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SU(2) LATTICE GAUGE THEORY AND MONTE CARLO CALCULATIONS

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A B S T R A C T

Monte Carlo results for the SU(2) lattice gauge theory in four dimensions are presented. The string tension is measured with high statistics and also the mass of the perimeter term is determined. Wilson loop-plaquette correlations, which are related to roughening, are measured.

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1. - INTRODUCTION

We consider standard $SU(2)$ lattice gauge theory ¹⁾ in four dimensions with lattice spacing a . The random variables $U(b) \in SU(2)$ are attached to links b of the lattice and the expectation value of a gauge invariant observable $O\{U\}$ is given by

$$\langle O \rangle = Z^{-1} \int \prod_b dU(b) O\{U\} \exp - \frac{\beta}{2} \sum_p \text{Tr}[1 - U(p)] \quad (1)$$

Here dU is the $SU(2)$ Haar measure. Summation is over all unoriented plaquettes p of the lattice and $U(p) = U(b_4) \dots U(b_1)$ is the parallel transporter around the plaquette p with boundary $b_1, \dots, b_4$. The partition function Z ensures the normalization $\langle 1 \rangle = 1$. β is related to the usual bare coupling constant by $\beta = 4/g^2$.

In the weak coupling (= low temperature) limit $\beta \rightarrow \infty$ the theory is supposed to approach a critical point and to define a quantum field theory in the continuum. The lattice spacing a is a cut-off for small distances and therefore a^{-1} is an ultra-violet cut-off in momentum space. In the limit $\beta \rightarrow \infty$ any physical mass m has to obey the weak coupling renormalization group equation, i.e., for the $SU(2)$ theory

$$m = \frac{\text{const}}{a} \beta^{\frac{51}{121}} \exp(-\frac{3\pi^2}{11}\beta) = \text{const} \Lambda^L \quad (2)$$

Λ^L defines the mass scale within the lattice regularization. Hasenfratz and Hasenfratz ²⁾ have worked out the relation between Λ^L and the scale Λ^M , defined by momentum space subtraction of conventional perturbation theory. For $SU(2)$:

$$\Lambda^M = 57.5 \Lambda^L \quad (3)$$

It is commonly believed that dynamical mass generation takes place in four-dimensional $SU(N)$ gauge theories ($N \geq 2$) and that the lowest state in the spectrum - called glueball - acquires a mass $m_g > 0$. The mass of the glueball sets the natural scale of the theory. The correlation length ξ is defined as the inverse glueball mass $\xi = 1/m_g$. If everything is measured in units of $[m_g]$, we see from (2) that the lattice spacing $a \rightarrow 0$ in the limit $\beta \rightarrow \infty$, whereas the

the correlation length ξ stays constant. It is in this limit $\xi = \text{const.}$, $a \rightarrow 0$ that Euclidean invariance of the continuum theory is believed to be restored.

In the strong coupling (= high temperature) expansion (β small) lattice gauge theories confine static quarks¹⁾. Large Wilson loops^{*)} fall off like $\exp\{-K \cdot A\}$, where A is the area of the loop and K is the string tension. With respect to confinement, the strong coupling limit does not distinguish between $U(1)$ and $SU(N)$ ($N \geq 2$) gauge theories. For the $U(1)$ gauge theory the existence of a non-confining phase is known⁴⁾ for sufficiently large β , whereas no such phase transition is expected for $SU(N)$ ($N \geq 2$) gauge theories. In the latter case, the weak coupling behaviour of the string tension is obtained from (2), because the string tension has dimension $[m^2]$.

Recently there has been some progress in investigating properties of the continuum theory by means of Monte Carlo calculations on finite lattices. Monte Carlo calculations were first advocated by Wilson⁵⁾ using the block spin recursion. Creutz⁶⁾ made the remarkable discovery that there is a region in β , where Monte Carlo data from small lattices up to size 10^4 , already exhibit the expected asymptotic weak coupling behaviour of the string tension K . This is quite surprising, as the size of the involved Wilson loops has to be large compared with the correlation length ξ and the continuum theory is only approached for $\xi \rightarrow \infty$. Self-consistency arguments⁷⁾ require a correlation length less than one lattice spacing in the region $\beta \gtrsim 2.0$, where the asymptotic weak coupling behaviour starts. Direct measurements of plaquette-plaquette correlations⁷⁾ indicate the relation

$$m_g = (3.7 \pm 1.2) \sqrt{K} \quad (4)$$

This gives $\xi \approx 0.4$ at $\beta = 2.1$. A similar order of magnitude ($m_g = (3 \pm 1)\sqrt{K}$) was independently obtained by Bhanot and Rebbi⁸⁾, who considered the maximal discrete subgroup of $SU(2)$. For definiteness we use in the present paper the value $m_g = 3.7\sqrt{K}$, but one has to be aware of the rather large error bars. In view of Ref. 8) and very recent results of Münster⁹⁾ the true value of m_g is presumably smaller. Details of the structure of the $SU(2)$ confinement mechanism are illuminated by the Monte Carlo results of Mack and Pietarinen^{10),11)}.

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*) They were first introduced by Wegner³⁾ as non-local order parameters.

High temperature expansion of the string tension up to order β^{12} ¹²⁾ exhibits a rapid cross-over from strong to weak coupling in the region $1.8 \leq \beta \leq 2.2$. The applicability of high temperature expansion beyond $\beta \approx 1.9$ is, however, questionable by the phenomenon of roughening ^{13),14)}. For $\beta > \beta_r$, where $\beta_r \approx 1.9$ determines the roughening transition, the thickness of chromo-electric flux tubes increases logarithmically with the separation L of the static quark pair, whereas it approaches a constant for $\beta < \beta_r$.

Let us consider

$$\epsilon_n(I, J) = \frac{\langle W(I, J) \cdot W(1, 1)_x \rangle - \langle W(I, J) \rangle \langle W(1, 1) \rangle}{\langle W(I, J) \rangle} \quad (5)$$

in the Euclidean region. Here $W(I, J) = \frac{1}{2} \text{Tr}(U(\dot{C}_{IJ}))$, $I, J = 1, 3, 5, \dots$, where C_{IJ} is a rectangular Wilson loop, which is chosen to be in the (x_3, x_4) plane with $x_1 = x_2 = 0$ and the centre-plaquette is located at $x_3 = x_4 = 0$. $U(\dot{C}_{IJ})$ is the ordered product of link variables along the boundary $\dot{C}_{IJ}$ of C_{IJ} . For the special case $I = J = 1$ we obtain $\frac{1}{2} \text{Tr}(U(\dot{p}))$. By $x = (x_1, x_2) = (n_1 a, n_2 a) \equiv na$ the separation of $W(1, 1)_x$ from $W(I, J)$ is fixed.

$\epsilon_n(I, J)$ is a form factor which measures the energy density inside the Wilson loop compared with the vacuum. For $|n| = (n_1^2 + n_2^2)^{\frac{1}{2}} \rightarrow \infty$, $\epsilon_n(I, J)$ falls off exponentially. For $|n| < I, J$, the width is the Euclidean pedant for the thickness of the chromo-electric flux tube in the Minkowski region. In Ref. 14)

$$\epsilon_n^\infty = \lim_{I, J \rightarrow \infty} \epsilon_n(I, J) \quad (6)$$

was considered by means of high temperature expansion. Above β_r the width of $\epsilon_n(I, J)$ diverges for large I and J logarithmically with $\min\{I, J\}$, whereas it approaches a constant for $\beta < \beta_r$.

In Section 3 of the present paper we consider $\epsilon_n^3 = \epsilon_n(3, 3)$ for $n = 0 \equiv (0, 0)$ and $n = 1 \equiv (1, 0)$ or $(0, 1)$. In a region of β , where we have a correlation length $\xi < a$, this is not a bad approximation of ϵ_n^∞ . Unfortunately the numerical smallness of ϵ_n^3 for $|n| > 1$ and ϵ_n^5 for all n does not allow a direct Monte Carlo measurement of the conjectured roughening transition. Nevertheless, the comparison of ϵ_n^3 with ϵ_n^1 provides some modest support for the existence of the roughening transition.

Our data for the string tension are presented in Section 2. The result is based on the rather high statistic of altogether more than 8000 sweeps through an 8^4 lattice. As far as quantitative comparison is possible, our values are in very good agreement (within the expected statistical errors) with the values published previously by Creutz^{6),15)}.

We have also computed the mass of the perimeter term in the asymptotic behaviour of the Wilson loops. In leading order perturbation theory [e.g., Ref. 16)] this mass is expected to be linear divergent. We find that this leading behaviour fits already quite well our Monte Carlo results.

Finally, some technical details of the statistical analysis of our data are relegated to the Appendix. All our data are obtained by means of the heat bath method, which is, for the $SU(2)$ gauge theory, described in detail in Ref. 6).

2. - STRING TENSION AND PERIMETER MASS

The string tension K was first measured by Creutz⁶⁾ by fitting square Wilson loops of size 5×5 (up to $S = 5$) to the behaviour $\langle W(S) \rangle = \exp\{-(c_1 + c_2 S + c_3 S^2)\}$. Later¹⁵⁾, Creutz refined his analysis by introducing the quantity

$$\chi(I, J) = - \ln \left(\frac{\langle W(I, J) \rangle \langle W(I-1, J-1) \rangle}{\langle W(I, J-1) \rangle \langle W(I-1, J) \rangle} \right) \quad (7)$$

Here $W(I, J)$ is a rectangular Wilson loop of area $A = a^2 I \cdot J$ and perimeter $P = 2a(I+J)$. Provided the Wilson loops behave according to

$$\langle W(I, J) \rangle = \text{const.} \exp(-K \cdot A - m_p \cdot P) \quad (8)$$

the quantity $\chi(I, J)$ measures directly the string tension. The behaviour (8) is expected to be the dominant term for $I, J \gg \xi$, where ξ is the correlation length. Relying on the estimate of the correlation length in Ref. 7), this means numerically $I, J \gtrsim 2\xi$.

Results in Monte Carlo calculations are obtained in units $[a^{\pm n}]$ ($n = 0, 1, 2, \dots$) of the lattice spacing a . In tables and figures we set $a = 1$. The power n is easily reconstructed by counting the engineering dimension of the considered physical quantity.

Figure 1 summarizes the data of Creutz¹⁵⁾ and of the present investigation for the string tension. In lowest order high temperature expansion the string tension is given by $K = -\ln\beta/4$. Above $\beta = 1.8$ a rapid cross-over from strong to weak coupling is observed. The estimate of the string tension appears in the graph as a band corresponding to

$$\Lambda^L = (1.27 \pm 0.19) \cdot 10^{-2} \sqrt{K} \quad (9)$$

As is seen from the graph, this estimate relies on data for $X(3,3)$ in the region $\beta = 2.2-2.5$. For self-consistency, $2a$ has to be large compared with the correlation length ξ in this region. At $\beta = 2.5$ we have, according to $m_g \approx 3.7 \sqrt{K}$, $\xi \approx 1$ and for $\beta \gtrsim 2.5$, the data of $(3,3)$ are not measuring the string tension any more.

Our data and the data of Creutz for $X(3,3)$ are in agreement within the statistically expected errors. Data for $X(2,2)$ cannot give information on the string tension above $\beta = 2.1$, because $W(1,1)$ is then too small compared with the correlation length.

The correlation length is $\xi \approx 1.5$ at $\beta = 2.55$ and $\xi \approx 2$ at $\beta = 2.75$. Therefore, data for $X(4,4)$ are expected to be in the estimated band for K up to $\beta \approx 2.55$ and data for $X(5,5)$ are expected to be in the band up to $\beta \approx 2.75$. We have attempted to measure $X(5,5)$ on a 12^4 lattice. Due to the numerical smallness of the involved Wilson loops, a very high statistic is needed to get sensitive results. So far our error bars do not admit any reliable conclusions. For illustration we have included one value for $X(5,5)$ at $\beta = 2.5$. With increasing β , Wilson loops are easier to measure, because they become numerically larger. At $\beta = 2.8$ the correlation length is, however, already larger than $2a$ and the value of Creutz for $X(4,4)$ cannot be expected to be inside the band for K .

Our data for the Wilson loops $W(1,1)$, $W(2,2)$, $W(3,2)$ and $W(3,3)$ are given in Table 1. They are altogether based on 8,172 sweeps through a 8^4 lattice. After each sweep, all the Wilson loops of the lattice are measured. For a 8^4 lattice this means $6 \cdot 8^4 = 24,576$ per sweep. We have used the heat bath method as described in Ref. 6). Approximately half of the data are obtained with a disordered (random) start and half of the data are obtained with an ordered $[a_0 = 1, \vec{a} = 0$ in the parametrization of Ref. 6)] start. The error bars for the Wilson loops are obtained from the variance, which is computed by comparing the mean value of all sweeps with the results of the single sweeps. The error

bars for the string tension rely on assuming statistical independence of the error-bars of the Wilson loops on which K depends. More details and some subtle points of the error estimate are discussed in the Appendix.

For Wilson loops of small size (e.g., up to 3×3), the cancellation of the perimeter dependence in (7) is a subtle point, because the perimeter is larger than the area. It can only work if the asymptotic behaviour (8) is indeed a good approximation for all involved loops. Therefore the data of Table 1 also enable us to determine the constant and the perimeter mass in (8).

The perimeter mass is a cut-off dependent quantity. In leading order weak coupling expansion, one expects [e.g., Ref. 16)] $m_p \propto a^{-1}$. One obtains m_p and the constant of (8) by fitting the Wilson loops, which enter in $X(3,3)$, to the behaviour (8). Our results for m_p are given in Fig. 2. The error bars are fitted as for the string tension. From the three different possibilities we have taken the smallest error bar. Roughly, m_p stays constant, leading to

$$m_p = (1.3 \pm 0.1) \cdot a^{-1} \quad (10)$$

It is quite remarkable that the small lattice already gives an approximation of the leading order cut-off dependence and that contributions of order g^2 seem to be suppressed. The constant in Eq. (8) also changes only slowly (and unsystematically) in the considered region of β . A good fit is $\text{const} \approx 1.2$. The values cannot be directly compared with the weak coupling value of Ref. 16), because the relation of the lattice scale to the scale implied by the regularization used in Ref. 16) is not clear (at least to us). The order of magnitude is that suggested by the relation (3) between Λ^M and Λ^L .

3. - WILSON LOOP PLAQUETTE CORRELATIONS

In this Section we mainly consider the quantity ϵ_n^3 as defined by (5) for $n = 0 \equiv (0,0)$ and $n = 1 \equiv (1,0)$ or $(0,1)$. For $|n| = (n_1^2 + n_2^2)^{\frac{1}{2}} > 1$, ϵ_n^3 becomes very small and we therefore did not attempt to measure ϵ_n^3 for $|n| > 1$.

Motivated by the high temperature results of Ref. 14), we are interested in ϵ_n^∞ . At first sight, ϵ_n^3 seems to be a very poor approximation to ϵ_n^∞ . One has, however, to be aware of the fact that ϵ_n^L ($L=1,3,5,\dots$) is a good approximation

to ϵ_n^∞ as long as $L \gg \xi$. Our experience with Wilson loops from the previous Section suggests that this numerically means $L \gtrsim 2\xi$. Up to $\beta = 2.6$ the correlation length is less than $1.5a$ and therefore ϵ_n^3 is not such a bad approximation of ϵ_n^∞ .

In Fig. 3, our data for ϵ_n^3 ($n=1,2$) and the high temperature expansion of Ref. 14) for ϵ_n^∞ ($n=1,2$) are represented. In fact our data for ϵ_n^3 are in good agreement with the high temperature expansion for $\beta = 1.8$ and $\beta = 1.9$, especially if one is aware of the fact that the high temperature expansion ceases to be reliable at $\beta \approx 1.9$. It would be desirable to have additional Monte Carlo data for $\beta < 1.8$. This is, however, difficult as $W(3,3)$ becomes numerically very small [like $(\beta/4)^9$] for $\beta \rightarrow 0$.

Our values for ϵ_n^3 ($n=0,1$) rely on measurements of

$$\rho(L,n) = \langle W(L,L) \cdot W(1,1)_x \rangle - \langle W(L,L) \rangle \langle W(1,1) \rangle, \quad x=na \quad (11)$$

for $L = 3$ and $n = 0,1$. Table 2 summarizes the directly measured data for $\rho(3,n)$. ϵ_n^3 is obtained by dividing through $\langle W(3,3) \rangle$ and assuming statistically independent errors. The error bars of $\rho(3,n)$ and $W(3,3)$ are obtained by the method explained in Section 1 (cf. also the Appendix).

Below the expected roughening transition $\beta_r \approx 1.9$, our data ($\beta = 1.8, 1.9$) are consistent with a drastic increase of $q^3 = \epsilon_1^3/\epsilon_0^3$. But in view of large error bars for ϵ_1^3 the data are not very restrictive.

Above $\beta_r \approx 1.9$ one would like to compare q^3 with $q^5 = \epsilon_1^5/\epsilon_0^5$. In the region up to $\beta \approx 2.2$, where $\xi \approx 0.5a$ one would expect the values of q^3 and q^5 to be sensitive with respect to a logarithmic increase of the width of ϵ_n^L (in $x = na$) as $L \rightarrow \infty$. Unfortunately, $W(5,5)$ is, in this region, numerically too small to be measured with the present Monte Carlo method. Therefore, we are sticking to presenting an amazing observation, which is obtained by comparing q^3 with $q^1 = \epsilon_1^1/\epsilon_0^1$. In a region where $\xi \lesssim 0.5a$ this may already give a qualitatively correct picture.

In Table 3, data ⁷⁾ for $\rho(n) = \rho(1,n)$ are given. In a region where the correlation length ξ is very small compared with $1a$, plaquette-plaquette correlations fall immediately off with the glueball mass :

$$\xi(n) = \text{const} \cdot \exp(-m_q \cdot |x|) \quad (12)$$

Above β_r , roughening gives the prediction

$$\xi(L, n) = \text{const}_L \cdot \exp\left(-\frac{m_1}{\ln(m_2 \cdot L)} x^2\right) \quad (13)$$

for $\xi \ll x \ll L$. For the assumed very small correlation length it is reasonable to argue that the logarithmic growth of the width starts immediately. One is then led to the formula

$$\xi(L, n) = \text{const}_L \cdot \exp\left(-\frac{m_q}{\ln(2 \cdot \alpha^{-1} \cdot L)} x^2\right) \quad (14)$$

for $x = na$ with $n = 0, 1$ and $L = 1, 3$.

At the roughening transition $\beta = \beta_r \approx 1.9$ the correlation length is estimated to be $\xi \approx 0.3$. It is increasing to $\xi \approx 0.4$ at $\beta = 2.1$. This is, of course, not very small but still small. Furthermore, for small n , the asymptotic behaviour (12) can become modified by an unknown β dependent power law. As outlined in Ref. 7), such a power law seems to become important around $\beta \approx 2.0$. In conclusion, only a great optimist can hope that Eq. (14) is a reasonable approximation for $L = 1, 3$, $n = 0, 1$ and β directly above β_r . In Table 4 we compare the naive prediction from (13) for q^3/q^1 with the measured value *) for q^3/q^1 in the region $1.8 \leq \beta \leq 2.1$.

The result is precisely what the great optimist would expect. At $\beta = 2.0$ the naive prediction is $q^3/q^1 = 5.7$ and the measured value $q^3/q^1 \approx 5.5$ is very close to it. However, for other values of β the discrepancy is much larger. This is expected, because for $\beta \leq \beta_r \approx 1.9$ one is still below roughening, whereas for $\beta > 2.0$ the correlation length increases rapidly. In conclusion we take the result as support - in a modest sense - for the conjecture of a roughening transition.

Finally, for large β one observes $q^3 \rightarrow \text{const}$, which is expected from a dominant power law behaviour.

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*) Because of the large error bars in ϵ_1^3 up to $\beta = 2.0$ the measured value of ϵ_1^3 is actually replaced by the high temperature expansion and its continuation by the broken line in Fig. 3, which seems to be the most reasonable fit.

4. - SUMMARY AND CONCLUSIONS

Our data for the string tension confirm the result of Creutz ^{6),15)} with high accuracy. Furthermore, the mass m_p of the perimeter term in the asymptotic behaviour of the Wilson loops is found to stay roughly constant. This is what would be expected if the leading linear cut-off dependence is dominant. Our measurements of the Wilson loop-plaquette correlations ϵ_0^3 and ϵ_1^3 are consistent with the high temperature expansion (where they should) and provide some modest support for the roughening transition ^{13),14)}.

A puzzling point for the whole investigation, but especially for the last argument in the previous Section, is that features of the continuum theory already show up at a very small correlation length ($<1a$). To get a physical understanding of this point seems to be a major challenge at present.

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APPENDIX

In this Appendix we give some details of the statistical analysis. Let us denote by $\bar{O}$ the mean value of an observable over all sweeps and by $O(i)$ the result of one sweep i , which is of course still the mean value of many measurements. For directly measurable observables we have obtained the error bar δ by means of

$$\delta = \sqrt{\frac{\sigma}{N-1}} \quad (A1)$$

Here the variance σ is defined by

$$\sigma = \frac{1}{N} \sum (\bar{O} - O(i))^2 \quad (A2)$$

N is the number of sweeps performed. Let $F = F(O_1, \dots, O_n)$ be a composite quantity, depending on several directly measurable observables $O_1, \dots, O_n$ with error bars $\delta_1, \dots, \delta_n$. If the quantities are statistically independent, the error bar of F is given by the Gaussian law

$$\delta_F = \left\{ \sum_{i=1}^n \left(\frac{\partial \bar{F}}{\partial \bar{O}_i} \delta_i \right)^2 \right\}^{1/2} \quad (A3)$$

where $\bar{F} = F(\bar{O}_1, \dots, \bar{O}_n)$. We have applied (A3) for the string tension K and the perimeter m_p , however, not for the correlation function $\rho(3, n)$ ($n=0, 1$). For $\rho(3, n)$ we have instead used directly (A1), (A2) again.

The applicability of our procedure may be doubted because data of successive sweeps are of course correlated. For control we have for each value of β divided our data set into blocks of 20 sweeps and repeated our analysis with $O(i)$ replaced by the mean value of block i , and N replaced by the number of blocks. The result is summarized in Table 5. For numerically large quantities, correlations between successive sweeps become important and the error bars are larger by factors up to 4.4. Numerically small quantities are, however, not strongly correlated between subsequent sweeps and their error bars stay approximately constant. This applies especially for $\rho(3, n)$ ($n=0, 1$). It is remarkable that $\langle W(3, 3) W(1, 1) \rangle_x$ and $\langle W(3, 3) \rangle \langle W(1, 1) \rangle$ are strongly correlated in the sense that a Gaussian estimate (A3) of the error bar of $\rho(3, n)$ from these quantities would strongly overestimate the error.

For the directly measured Wilson loops the error bars multiplied by the values of Table 5 are presumably the reliable ones. For the estimate of the errors in the string tension and the perimeter mass, we have nevertheless used the error bars of Table 1 because otherwise the errors of the dependent quantities would have been overestimated. This is an empirical fact, which shows up in a crude analysis of the confidence level.

We divide our data for a given β randomly in two equal parts. Let us denote by δ the error bar of the mean value $\bar{O}$ of the whole set and by $\bar{O}_1$ and $\bar{O}_2$ the mean values of the two subsets. By the central limit theorem we expect for mean values of large sets of measurements a Gaussian distribution. For a Gaussian distribution it is easily checked that one expects

$$|\bar{O}_1 - \bar{O}_2| \leq 2\delta \quad \text{with} \quad 68\% \quad \text{probability}$$

$$|\bar{O}_1 - \bar{O}_2| \leq 2(2\delta) \quad \text{with} \quad 95.5\% \quad \text{probability}$$

$$|\bar{O}_1 - \bar{O}_2| \leq 2(2\delta) \quad \text{with} \quad 99.7\% \quad \text{probability}$$

etc. In Table 6, the result of this check is given for the observables discussed in this paper. (Error bars δ are taken from Tables 1 and 2.) A value n means $|\bar{O}_1 - \bar{O}_2| \leq n(2\delta)$. For all observables together we get $n = 1$ for 73%, $n \leq 2$ for 99% and $n \leq 3$ for 100%. In view of a total number of 88 values this is very reasonable.

If one does not make a random choice of the two subsets, but does a division following the chronological order, strong correlations show up for the Wilson loops if $\beta \geq 2.2$ (n up to 6), whereas the values for K , m_p and $\rho(3,n)$ remain quite reasonable. We conclude that the error bars of K , m_p and $\rho(3,n)$ are O.K., whereas the error bars of the Wilson loops should be multiplied with the values of Table 5 for a comparison with other people's data.

Table 1: Data for the Wilson loops

β	$\langle W(1,1) \rangle$	$\langle W(2,2) \rangle$	$\langle W(3,2) \rangle$	$\langle W(3,3) \rangle$	#Sweeps
1.8	0.44075 ± 0.00010	0.04083 ± 0.00011	0.00845 ± 0.00011	0.00083 ± 0.00010	1009
1.9	0.46980 ± 0.00012	0.05460 ± 0.00012	0.01345 ± 0.00012	0.00180 ± 0.00010	991
2.0	0.50095 ± 0.00025	0.07315 ± 0.00014	0.02112 ± 0.00011	0.00338 ± 0.00010	920
2.1	0.53461 ± 0.00016	0.09998 ± 0.00017	0.03484 ± 0.00014	0.00771 ± 0.00012	861
2.2	0.56877 ± 0.00013	0.13586 ± 0.00022	0.05677 ± 0.00019	0.01727 ± 0.00014	864 / 813 ⁺
2.3	0.60183 ± 0.00014	0.17961 ± 0.00025	0.08820 ± 0.00023	0.03446 ± 0.00018	755 / 668
2.4	0.63032 ± 0.00012	0.22289 ± 0.00028	0.12406 ± 0.00027	0.06077 ± 0.00024	826 / 597
2.5	0.65135 ± 0.00034	0.25708 ± 0.00034	0.15431 ± 0.00037	0.08159 ± 0.00026	645 / 328
2.6	0.67012 ± 0.00012	0.28830 ± 0.00023	0.18290 ± 0.00025	0.10644 ± 0.00025	531
2.7	0.68574 ± 0.00013	0.31436 ± 0.00028	0.20703 ± 0.00031	0.12675 ± 0.00030	432
2.8	0.69957 ± 0.00014	0.33742 ± 0.00031	0.22867 ± 0.00035	0.14507 ± 0.00035	338

⁺) If two values for the number of sweeps are given, the first corresponds to W_{11} , W_{33} and the second to W_{22} , W_{32} .

Table 2: Data for correlations between W_{33} and W_{11}

β	$\rho(3,0) \cdot 10^{+2}$	$\rho(3,1) \cdot 10^{+3}$	#Sweeps
1.8	0.0257 ± 0.0036	0.035 ± 0.034	1009
1.9	0.0445 ± 0.0033	0.009 ± 0.041	991
2.0	0.0728 ± 0.0036	0.040 ± 0.039	920
2.1	0.1043 ± 0.0041	0.198 ± 0.042	861
2.2	0.1411 ± 0.0040	0.268 ± 0.035	864
2.3	0.1838 ± 0.0042	0.414 ± 0.031	755
2.4	0.1847 ± 0.0041	0.432 ± 0.032	826
2.5	0.1903 ± 0.0061	0.380 ± 0.035	645
2.6	0.1766 ± 0.0034	0.340 ± 0.037	531
2.7	0.1692 ± 0.0032	0.357 ± 0.040	432
2.8	0.1731 ± 0.0032	0.335 ± 0.043	338

Table 3: Some data for plaquette - plaquette correlations

β	$g(0)$	$g(1) \cdot 10^{+2}$
1.8	0.160	0.355 ± 0.003
1.9	0.148	0.387 ± 0.003
2.0	0.136	0.404 ± 0.003
2.1	0.122	0.396 ± 0.003
2.2	0.106	0.362 ± 0.003
2.3	0.092	0.293 ± 0.003
2.4	0.081	0.244 ± 0.004
2.5	0.072	0.203 ± 0.003
2.6	0.065	0.179 ± 0.003
2.7	0.060	0.157 ± 0.003

Table 4: Prediction and measured value for q_3/q_1

β	prediction	measured
1.8	6.5	4.5
1.9	6.1	5.2
2.0	5.7	5.5
2.1	3.7	5.7

Table 5: Factors which relate the errorbars of table 1 and table 2 to the errorbars obtained by the collecting of 20 sweeps into one block as explained in the appendix

β	w_{11}	w_{22}	w_{32}	w_{33}	$\rho^{(0)}$	$\rho^{(1)}$
1.8	2.0	1.1	1.0	1.0	0.9	0.8
1.9	2.3	1.5	1.2	1.1	0.8	1.0
2.0	2.5	1.8	1.3	0.9	0.9	1.0
2.1	3.0	1.8	1.4	1.3	1.1	1.1
2.2	2.7	2.7	2.4	1.8	1.0	1.0
2.3	3.0	2.6	2.5	2.5	1.1	0.9
2.4	2.5	2.8	2.9	3.3	1.1	1.0
2.5	4.4	2.5	2.6	2.9	0.8	1.0
2.6	2.3	1.8	1.9	2.0	0.9	1.1
2.7	2.2	2.0	1.9	1.9	0.8	1.1
2.8	2.8	2.2	2.2	2.1	0.8	1.1

Table 6: Random check of the confidence level

β	w_{11}	w_{22}	w_{32}	w_{33}	K	m_p	$\rho^{(0)}$	$\rho^{(1)}$	ε_0	ε_1
1.8	1	1	2	1	1	1	1	2	1	2
1.9	1	1	1	1	1	1	1	2	1	1
2.0	2	3	1	1	1	1	1	2	1	2
2.1	1	2	2	1	1	1	1	1	1	1
2.2	1	1	1	1	1	1	2	1	1	1
2.3	1	1	1	1	1	1	2	1	2	1
2.4	1	1	1	1	1	1	1	1	1	1
2.5	1	1	2	2	1	1	1	1	1	1
2.6	1	2	2	1	1	1	1	1	1	2
2.7	1	2	1	1	1	1	1	2	1	2
2.8	1	2	1	1	1	1	2	1	2	1

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FIGURE CAPTIONS

- Figure 1 String tension. Data for $X(I,I)$ in units $[a^{-2}]$: $\bullet$ (I=3) and $\blacksquare$ (I=5) from our measurements and from Creutz ¹⁵⁾ ; $\times$ (I=2), $\circ$ (I=3) and $\square$ (I=4).
- Figure 2 Mass of the perimeter term m_p as function of β . (Units $[a^{-1}]$).
- Figure 3 Monte Carlo data for $\epsilon_0^3, \epsilon_1^3$ and high temperature expansion for $\epsilon_0^\infty, \epsilon_1^\infty$.

FIG. 1

FIG. 2

FIG. 3