 + 216 + \text{h.c.}$
2	$70 + 70 + 420 + \bar{420} + 720$
3/2	$56 + 504 + 1344 + \text{h.c.}$
1	$(28 + 378 + 1512 + \text{h.c.}) + 2352'$
1/2	$8 + 168 + 1008 + 2352 + \text{h.c.}$
0	$1 + 1 + (36 + 336 + 1176 + \text{h.c.}) + 1764$

- TABLE II -

TABLE CAPTIONS

TABLE I Sets of supermultiplets fulfilling our requirements. Notice, however, that for $N \neq 8$ the solutions are not unique. Those given here contain the maximum number of SU(5) families and of superfamilies, with the minimum number of supermultiplets. In some cases the same number of $(10+\bar{5})$ families can be obtained with other choices for the superfamilies (e.g., one superfamily for $N = 9,10$). For $N = 8$, however, the solution is unique. The inclusion of the supercurrent multiplet would require adding more multiplets in some cases (see text).

TABLE II SU(8) representations corresponding to each spin J state of the irreducible massive representation $[J_{\text{MAX}} = 4, \frac{1}{2}]$ of $N = 8$ extended supersymmetry, characterized by a maximum spin state with $J_{\text{MAX}} = 4$ in an SU(8) singlet. SU(8) representations for a given spin J correspond to an irreducible representation of USp(16).

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