CERN/EP/81-13
19 February 1981THE REACTION $\gamma p \rightarrow p \pi^+ \pi^- \pi^+ \pi^-$ FOR PHOTON ENERGIES FROM 25 TO 70 GeV*Bonn¹-CERN²-Ecole Polytechnique³-Glasgow⁴-Lancaster⁵-Manchester⁶-Orsay⁷-Paris VI⁸-Paris VII⁹-Rutherford¹⁰-Sheffield¹¹ Collaboration*

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(Submitted to Nuclear Physics B)

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ABSTRACT

Measurements of the reaction $\gamma p \rightarrow p \pi^+ \pi^- \pi^+ \pi^-$ are presented, in which $\pi^+ \pi^- \pi^+ \pi^-$ systems with masses up to 3 GeV are produced from fragmentation of the incident photon. The reaction is dominated by production of the large peak of the $\rho'(1600)$ -meson and, at higher masses $\gtrsim 2$ GeV, by production of jet-like 4π systems. The $\rho'(1600)$ -meson is produced by a predominantly s-channel helicity conserving mechanism. At higher masses there are also indications of $\rho\pi$ peaks, of masses 1.3 GeV (the A_2 -meson) and 1.75 GeV, produced with a recoiling π -meson by a mechanism consistent with the Deck effect.

1. INTRODUCTION

We report measurements of the reaction

$$\gamma p \rightarrow p \pi^+ \pi^- \pi^+ \pi^- \quad (1)$$

for incident photon energies of 25-70 GeV and for masses of the $\pi^+ \pi^- \pi^+ \pi^-$ system up to 3 GeV. A large peak is found in the $\pi^+ \pi^- \pi^+ \pi^-$ mass spectrum at a mass ~ 1.5 GeV, due to the $\rho'(1600)$. At these high photon energies one can study this peak in detail as the beam fragmentation region is clearly separated from the target fragmentation region. This is in contrast with earlier experiments at lower energies where these regions overlapped, so that it was necessary to deduce indication of the $\rho'(1600)$ by angular momentum analysis [1] or by kinematic cuts [2,3].

Analysis of the $\pi^+ \pi^- \pi^+ \pi^-$ system in Sections 3 and 5 indicates an important contribution from the $\rho'(1600)$ and explores its structure. These sections also show how fragmentation of the photon into higher 4π masses produces jet-like 4π systems similar to those produced when a photon fragments into 6π systems [4] and into 5π systems [5] of similar masses. Cross-sections are presented in Section 4, showing evidence for a major contribution due to diffractive dissociation of the photon. Section 5 shows evidence, at the higher 4π masses, for peaks in 3π systems at masses of 1.3 and 1.75 GeV and that there is, otherwise, no strong leading ρ^0 -meson effect in photon-Pomeron collisions in this mass range. Section 7 compares production of the $\rho'(1600)$ in reaction (1) with related production in electron-positron annihilation and makes further comparisons with the Vector Dominance Model. The conclusions are summarized in Section 8.

2. EXPERIMENTAL DETAILS

These results form part of a study of photoproduction by photons of energy 25-70 GeV. A full account of the experimental arrangements and of the general principles of the data analysis has been reported elsewhere [4], so only a brief account will be presented here.

2.1 Apparatus

An electron beam of momentum 81 GeV/c, with a momentum spread of $\pm 2\%$, was produced by conversion of high-energy photons derived from bombardment of a beryllium target by 210 GeV protons from the CERN SPS. The electrons were transported by the EI beam, and produced photons by bremsstrahlung in a tungsten radiator of thickness 0.076 radiation lengths. The photon energy was determined by a tagging system from measurement of the momentum of the electron before and after radiating. The photons impinged on a liquid hydrogen target of length 67 cm, interacting to produce particles which were detected in the spark chambers, drift chambers and Čerenkov counter of the Omega spectrometer. The trigger required detection of at least 4 particles in detectors around the target and of between 4 and 9 charged particles in a detector 1.5 m downstream of the target. The resulting trigger cross-section was $\sim 70 \mu\text{b}$ and $\sim 80\%$ of the triggers were due to production of hadrons.

2.2 Data

The data were passed through two programs, of which the first (ROMEO) carried out pattern recognition and geometry for the tracks of the charged particles detected in the Omega spectrometer, and the second handled information from the beam tagging system and threshold Čerenkov counter, and wrote data summary tapes.

Events were selected by the following cuts:

- i) The incident photon energy had to be in the range $25 < E_{\gamma} < 70$ GeV.
- ii) Only one vertex was found by the ROMEO program, and it was located within the hydrogen target.
- iii) None of the tracks was signalled as a $K^{\pm}$ -meson or proton or anti-proton by the threshold Čerenkov counter.
- iv) Events were required to have two negatively charged tracks and either two or three positively charged tracks. When four charged particles were detected these were taken to be $\pi^{\pm}$ -mesons, with the proton not detected. When five charged particles were detected that positive particle with the smallest momentum was taken to be the proton, and the event was only accepted if the proton candidate had a momentum less than 1.5 GeV/c and it made an angle with respect to the incident photon greater than 0.5 radian. Monte Carlo simulations of reaction (1) for the range of 4π masses studied (≤ 3 GeV) show that this proton selection procedure is reliable.

- v) Events corresponding to reaction (1) were selected by a cut on:

$$\Delta E = E_{\gamma} - \sum_{i=1}^4 E_{\pi i} \quad (- \text{ proton K.E. when the proton is detected}) .$$

Observed distributions of ΔE are shown in Fig. 1 for two ranges of 4π mass. These show that reaction (1) is not completely separated from other reactions in which further particles have been produced. The background from such other reactions was investigated by studying transverse momentum balance in the 30% of events where five charged particles were detected. The background deduced for these events is shown in Fig. 1 with a normalization assuming that it has the same dependence on ΔE for the 70% of

events where the proton is not detected. Events were selected as reaction (1) by requiring:

$$-1.5 \Delta E < 0.6 \text{ GeV}$$

giving a background from other reactions which is estimated to be $\sim 10\%$ for 4π masses $< 1.8 \text{ GeV}$ and $\sim 30\%$ for masses from 1.8 to 3.0 GeV .

vi) A background, which was found to be due to purely electromagnetic shower processes, was found on studying distributions of the angle (θ_{++}^t) in the 4π CM-system between the resultant of the momenta of the two π^+ -mesons and the direction of the incident photon (the t-channel axis). This background is indicated by sharp peaks at $|\cos \theta_{++}^t| > 0.95$. The distribution of dip angles of tracks from the corresponding events at $|\cos \theta_{++}^t| > 0.95$ peaks strongly at small angles, confirming their electromagnetic nature. This background is responsible for $\sim 40\%$ of the events at 4π masses $< 1 \text{ GeV}$ and then falls rapidly with increasing 4π mass, reaching zero at a mass $\sim 2 \text{ GeV}$. A cut requiring $|\cos \theta_{++}^t| < 0.95$ was normally applied to remove this background.

2.3 Acceptance

To determine the acceptance of the apparatus each event was rotated in steps around the beam axis and that fraction of the rotation for which the event would trigger was determined, including at each step the effect of the measured electronic efficiencies of the detectors. The inverse of the fraction was taken as the acceptance weight so that the event was added into any histogram with this weight, and a contribution of $(\text{weight})^2$ was added to the corresponding squared error. This procedure indicates that the over-all acceptance is approximately half the geometrical acceptance.

The acceptance as a function of 4π mass is shown in Fig. 2. We limit our attention to 4π masses < 3.05 GeV for which the average geometrical acceptance is $> 1/2$, so that uncertainties in this acceptance correction procedure should not be large.

3. STRUCTURE OF $\pi^+\pi^-\pi^+\pi^-$ SYSTEM

This section describes an introductory study of the structure of the 4π system. The results guide the more detailed analyses in following sections.

Figure 2 presents the 4π mass spectrum showing a broad peak at a mass of ~ 1.5 GeV which merges into a continuum at masses ≥ 2 GeV. A similar mass spectrum has been reported [6] from diffractive photoproduction from a beryllium target at higher incident photon energies.

The $\pi^+\pi^-$ mass spectrum (with four entries/event) is shown in Fig. 3(a) and the $\pi^\pm\pi^\pm$ mass spectrum (with two entries/event) is shown in Fig. 3(b). Disregarding possible interference effects (such as will be discussed in Section 5) the peak in Fig. 3(a) indicates ~ 1 ρ^0 -meson/event. If one selects $\pi^+\pi^-$ pairs in the ρ^0 -meson peak $[0.6 < M(\pi^+\pi^-) < 0.9]$ and then examines the mass spectrum of the other $\pi^+\pi^-$ pair in the event, it is found that the ρ^0 -meson intensity in this mass spectrum is small and consistent with the expected absence of $\rho^0\rho^0$ events in diffractive dissociation.

Distributions of t , the squared four-momentum transfer from photon to 4π system, fit the form $A \exp(bt)$ over the range $0.05 < -t < 1.0$ (GeV/c)². Values of b , as a function of 4π mass, are presented in Fig. 4, showing that production of the 4π system is peripheral at all 4π masses studied.

Following Smadja et al. [1] we study the experimental distributions of θ_{++}^s , the angle in the 4π CM-system between the resultant of the momenta of the two π^+ -mesons and the momentum of the recoil proton (the axis in the s-channel or helicity system). Smadja et al. [1] have shown how, at low masses where the internal angular momenta are restricted to their lowest values, the alignment of the 4π system will be displayed by the distributions of θ_{++}^s . Assuming an s-channel helicity conserving (SCHC) mechanism (as has been found to be typical for photoproduction of lighter vector mesons [7] and indicated for photoproduction of the $\rho'(1600)$ by 9.3 GeV photons [1]), one expects at low masses:

$$\langle Y_0^0(\theta_{++}^s) \rangle = -5^{1/2} \langle Y_2^0(\theta_{++}^s) \rangle .$$

At higher masses higher internal angular momenta result in $\langle Y_2^0(\theta_{++}^s) \rangle$ becoming relatively smaller, as is shown in Ref. 1.

As was found for the distributions of θ_{++}^t there is residual electromagnetic background at large values of $|\cos \theta_{++}^s|$. Fits of the form $\sum_i A_i P_i(\cos \theta_{++}^s)$ were therefore made to experimental distributions of θ_{++}^s for $|\cos \theta_{++}^s| < 0.9$ (the cut on $|\cos \theta_{++}^t|$ was not applied to the data sample used here). To obtain good fits there was no need to include terms with $i > 2$. Moments defined by $N\langle Y_i^0(\theta_{++}^s) \rangle = A_i / [4(2i+1)\pi]^{1/2}$ are shown in Fig. 5. Figure 5(a) shows that $\langle Y_1^0(\theta_{++}^s) \rangle$ is consistent with zero throughout as, for example, would be expected if only a vector meson is being photoproduced. In Fig. 5(b), $(4\pi)^{1/2} N\langle Y_0^0(\theta_{++}^s) \rangle$ is compared with $-(20\pi)^{1/2} N\langle Y_2^0(\theta_{++}^s) \rangle$. The values of $(4\pi)^{1/2} N\langle Y_0^0(\theta_{++}^s) \rangle$ are a measurement of the 4π mass spectrum corrected for the events lost by the cut on $|\cos(\theta_{++}^s)|$. Comparison of the peak in $-(20\pi)^{1/2} N\langle Y_2^0(\theta_{++}^s) \rangle$, with the peak in $(4\pi)^{1/2} N\langle Y_0^0(\theta_{++}^s) \rangle$

indicates that at least $\sim 1/2$ of the production of the peak is due to an SCHC mechanism.

Figure 5(b) also shows that the continuum at 4π masses $\gtrsim 2$ GeV is due to a different mechanism with $Y_2^0(\theta_{++}^S)$ positive. This change in the angular distribution indicates a major change in the alignment of the 4π system, which can correspond to the onset of higher angular momenta; the $\pi^+\pi^-\pi^+\pi^-$ distribution is similar to the jet-like structure found to occur for masses > 2 GeV in photoproduced $\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ systems [4] and $\omega^0\pi^+\pi^-$ systems [5]. Details of this similarity are now established by showing the behaviour of p_T , the transverse component of momentum of mesons in the 4π CM-system with respect to the t-channel axis, for the $\sim 55\%$ of these events which are $\rho\pi\pi$. Figure 6 shows the average value of p_T as a function of 4π mass, separately for the ρ^0 -meson and for the other two π -mesons from the $\rho^0\pi^+\pi^-$ system. The distributions for these different mesons were estimated by taking the ρ^0 -meson contribution to be given by those $\pi^+\pi^-$ pairs in a peak region $[0.6 < M(\pi^+\pi^-) < 0.9 \text{ GeV}]$ and then subtracting a background deduced from side-band regions $[0.45 < M(\pi^+\pi^-) < 0.6 \text{ GeV}$ or $0.9 < M(\pi^+\pi^-) < 1.05 \text{ GeV}]$. The measured values of p_T in Fig. 6 are compared with estimates for p_T -limited $\rho^0\pi^+\pi^-$ phase space. To do this events were generated, using the Monte Carlo method and the intensity described in the Appendix, and were analysed by the same peak and side-band procedure as was used for the data. Figure 6 shows how the data disagree with estimates for $\rho^0\pi^+\pi^-$ phase space (truncation parameter, $A = 0$) and agree well with $A \sim 2.5-3$, close to the values of $A \sim 3-3.5$ found [4] to fit photoproduced $\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ systems of masses 2-5 GeV.

We therefore conclude that there is indication of two major contributions to the photoproduction of $\pi^+\pi^-\pi^+\pi^-$ systems:

- i) a peak at a mass ~ 1.5 GeV, with at least $\sim 1/2$ of its production being by an SCHC mechanism;
- ii) a jet-like structure produced with a nearly flat mass spectrum for 4π masses ≥ 2 GeV.

4. CROSS-SECTIONS

The normalization of the experiment was based on direct counting of the numbers of scattered electrons in the tagging system. To determine the cross-sections detector acceptance was taken into account as described in Section 2. Further corrections were made for:

- i) The acceptance of, and background remaining after, the ΔE cut, which was determined by repeating the estimation of the peak in ΔE due to reaction (1) for separate bites of E_γ of width 5 GeV.
- ii) The shape of the bremsstrahlung spectrum, as determined from special measurements which triggered only on an electron in the tagging system.
- iii) The efficiency of the ROMEO program (for pattern recognition, geometry and vertex finding) which was studied with simulation programs. There was appreciable uncertainty in the simulation of backgrounds, so this correction provides the dominant contribution to the errors on cross-sections.
- iv) Double bremsstrahlung in the radiator and for effects of various veto counters in the tagging system.

Cross-sections were deduced for production of 4π systems in two ranges: 1.2-1.8 GeV and 1.8-3.0 GeV. For each of these ranges the cross-section was found to vary according to $\sigma \sim E_Y^{-n}$ with $n = 0.35 \pm 0.15$ in both cases. The cross-sections for photon fragmentation to $\pi^+ \pi^- \pi^+ \pi^-$ states [4] and to $\omega^0 \pi^+ \pi^-$ and $\eta^0 \pi^+ \pi^-$ states [5] are observed to fall slowly with energy in a similar way.

These rates of fall of cross-section with increasing energy are slower than is regarded as typical of exchange reactions other than diffractive dissociation [8], while such rates of fall have been reported for production of states of similarly high mass by beams of similar energy in cases where diffractive dissociation is thought to be important. Cross-sections for the reactions $\pi p \rightarrow A_1 p$ or $A_3 p$ are proportional to p_π^{-n} with $n \sim 0.4$ [9,10]. (For the reaction $\pi p \rightarrow A_2 p$ a similar value of $n = 0.40 \pm 0.03$ is reported [11], indicating that this reaction is similarly diffractive.) Therefore it is plausible to conclude that, in the production of $\pi^+ \pi^- \pi^+ \pi^-$ states reported here, there is a major contribution due to diffractive dissociation. This conclusion is supported by the observation [6] that a similar 4π mass spectrum is found for coherent photoproduction from beryllium nuclei.

It is interesting that this fall of the cross-section for photoproduction of the $\rho'(1600)$ with increasing E_Y agrees with a suggested [12] scaling law for vector meson total cross-sections:

$$\sigma_{\text{tot}}(VN) \sim f(s/m_V^2)$$

as, if this were true, the $\rho'(1600)$ cross-section for $E_Y = 25-70$ GeV should vary as does the ρ -meson cross-section for $E_Y = 6-17$ GeV, over which range it falls [13] as $E_Y^{-0.4}$.

The average value of the cross-sections for reaction (1), which (given the above rate of fall) are estimates of the cross-sections at $E_Y = 40$ GeV, are:

$$\begin{aligned} 1.2 < M(4\pi) < 1.8 \text{ GeV} & : 0.7 \pm 0.2 \text{ } \mu\text{barn} \\ 1.8 < M(4\pi) < 3.0 \text{ GeV} & : 1.15 \pm 0.35 \text{ } \mu\text{barn} . \end{aligned}$$

5. STRUCTURE OF THE $\rho'(1600)$

In this section we report more detailed analyses of the structure of the 4π system, and particularly of the $\rho'(1600)$. As Section 3 showed evidence for major contribution of $\rho\pi\pi$ we particularly study $\rho\pi\pi$ models, which have the problem that, because of the large width of the ρ^0 -meson, one cannot identify which $\pi^+\pi^-$ pair is the ρ^0 -meson. We therefore follow the approach of Smadja et al. [1], where each of the $\pi^+\pi^-$ pairs in an event is considered to be the ρ^0 -meson and the corresponding probability amplitude is calculated. Then the resulting four amplitudes are added and their sum squared. The resulting intensity, multiplied by a phase space weight, is taken to be the probability of the event occurring. We then use a number of models, of which an account is given here, but technical details are postponed to the Appendix.

Simple $\rho\pi\pi$ or $\rho\epsilon$ models, with s-wave relative motion, give poor fits to the $\pi^+\pi^-$ mass spectra for $M(4\pi) \lesssim 1.45$ GeV and to 3π mass spectra for higher 4π masses, as is shown in Figs. 7 and 8. It is found that fits can be obtained by suitable deformation of the $\rho\pi\pi$ system. Two particular models which do this successfully have been found and are now described:

i) As well as s-wave relative motion of $\rho\epsilon$, d-wave relative motion is also allowed, with the resulting centrifugal repulsion between ρ and ϵ

deforming the 4π structure. (For the ϵ we use a final state interaction, with the measured phase shift.)

ii) The $\rho'(1600)$ is assumed to decay through the chain

$$\rho' \rightarrow A_1^\pm \pi^\mp \rightarrow \rho^0 \pi^\pm \pi^\mp.$$

Acceptable fits are found with $M(A_1) \sim 1.3$ GeV, $\Gamma(A_1) \sim 0.3$ GeV with s-wave relative motions, but not with lower masses. Some improvement of the fit is obtained by reducing the probability of the low mass tail of the A_1 -meson by an empirical factor. Such a change in shape could either be a feature of the A_1 -meson itself or could be due to an interference with non-resonant background (such as is found for photoproduced ρ^0 -mesons [7]).

It must be emphasized that, although either of these models is successful in fitting the data, in neither case is the basic input directly demonstrated by the data. The effect of d-wave $\rho\epsilon$ could be expected to show directly in angular distributions. However, the interference between s-wave and d-wave amplitudes cancels out the effect of the d-wave intensity so that the resulting θ_{++}^S distributions are indistinguishable from the pure s-wave result. Similarly an A_1 -meson could be expected to show directly in the 3π mass distribution, but is hidden for three reasons: the large width of the A_1 -meson, the peaking of the phase space distribution and because there are three other 3π combinations in each event. Since both models are indicated indirectly one can only regard their success as pointers which are worthy of exploration in other channels, noting that it is perfectly possible that other models might also fit the data.

Some typical calculated $\pi^+\pi^-$ mass spectra are shown in Fig. 7 and 3π mass spectra in Fig. 8, where they are compared with data. Distributions

of θ_{++}^S and of $\phi_\rho - \phi_+$ (where ϕ_ρ denotes the azimuth of the ρ^0 -meson in the 4π CM-system, and ϕ_+ denotes the azimuth of the π^+ -meson from decay of the ρ^0 -meson in the ρ^0 -meson CM-system), the latter being histogrammed for all four identifications of the ρ^0 -meson, have also been calculated showing that closely similar results are found for both successful models.

Maximum likelihood fits were then used to study in detail how these models fitted the data, in 100 MeV bins of 4π mass. In each fit several contributions were included and their relative intensities, α_i , were deduced as those which gave a minimum of

$$L(\alpha_1, \alpha_2, \dots, \alpha_n) = - \sum_j W_j \ln \left[\sum_i \frac{\alpha_i P_i(j)}{\int P_i(j) d\phi} \right].$$

Here $P_i(j)$ denotes the probability intensity of the j th event in the data for the i th contribution, and W_j denotes the acceptance weight of the event. The integrals over phase space, $\int P_i(j) d\phi$, were calculated by the Monte Carlo method. The minimization of L was done by the program MINUIT [14] and the errors on the α_i presented by that program are quoted. For each fit, the calculated $\pi^+\pi^-$ mass spectrum, 3π mass spectrum and $\cos \theta_{++}^S$ distribution were compared with the corresponding data. For all acceptable fits these comparisons were good. A set of fits was made, each including one of the deformed $\rho\pi\pi$ models, with SCHC alignment, together with several background contributions:

- i) 4π phase space.
- ii) 4π p_T -limited phase space.
- iii) $\rho\pi\pi$ with SCHC alignment.
- iv) $\rho\pi\pi$ p_T -limited phase space.

For the present exploration interferences between any of these contributions were ignored. One set of results typical of acceptable fits with $A_1\pi$ models is shown in Fig. 9, while the result of an acceptable fit combining $\rho\pi$ with s- and d-waves with backgrounds (i) and (iv) is shown in Fig. 10. In the latter case not only is the total intensity of the $\rho\pi\pi$ s- and d-wave model estimated by the maximum likelihood calculation but also the relative amplitude d/s, with the result shown. In all acceptable fits one finds a strong $\rho'(1600)$ peak in the intensity of the successful $\rho\pi\pi$ model, with $\geq 45\%$ of the events with $1.2 < M(4\pi) < 1.8$ GeV being in this peak. Further fits were made with two $A_1\pi$ models, differing only in that one had SCHC alignment and the other isotropic alignment, together with background contributions. These showed that the $\rho'(1600)$ peak is produced with $\geq 80\%$ SCHC alignment. For all these fits the intensities of the backgrounds and their variation with 4π mass are very similar. In accord with the results in Section 3 it is found that p_T -limited phase space contributions dominate for 4π masses > 2 GeV, with $\sim 55\% \rho\pi^+\pi^-$. It is interesting that a relatively large background is consistently found at low 4π masses ($\geq 50\%$ of the observed events for $M(4\pi) \leq 1.2$ GeV), which is presumably indicating some further contribution as yet unaccounted for. We find that, at these low 4π masses, this background is equally well fitted by $\rho\pi\pi$ p_T -limited phase space or by 4π p_T -limited phase space of similar intensity. A fit was also made with only a $\rho\pi$ s- and d-wave model, with no backgrounds. This gave $|d/s| < 0.05$ for all 4π masses < 1.8 GeV, and the fits to the mass spectra and $\cos \theta_{++}^s$ distributions did not agree in any acceptable way with the data. Therefore, at least for this model, the background contributions are needed.

Estimates of the properties of the $\rho'(1600)$ are made, by assuming a single resonance as indicated by the peaks found for the intensities of the successful $\rho\pi\pi$ models. The peak from each maximum likelihood fit was fitted with a relativistic Breit-Wigner formula with a constant width. Taking the average of the results from the different maximum likelihood fits and estimating an error from the spread of the results and their statistical errors, we conclude

$$M(\rho') = 1.52 \pm 0.03 \text{ GeV}$$

$$\Gamma(\rho') = 0.40 \pm 0.05 \text{ GeV} .$$

A fit with two interfering resonances, such as has been made by Barber et al. [3] to their data, would obviously allow a wider range of parameters and, in particular, could accommodate a narrower width for the $\rho'(1600)$. On the single resonance picture and allowing for the possibility that there could be some underestimate due to imperfections of the parametrization of the models, an estimate is made of 0.6 ± 0.1 for the fraction of the events in the range $1.2 < M(4\pi) < 1.8 \text{ GeV}$ which are due to the decay of $\rho'(1600)$ to $\rho^0\pi^+\pi^-$. Combining this with the cross-section from Section 4, and correcting for the fraction of the $\rho'(1600)$ which is outside this mass range and for the fact that 10% of the events (presumed to be in the background) are not due to reaction 1, we deduce a cross-section for production by photons of energy 40 GeV of the $\rho'(1600)$, followed by its decay into $\rho^0\pi^+\pi^-$, of $0.54 \pm 0.17 \text{ } \mu\text{barn}$. This is smaller than the cross-section of $1.6 \pm 0.4 \text{ } \mu\text{barn}$ reported [1] for production by photons of energy 9.3 GeV, by an amount consistent with the rate of fall with photon energies seen in this experiment.

We see that the structure of the 4π system from decay of the $\rho'(1600)$ has to be deformed from the simplest $\rho\pi\pi$ models, with two possibilities having been found for this deformation. One possibility is a $\rho\pi$ model, with interference between s-wave and d-wave relative motion with relative amplitude $d/s \sim 0.25$ (though this result is dependent on an understanding of the background). Such a d-wave admixture is plausible, as a similar admixture is found in $B \rightarrow \omega\pi$ decay. The alternative possibility is decay through an $A_1\pi$ state. For such a model the data require $M(A_1) \sim 1.3$ GeV, $\Gamma(A_1) \sim 0.3$ GeV, which are consistent with the A_1 -meson reported by Daum et al. [9]. As is shown by examples of calculations plotted on Figs. 7 and 8, the data do not indicate a major contribution from an A_1 -meson of lower mass. Examples include the mass of 1.15 GeV deduced [15] from observations of $\tau \rightarrow \rho\pi\nu$ decay (however it has been shown [16] how these results are also consistent with a much broader range of parameters for the A_1 -meson), and masses around 1.05 GeV reported in other experiments [17]. Resolution of the question of the relative contributions of these two models for $\rho'(1600) \rightarrow 4\pi$ should be helped by study of $\rho'(1600) \rightarrow \pi^+\pi^-\pi^0\pi^0$, as there any $\rho\pi$ decay should be into $\rho^0\pi^0\pi^0$ while any $A_1\pi$ decay should be into $\rho^\pm\pi^\mp\pi^0$.

Our measurements of the $\rho'(1600)$ decay to 4π indicate a lower mass and larger width than have been reported for a $\pi^+\pi^-$ decay mode:

Atiya et al. [6] : $M = 1.60 \pm 0.01$ GeV , $\Gamma = 0.282 \pm 0.014$ GeV

Aston et al. [18] : $M = 1.59 \pm 0.02$ GeV , $\Gamma = 0.23 \pm 0.08$ GeV .

If these 2π and 4π peaks are due to decay modes of the same particle, it would seem that at least one of these states must be interfering with some

background. Particular interference models which fit the over-all 4π mass spectrum have been proposed [19,20], in which a relatively weak resonance interferes with a background from the non-resonant $A_1\pi$ model of Pham et al. [21]. As our data indicate a more definite $\rho' \rightarrow 4\pi$ peak and a higher A_1 mass than is required by these models we apparently disagree with them. We note that a major foundation of our conclusion is the complete change of alignment shown in Fig. 5. We cannot rule out the possibility that such interference models could produce an alternative explanation of this result.

6. FEATURES OF HIGH MASS 4π SYSTEM

More detailed study of the data for higher 4π masses shows indication of two peaks in the 3π mass spectrum, particularly for the 3π systems produced in the forward direction in a photon-Pomeron collision. This is demonstrated in Fig. 11, which shows the $(3\pi)^\pm$ mass spectrum for unweighted events for $2.05 < M(4\pi) < 3.05$ GeV, for 3π combinations with $\cos_{3\pi} > 0.7$ (where $\theta_{3\pi}$ is the angle between the resultant momentum of the 3π system and the t-channel axis in the 4π CM-system). Fits were made to this mass spectrum, as shown in Fig. 11, with a second order polynomial only and with a second order polynomial + two relativistic Breit-Wigner formulas with constant widths. The results of the fits to unweighted data were:

	$\chi^2/\text{d.o.f.}$	Significance of	
		1.3 GeV peak	1.75 GeV peak
polynomial	82.8/24	3.3 σ	5.0 σ
polynomial + Breit-Wigners (widths fixed at 0.1 GeV)	28.0/20	4.7 σ	6.6 σ
Mass found (GeV)		1.292 ± 0.015	1.759 ± 0.016

The widths of the peaks were not well determined, but were ~ 0.1 GeV (resolution was such that a narrow peak would appear to have a width ~ 0.05 GeV). Examination of $\pi^+\pi^-$ spectra, particularly selecting events in the peak regions and subtracting backgrounds deduced from sidebands, indicate these peaks have substantial $\rho\pi$ decay modes.

No peaking is found in any mass spectra for $\cos \theta_{3\pi} < 0.7$, indicating that these peaks are produced in the forward direction in a photon-Pomeron collision. This is consistent with the reaction

$$\gamma p \rightarrow (A \rightarrow 3\pi)\pi p$$

proceeding by a Deck mechanism [22], with the π -meson being the exchanged particle as would be expected from its low mass.

The 1.3 GeV peak is consistent with being the A_2 -meson, for which the Particle Data Group [23] quote a mass of 1.315 ± 0.005 GeV and a width of 0.102 ± 0.005 GeV. They offer no candidate for the 1.75 GeV peak. Estimates of average cross-sections for production of these peaks are: 26 nb for the 1.3 GeV peak and 33 nb for the 1.75 GeV peak.

No indication of any appreciable $A_2\pi$ production is seen for 4π masses $\lesssim 1.9$ GeV, which is well above the threshold at a mass of 1.45 GeV. This lack of $A_2\pi$ production at lower masses would be expected from the threshold effect due to the d-wave relative motion of $A_2\pi$ when produced from a photon, as has been discussed [24] for the corresponding production in electron-positron annihilation. As production of the 1.75 GeV peak is found to be relatively strong at 4π masses closer to its threshold one speculates that the relative motion of this state and the other π -meson is s-wave or possibly p-wave.

Other than the forward production of these 3π peaks the data show forward-backward symmetry in photon-Pomeron collisions producing 4π systems of masses > 2 GeV. To check that particularly the ρ^0 -meson in the $\rho^0\pi^+\pi^-$ events is produced with this symmetry, comparisons have been made of 3π mass spectra for production of the 3π system in ranges of forward and backward angles in the 4π CM-system, both for selection on the ρ^0 -meson region in the $\pi^+\pi^-$ mass spectrum and for $\rho\pi\pi$ events deduced by subtracting a background from sideband regions in the $\pi^+\pi^-$ mass spectrum. In all these cases the corresponding 3π mass spectra for forward and backward angles were consistent with being the same outside the regions of the 1.3 and 1.75 GeV peaks. One therefore concludes that there is no strong leading ρ^0 -meson effect in photon-Pomeron collisions other than those due to the Deck effects demonstrated in this section. Similar forward-backward symmetry has been seen in the jet-like structure produced in $\omega^0\pi^+\pi^-$ [5].

7. COMPARISON WITH ELECTRON-POSITRON ANNIHILATION AND VECTOR DOMINANCE MODEL

Cross-sections for the reaction

$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$$

for 4π masses from 1.0 to 2.0 GeV have been reported from VEPP-2M [25], Adone [26] and DCI [27], with good agreement between these measurements. As a parametrization of them we use the results of a fit which has been made [26] with a Breit-Wigner formula with a varying width. Figure 12 shows a comparison of the shapes $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-)$ and of $[M(4\pi)]^2\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-)$ with the 4π mass spectrum from this experiment. In a Vector Dominance Model (VDM) framework (with the diagonal assumption that the diffractive production is due to a virtual 4π system in the photon scattering to become real) agreement of the photoproduced mass spectrum with

$M^2\sigma(e^+e^- \rightarrow 4\pi)$ would indicate that the cross-section for elastic scattering of the 4π system from the proton, $\sigma_{\text{elastic}}(4\pi+p)$, does not vary with M , while agreement with $\sigma(e^+e^- \rightarrow 4\pi)$ would indicate $\sigma_{\text{elastic}}(4\pi+p) \sim M^{-2}$. We see better agreement with a constant cross-section, and so use the diagonal VDM to deduce a value of

$$\sigma_{\text{tot}}(\rho'p) = \left[\frac{32\pi^3\alpha b}{M(\rho')} \cdot \frac{\sigma(\gamma p \rightarrow \rho'p)}{\int \sigma(e^+e^- \rightarrow \rho') dE} \right].$$

The comparison in Fig. 12 shows similar large widths in the over-all mass spectra and so suggests similar backgrounds in the two cases. As an exploration we therefore assume the $\rho'(1600)$ contributes the same fraction (which means that we are assuming these backgrounds have $J^\pi = 1^-$, even though our model fits ignore interferences) to the range $1.2 < M(4\pi) < 1.8$ GeV for electron-positron annihilation as it does for photoproduction and, making the diagonal assumption, deduce:

$$\sigma_{\text{tot}}[\rho'(1600)-p] = 16.7 \pm 3.4 \text{ mbarn}.$$

As our $\rho'(1600)$ cross-section falls with energy, it can be an over-estimate of the diffractive cross-section needed for this calculation. Assuming the scaling law of Greco [12] an estimate of a correction can be made from the decrease of ρ -meson photoproduction [13] from $E_\gamma = 10$ GeV to 200 GeV, which reduces $\sigma_{\text{tot}}(\rho'p)$ to 13.5 mbarn. However, all these calculations assume that the γ - ρ' coupling does not change as M_γ changes from 0.0 to 1.52 GeV. Any estimate of a correction for this effect (such as have been made [28] for photoproduction of the J/ψ -meson) would be such as to raise the value of $\sigma_{\text{tot}}(\rho'p)$, so the values quoted should be regarded as indicating a lower limit. These results are inconsistent with a broad range of generalized VDM (GVDM) models (see typically Close et al. [29])

which make the diagonal approximation and explain the constant value of R in electron-positron annihilation and scaling in deep inelastic scattering as the result of a sequence of vector mesons with $\Delta M^2 \sim 1 \text{ GeV}^2$. Such models require $\sigma_{\text{tot}}(\gamma p) \sim M_V^{-2}$, so (extrapolating from the ρ^0 -meson) would expect $\sigma_{\text{tot}}[\rho'(1600)-p] \sim 5\text{-}6 \text{ mbarn}$. This disagreement is presumably related to a further failure of these GVDM models, noted by Spinetti [26], who shows that the contribution of any $\rho'(1250)$ to the electron-positron annihilation cross-section is much smaller than predicted. These problems could presumably be due to a failure of the diagonal assumption.

As the $\rho'(1600)$ would have $I = 1$ it is interesting to consider possible candidates for the $I = 0$ members of the $SU(3)$ octet. To make a suitable quantitative comparison one has to estimate branching ratios, so as to deduce total cross-sections for photoproduction of these states. At this time such estimates can only provide orders of magnitude, but these do have interest:

a) $(\rho' \rightarrow \pi^+\pi^-\pi^0\pi^0)/(\rho' \rightarrow \pi^+\pi^-\pi^+\pi^-)$ is 0.5 if through $\rho\pi$ or 1.0 if through $A_1\pi$; there is an uncertainty in this range. There are also indications of smaller branching ratios into $\pi^+\pi^-$ [9] and into $\eta^0\pi^+\pi^-$ [5]. An estimate of $(\rho' \rightarrow \text{all})/(\rho' \rightarrow \pi^+\pi^-\pi^+\pi^-) \sim 2$ would seem to be in the middle of the possible range, so $\sigma(\gamma p \rightarrow \rho' p) \sim 1.1 \text{ } \mu\text{b}$.

b) A broad candidate for an $\omega' \rightarrow \omega^0\pi^+\pi^-$ is seen at a mass $\sim 1.7 \text{ GeV}$ [5]. Other decay modes must include $\omega^0\pi^0\pi^0$, and there could be others. Therefore, making similar assumptions as for the $\rho'(1600)$, that $(\omega' \rightarrow \text{all})/(\omega' \rightarrow \omega^0\pi^+\pi^-) \sim \sim 2$ and the fraction of ω' events in the peak at masses $< 2 \text{ GeV}$ is ~ 0.6 , one estimates $\sigma(\gamma p \rightarrow \omega' p) \sim 0.1 \text{ } \mu\text{b}$. Having made similar assumptions about

ρ' and ω' , a cross-section ratio consistent with the 1/9 expected for an ideally mixed nonet can be regarded as interesting.

c) Two possible candidates for ϕ' are indicated: a narrow peaking in $\pi^+\pi^-\pi^+\pi^-\pi^0$, $K_S^0 K^\pm \pi^\mp$ and $K^+ K^-$ at a mass of 1650 MeV [30] and a broad candidate in $(K^{*0} \rightarrow K^\pm \pi^\mp) K^\mp \pi^\pm$ at a mass of ~ 1.9 GeV [31]. We will discuss only the latter possibility here. If such a ϕ' decays into two $K\pi$ pairs each with $I = 1/2$ one has $(\phi' \rightarrow K\bar{K}\pi\pi)/(\phi' \rightarrow K^+ K^- \pi^+ \pi^-) = 4.5$. Assuming as for the $\rho'(1600)$, $\sim 25\%$ for other decays and that the fraction of ϕ' events with mass < 2.5 GeV is ~ 0.6 , one estimates $\sigma(\gamma p \rightarrow \phi' p) \sim 0.1 \mu\text{b}$. Again this estimate is close to the expected fraction of the $\rho'(1600)$ cross-section, taking into account the smaller cross-section for strange quarks.

8. CONCLUSIONS

The results divide into two ranges of 4π mass, at a mass ~ 2 GeV.

The lower mass range is dominated by a large peak at 1.5 GeV in the mass spectrum. This peak, which is responsible for ≥ 0.6 of the events in this mass range, is due to production of $\rho^0 \pi^+ \pi^-$ systems with a distribution deformed from phase space. Models for this deformation were discussed in Section 5. The peak is produced by a mechanism which is $\geq 80\%$ s-channel helicity conserving. It is at a mass of 1.52 ± 0.03 GeV and has a width of 0.40 ± 0.05 GeV. As well as the broad peak there are indications of other processes at lower masses, and particularly of a relatively strong contribution at low 4π masses with alignment differing strongly from SCHC.

At 4π masses above 2 GeV jet-like systems are dominant, with $\sim 55\%$ being $\rho^0 \pi^+ \pi^-$. There are also small but interesting contributions from

$\gamma p \rightarrow (\rho\pi)\pi p$ produced by mechanisms consistent with the Deck effect [22] with $\rho\pi$ peaks at masses of 1.292 ± 0.015 GeV and 1.759 ± 0.016 GeV, each with a width ~ 0.1 GeV. The lower mass peak is presumably the A_2 -meson. Other than these Deck mechanism processes there is forward-backward symmetry in photon-Pomeron collisions at these higher 4π masses with no indication of any major leading ρ^0 -meson.

Acknowledgements

We are grateful to CERN for providing the facilities and especially to the Omega Group for their help in operating the spectrometer, and the DD Division for providing on-line and off-line software. We have also benefited from the work of technical staff in our home institutions. We acknowledge financial support from the Science Research Council, from the Institut National de Physique Nucléaire et de Physique des Particules, and from the Bundesministerium für Wissenschaft und Forschung.

APPENDIX

Details are given of the models used for fitting the 4π system. In principle a complete analysis could be attempted, following the principles used by Zemach [32] for the 3π system. In practice this would be very complicated so attention has been restricted to a narrower region by the indication that the contribution of $\rho^0\pi^+\pi^-$ dominates, and that there is an important contribution from diffractive dissociation so that the 4π state has $C = -1$ and therefore $I = 1$. Therefore the isotopic spin wavefunctions to be considered (after suitably permuting the indices) are

$$(\pi_i \times \pi_j)(\pi_k \cdot \pi_\ell) \quad (A1)$$

$$[(\pi_i \times \pi_j) \cdot \pi_k] \pi_\ell \quad (A2)$$

and

$$[(\pi_i \times \pi_j) \times \pi_k] \times \pi_\ell . \quad (A3)$$

The second of these, corresponding to an $I = 0$ 3π state, can only contribute to $\pi^+\pi^-\pi^0\pi^0$ and so does not concern us here. Then as

$$\begin{aligned} & [(\pi_i \times \pi_j) \times \pi_k] \times \pi_\ell + [(\pi_i \times \pi_j) \times \pi_\ell] \times \pi_k \\ &= [(\pi_i \times \pi_j) \cdot \pi_\ell] \pi_k + [(\pi_i \times \pi_j) \cdot \pi_k] \pi_\ell - 2(\pi_i \times \pi_j)(\pi_k \cdot \pi_\ell) \end{aligned}$$

we see that (A1) and (A3) only differ in their relative contributions to the $\pi^+\pi^-\pi^0\pi^0$ state. Hence for the analysis of the present experiment any difference between them can only be due to dynamical effects which cause clusterings of the π -meson momenta.

In general therefore a model is represented by an amplitude

$$A(p_i, p_j, p_k, p_\ell)$$

such that i, j denote a pair of π -mesons of opposite charge which are taken to be the ρ^0 -meson. $A(p_i, p_j, p_k, p_l)$ must therefore be antisymmetric under interchange of p_i and p_j . For each event this amplitude is calculated for all permutations [4 if of the type (A1), or 8 if of the type (A3)], these contributions are added and the sum squared. The result is multiplied by the phase-space density to give the probability of the event. Angular distributions depend on the alignment of the 4π system. Where this matters we assume s-channel helicity conservation (SCHC), so the 4π system has $m = \pm 1$ with respect to the s-channel axis. However, we will only present formulas here for the $m = +1$ substate, noting that there is no real difference for the $m = -1$ substate.

Factors contributing to the amplitudes were:

i) ρ amplitude: $A_\rho = \exp(i\delta_\rho) \sin \delta_\rho / q_\rho^2$

(this and similar factors in later amplitudes are due to the Watson final state interaction theorem [33]) where

$$\tan \delta_\rho = M_\rho \Gamma_\rho (M_\rho^2 - M_{ij}^2)$$

$$\Gamma_\rho = 2\Gamma_{\rho 0} (q_\rho / q_{\rho 0})^3 / [1 + (q_\rho / q_{\rho 0})^2]$$

with

$$q_{\rho 0} = [1/4 M_\rho^2 - M_\pi^2]^{1/2}$$

$$q_\rho = [1/4 M_{ij}^2 - M_\pi^2]^{1/2}$$

M_{ij} is the mass of pair ij , $M_\rho = 0.785$ GeV, $\Gamma_\rho = 0.150$ GeV (these values were found to give a good fit), $M_\pi = \pi^\pm$ -meson mass.

ii) For an SCHC alignment a factor was included in the amplitude

$$A_{\text{SCHC}} = - \sin \theta_+ \exp(i\phi_+)$$

where θ_+ , ϕ_+ are angles of the π^+ -meson from the ρ^0 -meson, with respect to the s-channel axis in the ρ^0 -meson CM-system.

iii) When $\rho\pi\pi$ was given a p_T -limited angular distribution a factor was included in the amplitude:

$$A_{p_T} = \exp \left[-\frac{1}{2} A_\rho p_{Tij} \right] \exp \left[-\frac{1}{2} A_\pi (p_{Tk} + p_{Tl}) \right]$$

where p_{Tij} denotes the transverse momentum of pair ij and p_{Tk} , p_{Tl} denote the transverse momenta of π -mesons k and l , all with respect to the t-channel axis in the 4π CM-system. For the maximum likelihood fits $A_\rho = 2.0$, $A_\pi = 2.5$ was assumed, while for the analysis leading to Fig. 6 $A_\rho = A_\pi = A$, with the values of A shown, was assumed.

iv) ϵ amplitude

$$A_\epsilon = \exp(i\delta_\epsilon) \sin \delta_\epsilon / q_\epsilon$$

with

$$\delta_\epsilon = 3.4 q_\epsilon$$

(where δ_ϵ is in radians, with q_ϵ in GeV/c) where

$$q_\epsilon = \left[\frac{1}{4} M_{k\ell}^2 - M_\pi^2 \right]$$

chosen as an approximate mean of the s-wave $\pi\pi$ phase-shifts compiled by the Particle Data Group [23]. Other models of this phase-shift have also been tried and did not make for important differences. $M_{k\ell}$ is the mass of pair $k\ell$.

v) $\rho\epsilon$ angular distribution:

If the relative motion of ρ and ϵ is s-wave this is

$$A_s = A_{SCHC}$$

If s-d interference

$$A_{sd} = A_{SCHC} + Dkp^2 \left\{ 3 \sin^2 \theta_\rho \sin \theta_+ \exp [2i(\phi_\rho - \phi_+)] + 6 \sin \theta_\rho \cos \theta_\rho \cos \theta_+ \exp [i(\phi_\rho - \phi_+)] - (3 \cos^2 \theta_\rho - 1) \sin \theta_+ \right\} \exp (i\phi_+)$$

where θ_ρ, ϕ_ρ denote the direction of the resultant momentum of pair ij with respect to the s-channel axis in the 4π CM-system and θ_+, ϕ_+ denote the direction of the momentum of the π^+ -meson (i or j) in the ij CM-system. D is the d-wave amplitude, relative to the s-wave, k is a normalization factor chosen so that other than the factor D the integrals over s-wave and d-wave intensities are equal. p is the momentum of ρ^0 -meson or ϵ in the 4π CM-system, so that p^2 is the simplest model for d-wave centrifugal repulsion.

vi) $A_1\pi$ system: angular momenta, except for the ρ^0 -meson decay, are taken to be s-wave. The simplest amplitude for the shape of the A_1 -meson which makes a good fit is

$$A_{A\pi} = \exp (i\delta_{ijk}) \sin \delta_{ijk}/q_k$$

where

$$\tan \delta_{ijk} = M_A \Gamma_A / (M_A^2 - M_{ijk}^2)$$

$$\Gamma_A = \Gamma_o (q_k/q_{ok}) .$$

Here M_{ijk} is the mass of the three π -mesons ijk, q_k is the momentum of π -meson k in the 3π CM-system, and q_{ok} is the value of q_k for $M_{ijk} = M_A$, $M_{ij} = M_\rho$. An empirical modification which is found to improve the fit,

particularly to the $\pi^+\pi^-$ mass spectra for $1.15 < M(4\pi) < 1.45$ GeV, is to include a further factor of $q_\ell^{-1/2}$, where q_ℓ is the momentum of π -meson ℓ in the 4π CM-system.

Given these factors the various amplitudes are:

Phase space	1
$\rho\pi\pi$ phase space	A_ρ
$\rho\pi\pi$ SCHC alignment	$A_\rho A_{\text{SCHC}}$
$\rho\varepsilon$ SCHC alignment	$A_\rho A_\varepsilon A_{\text{SCHC}}$
$\rho\varepsilon$, s-d waves	$A_\rho A_\varepsilon A_{sd}$
$\rho\pi\pi$ p_T -limited phase space	$A_\rho A_{p_T}$

For 4π p_T -limited phase-space events were assumed to have a probability of $\exp [-A(p_{Ti} + p_{Tj} + p_{Tk} + p_{T\ell})]$ as there was no π -meson identification problem. For the maximum likelihood fits we assumed $A = 3.25$.

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Figure captions

- Fig. 1 : a) Observed spectrum of events, as a function of $\Delta E = E_Y - \sum_{i=1}^4 E_{\pi i}$ (- proton K.E. if proton is detected) for events with $1.2 < M(4\pi) < 1.8$ GeV. The dashed line shows the background for events in which more, unobserved, particles have been produced, deduced from transverse momentum balance in 5-prong events and then normalized to the spectrum for $\Delta E > 4$ GeV.
- b) Similar spectrum and background for events with $1.8 < M(4\pi) < 3.0$ GeV.

- Fig. 2 : Observed 4π mass spectrum. The curve indicates the average acceptance for events as a function of 4π mass, as determined by the procedure described in the text.

- Fig. 3 : a) Observed $\pi^+\pi^-$ mass spectrum for $M(4\pi) \leq 3.05$ GeV.
b) Observed $\pi^\pm\pi^\pm$ mass spectrum for $M(4\pi) \leq 3.05$ GeV.

- Fig. 4 : Measured values of b , in $(\text{GeV}/c)^{-2}$, resulting from fitting t -distributions, for $0.05 < -t < 1$ $(\text{GeV}/c)^2$, with $A \exp(bt)$, as a function of 4π mass.

- Fig. 5 : Measured moments of θ_{++}^S , the angle in the 4π CM-system between the resultant momentum of the two π^+ -mesons and the s -channel axis:

a) Values of $(12\pi)^{1/2} N\langle Y_1^0(\theta_{++}^S) \rangle$ as a function of 4π mass.

b) Comparison of values of

$$\begin{array}{l} \text{---} \quad (4\pi)^{1/2} N\langle Y_0^0(\theta_{++}^S) \rangle \\ \text{---} \quad - (20\pi)^{1/2} N\langle Y_2^0(\theta_{++}^S) \rangle \end{array}$$

as a function of 4π mass.

Fig. 6 : Measured values of p_T , with respect to t-channel axis in the 4π CM-system, for (a) ρ^0 -mesons, (b) other $\pi^+\pi^-$ mesons, from the dominant $\rho^0\pi^+\pi^-$ systems in the range $2.05 < M(4\pi) < 3.05$ GeV. The curves are calculated for p_T -limited phase-space, the degree of truncation being indicated by the parameter A.

Fig. 7 : Measured $\pi^+\pi^-$ mass spectra for 4π mass ranges:
(a) $1.25 < M(4\pi) < 1.35$ GeV, (b) $1.35 < M(4\pi) < 1.45$ GeV.
These are compared with calculations, suitably normalized, from:

_____	4π phase space
_____	$\rho\pi\pi$ phase space
- - - - -	$\rho\pi$, SCHC alignment
- . - . - . - . - . -	$\rho\pi$, s + d waves, d/s = 0.25
.....	$A_1\pi$, $M(A_1) = 1.3$ GeV, SCHC alignment
- . - . - . - . - . -	$A_1\pi$, $M(A_1) = 1.15$ GeV, SCHC alignment.

Fig. 8 : Measured 3π mass spectra for 4π mass ranges:
(a) $1.45 < M(4\pi) < 1.55$ GeV, (b) $1.55 < M(4\pi) < 1.65$ GeV.
These are compared with calculations, suitably normalized, from:

_____	4π phase space
_____	$\rho\pi\pi$ phase space
- - - - -	$\rho\pi$, SCHC alignment
- . - . - . - . - . -	$\rho\pi$, s + d waves
.....	$A_1\pi$, $M(A_1) = 1.3$ GeV, SCHC alignment
- . - . - . - . - . -	$A_1\pi$, $M(A_1) = 1.15$ GeV, SCHC alignment.

Fig. 9 : Results of maximum likelihood fit assuming $A_1\pi$ model. Intensities are shown resulting from fitting 4 contributions [5 for $M(4\pi) > 2.1$ GeV] to data in 100 MeV bins of 4π mass:

- (a) $A_1\pi$ with modified shape for A_1 , SCHC alignment.
- (b) $\rho\pi\pi$ p_T -limited phase-space.
- (c) 4π phase-space.
- (d) 4π p_T -limited phase-space (only included for $M(4\pi) > 2.1$ GeV).
- (e) $\rho\pi\pi$ phase-space, SCHC alignment.

Fig. 10 : Results of maximum likelihood fit assuming $\rho\pi$ s- and d-wave model. Intensities are shown resulting from fitting 3 contributions to data in 100 MeV bins of 4π mass:

- (a) $\rho\pi$, s- and d-wave, SCHC alignment [(b) shows the amplitude ratio d/s].
- (c) 4π phase-space.
- (d) $\rho\pi\pi$ p_T -limited phase-space.

Fig. 11 : 3π mass spectra for $2.05 < M(4\pi) < 3.05$ GeV for 3π combinations with $\cos \theta_{3\pi} > 0.7$ (where $\theta_{3\pi}$ is the angle between the resultant momentum of the 3π and the t-channel axis, in the 4π CM-system) for unweighted events. The full curve is a 2nd order polynomial fit to the data over the range shown and the dashed histogram is the result of a fit with a 2nd order polynomial + two Breit-Wigner formulas.

Fig. 12 : Comparison of measured 4π mass spectrum with the shapes of

----- $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-)$

and

$$\text{-----} \quad [M(4\pi)]^2 \sigma(e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-)$$

The electron-positron annihilation cross-section is represented by a Breit-Wigner fit to the measured cross-sections.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12