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METRIC FROM MATTER

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ABSTRACT

We propose a general relativistic invariant theory in terms of only fundamental fermions without any reference to a metric. The integration over matter quantum fluctuations generates spontaneously a scale Λ , to be later identified with the Planck mass. For energies $E \ll \Lambda$ the theory produces a metric in terms of which we find the usual Einstein action with vanishing cosmological constant and with a relation between electromagnetic and gravitational couplings. For $E \sim \Lambda$ the theory departs from usual Einstein gravitation - not even permitting an unambiguous definition of a metric - thus allowing the conjecture of a better ultra-violet behaviour.

This paper is dedicated to Andrei Sakharov
pioneer in the field of induced gravity,
on his 60th birthday

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1. Several attempts have been devoted¹⁾ to the idea of generating Einstein's theory of gravity from the interaction of matter with a background metric field. Our aim here will be different. We will start from a general relativistic invariant theory written in terms of only fundamental fermions without any reference to a metric. We shall then show that the fluctuations of the matter field generate a metric - both as a background field and its quantum fluctuations - as a valid concept at large distances.

Our original action will be free of dimensional parameters and the regularization of the theory will preserve all the invariances. The theory will be shown to generate spontaneously a mass scale Λ in terms of which all dimensional quantities will be obtained. For energies $E \ll \Lambda$ the theory will be shown to give rise to the usual Einstein action with a vanishing cosmological term. All parameters are then given by the theory in terms of Λ . In particular the induced Newton constant G allows to identify Λ with the Planck mass.

A $U(1)$ gauge symmetry (electromagnetism) can be easily added without changing the pure fermionic content of the theory²⁾. The corresponding electromagnetic coupling constant is then related to G by the often conjectured relation:

$$\alpha(q^2) = \frac{N}{3\pi} \log \frac{1}{G q^2} + O(1) ; m_i^2 \lesssim q^2 \ll \frac{1}{G} \quad (1)$$

for N fundamental fermions of mass m_i .

The introduction of other internal gauge symmetries can follow the methods of Ref. 2) thus allowing a unification of all interactions^{*)} at the spontaneously generated Planck mass.

2. In order to find an invariant action, we start constructing a covariant derivative in terms of fermionic fields following the method of Ref. 2). For a Lorentz transformation this is given by

$$D_\mu \equiv \partial_\mu - \frac{i}{4} \sigma^{\beta\gamma} \text{Tr}(\sigma^{\beta\gamma} M_\mu M^{-1}) \quad (2)$$

^{*)} We recall nevertheless that the treatment of Ref. 2) was applicable to unbroken gauge theories with a real fermion content, because otherwise we lack a correct regularization prescription.

where

$$M = \psi \bar{\psi} \quad M_\mu = \psi \partial_\mu \bar{\psi} \quad (3)$$

are matrices in Dirac index space. The quantities $\bar{\psi} \gamma_\alpha D_\mu \psi$ form a set of four covariant vectors (like the usual vierbein) so that

$$A = \int d^4x \det \left[\frac{1}{K_0} \bar{\psi} \gamma_\alpha i D_\mu \psi \right] \quad (4)$$

where the determinant refers to the 4×4 matrix with elements (α, μ) , is a general relativistic invariant^{*)}. Notice that, in spite of its highly non-linear structure, this action is only of first order in the time derivative.

In the action (4) ψ , $\bar{\psi}$ and the constant K_0 are all dimensionless. Although K_0 could always be set equal to one by a rescaling of ψ , $\bar{\psi}$, we have not done that for the following reason: the presence of M^{-1} in Eq. (2) calls for a non-zero v.e.v. of M . A Lorentz invariant vacuum implies $\langle M \rangle$ proportional to the unit matrix. The proportionality constant, as long as it does not vanish, is irrelevant because it may be normalized to one by a rescaling of the fields thus changing K_0 accordingly. In other words, if $\langle M \rangle \neq 0$ we can set

$$\langle M \rangle = 1 \quad (5)$$

and the K_0 parameter in (4) acquires physical significance. We shall enforce (5) into the theory. This may be done either by a potential or a constraint. We stress that by enforcing (5), i.e., a non-zero v.e.v. of M , we have preserved all the required invariances of the theory, breaking only the invariance under $\psi \rightarrow c\psi$, $\bar{\psi} \rightarrow c\bar{\psi}$ which were preserved with $\langle M \rangle = 0$.

Our strategy for analyzing the theory will be to render the action bilinear in the fermion fields by the introduction of a (large!) set of auxiliary bosonic fields. We shall then integrate over the fermions. The resulting action, written in terms of the auxiliary fields, is then analyzed by finding a stationary

^{*)} External sources, needed to define the generating functional, are introduced in the usual way and are therefore not explicitly mentioned in this note.

solution and expanding around it^{*)}. In order to be able to perform this programme we need to regularize the theory in an invariant way. Having only fermions to start with, the obvious choice is the Pauli-Villars (PV) regularization.

Following the aforementioned procedure and including also a $U(1)$ gauge symmetry we arrive at the following effective bosonic action after integration over fermions.

$$\begin{aligned}
 A = & \int d^4x (\det W_{\mu\alpha}) \left[1 - \xi^{\mu\alpha} W_{\mu\alpha} - \lambda^\mu A_\mu - \lambda_{\alpha\beta}^\mu A_{\mu}^{\alpha\beta} + T(\sigma\phi) - V(\phi) \right] + \\
 & + \sum_i C_i \text{tr} \log \left[K_i^{-1} \xi^{\mu\alpha} \gamma_\alpha (i\partial_\mu - A_\mu - \sigma_{\beta\delta} A_{\mu}^{\beta\delta}) - \sigma + \right. \\
 & \left. + \frac{1}{4} \lambda^\mu \phi^{-1} i\partial_\mu + \frac{1}{8} (\sigma^{\beta\delta} \phi^{-1} \partial_\mu^\leftarrow - \phi^{-1} \sigma^{\beta\delta} \partial_\mu^\rightarrow) \lambda_{\beta\delta}^\mu \right]
 \end{aligned} \quad (6)$$

Here the auxiliary fields $\lambda^\mu, \lambda_{\alpha\beta}^\mu$ [of dimension $(\text{length})^1$] are setting the connections $A_\mu, A_{\mu}^{\alpha\beta}$ [of dimension $(\text{length})^{-1}$] equal to their expression in terms of fermionic fields as in Eq. (2).

In addition, in order to get rid of the non-linearity of $\det(\psi\gamma D\psi)$, we have introduced $W_{\mu\alpha} = \bar{\psi} \gamma_\alpha D_\mu \psi$ using the multiplier field $\xi^{\mu\alpha}$. Finally, σ and ϕ are dimensionless Dirac matrix fields, with σ imposing $\phi = \sum_i M_i$ and $V(\phi)$ being an arbitrary scalar potential introduced in order to enforce $\langle\phi\rangle = 1$. The index $i = 0, 1, 2, 3$ runs over the original fermion ($i=0$) and over three PV regulators whose parameters C_i, K_i have to satisfy

$$\sum_{i=0}^3 C_i K_i^\mu = 0 ; \quad \mu = 0, 2, 4 ; \quad C_0 \equiv 1 \quad (7)$$

in order to eliminate all the ultra-violet divergences encountered while expanding the $\text{tr} \log$ term in Eq. (6).

^{*)} This procedure is the (fermion) loop expansion justified by a $1/N$ expansion had we started with N fermions. This extension is a triviality in our treatment and we avoid it for simplicity.

3. Our first task will be to find stationary solutions by setting to zero all first derivatives in the fields. As expected from Lorentz invariance we find that:

$$\langle \lambda^\mu \rangle = \langle A_\mu \rangle = \langle \lambda_{\alpha\beta}^\mu \rangle = \langle A_{\mu}^{\alpha\beta} \rangle = 0 \quad (8)$$

is a solution while, -having imposed $\langle \phi \rangle = 1 - \sigma, \xi$ and W , need to acquire v.e.v. The condition $\delta\Gamma/\delta W = 0$ implies the relation

$$\langle \xi \rangle^{\mu\alpha} = (\langle W \rangle^{-1})^{\mu\alpha} [1 - \langle \xi \rangle^{\nu\beta} \langle W \rangle_{\nu\beta} + 4\langle \sigma \rangle] \quad (9)$$

We note that Eq. (6) contains two independent fields $\xi^{\mu\alpha}$ and $W_{\mu\alpha}$ with dimensions ± 1 and with the index structures of vierbeins. They do not give rise to a definite metric unless one is identified with the inverse of the other. Equation (9) shows that this is the case at the classical level and, as we shall see later, this fact will be preserved by quantum fluctuation at low energies.

The remaining minimization conditions $\delta\Gamma/\delta\sigma = \delta\Gamma/\delta\xi = 0$ are easy to analyze for constant field solutions. In this case, with usual techniques, we obtain the conditions

$$\langle \sigma \rangle = 1, \quad \sum_i \frac{C_i K_i^2}{(2\pi)^4} \left(\frac{\mathcal{A}^4 \{ \}}{\{ \}^2 - K_i^2} \right) = 1 \quad (10)$$

which, together with Eq. (9), imply without loss of generality

$$\langle W \rangle_{\mu\alpha} = \Lambda \delta_{\mu\alpha}, \quad \langle \xi \rangle^{\mu\alpha} = \frac{1}{\Lambda} \delta^{\mu\alpha} \quad (11)$$

where

$$\{ \}^2 = \gamma_{\alpha\beta} \{ \}_\alpha \{ \}_\beta \quad ; \quad \gamma_{00} = -\gamma_{11} = -\gamma_{22} = -\gamma_{33} = 1$$

and Λ is an arbitrary scale which defines the energy unit of the theory.

The flat solution (11) gives rise to the usual Dirac propagator in the expansion of the trace log in (6). This allows the determination of the fermion masses that turn out to be

$$m_i = K_i \Lambda \quad (12)$$

Wishing to keep the original fermion much lighter than the PV regulators, we have to take K_i ($i \neq 0$) much larger than K_0 but all of the same order. In this case Eqs (7) and (10) imply

$$K_i = (32\pi^2)^{1/4} + O(K_0), \quad K_0 \ll 1 \quad (13)$$

thus finding the regulators around a common mass

$$m_{PV} \approx 2^{5/4} \sqrt{\pi} \Lambda \quad \text{and} \quad m_f = K_0 \Lambda \ll \Lambda \quad (14)$$

We notice the interesting fact that we have introduced the PV regulators without affecting the co-ordinate transformation invariance and in particular, dilatations. This is why the scale Λ was considered as spontaneously generated^{*)}.

At first sight Eq. (13) - fixing a parameter of the Lagrangian as a condition to find a classical solution - seems surprising. It generates the relation (4) between the PV regulator masses and Λ , thus providing a realization of the often claimed analogy between regularization and spontaneous breaking of dilatation.

4. We are now ready to discuss the quadratic terms of the action in terms of the fluctuations of our auxiliary fields. These quadratic quantities are expanded in a local part (i.e., a mass term) plus terms containing derivatives of fields among which we will find induced kinetic terms^{**)}. Of course, bilinear terms in different fields appear and therefore a diagonalization must be performed in order to find the eigenmodes.

^{*)} The zero mode of this spontaneously broken invariance will turn out to be the trace of a massless vierbein that will be identified with that of the usual Einstein theory.

^{**) Square masses are defined as the ratio of the coefficient of the quadratic local term and the kinetic one. We stress that the only auxiliary field whose mass is not determined uniquely by the action (6) is $\text{Tr } \phi$ due to the fact that it depends on $V(\phi)$, i.e., on the way in which $\langle \phi \rangle = 1$ is implemented.}

We will present here only the result: the interesting structures of this lengthy calculation will be given elsewhere. We find that the field combinations.

$$\phi, \sigma, \lambda^\mu, \lambda_{\alpha\beta}^\mu, \xi - W^{-1}, \quad (15)$$

and $A_\mu^{\beta\gamma} = \frac{\Lambda}{4} \partial_\mu (\xi^{\beta\gamma} - \xi^{\gamma\beta})$

have masses of order Λ , while

$$A_\mu, \quad V^{\mu\alpha} = \Lambda \left(\xi^{\mu\alpha} + (W^{-1})^{\mu\alpha} \right) \quad (16)$$

are massless.

Therefore as long as $E \ll \Lambda$ all fields in (15) will be frozen around their v.e.v. In particular we find that the classical result

$$\xi = W^{-1} \quad (17)$$

is not modified by quantum fluctuations as long as $E \ll \Lambda$. And, as discussed before, Eq. (17) is what allows the definition of a metric which therefore appears to stem from the theory as a low energy concept.

Using (17), the freezing of the last combination in (15) implies that for $E \ll \Lambda$

$$\sigma^{\beta\gamma} A_\mu^{\beta\gamma} = \frac{1}{2} (V^{-1})_{\nu\beta} \partial_\mu V^{\nu\delta} \sigma^{\beta\delta} \quad (18)$$

where V is given in (16). For our low energy physics, we are therefore left with a massless photon and a massless vierbein besides the light arbitrary mass fermion. We could have introduced many light fermions and as noted before, further gauge symmetries. To our view²⁾ their possibility of being light is a consequence of a chiral protection that would be inoperative for fundamental scalars.

The induced kinetic terms of the photon and the vierbein are given respectively by

$$-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} \cdot \frac{1}{3\pi} \log \frac{\Lambda^2}{q^2} \quad \text{and} \quad (\det V^{-1}) R \cdot \frac{\Lambda^2}{6\sqrt{32}\pi}$$

where R is the usual scalar curvature as written in terms of $V^{\alpha\mu}$. On the other hand, Eq. (18) represents the usual way of writing the Lorentz connection in terms of the vierbein. Therefore, the effective low energy action is totally equivalent to that of a fermion matter field in interaction with an electromagnetic and a metric field as expressed by the usual Einstein action in which the cosmological constant vanishes^{*)} while the electromagnetic and gravitational couplings are given by

$$\alpha(q^2) = \left[\frac{1}{3\pi} \log \frac{\Lambda^2}{q^2} + O(1) \right]^{-1}, \quad g = \frac{3}{\sqrt{2}} \Lambda^{-2} \quad (19)$$

The last relation allows to connect Λ with the Planck mass. For N light fermions (19) gives the anticipated relation (1) between α and G .

In all our preceding discussion, the quadratic terms in the fluctuations gave rise to the corresponding quadratic terms in all equations we wrote down, as Eq. (18) or the kinetic terms. Higher powers in the fluctuations in those expressions, originate of course from higher powers of same dimensionality in the expansion of the action. It is not astonishing that our action generates these forms: they are dictated by Lorentz and gauge invariance. These invariances, which were present in our original action (4), are of course, transferred to the action as written in terms of the auxiliary fields (6), and its subsequent expansion, due to the fact that they were preserved by all our manipulations.

Of course, besides the terms we have discussed, in the development of the action we will find higher dimensional terms appearing always in invariant forms. For instance, a term of the form $R^2 - 3R_{\mu\nu} R^{\mu\nu}$ will be generated with a coefficient of order $\log \Lambda^2/q^2 \propto 1/\alpha(q^2)$. All these terms will be irrelevant at $E \ll \Lambda$; the dimensionality of fields and derivatives being always compensated by powers of $1/\Lambda$.

^{*)} The absence of a cosmological term is due to the fact that the flat background metric appears as a classical solution. This is of course not the case in the approaches of Ref. 1).

5. We see therefore that the quantum fluctuations of matter fields give rise to an action in terms of composite fields which for energies $E \ll \Lambda$ coincides with the Einstein one.

The action in terms of composite fields must now be evaluated, i.e., quantum fluctuation in those fields must be resummed. In so doing we will find a basic difference between the ultra-violet behaviour of our theory and the usual one. Indeed, as long as $q^2 \ll \Lambda^2$ the two coincide, but for virtual square momenta $q^2 \sim \Lambda^2$ the two theories differ. We conjecture that our approach will not encounter the ultra-violet problems that the Einstein theory did. This conjecture is, for the time being, based on very thin ice and we plan to understand better at least the first corrections in q^2/Λ^2 so as to put it on more solid grounds. A hint in this direction is that vertices of auxiliary fields were provided by loops of heavy fermions (regulators with masses of order Λ). This implies a necessary smoothing for $q^2 \sim \Lambda^2$. Another hint is that around $q^2 \sim \Lambda^2$ all fields contribute analogously. Masses (except those of fundamental fermions and ϕ) and couplings being all related, strong cancellations can be at work. These cancellations are also needed in order to avoid that the regulators, whose masses are given by Eq. (14), could jeopardize unitarity at very high energies.

Another mechanism of regularization can be provided by the scalar field ϕ whose mass was free because it was generated by the implementation of $\langle \phi \rangle = 1$. Indeed, as shown in Ref. 3), the resummation of a development of $1/(1 + \hat{\phi})$ may give rise to mild short distance behaviour even if each term of the expansion has strong ultra-violet singularities. Our conjecture, if justified, would imply that many of the ultra-violet diseases of Einstein's theory of gravitation are due to have adopted a phenomenological action beyond its limit of validity.

6. We have analyzed our theory by developing it around a constant v.e.v. This is surely the relevant vacuum for long range effects and therefore for the low energy analysis we have performed. It is nevertheless not clear to us whether in discussing short distance effects for large mass densities or very high temperature, other eventual solutions localized in finite region of space time could be relevant. Their treatment in our framework meets the high complexity of the classical equations which are given as traces of operators in the presence of self-consistent classical fields. This could nevertheless open the possibility of investigating the possible superposition of different cosmologies at very short distances.

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