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ANOTHER CONNECTION BETWEEN THE Λ PARAMETERS
OF THE EUCLIDEAN LATTICE AND CONTINUUM QCD

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A B S T R A C T

We compute the relation between the lattice and the short distance $q\bar{q}$ force Λ parameters. The result of this perturbative computation is in striking agreement with the Creutz Monte Carlo estimate.

There is a general belief that the Euclidean continuum $SU(N)$ theory can be obtained from a latticized version in the limit lattice spacing $a \rightarrow 0$ and bare coupling $g_0 \rightarrow 0$ so that the renormalization group invariant mass parameter

$$\Lambda_L = \frac{1}{a} e^{-\frac{1}{2\beta_0 g_0^2} \left(\beta_0 g_0^2 \right)^{-\frac{\beta_1}{2\beta_0^2} (1 + O(g_0^2))}} \quad (1)$$

tends to a finite non-zero value. β_0 and β_1 are the famous first two coefficients of the Callan-Symanzik β function

$$\beta_0 = \frac{1}{(4\pi)^2} \left(\frac{11}{3} C_2(G) - \frac{4}{3} T(R) \right) \quad (2)$$

$$\beta_1 = \frac{1}{(4\pi)^4} \left(\frac{34}{3} C_2(G)^2 - \frac{20}{3} C_2(G) T(R) - 4 C_2(R) T(R) \right)$$

Λ_L sets the mass scale of the continuum theory. All physical masses are expressed as a constant times Λ_L . For example, Creutz's Monte Carlo data indicate that, in the continuum limit, the string tension (for a pure gauge system latticized à la Wilson) is equal to¹⁾

$$\sqrt{K} = \frac{1}{(1.3 \pm 0.2) 10^{-2}} \Lambda_L \quad (N=2)$$

$$\sqrt{K} = \frac{1}{(5.0 \pm 1.5) 10^{-3}} \Lambda_L \quad (N=3) \quad (3)$$

The problem is that the Λ parameter is scheme dependent. To exploit the above results one needs the relation between Λ_L and a more conventional continuum theory parameter, e.g., Λ_{MOM} . The necessary lengthy computation has been done by Hasenfratz and Hasenfratz²⁾, for the $SU(2)$ and $SU(3)$ pure gauge theory, with the results

$$\begin{aligned} \Lambda_{\text{MOM}}^{(\alpha=1)} &= 57.5 \Lambda_L & (N=2) \\ \Lambda_{\text{MOM}}^{(\alpha=1)} &= 83.5 \Lambda_L & (N=3) \end{aligned} \quad (4)$$

Now, analysis of deep inelastic experiments suggests that $\Lambda_{\overline{\text{MS}}} \approx 400 \text{ MeV}$, i.e.,

$$\Lambda_{(\text{MOM})}^{(\alpha=1)} = 2.9 \Lambda_{\overline{\text{MS}}} \approx 1150 \text{ MeV} \quad (5)$$

The Monte Carlo lattice gauge theory prediction is thus that³⁾

$$\sqrt{K} \approx 2.8 \text{ GeV} \quad (6)$$

in a quarkless $\text{SU}(3)$ world. This is in disagreement with the two phenomenological determinations of the string tension currently available, the first one being the string model connection to the Regge slope⁴⁾,

$$K = \frac{1}{2\pi \alpha'} \quad (7)$$

which, using $\alpha' \approx 1 \text{ GeV}^2$, leads to

$$\sqrt{K} \approx 400 \text{ MeV} \quad (8)$$

The second one comes from heavy quark bound state phenomenology. This uses non-relativistic quantum mechanics with a confining potential $V \underset{r \rightarrow \infty}{\sim} \sqrt{K} r$. Attempts to reproduce energy levels and transition widths happen to require⁵⁾ $\sqrt{K}$'s between 400 and 500 MeV in remarkable agreement with the (ultra-relativistic) string model estimate, but in disagreement with the Monte Carlo result.

This disagreement can be decreased a little bit if one realizes that the value $\Lambda_{\overline{\text{MS}}} \approx 400 \text{ MeV}$ has been obtained by comparing experimental scaling violation data with a theoretical expression including light fermion effects. To be consistent one should compute $\Lambda_L / \Lambda_{\overline{\text{MS}}}$, taking into account three massless fermions. This computation has recently been done by Weisz⁶⁾. It gives

$$\Lambda_L = 0.20 \Lambda_{\overline{MS}} \quad (9)$$

$$\sqrt{K} = 1.6 \text{ GeV} \quad (10)$$

reducing the discrepancy. The most conservative explanation of this discrepancy is that it is due to the absence of quarks in the Monte Carlo.

The string tension refers to the long range $q\bar{q}$ static potential. Creutz has recently also investigated the short range potential^{7),1)}. To remove the divergent self energy of point-like sources he considers the force rather than the potential itself. His Monte Carlo data for SU(2) agree with the perturbation theory result

$$F(r) = - \frac{C_2(R) g^2}{4\pi r^2} = - \frac{C_2(R)}{4\pi r^2 \left(\beta_0 \ln \frac{1}{(r\Lambda_r)^2} + \frac{\beta_1}{\beta_0} \ln \ln \frac{1}{(r\Lambda_r)^2} \right)} \quad (11)$$

provided that

$$\Lambda_r \approx 22 \Lambda_L \quad (12)$$

The aim of this note is to give an independent determination of the ratio Λ_r/Λ_L . As stressed by Celmaster and Gonsalves⁸⁾, such a ratio is determined to all orders by a one loop perturbative computation. A straightforward g^2 expansion of the continuum theory Wilson loop gives, in the $\overline{MS}$ scheme^{9),10)},

$$V(r) = - \frac{C_2(R) g^2}{4\pi r} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[C_2(G) \left[\frac{22}{3} \ln(\mu r) + \gamma \right] + \frac{31}{9} \right] - T(R) \left[\frac{8}{3} \ln(\mu r) + \gamma + \frac{20}{9} \right] \right\} + O(g^4) \quad (13)$$

Specializing in the pure gauge system, one obtains the force as

$$\begin{aligned} \vec{F}(r) = - \frac{dV(r)}{dr} = - \frac{C_2(R) g^2}{4\pi r^2} \left\{ 1 + \frac{g^2}{(4\pi)^2} C_2(G) \right. \\ \left. \left[\frac{22}{3} \left(\ln(\mu r) + \gamma - \frac{1}{2} \right) + \frac{31}{9} \right] + O(g^4) \right\} \end{aligned} \quad (14)$$

Renormalization group analysis¹⁰⁾ tells us that this expression is invariant if one changes the renormalization mass scale μ into some new scale μ' , provided one changes g^2 into $g^2(\mu')$,

$$g^2(\mu') = \frac{1}{\beta_0 \ln \left(\frac{\mu'}{\Lambda_{\overline{MS}}} \right)^2 + \frac{\beta_1}{\beta_0} \ln \ln \left(\frac{\mu'}{\Lambda_{\overline{MS}}} \right)^2} \quad (15)$$

Comparing Eqs (11), (14) and (15), one obtains

$$\frac{\Lambda_r}{\Lambda_{\overline{MS}}} = \frac{1}{\mu r} e^{-\frac{1}{2\beta_0} \left(\frac{1}{g_r^2(r)} - \frac{1}{g_{\overline{MS}}^2(\mu)} \right)} (1 + O(g^2)) \quad (16)$$

This expression is valid for all r and μ 's so that perturbation theory holds. Now, setting $r \rightarrow 0$, $\mu \rightarrow \infty$, Eq. (16) gives

$$\frac{\Lambda_r}{\Lambda_{\overline{MS}}} = \frac{1}{\mu^r} e^{\left(\beta_r(\mu^r) + \gamma - \frac{35}{66} \right)} = e^{\gamma - \frac{35}{66}} = 1.05 \quad (17)$$

The perturbative prediction is thus

$$\frac{\Lambda_r}{\Lambda_L} = 20.78 \quad (18)$$

This is in incredible agreement with Creutz's Monte Carlo result, strengthening the conclusion that virtual quark effects are essential in order to give the string tension its actual value.

Equation (16) can be extended to any $SU(N)$ with N_f fermions using Weisz's recent generalization of the Hasenfratz and Hasenfratz result⁶⁾:

$$\frac{\Lambda_L}{\Lambda_{\overline{MS}}} = e^{\frac{1}{\beta_0} \left(\frac{1}{16N} - N \mathcal{P}_1 + \frac{N_f}{2} \mathcal{P}_3 \right)} \quad (19)$$

$$\mathcal{P}_1 = .0849780, \quad \mathcal{P}_3 = .006887$$

The general formula is thus

$$\frac{\Lambda_r}{\Lambda_{\overline{MS}}} = e^{\gamma - 1 + \frac{1}{\beta_0} \left(-\frac{1}{16N} + .095884N - .00696 N_f \right)} \quad (20)$$

The values for $N = 2, 3$, $N_f = 0, 1, 2, 3, 4$ are given in the Table.

Another possible extension of these results would be to treat the case of the alternative lattice actions¹¹⁾. It would be of great interest to know if the same remarkable agreement between Monte Carlo and perturbation theory is obtained.

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Table

Values of Λ_r/Λ_L

	0	1	2	3	4
N = 2	20.78	24.89	31.04	40.92	58.36
N = 3	30.19	34.76	40.79	49.03	60.68

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