Ref.TH-3094-CERN

SPIN SUM RULES IN EXTENDED SUPERSYMMETRY

S. Ferrara, C.A. Savoy

CERN--Geneva

and

L. Girardello

Gordon McKay Laboratory,
Harvard University, Cambridge, U.S.A.

ABSTRACT

We derive general helicity and spin sum rules for massless and massive supermultiplets using properties of the spinor representations of the $SO(2N)$ $[SO(4N)]$ group of automorphisms of the N extended supersymmetry algebra at fixed light-like (timelike) momenta. These include observed sum rules which are responsible for the softening of quantum divergences of model field theories with N extended supersymmetry. The connection of mass formulae obtained in N extended spontaneously broken supergravity models with these spin sum rules is pointed out.

Ref.TH-3094-CERN

27 May 1981

It has been recently argued that hidden symmetries could prevent the presence of ultra-violet quantum differences in N extended supersymmetric field theories, beyond naive symmetry arguments. Examples of hidden symmetries are already present in the maximally extended $N = 4$ supersymmetric Yang-Mills theory where explicit calculations have proven that the β function, related to charge renormalization, is zero at the first three loops of perturbation theory¹⁾. Conjectures have been made in order to further support the vanishing of the β function to all orders of perturbation theory²⁾. The relevance of hidden symmetries is even more dramatic in extended supergravity where explicit possible on-shell counterterms have been constructed at the three-loop level linearized from as well as full non-linear counterterms at the eight-loop order in $N = 8$ supergravity³⁾. Remarkably, Christensen et al.⁴⁾ have shown that in $N > 4$ extended supergravity with an $SO(N)$ Yang-Mills gauge group, charge renormalization is absent at the one-loop level. Since in these theories the Yang-Mills charge is related, through supersymmetry, to the cosmological constant⁵⁾, $\Lambda = -6e^2/\kappa^4$, this result also implies that the cosmological constant is one-loop finite and calculable in these models⁴⁾. This is a remarkable example of the absence of ultra-violet divergences due to a hidden symmetry since an infinite charge renormalization does not seem to be forbidden by any manifest symmetry argument.

Recently Curtright⁶⁾ has observed that the vanishing of the (one-loop) β function in $N = 4$ Yang-Mills theory as well as in $N > 4$ extended supergravity with the $SO(N)$ Yang-Mills gauge group is due to certain helicity sum rules which hold for arbitrary massless multiplets of N extended supersymmetry and that can be summarized as follows:

$$\sum_{\lambda} (-)^{2\lambda} d(\lambda) \lambda^p = 0, \quad p < N \quad (1)$$

$$\sum_{\lambda} (-)^{2\lambda} d(\lambda) \lambda^p C_2(\lambda) = 0, \quad p+2 < N \quad (2)$$

The sum is performed over the set of helicity states, $\lambda_{\text{MAX}}, \lambda_{\text{MAX}} - 1/2, \dots, \lambda_{\text{MAX}} - N/2$, and $d(\lambda)$ is a multiplicity factor for the state of helicity λ . The generic state $(\lambda_{\text{MAX}} - k/2)$ occurs with multiplicity $\binom{N}{k}$, i.e., in a totally antisymmetric representation of $SO(N)$ [or $SU(N)$]. The second sum rule contains $C_2(\lambda)$, the second order Casimir operator of the group $SO(N)$ [or $SU(N)$]. As a matter of fact, the sum rules (1) and (2) are valid for arbitrary supermultiplets. If the state with maximal helicity, λ_{MAX} , transforms as a non-trivial representation R of $SO(N)$ [or $SU(N)$], then:

$$d(\lambda_{\text{MAX}} - k/2) = D(R) \binom{N}{k}, \quad (3)$$

$$\text{Trace } C_2(\lambda_{\text{MAX}} - k/2) = \left[C_2(R) \binom{N}{k} + (N^2 - 1) \binom{N-2}{k-1} \right] D(R),$$

where $D(R)$ and $C_2(R)$ are the dimensionality and the C_2 eigenvalue of R , respectively. The sum rules in Eqs. (1) and (2) immediately explain the vanishing of the one-loop β function in $N = 4$ Yang-Mills theory and in $N > 4$ extended supergravity. Indeed, the one-loop contribution to the β function for a particle of helicity λ has the structure⁶⁾

$$\beta_\lambda(e) = C_2(\lambda) (a + b\lambda^2) (-)^{2\lambda}. \quad (4)$$

For N extended supersymmetric Yang-Mills theory $C_2(\lambda)$ is independent of λ and β vanishes for $N = 3, 4$ because of the sum rule (1). On the other hand, for N extended supergravity, $C_2(\lambda)$ is given by Eq. (3), and the β function vanishes for $N > 4$ because of the sum rule Eq. (2).

In the present paper we give a very simple explanation of (1) and (2) in N extended supersymmetry and, in fact, we derive general sum rules for arbitrary $SO(N)$ [or $SU(N)$] Casimir operators which are valid for both massless and massive supermultiplets. More specifically, we use some properties of the spinor representations of orthogonal groups to relate summations over the bosonic and fermionic states respectively.

Let us start with the massless representations of N extended supersymmetry. As is well known, the supersymmetry algebra at fixed light-like momentum $P^2 > 0$, $P^0 > 0$, reduces to an algebra of N creation and N annihilation fermionic operators

$$\{Q^i, Q_j^*\} = \delta_j^i, \quad \{Q^i, Q^j\} = 0. \quad (5)$$

A supermultiplet is obtained by applying products of Q_i^* 's to a Clifford vacuum Ω , defined by the condition

$$Q^i \Omega = 0 . \quad (6)$$

Then, if Ω corresponds to a massless particle with helicity λ_{MAX} , transforming according to an irreducible representation R of $SU(N)$, the generic state

$$Q_{i_1}^* Q_{i_2}^* \dots Q_{i_k}^* \Omega , \quad (7)$$

will have helicity $(\lambda_{\text{MAX}} - k/2)$ and will transform as the representation $R \times [\overline{k}]_N$ of $SU(N)$. The symbol $[\overline{k}]_N$ denotes the k fold antisymmetric representation of $SU(N)$. A physical multiplet is obtained by adding the PCT conjugate multiplet. This is necessary if $\lambda_{\text{MAX}} \neq N/4$ or $R \neq \bar{R}$. In this case a massless multiplet of PCT conjugate states is constructed by adding to the previous one the new set of states obtained from a vacuum state Ω' with $\lambda'_{\text{MAX}} = N/2 - \lambda_{\text{MAX}}$ and $R' = \bar{R}$. From now on we denote such a massless supermultiplet by the quantum number of the vacuum state, $\{\lambda_{\text{MAX}}, R\}$. Because of the PCT doubling, $\{\lambda_{\text{MAX}}, R\}$ and $\{N/2 - \lambda_{\text{MAX}}, \bar{R}\}$ will refer to the same PCT conjugate supermultiplet. A general massless supermultiplet will contain $D(R)2^{N+1}$ if $\lambda_{\text{MAX}} = N/4$ and $R = \bar{R}$ states. Since the states of the supermultiplet $\{\lambda_{\text{MAX}}, R\}$ are obtained by making the tensor product of the states of the fundamental representation $\{\lambda_{\text{MAX}}, 1\}$ by R , we will temporarily confine our discussion to the fundamental (2N dimensional) representation of the Clifford algebra, Eq. (5).

We introduce Γ matrices as follows

$$\begin{aligned} \Gamma_{2i-1} &= Q^i + Q_i^* , \quad \Gamma_{2i} = i(Q^i - Q_i^*) , \\ &\quad (i = 1, 2, \dots, N) , \end{aligned} \quad (8)$$

so that Eq. (5) can be rewritten in the real form

$$\{\Gamma_A, \Gamma_B\} = 2\delta_{AB} , \quad (A, B = 1, \dots, 2N). \quad (9)$$

This algebra manifestly exhibits $SO(2N)$ as the group of automorphisms. The $SO(2N)$ generators are

$$\Gamma_{AB} = \frac{1}{4i} [\Gamma_A, \Gamma_B] , \quad (10)$$

and the $SO(2N)$ invariant operator

$$\Gamma_{2N+1} = (i)^N \Gamma_1 \Gamma_2 \dots \Gamma_{2N} , \quad (11)$$

allows us to decompose the 2^N dimensional irreducible representation of the Clifford algebra, Eq. (5) or (9), into two inequivalent irreducible spinor representations of $SO(2N)$, each of dimension 2^{N-1} . These two representations correspond precisely to the bosons and fermions which are contained in the 2^N dimensional supermultiplet. The maximal subgroup $SU(N) \times U(1)$ of $SO(2N)$ is of particular relevance. This is also the maximal invariance group of the Clifford vacuum Ω . When combined with the $U(N)$ group of outer automorphisms of the relativistic supersymmetry algebra, this group is adequate to define super-Casimir operators which classify all irreducible representations of N extended supersymmetry. The $SU(N) \times U(1)$ generators can be written as follows:

$$T_j^i = \frac{1}{2} [Q^i, Q_j^*] - \frac{1}{2N} \delta_j^i [Q^k, Q_k^*] , \quad (12)$$

$$T = \frac{1}{4} [Q^i, Q_i^*] \quad (13)$$

The helicity operator, in a given supermultiplet, is related to T by⁷⁾

$$\Lambda = T + \lambda_{MAX} - N/4 \quad (14)$$

Note that $T\Omega = (N/4)\Omega$ and the spectrum of T is

$$N/4, N/4 - 1/2, \dots, N/4 - k/2, \dots, -N/4 \quad (15)$$

Obviously $\text{Trace } T = 0$ because $T \in SO(2N)$. The spectrum of Λ is given by $\lambda_{MAX}, \lambda_{MAX} - 1/2, \dots, \lambda_{MAX} - N/2$.

Our results which follow are derived from the obvious property:

$$\text{Trace } \Gamma_{2N+1} \Gamma_{A_1} \Gamma_{A_2} \dots \Gamma_{A_p} = 0, \quad p < 2N, \quad (16)$$

which implies that any operator O which is bilinear in the Γ s satisfies the identities

$$\text{Trace } \Gamma_{2N+1} O^p = 0, \quad p < N, \quad (17)$$

and introducing the projectors $\pi_{\pm} = 1/2(1 \pm \Gamma_{2N+1})$,

$$\text{Trace } \pi_+ O^p \pi_+ = \text{Trace } \pi_- O^p \pi_-, \quad (18)$$

($p < N$).

Notice that, in general, $\text{Tr } \pi_{\pm} O^p \neq 0$ because O^p is even in the Γ matrices. But since π_+ (π_-) projects into the bosonic (fermionic) states, respectively, then we also have the result

$$\text{Trace } \pi_+ O^p \pi_+ - \text{Trace } \pi_- O^p \pi_- = \text{Supertrace } O^p = 0, \quad (19)$$

$p < N$.

It is obvious that this property does not depend either on the quantum number of the Clifford vacuum Ω , nor on the fact that the PCT conjugate multiplet has been added. Therefore, the extension to these cases is straightforward. In order to get the sum rule in Eq. (1) we simply take $O = A$ in Eq. (19):

$$\text{Supertrace } A^p = 0, \quad p < N, \quad (20)$$

which, when computed for a massless supermultiplet, gives

$$\sum_{k=0}^N \binom{N}{k} (\lambda_{\text{MAX}} - k/2)^p (-)^k = 0, \quad p < N. \quad (21)$$

In order to obtain sum rules involving $SU(N)$ Casimir operators we apply Eq. (19) to the $SU(N)$ generators T_j^i as well, and we get:

$$\text{Supertrace } \Lambda^h T_{j_1}^{i_1} T_{j_2}^{i_2} \dots T_{j_n}^{i_n} = 0, \quad h+n < N. \quad (22)$$

This equation gives in particular sum rules in terms of Casimir operators

$$C_k = T_{i_2}^{i_1} T_{i_3}^{i_2} \dots T_{i_k}^{i_{k-1}} T_{i_1}^{i_k} \quad (2 \leq k \leq N) \quad (23)$$

as follows,

$$\text{Supertrace } C_{k_1}^{h_1} C_{k_2}^{h_2} \dots C_{k_p}^{h_p} \Lambda^h = 0 \quad (24)$$

for $h_1 k_1 + h_2 k_2 + \dots + h_p k_p + h < N$ [C_k^h is the h^{th} power of the k^{th} order Casimir operator of $SU(N)$]. The quantities C_k can be explicitly computed⁸⁾. For instance, for a totally antisymmetric representation $[n]_N$ of $SU(N)$ one has

$$C_k([n]_N) = \frac{N+1}{N^k} \frac{n(N-n)}{N-n+1} [(N+1)^{k-1} (N-n)^{k-1} - (-n)^{k-1}] \quad (25)$$

Equations (20) and (24) are independent of λ_{MAX} because of Eq. (14). Moreover, these equations hold for arbitrary supermultiplets $\{\lambda_{\text{MAX}}, R\}$, since one can factorize the dependence on R by using obvious relations like

$$\begin{aligned} \text{Trace } C_k(R_1 \times R_2) &= D(R_1) \text{Trace } C_k(R_2) + D(R_2) \text{Trace } C_k(R_1) \\ &+ \sum_{r_1 + \dots + r_p + s_1 + \dots + s_q < k} a^{r_1 \dots r_p s_1 \dots s_q} \text{Trace } C_{r_1}(R_1) \dots C_{r_p}(R_1) \cdot \\ &\quad \cdot \text{Trace } C_{s_1}(R_2) \dots C_{s_q}(R_2), \end{aligned} \quad (26)$$

and its generalizations to products of Casimir operators where the coefficients $a^{r_1 \dots s_q}$ are independent of R_1 and R_2 .

The sum rule in Eq. (2) is a particular case of Eq. (24) with $p = 1$, $h = 1$, $k = 2$, namely,

$$\text{Supertrace } C_2 \Lambda^h = 0, \quad h+2 < N. \quad (27)$$

Another interesting relation,

$$\text{Supertrace } A \Lambda^h = 0, \quad h+3 \leq N, \quad (28)$$

involves the group-theoretical factor of the axial anomalies for $SU(N)$ representations

$$A(R) = D(R) \left[C_3(R) - \frac{N}{2} C_2(R) \right] = \frac{1}{2} \text{Trace}_{(R)} T_j^i \{ T_k^j, T_i^k \}. \quad (29)$$

The physical relevance of Eq. (28), in view of the difficulty in defining an axial anomaly for higher spin particles, remains unclear to us.

These sum rules can be immediately generalized to the massive supermultiplets of N extended supersymmetry⁹⁾. The rest frame supersymmetry algebra (without central charges) is

$$\{ Q_\alpha^i, Q^{*\beta}_j \} = \delta_\alpha^\beta \delta_j^i, \quad \{ Q_\alpha^i, Q_\beta^j \} = 0 \quad (30)$$

(mass = 1)

In this case, the group of automorphisms of the rest frame supersymmetry algebra is $SO(4N)$ rather than $SO(2N)$, so we get, instead of Eq. (18), the condition

$$\text{Supertrace } O^P = 0, \quad p < 2N \quad (31)$$

If we apply Eq. (31) to the spin operator $S^2 = \vec{S} \cdot \vec{S}$ we get the sum rule

$$\text{Supertrace } (S^2)^P = 0, \quad p < N \quad (32)$$

which can be explicitly written in terms of the contributions of states of definite spin s as follows:

$$\sum_{s=0}^{N/2} (2s+1) s^P (s+1)^P (-)^{2s} D(N, s) = 0 \quad (33)$$

where $d(N, s)$ denotes the degeneracy of spin s states in the 2^{2N} dimensional fundamental massive representation of N extended supersymmetry. As a matter of fact, $D(N, s) = (2s+1/N+1) \binom{2N+2}{N-2s}$ is the dimensionality of the irreducible anti-symmetric (traceless) representation $[N - 2s]$ of $USp(2N)$. Its decomposition according to the subgroup $SU(N) \times U(1)$ is easily found by the diagrammatic rule given in the Table. Whenever the vacuum state Ω is not an $SU(N)$ singlet spinless particle, but has spin L and transforms according to an irreducible representation R of $SU(N)$, the states of definite spin and $SU(N)$ quantum numbers are obtained by making the tensor product of the quantum numbers in the fundamental representation with the quantum numbers of the vacuum, (L, R) . Equation (31), which is the generalization to massive representations of the helicity sum rule, Eq. (20), remains valid for these (irreducible) supermultiplets.

These sum rules can be extended to the generators of the $SU(N) \times U(1)$ algebra as in the massless case. The generators of the $SO(4N)$ algebra of automorphisms of Eq. (30) are

$$\begin{aligned} \Lambda_{j\alpha}^{i\beta} &= \frac{1}{2} [Q_{\alpha}^i, Q^{*\beta}_j] \\ K_{\alpha\beta}^{ij} &= \frac{1}{2} [Q_{\alpha}^i, Q_{\beta}^j], \quad \bar{K}_{ij}^{\alpha\beta} = [Q^{*\alpha}_i, Q^{*\beta}_j] \end{aligned} \quad (34)$$

This $SO(4N)$ algebra contains an $SU(2)_{\text{spin}} \times USp(2N)$ subalgebra, with generators

$$S_k = \frac{1}{4} (\sigma_k)_{\beta}^{\alpha} [Q_{\alpha}^i, Q^{*\beta}_j] = \frac{1}{2} (\sigma_k)_{\beta}^{\alpha} \Lambda_{i\alpha}^{j\beta}, \quad (35)$$

$$T_j^i = \Lambda_{j\alpha}^{i\alpha} - \frac{1}{N} \delta_j^i \Lambda_{k\alpha}^{k\alpha}, \quad W = \frac{1}{2} \Lambda_{i\alpha}^{i\alpha} \quad (36)$$

$$K_{ij}^{kl} = \frac{1}{2} \epsilon^{\alpha\beta} K_{\alpha\beta}^{ij}, \quad \bar{K}_{ij}^{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta} \bar{K}_{ij}^{\alpha\beta}$$

The generators of the $SU(N) \times U(1)$ subalgebra of $USp(2N)$ are then,

$$T_j^i = \frac{1}{2} [Q_\alpha^i, Q^{*\alpha}_j] - \frac{1}{2N} \delta_j^i [Q_\alpha^k, Q^{*\alpha}_k], \quad (37)$$

$$W = \frac{1}{4} [Q_\alpha^i, Q^{*\alpha}_i]. \quad (38)$$

This $SU(N) \times U(1)$ algebra can be combined with the outer automorphism of the relativistic supersymmetry algebra to give an $SU(N) \times U(1)$ which commutes with the spinorial charges Q_α^i .

In terms of the $SU(N)$ Casimir operators, $C_k (k = 2, \dots, N)$, and the W generator of the $U(1)$ factor we derive the sum rule

$$\text{Supertrace } (S^2)^h W^q C_{k_1}^{h_1} \dots C_{k_p}^{h_p} = 0, \quad (39)$$

for $2h + q + h_1 k_1 + \dots + h_p k_p < 2N$.

These sum rules can be extended to the case where central charges are present. The corresponding supermultiplets for general central charge matrices have been given¹⁰⁾. The most interesting case is obtained by dimensional reduction from massless representations in higher dimensions. For even N , the rest frame supersymmetry algebra has $SO(2N)$ as a group of automorphisms as in the massless case, while the $U(N)$ subgroup is broken down to $USp(N)$. The sum rule, Eq. (39), is then replaced by the following one

$$\text{Supertrace } (S^2)^h T^{i_1 j_1} \dots T^{i_n j_n} = 0, \quad (40)$$

for $2h + n < N$, where the T^{ij} s are now generators of $USp(N)$.

If, in a given model, the mass operator transforms as one of the generators which commute with the spin, Eqs. (39) and (40) lead to mass sum rules. This is indeed what happens in extended supergravity models where masses are introduced via generalized dimensional reduction¹¹⁾. For instance, in the "spontaneously" broken $N = 8$ supergravity, the mass operator, M , is proportional to the Cartan generators of $USp(8)$ so that one obtains for M a sum rule from Eq. (40)

$$\text{Supertrace } (S^2)^h M^{2k} = 0 \quad (41)$$

for $h + k < 4$. For $h = 0$, we reproduce three sum rules which were already known¹²⁾:

$$\sum_s (-)^{2s} (2s+1) d(s) m^{2k}(s) = 0 \quad (k=1,2,3), \quad (42)$$

while for $h \neq 0$ we obtain three new sum rules:

$$\sum_s (-)^{2s} (2s+1) d(s) s^p (s+1)^p m^2(s) = 0 \quad (p=1,2), \quad (43)$$

$$\sum_s (-)^{2s} (2s+1) d(s) s (s+1) m^4(s) = 0.$$

It is well known that the sum rules, Eq. (42), for $k = 1, 2$ ensure that the one-loop induced cosmological term is finite and calculable in these theories in contrast with non-supersymmetric theories where it is quartically divergent¹³⁾. The sum rules in Eq. (42) for $k = 3$ and in Eq. (43) have not yet an analogous physical interpretation.

We conclude with some remarks. The sum rules we have derived can be further generalized to cases in which the supertrace in (16) does not vanish. In fact, the supertrace of operators like in (22) can be computed for $h \geq N$ using properties of Γ matrices. In particular, this trick can be used to evaluate the one-loop contribution to the charge renormalization and to the cosmological constant in cases where they do not vanish. Eventually, the identities derived with the method outlined in this paper could be relevant for higher loop calculations reveal cancellations which could not be deduced from naive symmetry arguments.

$$2s \left\{ \begin{array}{c} \boxed{\vdots} \\ \boxed{\vdots} \\ \boxed{\vdots} \end{array} + \begin{array}{c} \boxed{} \boxed{} \\ \boxed{\vdots} \\ \boxed{\vdots} \\ \boxed{\vdots} \end{array} + \begin{array}{c} \boxed{} \boxed{} \boxed{} \\ \boxed{\vdots} \\ \boxed{\vdots} \\ \boxed{\vdots} \end{array} + \cdots + \begin{array}{c} \boxed{} \boxed{} \boxed{} \boxed{} \\ \boxed{\vdots} \\ \boxed{\vdots} \\ \boxed{\vdots} \end{array} + \cdots + \begin{array}{c} \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \\ \boxed{\vdots} \\ \boxed{\vdots} \\ \boxed{\vdots} \end{array} \right\} \begin{array}{l} N-2s \\ 2s \end{array}$$

$$W = \left(s - \frac{N}{2}\right), \left(s + 1 - \frac{N}{2}\right), \left(s + 2 - \frac{N}{2}\right), \dots, \left(s + p - \frac{N}{2}\right), \dots, \left(\frac{N}{2} - s\right).$$

$$= \sum_{p=0}^{N-2s} \left\{ \begin{array}{c} \boxed{} \boxed{} \boxed{} \\ \boxed{\vdots} \\ \boxed{\vdots} \\ \boxed{\vdots} \end{array} \right\} \begin{array}{l} p \\ 2s \end{array} \quad \left(W = s + p - \frac{N}{2}\right)$$

$$D(N, s, p) = \frac{2s+1}{2s+p+1} \binom{N+1}{p} \binom{N}{2s+p}$$

$$D(N, s) = \sum_{p=0}^{N-2s} D(N, s, p) = \frac{2s+1}{N+1} \binom{2N+2}{N-2s}$$

$$2^{2N} = \sum_{s=0}^{N/2} (2s+1) D(N, s)$$

Table: U(N) content of the fundamental massive representation of the supersymmetry algebra. The SU(N) decomposition of states at spin s are given in terms of Young tableaux, with at most two columns, characterized by s and another number, p ($p = 0, 1, \dots, N - 2s$). The corresponding eigenvalue of W , the generator of the U(1) factor, is also given as well as the dimensionality $D(N, s, p)$ of the SU(N) representations. The states at spin s make altogether the irreducible representation $[N - 2s]$ of USp(2N), i.e., its $(N - 2s)$ -fold antisymmetric traceless representation with dimensionality $D(N, s)$. General massive multiplets are obtained by making the tensor product of the states with the set of vacuum states, characterized by an (internal) spin L and a representation R of SU(N).

REFERENCES

- 1) One loop : S. Ferrara and B. Zumino, Nucl. Phys. B79 (1974) 413;
Two loops : D.R.T. Jones, Phys. Lett. B72 (1977) 199;
H.N. Pendleton and E.C. Poggio, Phys. Lett. B72 (1977) 200;
Three loops : D.V. Tarasov and A.A. Vladimirov, Dubna preprint E2-80-483 (1980);
M.T. Grisaru, M. Rocek and W. Siegel, Phys. Rev. Lett. 45 (1980) 1063;
W.E. Caswell and D. Zanon, Phys. Lett. B100 (1980) 152.
- 2) S. Ferrara and B. Zumino, unpublished;
M.F. Sohnius and P.C. West, Phys. Lett. B100 (1981) 245.
- 3) R. Kallosh, Phys. Lett. B99 (1981) 122;
P.S. Howe and U. Lindström, Nucl. Phys. B181 (1981) 487;
P.S. Howe, K.S. Stelle and P.K. Townsend, CERN preprint TH-3065 (1981).
- 4) S.M. Christensen, M.J. Duff, G.W. Gibbons and M. Rocek, Phys. Rev. Lett. 45 (1980) 161.
- 5) D.Z. Freedman and A. Das, Nucl. Phys. B170 (1977) 221;
E.S. Fradkin and M.A. Vasiliev, Lebedev Institute preprint (1977).
- 6) T.L. Curtright, University of Florida preprint 80-23 (1981).
- 7) S. Ferrara, CERN preprint TH-2957 (1980), plenary talk at the 9th International Conference on General Relativity and Gravitation (Jena, GDR, July 1980).
- 8) A.M. Peremolov and V.S. Popov, JETP Lett. 1 (1965) 160.
- 9) A. Salam and J. Strathdee, Nucl. Phys. B76 (1974) 477; B84 (1975) 127.
- 10) S. Ferrara, C.A. Savoy and B. Zumino, Phys. Lett. 100B (1981) 393;
- 11) J. Scherk and J. Schwarz, Nucl. Phys. B153 (1979) 61;
E. Cremmer, J. Scherk and J. Schwarz, Phys. Lett. 84B (1979) 83.
- 12) S. Ferrara and B. Zumino, Phys. Lett. 86B (1979) 279.
- 13) B. Zumino, Nucl. Phys. B89 (1975) 535.