Ref.TH.3109-CERN

GIBBS AVERAGE IN THE FUNCTIONAL FORMULATION
OF QUANTUM FIELD THEORY

V. de Alfaro^{*)}, S. Fubini
CERN -- Geneva

and

G. Furlan⁺⁾
Istituto di Fisica Teorica dell' Università
Trieste, Italy

A new formulation of the Euclidean functional approach to quantum field theory is proposed. It is based on functional integration over a Gibbs ensemble. This automatically ensures invariance with respect to general field transformations. The new momenta conjugate to the field variables perform naturally the role of the ghost fields required by the general consistency of the theory.

^{*)} On leave from Istituto di Fisica Teorica dell' Università, Torino and INFN, Torino, Italy.

⁺⁾ and Istituto Nazionale di Fisica Nucleare, Sezione de Trieste and International Centre for Theoretical Physics, Trieste, Italy.

The Euclidean functional formulation of quantum field theory has assumed a very important role in recent developments¹⁾. Let us mention the most relevant reasons for this.

This formulation gives the covariant perturbation theory in the most direct way since it leads straight to Feynman rules. Furthermore, for gauge theories the functional approach offers the simplest way of obtaining the general rules, avoiding many of the problems encountered in the canonical quantization of those systems. Finally, it is a very important tool for the investigation of non-perturbative properties and has been the basis of the recent lattice approach. Let us recall the main points.

Let us recall the main points. At the start one has a Lagrangian function of the fields (and of their space and time derivatives)

$$\begin{aligned} \mathcal{L} &= \mathcal{L}(\varphi, \partial_\mu \varphi) \\ &= \frac{1}{2} (\partial_\mu \varphi)^2 + V(\varphi) \end{aligned} \quad (1)$$

Then the fundamental generating functional is given by

$$Z = \int d\Omega \exp \left\{ -\frac{1}{\hbar} \int d^4x (\mathcal{L} + J^{\text{ext}} \varphi) \right\} \quad (2)$$

where $J^{\text{ext}}(x)$ is an external field. In what follows this factor will be omitted for simplicity of notation.

The analogy with classical statistical mechanics is well known. The field φ plays the role of a Lagrangian set of co-ordinates defined over a four dimensional Euclidean manifold. The action is the analogue of the Hamiltonian and $\hbar$ represents the temperature.

One still open question concerns the definition of the functional integration measure $d\Omega$. There is to our knowledge no completely explicit rule that defines this measure and correspondence with the canonical method must be used in order to determine it.

In this letter we want to discuss an improved functional formulation where the similarity to classical statistical mechanics becomes a complete equivalence.

This identification with classical statistical mechanics leads to an entirely unambiguous definition of the functional integration. The prescription is unique and coincides with the conventional results in the cases already known.

If we examine the functional integral (2) we note a fundamental difference from the functional integral of classical statistical mechanics

$$\int dV_p dV_q \exp \left\{ -\beta H(p_i, q_i) \right\} \quad (3)$$

In the above formula, H depends not just on the co-ordinates q_i , it is also a function of the conjugate momenta p_i and the integration measure is the elementary volume in phase space (with well-known canonical invariance properties) and not just the volume element of configuration space.

Our functional formulation is based on the principle of substituting for the conventional definition of Z given by (2) a new definition that generalizes the Gibbs integral (3) to the present case. The procedure goes as follows. One must first introduce a fictitious 5th time t and consider the fields as functions of our four Euclidean variables x_μ and of t (2):

$$\varphi = \varphi(x_1, x_2, x_3, x_4, t) \quad (4)$$

Secondly the conventional Lagrangian is interpreted as a potential energy term

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2 + U(\varphi) \equiv V(\varphi)$$

and therefore in order to get the new Lagrangian we add a kinetic term $\propto (\partial\varphi/\partial t)^2$ so the new Lagrangian is

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 - \mathcal{L} = \frac{1}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 - V \quad (5)$$

We may now introduce the conjugate momentum $\pi(x)$

$$\pi(x) = \frac{\partial L}{\partial \dot{\varphi}} = \frac{\partial \varphi}{\partial t} \quad (6)$$

The Hamiltonian is thus

$$H = \frac{1}{2} \pi^2 + V \quad (7)$$

We now define Z as the well-known Gibbs average

$$Z = \int d\pi d\varphi \exp \left\{ - \int d^4x \left(\frac{1}{2} \pi^2 + V \right) \right\} \quad (8)$$

Equation (8) will represent our general definition of the functional integral Z .

We note that because of the Liouville theorem the volume element and thus the functional Z will be independent of t which is thus a purely convenient device for getting the fundamental definition (8).

We define the classical (equal t) Poisson bracket

$$\left[\varphi(x), \pi(y) \right]_{\text{P.B.}} = \delta^4(x-y) \quad (9)$$

together with the canonical transformations which leave this Poisson bracket invariant. This will lead to a corresponding invariance property of the functional Z .

We can now go on to show how our procedure applies to several cases of physical interest.

1 - Scalar Theory

Since in this case the dependence of the exponent in (8) on the momenta is purely Gaussian, the integration over π can be performed elementarily, leading to the conventional form of Z :

$$Z = (\text{const.}) \cdot \int d\varphi \exp \left\{ - \int d^4x \, U \right\} \quad (10)$$

This result seems to show that the addition of momenta to the formalism is just a useless complication. Actually even in this elementary case the form (8) is advantageous due to the invariance properties of the Gibbs measure. Let us perform the transformation of variables

$$\begin{aligned} \varphi &= F(\phi) \\ \frac{\partial \varphi}{\partial t} &= \dot{F}(\phi) \frac{\partial \phi}{\partial t} \end{aligned} \quad (11)$$

The Lagrangian becomes

$$\begin{aligned} L &= \frac{1}{2} \dot{F}^2 \left(\frac{\partial \phi}{\partial t} \right)^2 - U(\phi) \\ U(\phi) &= \frac{1}{2} \dot{F}^2 \left(\frac{\partial \phi}{\partial x_\mu} \right)^2 + V(\phi) \end{aligned} \quad (12)$$

The new momentum is defined by

$$\Pi_\phi = \dot{F}^2 \frac{\partial \phi}{\partial t} \quad (13)$$

and the Hamiltonian is now

$$H = \frac{1}{\dot{F}^2} \Pi_\phi^2 + U(\phi) \quad (14)$$

If we now integrate over momenta in order to reduce the expression of Z to its conventional form we obtain

$$Z = \int d\phi \, dt \, \dot{F}(\phi) \exp \left\{ - \int d^4x \, U(\phi) \right\} \quad (15)$$

The definition of Z has automatically produced the functional determinant of the transformation from φ to Φ . Of course it is better, in the framework of perturbation theory, not to perform the integration on the field π , which plays the role of the fictitious field introduced in the conventional expression most usually through fermion fields. The equivalence is obtained by the formula

$$\det A = \int d\psi d\psi^+ e^{\psi^+ A \psi} = \int d\phi e^{-\frac{1}{2} \phi A^{-2} \phi} \quad (16)$$

2 - Chiral Lagrangians

A second example of the advantages of the procedure proposed here is given by the chiral theories for which we refer to the classical paper by Gerstein, Jackiw, Lee and Weinberg³⁾. We start from the form of the conventional Lagrangian

$$\mathcal{L} = \frac{1}{2} G_{ab}(\phi) \partial_\mu \phi_a \partial_\mu \phi_b = \mathcal{V} \quad (17)$$

where $G_{ab}(\phi)$ is a function of the fields whose specific form depends, as shown in Ref. 3), on the definition of the fields ϕ_a .

In the present approach the Lagrangian is defined as

$$\mathcal{L} = \frac{1}{2} G_{ab} \frac{\partial \phi_a}{\partial t} \frac{\partial \phi_b}{\partial t} - \mathcal{V} \quad (18)$$

where the kinetic term is unambiguously determined by chiral invariance.

Now the conjugate momentum is again

$$\pi_a = G_{ab} \frac{\partial \phi_b}{\partial t} \quad (19)$$

and the Hamiltonian and generating functional are

$$H = \frac{1}{2} \pi_a G^{-1}_{ab} \pi_b + \mathcal{V} \quad (20)$$

$$Z = \int d\phi_a d\pi_a \exp \left\{ - \int d^4x H \right\} \quad (21)$$

The Gibbs measure is chiral invariant and invariant under general field transformations. The equivalence with the conventional formulation is obtained by integration over the momenta that leads to the well-known functional determinant

$$\begin{aligned} \int d\pi \exp \left\{ - \frac{1}{2} \int d^4x \pi_a G^{-1}_{ab} \pi_b \right\} = \\ = \det G^{1/2} = \exp \left\{ \frac{1}{2} \delta^4(0) \text{Tr} \ln G \right\} \end{aligned} \quad (22)$$

Again the momenta π_a are the scalar equivalent of the fermionic ghost fields usually introduced.

3 - Gauge Theories

We now arrive at the most subtle case, namely gauge theories. Let us investigate explicitly the case of Yang-Mills SU(2) gauge theory.

We start from the three gauge fields A_μ^a and define

$$A_\mu = \frac{1}{2i} A_\mu^a \sigma^a \quad (23)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g [A_\mu, A_\nu]$$

The conventional Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a = -\frac{1}{2} \text{Tr} F_{\mu\nu} F_{\mu\nu} \equiv \mathcal{V} \quad (24)$$

is of course invariant under the group of gauge transformations

$$A_\mu \rightarrow \Omega^{-1} A_\mu \Omega + \frac{1}{g} \Omega^{-1} \partial_\mu \Omega \quad (25)$$

Our procedure consists in considering $A_\mu(x,t)$ as a function of the 5th time' and adding to the potential term $\mathcal{V}$ a kinetic term of the form

$$T = \frac{1}{2} \frac{\partial A_\mu^a}{\partial t} \frac{\partial A_\mu^a}{\partial t} = -T_\mu \frac{\partial A_\mu}{\partial t} \frac{\partial A_\mu}{\partial t} \quad (26)$$

Since the gauge matrix Ω is only a function of x , T is gauge invariant and is the unique quadratic expression in the derivative that is so.

The conjugate momentum is

$$P_\mu^a = \frac{\partial A_\mu^a}{\partial t} \quad (27)$$

$$P_\mu = \frac{\partial A_\mu}{\partial t}$$

and the Poisson brackets are given by

$$\left[A_\mu^a(x), P_\nu^b(y) \right]_{\text{P.B.}} = \delta_{\mu\nu} \delta_{ab} \delta^4(x-y) \quad (28)$$

The Hamiltonian is

$$H = T + \mathcal{V} = \frac{1}{2} P_\mu^a P_\mu^a + \mathcal{V} \quad (29)$$

We write the expression for the generator of local gauge transformations

$$G_a(x) = \partial_\mu P_\mu^a + g \epsilon_{abc} A_\mu^b P_\mu^c = D_\mu^{ab} P_\mu^b$$

$$G \equiv \frac{1}{2i} \sigma_a G_a = \partial_\mu P_\mu + g [A_\mu, P_\mu] \quad (30)$$

It is easy to check that its Poisson brackets generate the gauge variations of fields and momenta. The Gibbs representation of the functional Z is

$$Z = \int dA_\mu dp_\mu \exp\{-\int H d^4x\} \quad (31)$$

where the integration measure is gauge invariant and of course invariant under canonical transformations.

As a consequence of gauge invariance, some combinations of the dynamical variables do not appear in $F_{\mu\nu}^a F_{\mu\nu}^a$ (in the Abelian case $\partial_\mu A_\mu$ is absent). In order to separate those variables it is advisable to parametrize the original set of variables A_μ^a into a gauge invariant subset plus a part that depends on the group variables⁴⁾:

$$A_\mu = U^{-1} B_\mu U + \frac{1}{g} U^{-1} \partial_\mu U \quad (32)$$

where B_μ is by definition gauge invariant and the redundancy is eliminated by a subsidiary condition like $B_4 = 0$ or $\partial_\mu B_\mu = 0$ for instance. The matrix U that contains the group parameters under the gauge transformation (25) behaves as

$$U \rightarrow U' = U \Omega \quad (33)$$

As a consequence of gauge invariance, the potential V depends on B_μ but not on the parameters defining U .

Our aim is now to transform our expressions from the A_μ set of variables to the set B_μ and U . Let us define the gauge invariant four vectors $\bar{p}_\mu$ by

$$\bar{p}_\mu = U p_\mu U^{-1} = U \frac{\partial A_\mu}{\partial t} U^{-1} \quad (34)$$

Introduce $\bar{p}_\mu$ in the expression (29) for H :

$$H = \frac{1}{2} \bar{p}_\mu^a \bar{p}_\mu^a + V \quad (35)$$

Now consider the local gauge generator [Eq. (30)]. We introduce $\bar{G}$ defined by

$$\bar{G} = U G U^{-1} \quad (36)$$

$\bar{G}$ is given by

$$\bar{G} = \partial_\mu \bar{p}_\mu + g [B_\mu, \bar{p}_\mu] = \frac{1}{2i} \sigma_a \bar{G}_a \quad (37)$$

and

$$\bar{G}_a = \partial_\mu \bar{p}_\mu^a + g \epsilon_{abc} B_\mu^b \bar{p}_\mu^c \quad (38)$$

The quantities $\bar{p}_\mu$ are not yet the momenta conjugate to the new variables B_μ and U but are closely related to such momenta. This can be seen by expressing $\bar{p}_\mu$ in terms of the time derivatives of the new variables B_μ and U . We can easily obtain

$$\bar{p}_\mu = \dot{B}_\mu + g [B_\mu, \Lambda] + \partial_\mu \Lambda \quad (39)$$

where the gauge variables of U enter Eq. (39) through the single operator

$$\Lambda \equiv \frac{1}{2i} \sigma_a \lambda_a = \frac{1}{g} \frac{\partial U}{\partial t} U^{-1} \quad (40)$$

We can now use Eq. (39) to obtain the momenta conjugate to B_μ and to U . Indeed, using the form (35) of H we get

$$\Pi_\mu^{(B)} = \bar{p}_\mu \hat{\delta}_{\alpha\mu} \quad (41)$$

where

$$\hat{\delta}_{\alpha\mu} = \frac{\partial \bar{p}_{\alpha}}{\partial \dot{B}_{\mu}} \quad (42)$$

is the projector on the transverse space [i.e., on the subspace orthogonal to the components of B_{μ} that have been set to zero^{*)}].

The momenta corresponding to the parameters contained in U are obtained through the use of the formula

$$\frac{\partial H}{\partial \lambda_r} = - \bar{G}_r \quad (43)$$

If one expresses U in terms of a well-defined choice of gauge parameters (as shown in Appendix A), Eq. (43) leads to a corresponding expression of the momenta conjugate to the gauge parameters.

We finally discuss the form of the Gibbs measure in the new variables. We can write

$$dV_{\text{Gibbs}} = dV_t dV_{\text{long}} \quad (44)$$

For transverse components we simply have

$$dV_t = dB_{\mu} d\pi_{\mu}^{(B)} \quad (45)$$

For longitudinal variables we get, as shown in Appendix A

$$dV_{\text{long}} = d\Omega_g d\bar{G}_i \quad (46)$$

where $d\Omega_g$ is the invariant group measure expressed in terms of the group parameters. On the other hand, the momentum part is simply expressed in terms of the generators $\bar{G}$ of the group transformations. We note that dV_{long} is also clearly invariant under gauge transformations.

^{*)} For instance, the subspace of components B_{μ} , $\mu = 1, 2, 3$ if we set $B_4 = 0$, or the subspace orthogonal to k_{μ} if we set $\delta_{\mu} B_{\mu} = 0$.

We are now ready to discuss our functional integral in terms of the new variables. We have from Eq. (35)

$$H = \frac{1}{2} (\Pi^{(B)})^2 + \frac{1}{2} (\bar{p}^L)^2 + V(B) \quad (47)$$

The integration over transverse momenta Π^B can be carried out elementarily and gives, as in the scalar case, a simple multiplicative constant which disappears from all physical results.

The integral over the group variables gives the group volume that is again an (infinite) constant which can likewise be eliminated. The important feature is that $\bar{p}^L$ appears in the exponent while $\bar{G}$ appears in the measure. So the (linear) expression of $\bar{G}$ in terms of $\bar{p}^L$ plays a fundamental role. We can write Eq. (38) in the form

$$\bar{G}_a = C_{ab} \bar{p}_b^L + S_b \quad (48)$$

where C_{ab} depends on the co-ordinates B_μ and S_b depends on the B_μ and on transverse momenta. If we use the expression (48) to write the measure $d\bar{G}$ in terms of $\bar{p}^L$ and then perform the integration over those variables, a functional determinant appears which is the one appearing in the customary expression of the generating function of gauge theory (see Appendix B). On the other hand, we may use the inverse of Eq. (48)

$$\bar{p}_a^L = A_{ab} G'_b$$

$$G'_b = \bar{G}_b - S_b$$

and substitute in the expression of Z . It is then clear that G' plays the role of a scalar fictitious particle field which replaces the usual Faddeev-Popov ghost.

CONCLUSION

The present Gibbs formulation of quantum field theory has the advantage of providing a compact and unified formulation of the theory. The Gibbs measure in phase space, defined in a unique way, is invariant under any co-ordinate transformation. The only ambiguity is in the choice of the dependence of our Lagrangian on the momenta conjugate to the fields. This freedom is, however, limited by the request that the canonical formalism be valid and consequently only quadratic terms in the derivatives of the fields are allowed.

Furthermore, in the case of chiral theories or gauge theories the invariance requirements determine in a practically unique way the choice of the additional terms in L .

As we have already said, the fundamental advantage of our formalism lies in its invariance under transformations of variables. It is just the lack of such invariance that compels one to introduce in the conventional treatment the fictitious particle fields to remedy the non-unitarity of naive Feynman rules.

As we have seen, in this formalism the conjugate momenta assume the role and give the physical effects of ghost fields. It will be very interesting to see how the BRS⁵⁾ transformations fit into the present scheme. The momenta appear in the theory right from the beginning. The cases where there are no ghosts in the conventional theory are those where the functional integrations over momenta can be trivially carried on.

We finally notice that the full equivalence between our procedure and classical statistical mechanics leads to a possible way of understanding some of the open problems in quantum field theory. In a certain sense we can say that quantum field theory inherits not only the advantages but also the disadvantages of classical statistical mechanics. The latter are of two sorts.

- a) The first problem is caused by the ultra-violet divergences that appear in the classical scaling laws, a most typical example of which is given by the classical black body law which is physically unacceptable at high frequencies.
- b) Another celebrated difficulty consists in the breakdown of statistical averages in the case of attractive forces (for instance in atoms) since the state with highest probability is the collapsed one.

These difficulties have their natural counterparts in quantum field theory in the well-known ultra-violet divergences and in the difficulties encountered in quantizing systems whose conventional Euclidean action (that corresponds to the potential in our scheme) is not positive definite, as happens for gravitation.

It is well known that in statistical mechanics these problems have been solved by quantization (p and q as non-commuting operators and averages defined as operator traces). This suggests that a possible regularization may follow a pattern of new quantization. For the classical Poisson brackets (9) one substitutes the commutator

$$\varphi(x) \pi(y) - \pi(y) \varphi(x) = i b \delta^4(x-y) \quad (49)$$

where the new constant b has the dimension of length. For the Gibbs integral one substitutes the trace in the operator H space.

We are presently investigating whether this new regularization procedure may be viable and may lead to a new scheme for field theory.

ACKNOWLEDGMENT

We wish to thank Stanley Deser for having read the manuscript and for his kind advice and criticism; we also enjoyed illuminating discussions with Daniele Amati, Claudio Rebbi and Gabriele Veneziano.

APPENDIX A

We parametrize the unitary unimodular matrix U as follows :

$$U = (1 + i \bar{\sigma} \cdot \bar{\phi}) (1 - i \bar{\sigma} \cdot \bar{\phi})^{-1} \quad (A1)$$

The variables ϕ_a are our gauge co-ordinates of A_μ . From formula (40) we easily compute the components of Λ

$$\lambda_a = -\frac{4}{g} \frac{1}{(1+\phi^2)^2} \left\{ 2 \bar{\phi} \dot{\bar{\phi}} \phi_a + (1-\phi^2) \dot{\phi}_a + 2 \epsilon_{abc} \dot{\phi}_b \dot{\phi}_c \right\} \quad (A2)$$

It is now easy to compute the canonical momenta $\pi_a^{(\phi)}$ conjugate to ϕ_a . We have

$$\pi_a^{(\phi)} = \frac{\partial H}{\partial \dot{\phi}_a} = \frac{\partial H}{\partial \lambda_a} \frac{\partial \lambda_a}{\partial \dot{\phi}_a} = -\bar{G}_a \frac{\partial \lambda_a}{\partial \dot{\phi}_a} \quad (A3)$$

Thus

$$\pi_a^{(\phi)} = \frac{4}{g} \frac{1}{(1+\phi^2)^2} \left\{ 2 \phi_i \bar{G}_i \phi_a + (1-\phi^2) \bar{G}_a + 2 \epsilon_{abc} \bar{G}_b \phi_c \right\} \quad (A4)$$

Note that Eq. (A4) can be expressed in matrix form as

$$\sigma_i \pi_i^{(\phi)} = -\frac{2i}{g} \frac{1}{1+\phi^2} v^{-1} \sigma_a \bar{G}_a v \quad (A5)$$

where

$$v = U^{1/2} = \frac{1}{\sqrt{1+\phi^2}} (1 + i \bar{\sigma} \cdot \bar{\phi})$$

We are now ready to compute

$$dV_{\text{long}} = d\phi d\pi^{(\phi)} \quad (A6)$$

First of all it is well known that

$$d^3\phi = (1+\phi^2)^3 d\Omega_g \tag{A7}$$

where $d\Omega_g$ is the invariant group measure. For $d^3\pi^{(\phi)}$ (computed at fixed ϕ) Eq.. (A5) leads immediately (apart from a multiplicative constant) to

$$d^3\pi^{(\phi)} = (1+\phi^2)^{-3} d^3\bar{G} \tag{A8}$$

Thus

$$dV_{\text{long}} = d\Omega_g d\bar{G} \tag{A9}$$

APPENDIX B

In this appendix we make some comments about the integration over longitudinal variables. The initial formulae are

$$Z = \int d\mathcal{B} d\pi^{(B)} \int d\Omega_g d\bar{G} \exp \left\{ - \int d^4x \left(\pi^{\mu 2} + \bar{p}^{\mu 2} + \mathcal{V}(\mathcal{B}) \right) \right\} \quad (B1)$$

$$\begin{aligned} \bar{G}_a &= \partial_\mu \bar{p}_\mu^a + g \epsilon_{abc} \mathcal{B}_\mu^b \bar{p}_\mu^c \\ &= C_{ab} \bar{p}_\mu^b + S_b \end{aligned} \quad (B2)$$

The important feature is the way $\bar{p}^L$ appears in (B2). Looking at the dependence of $\bar{G}_a$ on $\bar{p}^L$ we see two possibilities.

The simplest one is that in (B2) $\bar{p}^L$ appears only in the term $\partial_\mu \bar{p}_\mu$ and not in the term proportional to the coupling constant g . This of course happens in QED and in QCD is typical of the axial kind of gauge. Take for instance $B_4 = 0$, then $\bar{p}^L = \bar{p}_4$ is absent from the second term in the right-hand side of (B2). In this case the operator C_{ab} [Eq. (48)] does not contain other fields and it can be trivially integrated disappearing from the quantities of interest. If we parametrize with ghost fields they would remain decoupled from the physical fields.

In the more interesting case, $\bar{p}^L$ also appears in the g term in the right-hand side of (B2). In this case C_{ab} also contains the field $\mathcal{B}$ and it is not trivial to take into account the presence of $\bar{p}^{2L}$ at exponent in Eq. (B1). This case corresponds to essential (i.e., coupled to matter) Faddeev-Popov ghosts.

Let us take as an example the Lorentz gauge, $\partial_\mu \mathcal{B}_\mu = 0$. We separate

$$\bar{p}_\mu = \partial_\mu \bar{p}^L + \bar{p}_\mu^T \quad (B3)$$

So

$$\bar{G}_a = \square \bar{p}_a^L + g \epsilon_{abc} \mathcal{B}_\mu^b \left(\partial_\mu \bar{p}_\mu^L + \bar{p}_\mu^T \right) + \partial_\mu \bar{p}_{\mu a}^T \quad (B4)$$

and

$$C_{ab} = \delta_{ab} \square + g \epsilon_{pba} B_{\mu}^p \partial_{\mu} \quad (B5)$$

The functional determinant

$$\det C_{ab} = \det \frac{\partial \bar{G}_a}{\partial \bar{p}_b^L}$$

is exactly the Faddeev-Popov determinant.

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