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ON THE THRUST DISTRIBUTION IN e^+e^- ANNIHILATION

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A B S T R A C T

The thrust distribution calculated in perturbative QCD through to $O(\alpha_s^2)$ is presented. The results, which come from an elaboration of previously published formulae, show that the $O(\alpha_s^2)$ correction at present energies is large and positive in agreement with the previously obtained results of many groups.

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Last year we (ERT) presented details¹⁾ of a calculation of the event shape in e^+e^- annihilation into hadrons due to the radiation of massless quarks and gluons. Our calculation was presented in terms of the eigenvalues of an infrared finite tensor formed from the centre-of-mass 3 momenta P_a of the hadrons in the final state²⁾

$$\theta^{ij} = \frac{\sum_a P_a^i P_a^j}{\sum_a |P_a|} \bigg/ \sum_a |P_a| \quad (1)$$

From the tensor θ we defined the variable 'C' which is related to its eigenvalues, λ_i , by

$$C = 3(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1)$$

'C' provides an effective separation between two-jet and multi-jet events.

Several groups K³⁾, FSKS^{4),5)}, VGO⁶⁾, CW⁷⁾ have now presented results on the jet structure in e^+e^- annihilation. Of these two (FSKS,VGO) represent totally independent calculations, whilst the remaining two represent numerical elaborations of the analytic formula contained in ERT. Most of these groups have chosen to present their results in terms of the thrust⁸⁾, which is defined as

$$t = 2 \max\{n\} \sum_{\text{hemisphere}} \frac{n \cdot P_a}{\sqrt{S}} \quad (2)$$

The purpose of this report is twofold. The first is to point out that Ref. 1) contains sufficient information to allow the calculation of many other quantities characterizing the event shape in e^+e^- annihilation, including the thrust, by writing a simple numerical integration programme. This fact has already been exploited by the authors of Refs. 3) and 7).

Secondly, we want to try and clarify a disagreement which exists in the literature between FSKS, [who find that the $O(\alpha_s^2)$ pieces lead to small corrections to the thrust distribution at present energies] and the remaining groups who have calculated the thrust (K,VGO,CW). In order to do this we present detailed numerical results for the thrust distribution in such a way that the comparison with the work of other groups is possible.

To calculate the thrust distribution we use only the formulae of Ref. 1). These contain three component parts; (i) $d\sigma^{(3)}$, the cross-section for $e^+e^- \rightarrow q\bar{q}G$ calculated to order α_s^2 , which has been confirmed in Ref. 5); (ii) $d\sigma^{(4)}$ the cross-section for $e^+e^- \rightarrow q\bar{q}q\bar{q}$, $q\bar{q}GG$ first calculated in Refs 9) and 10) and confirmed in Ref. 1); (iii) $d\sigma^{(S)}$, a term containing singular parts which is added and subtracted and is required to render the two parts separately finite. Since it is added and subtracted its precise form is irrelevant as long as the singular parts are correct. We note in passing that comparison with the analytic results of VGO is not possible since they used a different method of regulating the infra-red and collinear singularities in intermediate stages.

Our results are shown in Figs. 1 and 2. We may write the perturbative expansion of the thrust distribution as follows ($T \neq 1$).

$$\frac{1}{\sigma_0} \frac{d\sigma}{dt} = A_0(t) \frac{\alpha_s(\sqrt{s})}{\pi} + A_1(t) \left(\frac{\alpha_s}{\pi}\right)^2 + \dots \quad (3)$$

where σ_0 is the lowest order e^+e^- annihilation cross-section

$$\sigma_0 = \frac{4\pi\alpha^2}{3s} N_c \sum_f e_f^2 \quad (4)$$

In Fig. 1 we have plotted the quantities $A_0(t)$ and $A_1(t)$ calculated using our Monte Carlo programme, together with the corresponding quantities (dotted line) taken from Fig. 8 of VGO⁶⁾. The agreement in the shape and normalization of both histograms is good. A similar curve which can be derived from Fig. 1 of CW⁷⁾ is also in good agreement. However, the histograms are in net disagreement with the curve derived from Fig. 2 of Ref. 4). Our histograms are also in qualitative agreement^{*)} with the results of Ref. 3).

The size of the correction may be characterized by an integral over the multi-jet region of the thrust distribution,

$$\frac{1}{\sigma_0} \int_{0.5}^{0.85} dt \frac{d\sigma}{dt} = B_0 \frac{\alpha_s(\sqrt{s})}{\pi} \left[1 + K \frac{\alpha_s}{\pi} + \dots \right]$$

^{*)} Owing to the limited statistics, the calculation of Ref. 3) contains large Monte Carlo errors (Z. Kunszt, private communication) which coupled with the different number of flavours used in Ref. 3) makes an exact quantitative comparison difficult.

From our numerical programme we find the following values for the parameters^{*)}

$$B_0 = 1.156 \pm 0.001, \quad K = 17.6 \pm 0.3.$$

In Fig. 2 we plot the thrust distribution in picobarns displaying separately the $O(\alpha_s)$ term and the total cross-section through to $O(\alpha_s^2)$. These curves are the pure QCD contribution and contain no fragmentation.

The salient points of our analysis are:

- (1) the perturbative corrections to the $O(\alpha_s)$ expressions for both the thrust and C distributions are large and positive at present energies in the multiplet region;
- (2) although the correction to the normalization of the thrust distribution is large, the shape of the thrust distribution is substantially the same as the lowest order result.

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^{*)} The corresponding quantities from the results of VGO are $B_0 = 1.156 \pm 0.001$, $K = 17.2 \pm 0.2$ (J.A.M. Vermaseren, private communication). New results from Z. Kunszt with five flavours and reduced Monte Carlo errors give $K = 17.6 \pm 0.3$ (private communication).

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FIGURE CAPTIONS

Fig. 1 : The quantities $A_0(t)$, $A_1(t)$ (solid histogram). The dotted histograms are the corresponding results from Ref. 6).

Fig. 2 : $d\sigma/dt$ (in picobarns) versus t . The lower dotted curve is the $O(\alpha_s(Q))$ result whereas the upper curve is the full $O(\alpha_s^2)$ result. The coupling constant $\alpha_s^{\overline{MS}}(Q)$ at $Q = \sqrt{s} = 35$ GeV is taken to be 0.144.

FIG. 1

FIG. 2